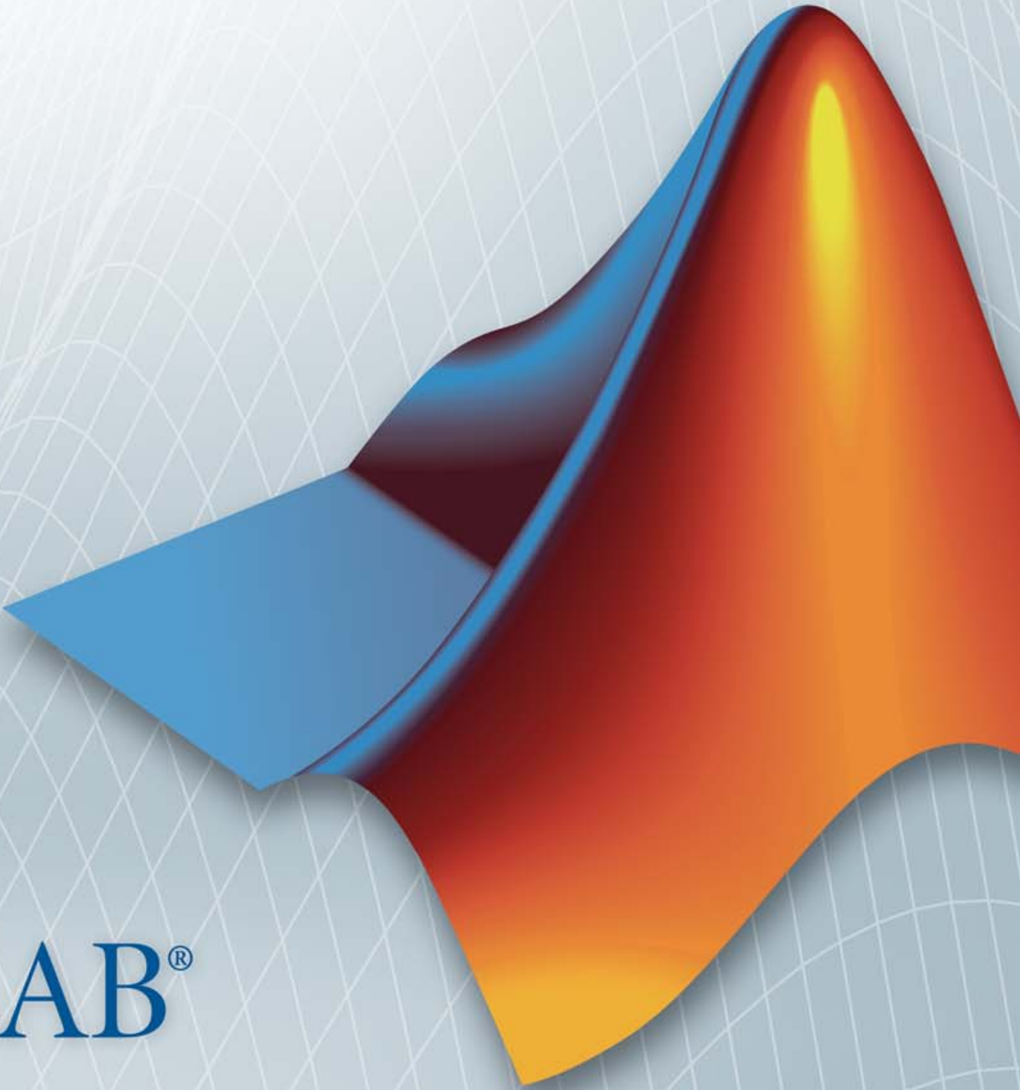
# Financial Instruments Toolbox™

## User's Guide

**R2012b**



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*Financial Instruments Toolbox™ User's Guide*

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# 1

# Getting Started

# Product Description

### Design, price, and hedge complex financial instruments

Financial Instruments Toolbox™ provides functions for pricing, modeling, and analyzing fixed-income, credit, and equity instrument portfolios. You can use the toolbox to perform cash flow modeling and yield curve fitting analysis, compute prices and sensitivities, view price evolutions, and perform hedging analyses using common equity and fixed-income modeling methods. The toolbox lets you create new financial instrument types and fit yield curves to market data using parametric fitting models and bootstrapping.

Financial Instruments Toolbox includes functions for pricing and analyzing fixed-income and equity instruments. Fixed-income modeling tools let you calculate price, yield, spread, and sensitivity values for several types of securities and derivatives, including mortgage-backed securities, treasury bills, bonds, bonds with embedded options, swaps, caps, floors, and floating-rate notes. For equities, the toolbox lets you compute price, implied volatility, and greek values of vanilla equity options and of several exotic equity derivatives such as Bermuda, basket, barrier, digital, and rainbow options.

## Key Features

- Yield curve fitting with bootstrapping and parametric fitting models, and term-structure analysis
- Black Scholes, Black, Garman-Kohlhagen, Roll-Geske-Whaley, Bjerksund-Stensland, Nengjiu Ju, Stulz, and Longstaff-Schwartz models
- Fixed-income and equity derivative calculations for price, yield, discount rate, cash-flow schedule, spread, implied volatility, option adjusted spread (OAS), and greeks
- Tree models: CRR, EQP, LR, ITT, HJM, BDT, BK, and HW
- Interest-rate instruments: bonds, stepped-coupon bonds, futures, vanilla options, Bermuda options, bonds with embedded options, vanilla swaps, forward swaps, amortizing swaps, swaptions, caps, floors, range notes, floating-rate notes, and collared floating-rate notes

- Equity instruments: stocks, vanilla options, Bermuda options, Asian options, lookback options, barrier options, digital options, rainbow options, basket options, compound options, and chooser options
- Credit instruments: mortgage pools, balloon mortgages, and credit default swaps

# Interest-Rate-Based Derivatives

The toolbox provides functionality that supports the creation and management of these interest-rate-based instruments:

- Bonds
- Bond options (puts and calls)
- Bond with embedded options
- Caps
- Fixed-rate notes
- Floating-rate notes
- Floors
- Swaps
- Swaption

Additionally, the toolbox provides functions to create *arbitrary cash flow instruments*. The toolbox provides pricing and sensitivity routines for these instruments. For more information, see “Pricing Using Interest-Rate Term Structure” on page 2-38 ,“Pricing Using Interest-Rate Tree Models” on page 2-67, and“Interest-Rate Derivatives Using Closed-Form Solutions” on page 2-90.

# Equity-Based Derivatives

The toolbox also provides functions to create and manage various equity-based derivatives, including the following:

- Asian options
- Barrier options
- Basket options
- Compound options
- Digital options
- Lookback options
- Rainbow options
- Vanilla stock options (put and call options)

The toolbox also provides pricing and sensitivity routines for these instruments. (See “Pricing Equity Derivatives Using Trees” on page 3-33, “Equity Derivatives Using Closed-Form Solutions” on page 3-51, and “Basket Option” on page 3-24.)

## Expected Users

In general, this guide assumes experience working with financial derivatives and some familiarity with the underlying models.

In designing Financial Instruments Toolbox documentation, we assume your title is similar to one of these:

- Analyst, quantitative analyst
- Risk manager
- Portfolio manager
- Fund manager, asset manager
- Financial engineer
- Trader
- Student, professor, or other academic

We also assume your background, education, training, and responsibilities match some aspects of this profile:

- Finance, economics, perhaps accounting
- Engineering, mathematics, physics, other quantitative sciences
- Bachelor's degree minimum; MS or MBA likely; Ph.D. perhaps; CFA
- Comfortable with probability theory, statistics, and algebra
- Understand linear or matrix algebra, calculus, and differential equations
- Previously done traditional programming (C, Fortran, etc.)
- Responsible for instruments or analyses involving large sums of money
- Perhaps new to MATLAB®

# Portfolio Creation

**In this section...**

## Introduction

The `instadd` function creates a set of instruments (portfolio) or adds instruments to an existing instrument collection. The `TypeString` argument specifies the type of the investment instrument. For interest-rate-based derivatives, the types are: `Bond`, `OptBond`, `CashFlow`, `Fixed`, `Float`, `Cap`, `Floor`, and `Swap`. For equity derivatives, the types are `Asian`, `Barrier`, `Compound`, `Lookback`, and `OptStock`.

The input arguments following `TypeString` are specific to the type of investment instrument. Thus, the `TypeString` argument determines how the remainder of the input arguments is interpreted. For example, `instadd` with the type string `Bond` creates a portfolio of bond instruments.

```
InstSet = instadd('Bond', CouponRate, Settle, Maturity, Period,
Basis, EndMonthRule, IssueDate, FirstCouponDate, LastCouponDate,
StartDate, Face)
```

## Interest-Rate-Based Derivatives

In addition to the bond instrument already described, the toolbox can create portfolios containing the following set of interest-rate-based derivatives:

- Bond option

  ```
  InstSet = instadd('OptBond', BondIndex, OptSpec, Strike, ExerciseDates, AmericanOpt)
  ```

- Arbitrary cash flow instrument

  ```
  InstSet = instadd('CashFlow', CFlowAmounts, CFlowDates, Settle, Basis)
  ```

- Fixed-rate note instrument

```
InstSet = instadd('Fixed', CouponRate, Settle, Maturity, FixedReset, Basis, Principal)
```

- Floating-rate note instrument

```
InstSet = instadd('Float', Spread, Settle, Maturity, FloatReset, Basis, Principal)
```

- Cap instrument

```
InstSet = instadd('Cap', Strike, Settle, Maturity, CapReset, Basis, Principal)
```

- Floor instrument

```
InstSet = instadd('Floor', Strike, Settle, Maturity, FloorReset, Basis, Principal)
```

- Swap instrument

```
InstSet = instadd('Swap', LegRate, Settle, Maturity, LegReset, Basis, Principal, LegType)
```

- Swaption instrument

```
InstSet = instadd('Swaption', OptSpec, Strike, ExerciseDates, Spread, ...
Settle, Maturity, AmericanOpt, SwapReset, Basis, Principal)
```

- Bond with embedded option instrument

```
InstSet = instadd('OptEmBond', CouponRate, Settle, Maturity, OptSpec, Strike, ...
ExerciseDates, 'AmericanOpt', AmericanOpt, 'Period', Period,'Basis', Basis, ...
'EndMonthRule', EndMonthRule,'Face',Face,'IssueDate', IssueDate, 'FirstCouponDate', ...
FirstCouponDate, 'LastCouponDate', LastCouponDate,'StartDate', StartDate)
```

## Equity Derivatives

The toolbox can create portfolios containing the following set of equity derivatives:

- Asian instrument

```
InstSet = instadd('Asian', OptSpec, Strike, Settle, ExerciseDates, AmericanOpt, ...
AvgType, AvgPrice, AvgDate)
```

- Barrier instrument

```
InstSet = instadd('Barrier', OptSpec, Strike, Settle, ExerciseDates, AmericanOpt, ...
BarrierType, Barrier, Rebate)
```

- Compound instrument

```
InstSet = instadd('Compound', UOptSpec, UStrike, USettle, UExerciseDates, UAmericanOpt, ...
COptSpec, CStrike, CSettle, CExerciseDates, CAmericanOpt)
```

- Lookback instrument

```
InstSet = instadd('Lookback', OptSpec, Strike, Settle, ExerciseDates, AmericanOpt)
```

- Stock option instrument

```
InstSet = instadd('OptStock', OptSpec, Strike, Settle, Maturity, AmericanOpt)
```

## Adding Instruments to an Existing Portfolio

To use the `instadd` function to add additional instruments to an existing instrument portfolio, provide the name of an existing portfolio as the first argument to the `instadd` function.

Consider, for example, a portfolio containing two cap instruments only:

```
Strike = [0.06; 0.07];
Settle = '08-Feb-2000';
Maturity = '15-Jan-2003';

Port_1 = instadd('Cap', Strike, Settle, Maturity);
```

These commands create a portfolio containing two cap instruments with the same settlement and maturity dates, but with different strikes. In general, the input arguments describing an instrument can be either a scalar, or a number of instruments (`NumInst`)-by-`1` vector in which each element corresponds to an instrument. Using a scalar assigns the same value to all instruments passed in the call to `instadd`.

Use the `instdisp` command to display the contents of the instrument set:

```
instdisp(Port_1)

Index Type Strike Settle      Maturity    CapReset Basis Principal
```

```
1     Cap  0.06   08-Feb-2000 15-Jan-2003 1        0     100
2     Cap  0.07   08-Feb-2000 15-Jan-2003 1        0     100
```

Now add a single bond instrument to `Port_1`. The bond has a 4.0% coupon and the same settlement and maturity dates as the cap instruments.

```
CouponRate = 0.04;
Port_1 = instadd(Port_1, 'Bond', CouponRate, Settle, Maturity);
```

Use `instdisp` again to see the resulting instrument set:

```
instdisp(Port_1)


Index Type Strike Settle         Maturity       CapReset Basis Principal
1     Cap  0.06   08-Feb-2000    15-Jan-2003    1        0     100
2     Cap  0.07   08-Feb-2000    15-Jan-2003    1        0     100

Index Type CouponRate Settle         Maturity     Period Basis EndMonthRule IssueDate ... Face
3     Bond 0.04       08-Feb-2000    15-Jan-2003  2      0     1            NaN       ... 100
```

# Pricing a Portfolio Using the Black-Derman-Toy Model

This example illustrates how the Financial Instruments Toolbox™ can be used to create a Black-Derman-Toy (BDT) tree and price a portfolio of instruments using the BDT model.

**Create the Interest Rate Term Structure**

The structure RateSpec is an interest rate term structure that defines the initial forward-rate specification from which the tree rates are derived. Use the information of annualized zero coupon rates in the table below to populate the RateSpec structure.

| From | To | Rate |
|---|---|---|
| 01 Jan 2005 | 01 Jan 2006 | 0.0275 |
| 01 Jan 2005 | 01 Jan 2007 | 0.0312 |
| 01 Jan 2005 | 01 Jan 2008 | 0.0363 |
| 01 Jan 2005 | 01 Jan 2009 | 0.0415 |
| 01 Jan 2005 | 01 Jan 2010 | 0.0458 |

```
StartDates = ['01 Jan 2005'];

EndDates =   ['01 Jan 2006';
              '01 Jan 2007';
              '01 Jan 2008';
              '01 Jan 2009';
              '01 Jan 2010'];

ValuationDate = ['01 Jan 2005'];
Rates = [0.0275; 0.0312; 0.0363; 0.0415; 0.0458];
Compounding = 1;

RateSpec = intenvset('Compounding',Compounding,'StartDates', StartDates,...
                     'EndDates', EndDates, 'Rates', Rates,'ValuationDate',
```

```
RateSpec =

           FinObj: 'RateSpec'
      Compounding: 1
```

```
             Disc: [5x1 double]
            Rates: [5x1 double]
         EndTimes: [5x1 double]
       StartTimes: [5x1 double]
         EndDates: [5x1 double]
       StartDates: 732313
    ValuationDate: 732313
            Basis: 0
     EndMonthRule: 1
```

#### Specify the Volatility Model

Create the structure VolSpec that specifies the volatility process with the following data.

```
Volatility = [0.005; 0.0055; 0.006; 0.0065; 0.007];
BDTVolSpec = bdtvolspec(ValuationDate, EndDates, Volatility)


BDTVolSpec =

             FinObj: 'BDTVolSpec'
      ValuationDate: 732313
           VolDates: [5x1 double]
           VolCurve: [5x1 double]
    VolInterpMethod: 'linear'
```

#### Specify the Time Structure of the Tree

The structure TimeSpec specifies the time structure for an interest rate tree. This structure defines the mapping between the observation times at each level of the tree and the corresponding dates.

```
Maturity = EndDates;
BDTTimeSpec = bdttimespec(ValuationDate, Maturity, Compounding)


BDTTimeSpec =
```

```
          FinObj: 'BDTTimeSpec'
    ValuationDate: 732313
         Maturity: [5x1 double]
      Compounding: 1
            Basis: 0
     EndMonthRule: 1
```

#### Create the BDT Tree

Use the previously computed values for RateSpec, VolSpec and TimeSpec to create the BDT tree.

```
BDTTree = bdttree(BDTVolSpec, RateSpec, BDTTimeSpec)
```

```
BDTTree =

      FinObj: 'BDTFwdTree'
     VolSpec: [1x1 struct]
    TimeSpec: [1x1 struct]
    RateSpec: [1x1 struct]
        tObs: [0 1 2 3 4]
        dObs: [732313 732678 733043 733408 733774]
        TFwd: {[5x1 double]  [4x1 double]  [3x1 double]  [2x1 double]  [4]}
      CFlowT: {[5x1 double]  [4x1 double]  [3x1 double]  [2x1 double]  [5]}
     FwdTree: {1x5 cell}
```

#### Observe the Interest Rate Tree

Visualize the interest rate evolution along the tree by looking at the output structure BDTTree. BDTTree returns an inverse discount tree, which you can convert into an interest rate tree with the cvtree function.

```
BDTTreeR = cvtree(BDTTree);
```

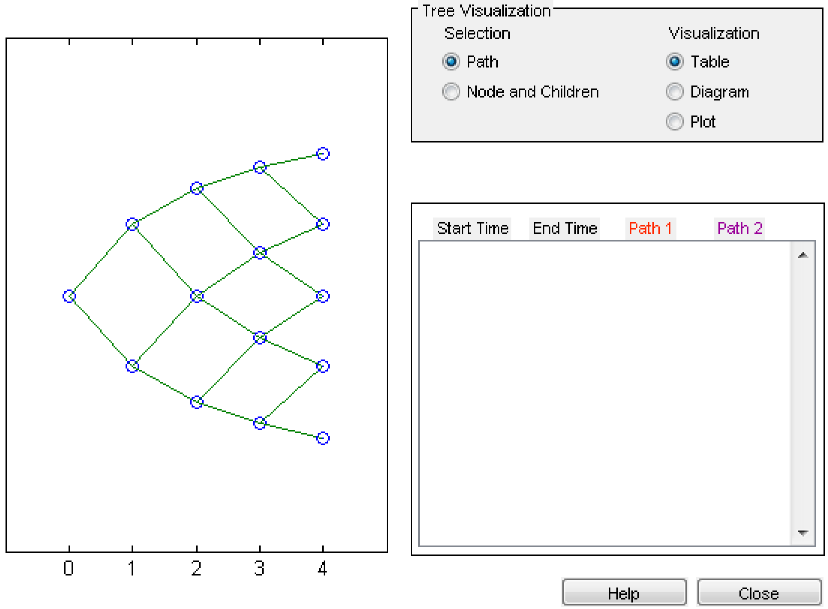
Look at the upper branch and lower branch paths of the tree:

```
%Rate at root node:
RateRoot     = treepath(BDTTreeR.RateTree, [0])

%Rates along upper branch:
RatePathUp   = treepath(BDTTreeR.RateTree, [1 1 1 1])

%Rates along lower branch:
RatePathDown = treepath(BDTTreeR.RateTree, [2 2 2 2])
```

```
RateRoot =

    0.0275


RatePathUp =

    0.0275
    0.0347
    0.0460
    0.0560
    0.0612


RatePathDown =

    0.0275
    0.0351
    0.0472
    0.0585
    0.0653
```

You can also display a graphical representation of the tree to examine interactively the rates on the nodes of the tree until maturity. The function treeviewer displays the structure of the rate tree in the left pane. The tree visualization in the right pane is blank, but by selecting Diagram and clicking on the nodes you can examine the rates along the paths.

```
treeviewer(BDTTreeR)
```



Tree Visualization

Selection

● Path

○ Node and Children

Visualization

● Table

○ Diagram

○ Plot

### Create an Instrument Portfolio

Create a portfolio consisting of two bond instruments and a option on the 5% Bond.

```
% Bonds
CouponRate = [0.04;0.05];
Settle = '01 Jan 2005';
Maturity = ['01 Jan 2009';'01 Jan 2010'];
Period = 1;

% Option
OptSpec = {'call'};
Strike = 98;
ExerciseDates = ['01 Jan 2010'];
AmericanOpt = 1;

InstSet = instadd('Bond',CouponRate, Settle,  Maturity, Period);
```

```
InstSet = instadd(InstSet,'OptBond', 2, OptSpec, Strike, ExerciseDates, Ame
```

Examine the set of instruments contained in the variable InstSet.

```
instdisp(InstSet)

Index Type CouponRate Settle         Maturity       Period Basis EndMonthRu
1     Bond 0.04       01-Jan-2005    01-Jan-2009    1      0     1
2     Bond 0.05       01-Jan-2005    01-Jan-2010    1      0     1

Index Type    UnderInd OptSpec Strike ExerciseDates  AmericanOpt
3     OptBond 2        call    98     01-Jan-2010    1
```

#### Price the Portfolio Using a BDT Tree

Calculate the price of each instrument in the instrument set.

```
Price = bdtprice(BDTTree, InstSet)


Price =

   99.6374
  102.2460
    4.2460
```

The prices in the output vector Price correspond to the prices at observation time zero (tObs = 0), which is defined as the Valuation Date of the interest-rate tree.

In the Price vector, the first element, 99.6374, represents the price of the first instrument (4% Bond); the second element, 102.2460, represents the price of the second instrument (5% Bond), and 4.2460 represents the price of the Option.

# Portfolio Management

**In this section...**

## Instrument Constructors

The toolbox provides constructors for the most common financial instruments. A *constructor* is a function that builds a structure dedicated to a certain type of object; in this toolbox, an *object* is a type of market instrument.

The instruments and their constructors in this toolbox are listed below.

| Instrument | Constructor |
|---|---|
| Asian option | `instasian` |
| Barrier option | `instbarrier` |
| Bond | `instbond` |
| Bond option | `instoptbnd` |
| Arbitrary cash flow | `instcf` |
| Compound option | `instcompound` |
| Fixed-rate note | `instfixed` |
| Floating-rate note | `instfloat` |
| Cap | `instcap` |
| Floor | `instfloor` |
| Lookback option | `instlookback` |
| Stock option | `instoptstock` |
| Swap | `instswap` |
| Swaption | `instswaption` |

Each instrument has parameters (fields) that describe the instrument. The toolbox functions let you do the following:

- Create an instrument or portfolio of instruments.
- Enumerate stored instrument types and information fields.
- Enumerate instrument field data.
- Search and select instruments.

The instrument structure consists of various fields according to instrument type. A *field* is an element of data associated with the instrument. For example, a bond instrument contains the fields `CouponRate`, `Settle`, `Maturity`, and so on. Additionally, each instrument has a field that identifies the investment type (bond, cap, floor, and so on).

In reality, the set of parameters for each instrument is not fixed. You have the ability to add additional parameters. These additional fields are ignored by the toolbox functions. They may be used to attach additional information to each instrument, such as an internal code describing the bond.

Parameters not specified when *creating* an instrument default to `NaN`, which, in general, means that the functions using the instrument set (such as `intenvprice` or `hjmprice`) will use default values. At the time of *pricing*, an error occurs if any of the required fields is missing, such as `Strike` in a cap or `CouponRate` in a bond.

## Creating Instruments or Properties

Use the `instaddfield` function to create a kind of instrument or to add new properties to the instruments in an existing instrument collection.

To create a kind of instrument with `instaddfield`, you must specify three arguments:

- `Type`
- `FieldName`
- `Data`

`Type` defines the type of the new instrument, for example, `Future`. `FieldName` names the fields uniquely associated with the new type of instrument. `Data` contains the data for the fields of the new instrument.

An optional fourth argument is `ClassList`. `ClassList` specifies the data types of the contents of each unique field for the new instrument.

Use either syntax to create a kind of instrument using `instaddfield`:

```
InstSet = instaddfield('FieldName', FieldList, 'Data', DataList,...
'Type', TypeString)
InstSet = instaddfield('FieldName', FieldList, 'FieldClass',...
ClassList, 'Data' , DataList, 'Type', TypeString)
```

To add new instruments to an existing set, use:

```
InstSetNew = instaddfield(InstSetOld, 'FieldName', FieldList,...
'Data', DataList, 'Type', TypeString)
```

As an example, consider a futures contract with a delivery date of July 15, 2000, and a quoted price of $104.40. Since Financial Instruments Toolbox software does not directly support this instrument, you must create it using the function `instaddfield`. Use these parameters to create instruments:

- Type: `Future`
- Field names: `Delivery` and `Price`
- Data: Delivery is July 15, 2000, and price is $104.40.

Enter the data into MATLAB software:

```
Type = 'Future';
FieldName = {'Delivery', 'Price'};
Data = {'Jul-15-2000', 104.4};
```

Finally, create the portfolio with a single instrument:

```
Port = instaddfield('Type', Type, 'FieldName', FieldName,...
'Data', Data);
```

Now use the function `instdisp` to examine the resulting single-instrument portfolio:

```
instdisp(Port)

Index Type   Delivery    Price
1     Future Jul-15-2000 104.4
```

Because your portfolio `Port` has the same structure as those created using the function `instadd`, you can combine portfolios created using `instadd` with portfolios created using `instaddfield`. For example, you can now add two cap instruments to `Port` with `instadd`.

```
Strike = [0.06; 0.07];
Settle = '08-Feb-2000';
Maturity = '15-Jan-2003';

Port = instadd(Port, 'Cap', Strike, Settle, Maturity);
```

View the resulting portfolio using `instdisp`.

```
instdisp(Port)

Index   Type   Delivery      Price
1       Future 15-Jul-2000   104.4

Index Type Strike Settle      Maturity    CapReset  Basis Principal
2     Cap  0.06   08-Feb-2000 15-Jan-2003 1         0     100
3     Cap  0.07   08-Feb-2000 15-Jan-2003 1         0     100
```

## Searching or Subsetting a Portfolio

Financial Instruments Toolbox software provides functions that enable you to:

- Find specific instruments within a portfolio.
- Create a subset portfolio consisting of instruments selected from a larger portfolio.

The `instfind` function finds instruments with a specific parameter value; it returns an instrument index (position) in a large instrument set. The `instselect` function, on the other hand, subsets a large instrument set into a portfolio of instruments with designated parameter values; it returns an instrument set (portfolio) rather than an index.

### instfind

The general syntax for `instfind` is

```
IndexMatch = instfind(InstSet, 'FieldName', FieldList, 'Data',...
DataList, 'Index', IndexSet, 'Type', TypeList)
```

`InstSet` is the instrument set to search. Within `InstSet` instruments categorized by type, each type can have different data fields. The stored data field is a row vector or string for each instrument.

The `FieldList`, `DataList`, and `TypeList` arguments indicate values to search for in the `FieldName`, `Data`, and `Type` data fields of the instrument set. `FieldList` is a cell array of field name(s) specific to the instruments. `DataList` is a cell array or matrix of acceptable values for the parameter(s) specified in `FieldList`. `FieldName` and `Data` (consequently, `FieldList` and `DataList`) parameters must appear together or not at all.

`IndexSet` is a vector of integer index(es) designating positions of instruments in the instrument set to check for matches; the default is all indices available in the instrument set. `TypeList` is a string or cell array of strings restricting instruments to match one of the `TypeList` types; the default is all types in the instrument set.

`IndexMatch` is a vector of positions of instruments matching the input criteria. Instruments are returned in `IndexMatch` if all the `FieldName`, `Data`, `Index`, and `Type` conditions are met. An instrument meets an individual field condition if the stored `FieldName` data matches any of the rows listed in the `DataList` for that `FieldName`.

**instfind Examples.** The examples use the provided MAT-file `deriv.mat`.

The MAT-file contains an instrument set, `HJMInstSet`, that contains eight instruments of seven types.

```
load deriv.mat
instdisp(HJMInstSet)

Index Type CouponRate Settle       Maturity     Period Basis ...  Name      Quantity
1    Bond 0.04       01-Jan-2000  01-Jan-2003    1     NaN   ... 4% bond     100
2    Bond 0.04       01-Jan-2000  01-Jan-2004    2     NaN   ... 4% bond      50

Index Type    UnderInd OptSpec Strike ExerciseDates  AmericanOpt Name        Quantity
3     OptBond 2        call    101    01-Jan-2003    NaN         Option 101   -50

Index Type  CouponRate Settle      Maturity     FixedReset Basis Principal Name     Quantity
4     Fixed 0.04       01-Jan-2000 01-Jan-2003    1        NaN    NaN     4% Fixed   80

Index Type  Spread Settle      Maturity   FloatReset  Basis Principal Name        Quantity
5     Float 20     01-Jan-2000 01-Jan-2003 1           NaN   NaN       20BP Float   8

Index Type Strike Settle         Maturity      CapReset Basis Principal Name     Quantity
6     Cap  0.03   01-Jan-2000    01-Jan-2004    1        NaN   NaN       3% Cap   30

Index Type  Strike Settle      Maturity     FloorReset Basis Principal Name       Quantity
7     Floor 0.03 01-Jan-2000 01-Jan-2004    1          NaN   NaN        3% Floor   40

Index Type LegRate   Settle     Maturity     LegReset Basis Principal LegType  Name   Quantity

8     Swap [0.06 20] 01-Jan-2000  01-Jan-2003  [1  1]   NaN   NaN     [NaN]  6%/20BP Swap  10
```

Find all instruments with a maturity date of January 01, 2003.

```
Mat2003 = ...
instfind(HJMInstSet,'FieldName','Maturity','Data','01-Jan-2003')

Mat2003 =

       1
       4
       5
       8
```

Find all cap and floor instruments with a maturity date of January 01, 2004.

```
CapFloor = instfind(HJMInstSet,...
'FieldName','Maturity','Data','01-Jan-2004', 'Type',...
{'Cap';'Floor'})

CapFloor =

     6
     7
```

Find all instruments where the portfolio is long or short a quantity of 50.

```
Pos50 = instfind(HJMInstSet,'FieldName',...
'Quantity','Data',{'50';'-50'})

Pos50 =

     2
     3
```

### instselect

The syntax for `instselect` is the same syntax as for `instfind`. `instselect` returns a full portfolio instead of indexes into the original portfolio. Compare the values returned by both functions by calling them equivalently.

Previously you used `instfind` to find all instruments in `HJMInstSet` with a maturity date of January 01, 2003.

```
Mat2003 = ...
instfind(HJMInstSet,'FieldName','Maturity','Data','01-Jan-2003')

Mat2003 =

     1
     4
     5
     8
```

Now use the same instrument set as a starting point, but execute the `instselect` function instead, to produce a new instrument set matching the identical search criteria.

```
Select2003 = ...
instselect(HJMInstSet,'FieldName','Maturity','Data',...
'01-Jan-2003')

instdisp(Select2003)
```

```
Index Type CouponRate Settle       Maturity   Period Basis .........    Name   Quantity
1     Bond 0.04       01-Jan-2000  01-Jan-2003  1      NaN .........    4% bond    100

Index Type  CouponRate Settle      Maturity  FixedReset Basis Principal Name    Quantity
2     Fixed 0.04       01-Jan-2000 01-Jan-2003    1      NaN   NaN       4% Fixed   80

Index Type  Spread Settle      Maturity   FloatReset  Basis Principal Name       Quantity
3     Float 20     01-Jan-2000 01-Jan-2003 1           NaN   NaN       20BP Float   8

Index Type LegRate   Settle     Maturity  LegReset Basis Principal LegType Name Quantity
4     Swap [0.06 20] 01-Jan-2000  01-Jan-2003  [1  1]  NaN   NaN  [NaN] 6%/20BP Swap 10
```

**instselect Examples.** These examples use the portfolio `ExampleInst` provided with the MAT-file `InstSetExamples.mat`.

```
load InstSetExamples.mat
instdisp(ExampleInst)

Index Type   Strike Price Opt  Contracts
1     Option  95    12.2  Call     0
2     Option 100     9.2  Call     0
3     Option 105     6.8  Call  1000

Index Type    Delivery       F     Contracts
4     Futures 01-Jul-1999    104.4 -1000

Index Type   Strike Price Opt  Contracts
5     Option 105     7.4  Put  -1000
6     Option  95     2.9  Put      0

Index Type  Price Maturity       Contracts
7     TBill 99    01-Jul-1999    6
```

The instrument set contains 3 instrument types: `Option`, `Futures`, and `TBill`. Use `instselect` to make a new instrument set containing only options struck at `95`. In other words, select all instruments containing the field `Strike` *and* with the data value for that field equal to `95`.

```
InstSet = instselect(ExampleInst,'FieldName','Strike','Data',95);

instdisp(InstSet)

Index Type   Strike Price Opt  Contracts
1     Option  95    12.2  Call     0
2     Option  95     2.9  Put      0
```

You can use all the various forms of `instselect` and `instfind` to locate specific instruments within this instrument set.

# 2

# Interest-Rate Derivatives

# Supported Interest-Rate Instruments

**In this section...**

## Bond

A *bond* is a long-term debt security with a preset interest-rate and maturity. At maturity you must pay the principal and interest.

The price or value of a bond is determined by discounting the expected cash flows of the bond to the present, using the appropriate discount rate. The following equation represents the relationship of the expected cash flows and discount rate:

$$B_0 = \frac{C}{2}\left[\frac{1-\left(1+\frac{r}{2}\right)^{-2t}}{\frac{r}{2}}\right] + \frac{F}{\left(1+\frac{r}{2}\right)^{2t}}$$

where:

$B_0$ is the bond value.

$C$ is the annual coupon payment.

$F$ is the face value of the bond.

$r$ is the required return on the bond.

$t$ is the number of years remaining until maturity.

Financial Instruments Toolbox supports the following for pricing and specifying a bond.

| **Function** | **Purpose** |
|---|---|
| `bondbybdt` | Price a bond using a BDT interest-rate tree. |
| `bondbyhw` | Price a bond using an HW interest-rate tree. |
| `bondbybk` | Price a bond using a BK interest-rate tree. |
| `bondbyhjm` | Price a bond using an HJM interest-rate tree. |
| `bondbyzero` | Price a bond using a set of zero curves. |
| `instbond` | Construct a bond instrument. |

## Stepped Coupon Bonds

A step-up and step-down bond is a debt security with a predetermined coupon structure over time. With these instruments, coupons increase (step up) or decrease (step down) at specific times during the life of the bond. For more information on options features (call and puts), see "Stepped Coupon Bonds with Calls and Puts" on page 2-7. The following functions have a modified `CouponRate` argument to support a new variable coupon schedule allowing pricing of stepped coupon bonds.

| Function | Purpose |
|---|---|
| `bondbyzero` | Price bonds using a term structure model. |
| `bondbybdt` | Price bonds using a BDT tree model. |
| `bondbyhjm` | Price bonds using an HJM tree model. |
| `bondbyhw` | Price bonds using an HW tree model. |
| `bondbybk` | Price bonds using a BK tree model. |
| `instbond` | Construct a bond instrument. |
| `instoptbnd` | Construct a bond option instrument. |
| `instdisp` | Display instruments stored in a variable. |

## Sinking Fund Bonds

A sinking fund bond is a coupon bond with a sinking fund provision. This provision obligates the issuer to amortize portions of the principal before maturity, affecting bond prices since the time of the principal repayment changes. This means that investors receive the coupon and a portion of the principal paid back over time. These types of bonds reduce credit risk, since it lowers the probability of investors not receiving their principal payment at maturity. For more information on options support for sinking fund bonds, see "Sinking Fund Bonds with an Embedded Option" on page 2-8. The following functions have a modified `Face` argument to support a variable face schedule for pricing bonds with a sinking provisions.

| Function | Purpose |
|---|---|
| `bondbyzero` | Price bonds using a term structure model. |
| `bondbybdt` | Price bonds using a BDT tree model. |

| Function | Purpose |
|---|---|
| `bondbyhjm` | Price bonds using an HJM tree model. |
| `bondbyhw` | Price bonds using an HW tree model. |
| `bondbybk` | Price bonds using a BK tree model. |
| `instbond` | Construct a bond instrument. |
| `instoptbnd` | Construct a bond option instrument. |
| `instdisp` | Display instruments stored in a variable. |

## Bonds with an Amortization Schedule

A bond with an amortization schedule repays part of the principal (face value) along with the coupon payments. An amortizing bond is a special case of a sinking fund bond when there is no market purchase option and no call provision.. The following functions have a modified `Face` argument to support an amortization schedule.

| Function | Purpose |
|---|---|
| `bondbyzero` | Price bonds using a term structure model. |
| `bondbybdt` | Price bonds using a BDT tree model. |
| `bondbyhjm` | Price bonds using an HJM tree model. |
| `bondbyhw` | Price bonds using an HW tree model. |
| `bondbybk` | Price bonds using a BK tree model. |

## Bond Options

Financial Instruments Toolbox software supports three types of put and call options on bonds:

- American option: An option that you exercise any time until its expiration date.
- European option: An option that you exercise only on its expiration date.

- Bermuda option: A Bermuda option resembles a hybrid of American and European options. You can exercise it on predetermined dates only, usually monthly.

Financial Instruments Toolbox supports the following for pricing and specifying a bond option.

| Function | Purpose |
|---|---|
| `optbndbybdt` | Price a bond option price using a BDT interest-rate tree. |
| `optbndbyhw` | Price a bond option price using an HW interest-rate tree. |
| `optbndbybk` | Price a bond option price using a BK interest-rate tree. |
| `optbndbyhjm` | Price a bond option price using an HJM interest-rate tree. |
| `instoptbnd` | Construct a bond option instrument. |

## Bond with Embedded Options

A bond with embedded options allows the issuer to buy back or redeem the bond at a predetermined price at specified future dates. Financial Instruments Toolbox software supports American, European, and Bermuda callable and puttable bonds.

The pricing for a bond with embedded options is as follows:

- For a callable bond: $PriceCallableBond = BondPrice - BondCallOption$
- For a puttable bond: $PricePuttableBond = PriceBond + PricePutOption$

Financial Instruments Toolbox supports the following for pricing and specifying a bond with embedded options.

| Function | Purpose |
|---|---|
| `optembndbybdt` | Price a bond with embedded options using a BDT interest-rate tree. |
| `optembndbyhw` | Price a bond with embedded options using an HW interest rate tree. |
| `optembndbybk` | Price a bond with embedded options using a BK interest-rate tree. |
| `optembndbyhjm` | Price a bond with embedded options using an HJM interest-rate tree. |
| `instoptembnd` | Construct a bond-with-embedded-options instrument. |

## Stepped Coupon Bonds with Calls and Puts

A step-up and step-down bond is a debt security with a predetermined coupon structure over time. For more information on stepped coupon bonds, see "Stepped Coupon Bonds" on page 2-4. Stepped coupon bonds can have options features (call and puts). The following functions have a modified `CouponRate` argument to support a new variable coupon schedule allowing pricing stepped coupon bonds with callable and puttable features:

| Function | Purpose |
|---|---|
| `optembndbybdt` | Price bonds with embedded options using a BDT model tree. |
| `optembndbyhjm` | Price bonds with embedded options using an HJM model tree. |
| `optembndbybk` | Price bonds with embedded options using a BK model tree. |
| `optembndbyhw` | Price bonds with embedded options using an HW model tree. |
| `instbond` | Construct a bond instrument. |
| `instoptbnd` | Construct a bond option instrument. |

| Function | Purpose |
|---|---|
| `instoptembnd` | Construct a bond with an embedded option instrument. |
| `instdisp` | Display instruments stored in a variable. |

## Sinking Fund Bonds with an Embedded Option

A sinking fund bond is a coupon bond with a sinking fund provision. For more information on sinking fund bonds, see "Sinking Fund Bonds" on page 2-4. The sinking fund bond can have a sinking fund option provision allowing the issuer to retire the sinking fund obligation either by purchasing the bonds to be redeemed from the market or by calling the bond via a sinking fund call, whichever is cheaper.

If interest rates are high, then the issuer buys back the required amount of bonds from the market since bonds will be cheap. But if interest rates are low (bond prices are high), then most likely the issuer buys the bonds at the call price. Unlike a call feature, however, if a bond has a sinking fund option provision, it is an obligation, not an option, for the issuer to buy back the increments of the issue as stated. Because of this, a sinking fund bond trades at a lower price than a nonsinking fund bond. The following functions have a modified `Face` argument to support a variable face schedule for pricing bonds with a sinking fund option provision.

| Function | Purpose |
|---|---|
| `optembndbybdt` | Price bonds with embedded options using a BDT model tree. |
| `optembndbyhjm` | Price bonds with embedded options using an HJM model tree. |
| `optembndbybk` | Price bonds with embedded options using a BK model tree. |
| `optembndbyhw` | Price bonds with embedded options using an HW model tree. |
| `instbond` | Construct a bond instrument. |

| **Function** | **Purpose** |
|---|---|
| `instoptbnd` | Construct a bond option instrument. |
| `instdisp` | Display instruments stored in a variable. |

## Fixed-Rate Note

A *fixed-rate note* is a long-term debt security with a preset interest rate and maturity, by which the interest must be paid. The principal may or may not be paid at maturity. In Financial Instruments Toolbox software, the principal is always paid at maturity.

| **Function** | **Purpose** |
|---|---|
| `fixedbybdt` | Price a fixed-rate note using a BDT interest-rate tree. |
| `fixedbyhw` | Price a fixed-rate note using an HW interest-rate tree. |
| `fixedbybk` | Price a fixed-rate note using a BK interest-rate tree. |
| `fixedbyhjm` | Price a fixed-rate note using an HJM interest-rate tree. |
| `fixedbyzero` | Price a fixed-rate note using a set of zero curves. |
| `instfixed` | Construct a fixed-rate instrument. |

## Floating-Rate Note

A *floating-rate note* is a security like a bond, but the interest rate of the note is reset periodically, relative to a reference index rate, to reflect fluctuations in market interest rates.

| Function | Purpose |
|---|---|
| `floatbybdt` | Price a floating-rate note using a BDT interest-rate tree. |
| `floatbyhw` | Price a floating-rate note using an HW interest-rate tree. |
| `floatbybk` | Price a floating-rate note using a BK interest-rate tree. |
| `floatbyhjm` | Price a floating-rate note using an HJM interest-rate tree. |
| `floatbyzero` | Price a floating-rate note using a set of zero curves. |
| `instfloat` | Construct a floating-rate note instrument. |

## Floating-Rate Note with an Amortization Schedule

A floating-rate note with an amortization schedule repays part of the principal (face value) along with the coupon payments. The following functions have a `Principal` argument to support an amortization schedule.

| Function | Purpose |
|---|---|
| `floatbyzero` | Price floating-rate note from set of zero curves. |
| `floatbybdt` | Price floating-rate note from Black-Derman-Toy interest-rate tree. |
| `floatbyhjm` | Price floating-rate note from Heath-Jarrow-Morton interest-rate tree. |
| `floatbyhw` | Price floating-rate note from Hull-White interest-rate tree. |
| `floatbybk` | Price floating-rate note from Black-Karasinski interest-rate tree. |

## Floating-Rate Note with Caps, Collars, and Floors

A floating-rate note with caps, collars, and floors This type of instrument can carry restrictions on the maximum (cap) or minimum (floor) coupon rate are paid. A cap is an unattractive feature for an investor, since they constrain the coupon rates from increasing. A floor, on the other hand, is an attractive feature, since it allows investors to get a minimum coupon rate when market rates decrease below a certain level. Also, a floating-rate note can have a collar which is a combination of a cap and a floor together. The following functions have a `CapRate` and `FloorRate` argument to support a capped, collared, or floored floating-rate note.

| **Function** | **Purpose** |
|---|---|
| `floatbybdt` | Price a capped floating-rate note from a Black-Derman-Toy interest-rate tree. |
| `floatbyhjm` | Price a capped floating-rate note from a Heath-Jarrow-Morton interest-rate tree. |
| `floatbyhw` | Price a capped floating-rate note from a Hull-White interest-rate tree. |
| `floatbybk` | Price a capped floating-rate note from a Black-Karasinskiinterest-rate tree. |
| `instfloat` | Create a capped floating-rate note instrument. |
| `instadd` | Add a capped floating-rate note instrument to a portfolio. |

## Cap

A *cap* is a contract that includes a guarantee that sets the maximum interest rate to be paid by the holder, based on an otherwise floating interest rate. The payoff for a cap is:

$$\max(CurrentRate - CapRate, 0)$$

| Function | Purpose |
|---|---|
| `capbybdt` | Price a cap instrument using a BDT interest-rate tree. |
| `capbyhw` | Price a cap instrument using an HW interest-rate tree. |
| `capbybk` | Price a cap instrument using a BK interest-rate tree. |
| `capbyhjm` | Price a cap instrument using an HJM interest-rate tree. |
| `capbyblk` | Price a cap instrument using the Black option pricing model. |
| `instcap` | Construct a cap instrument. |

## Floor

A *floor* is a contract that includes a guarantee setting the minimum interest rate to be received by the holder, based on an otherwise floating interest rate. The payoff for a floor is:

$$\max(FloorRate - CurrentRate, 0)$$

| Function | Purpose |
|---|---|
| `floorbybdt` | Price a floor instrument using a BDT interest-rate tree. |
| `floorbyhw` | Price a floor instrument using an HW interest-rate tree. |
| `floorbybk` | Price a floor instrument using a BK interest-rate tree. |
| `floorbyhjm` | Price a floor instrument using an HJM interest-rate tree. |
| `instfloor` | Construct a floor instrument. |

## Range Note

A *range note* is a structured (market-linked) security whose coupon-rate is equal to the reference rate as long as the reference rate is within a certain range. If the reference rate is outside of the range, the coupon-rate is 0 for that period. This type of instrument entitles the holder to cash flows that depend on the level of some reference interest-rate that are floored to be positive and gives the holder of the note direct exposure to the reference rate. This type of instrument is useful for cases where you believe that interest rates will stay within a certain range. In return for the drawback that no interest will be paid for the time the range is left, a range note offers higher coupon rates than comparable standard products, like vanilla floating notes.

| **Function** | **Purpose** |
|---|---|
| `instrangefloat` | Create a range note instrument. |
| `rangefloatbybdt` | Price range floating note using a BDT tree. |
| `rangefloatbybk` | Price range floating note using a BK tree. |
| `rangefloatbyhjm` | Price range floating note using an HJM tree. |
| `rangefloatbyhw` | Price range floating note using an HW tree. |

## Swap

A *swap* is contract between two parties obligating the parties to exchange future cash flows. This toolbox version handles only the vanilla swap, which is composed of a floating-rate leg and a fixed-rate leg.

| **Function** | **Purpose** |
|---|---|
| `swapbybdt` | Price a swap instrument using a BDT interest-rate tree. |
| `swapbyhw` | Price a swap instrument using an HW interest-rate tree. |
| `swapbybk` | Price a swap instrument using a BK interest-rate tree. |
| `swapbyhjm` | Price a swap instrument using an HJM interest-rate tree. |

| Function | Purpose |
|---|---|
| `swapbyzero` | Price a swap instrument using a set of zero curves. |
| `instswap` | Construct a swap instrument. |

## Swap with an Amortization Schedule

A swap with an amortization schedule repays part of the principal (face value) along with the coupon payments. A swap with an amortization schedule is used to manage interest rate risk and serve as a cash flow management tool. For this particular type of swap, the notional amount decreases over time. This means that interest payments will decrease not only on the floating leg but also on the fixed leg. The following swap functions have a `Principal` argument to support an amortization schedule.

| Function | Purpose |
|---|---|
| `swapbyzero` | Price swap instrument from set of zero curves. |
| `swapbybdt` | Price swap instrument from Black-Derman-Toy interest-rate tree. |
| `swapbyhjm` | Price swap instrument from Heath-Jarrow-Morton interest-rate tree. |
| `swapbyhw` | Price swap instrument from Hull-White interest-rate tree. |
| `swapbybk` | Price swap instrument from Black-Karasinski interest-rate tree. |
| `instswap` | Construct swap instrument. |

## Forward Swap

In a forward interest-rate swap, a fixed interest rate loan is exchanged for a floating interest rate loan at a future specified date The following functions have a `StartDate` argument to support the future date for the forward swap.

| Function | Purpose |
|---|---|
| `swapbyzero` | Price a forward swap from a zero curve. |
| `swapbybdt` | Price a forward swap from a Black-Derman-Toy interest-rate tree. |
| `swapbyhjm` | Price a forward swap from a Heath-Jarrow-Morton interest-rate tree. |
| `swapbyhw` | Price a forward swap from a Hull-White interest-rate tree. |
| `swapbybk` | Price a forward swap from a Black-Karasinskiinterest-rate tree. |
| `instswap` | Create a forward swap instrument. |
| `instadd` | Add a capped floating-rate note instrument to a portfolio. |

## Swaption

A swaption is an option to enter into an interest-rate swap contract. A call swaption allows the option buyer to enter into an interest-rate swap where the buyer of the option pays the fixed-rate and receives the floating-rate. A put swaption allows the option buyer to enter into an interest-rate swap where the buyer of the option receives the fixed-rate and pays the floating-rate.

| Function | Purpose |
|---|---|
| `swaptionbybdt` | Price a swaption instrument using a BDT interest-rate tree. |
| `swaptionbyhw` | Price a swaption instrument using an HW interest-rate tree. |
| `swaptionbybk` | Price a swaption instrument using a BK interest-rate tree. |
| `swaptionbyhjm` | Price a swaption instrument using an HJM interest-rate tree. |

| Function | Purpose |
|---|---|
| `swaptionbyblk` | Price swaptions using the Black model with a forward on a swap. |
| `instswaption` | Construct a swaption instrument. |

Use `swaptionbyblk` to price a swaption using the Black model. The Black model is standard model used in the swaption market when pricing European swaptions. This type of model is widely used by when speed is important to quickly obtain a price at settlement date, even if the price is less accurate than other swaption pricing models based on interest-rate tree models.

## Bond Futures

Bond futures are futures contracts where the commodity for delivery is a government bond. There are established global markets for government bond futures. Bond futures provide a liquid alternative for managing interest-rate risk.

In the U.S. market, the Chicago Mercantile Exchange (CME) offers futures on Treasury bonds and notes with maturities of 2, 5, 10, and 30 years. Typically, the following bond future contracts from the CME have maturities of 3, 6, 9, and 12 months:

- 30-year U.S. Treasury bond
- 10-year U.S. Treasury bond
- 5-year U.S. Treasury bond
- 2-year U.S. Treasury bond

The short position in a Treasury bond or note future contract must deliver to the long position in one of many possible existing Treasury bonds. For example, in a 30-year Treasury bond future, the short position must deliver a Treasury bond with at least 15 years to maturity. Because these bonds have different values, the bond future contract is standardized by computing a conversion factor. The conversion factor normalizes the price of a bond to a theoretical bond with a coupon of 6%. The price of a bond future contract is represented as:

$$InvoicePrice = FutPrice \times CF + AI$$

where:

$FutPrice$ is the price of the bond future.

$CF$ is the conversion factor for a bond to deliver in a futures contract.

$AI$ is the accrued interest.

You can reference these conversion factors at U.S. Treasury Bond Futures Contract. The short position in a futures contract has the option of which bond to deliver and, in the U.S. bond market, when in the delivery month to deliver the bond. The short position typically chooses to deliver the bond known as the Cheapest to Deliver (CTD). The CTD bond most often delivers on the last delivery day of the month.

Financial Instruments Toolbox software supports the following bond futures:

- U.S. Treasury bonds and notes
- German Bobl, Bund, Buxl, and Schatz
- UK gilts
- Japanese government bonds (JGBs)

The functions supporting all bond futures are:

| Function | Purpose |
|---|---|
| `convfactor` | Calculates bond conversion factors for U.S. Treasury bonds, German Bobl, Bund, Buxl, and Schatz, U.K. gilts, and JGBs. |
| `bndfutprice` | Prices bond future given repo rates. |
| `bndfutimprepo` | Calculates implied repo rates for a bond future given price. |

The functions supporting U.S. Treasury bond futures are:

| Function | Purpose |
|---|---|
| `tfutbyprice` | Calculates future prices of Treasury bonds given the spot price. |
| `tfutbyyield` | Calculates future prices of Treasury bonds given current yield. |
| `tfutimprepo` | Calculates implied repo rates for the Treasury bond future given price. |
| `tfutpricebyrepo` | Calculates implied repo rates given the Treasury bond future price. |
| `tfutyieldbyrepo` | Calculates implied repo rates given the Treasury bond future yield. |

For more information on bond futures, see “Bond Futures” on page 7-12.

# Overview of Interest-Rate Tree Models

**In this section...**

## Interest-Rate Modeling

Financial Instruments Toolbox software computes prices and sensitivities of interest-rate contingent claims based on several methods of modeling changes in interest rates over time:

- The interest-rate term structure

  This model uses sets of zero-coupon bonds to predict changes in interest rates.

- Heath-Jarrow-Morton (HJM) model

  The HJM model considers a given initial term structure of interest rates and a specification of the volatility of forward rates to build a tree representing the evolution of the interest rates, based on a statistical process.

- Black-Derman-Toy (BDT) model

  In the BDT model, all security prices and rates depend on the short rate (annualized 1-period interest rate). The model uses long rates and their volatilities to construct a tree of possible future short rates. The resulting tree can then be used to determine the value of interest-rate sensitive securities from this tree.

- Hull-White (HW) model

  The Hull-White model incorporates the initial term structure of interest rates and the volatility term structure to build a trinomial recombining tree of short rates. The resulting tree is used to value interest-rate dependent securities. The implementation of the HW model in Financial Instruments Toolbox software is limited to one factor.

- Black-Karasinski (BK) model

The BK model is a single-factor, log-normal version of the HW model.

For detailed information about interest-rate models, see:

- "Pricing Using Interest-Rate Term Structure" on page 2-38 for a discussion of price and sensitivity based on portfolios of zero-coupon bonds
- "Pricing Using Interest-Rate Tree Models" on page 2-67 for a discussion of price and sensitivity based on the HJM and BDT interest-rate models

---

**Note** Historically, the initial version of Financial Instruments Toolbox software provided only the HJM interest-rate model. A later version added the BDT model. The current version adds both the HW and BK models. This section provides extensive examples of using the HJM and BDT models to compute prices and sensitivities of interest-rate based financial derivatives.

The HW and BK tree structures are similar to the BDT tree structure. To avoid needless repetition throughout this section, documentation is provided only where significant deviations from the BDT structure exist. Specifically, "HW and BK Tree Structures" on page 2-62 explains the few noteworthy differences among the various formats.

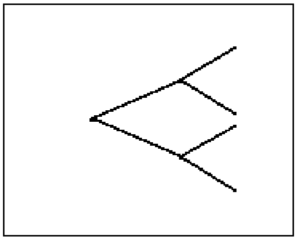
---

## Rate and Price Trees

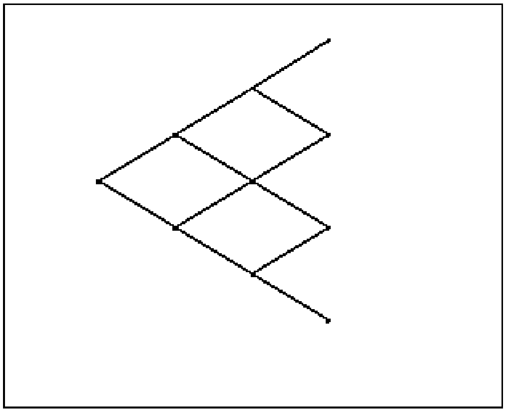
The interest-rate or price trees supported in this toolbox can be either *binomial* (two branches per node) or *trinomial* (three branches per node). Typically, binomial trees assume that underlying interest rates or prices can only either increase or decrease at each node. Trinomial trees allow for a more complex movement of rates or prices. With trinomial trees the movement of rates or prices at each node is unrestricted (for example, up-up-up or unchanged-down-down).

### Types of Trees

Financial Instruments Toolbox trees can be classified as *bushy* or *recombining*. A bushy tree is a tree in which the number of branches increases exponentially relative to observation times; branches never recombine. In this context, a recombining tree is the opposite of a bushy tree. A recombining tree has branches that recombine over time. From any given node, the node reached by taking the path up-down is the same node reached by taking the path down-up. A bushy tree and a recombining binomial tree are illustrated next.



Bushy Tree



Recombining Binomial Tree

In this toolbox the Heath-Jarrow-Morton model works with bushy trees. The Black-Derman-Toy model, on the other hand, works with recombining binomial trees.

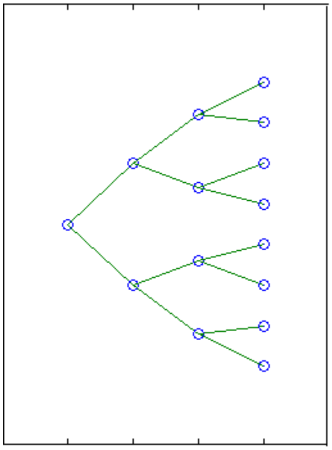
The other two interest rate models supported in this toolbox, Hull-White and Black-Karasinski, work with recombining trinomial trees.

## Viewing Rate or Price Movement

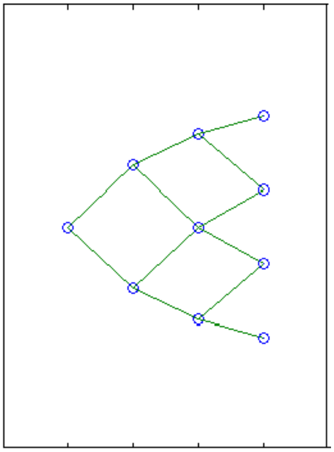
This toolbox provides the data file `deriv.mat` that contains four interest-rate based trees:

- `HJMTree` — A bushy binomial tree
- `BDTTree` — A recombining binomial tree
- `HWTree` and `BKTree` — Recombining trinomial trees

The toolbox also provides the `treeviewer` function, which graphically displays the shape and data of price, interest rate, and cash flow trees. Viewed with `treeviewer`, the bushy shape of an HJM tree and the recombining shape of a BDT tree are apparent.

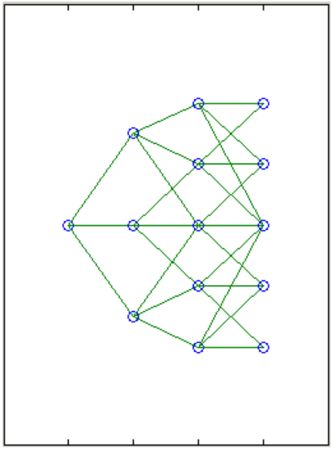


HJMTree (bushy)



BDTTree (recombining)

With `treeviewer`, you can also see the recombining shape of HW and BK trinomial trees.



HWTree and BKTree (recombining)

# Understanding the Interest-Rate Term Structure

**In this section...**

## Introduction

The *interest-rate term structure* represents the evolution of interest rates through time. In MATLAB software, the interest-rate environment is encapsulated in a structure called `RateSpec` (*rate specification*). This structure holds all information required to completely identify the evolution of interest rates. Several functions included in Financial Instruments Toolbox software are dedicated to the creating and managing of the `RateSpec` structure. Many others take this structure as an input argument representing the evolution of interest rates.

Before looking further at the `RateSpec` structure, examine three functions that provide key functionality for working with interest rates: `disc2rate`, its opposite, `rate2disc`, and `ratetimes`. The first two functions map between discount factors and interest rates. The third function, `ratetimes`, calculates the effect of term changes on the interest rates.

## Interest Rates Versus Discount Factors

*Discount factors* are coefficients commonly used to find the current value of future cash flows. As such, there is a direct mapping between the rate applicable to a period of time, and the corresponding discount factor. The function `disc2rate` converts discount factors for a given term (period) into interest rates. The function `rate2disc` does the opposite; it converts interest rates applicable to a given term (period) into the corresponding discount factors.

### Calculating Discount Factors from Rates

As an example, consider these annualized zero-coupon bond rates.

| From | To | Rate |
|---|---|---|
| 15 Feb 2000 | 15 Aug 2000 | 0.05 |
| 15 Feb 2000 | 15 Feb 2001 | 0.056 |
| 15 Feb 2000 | 15 Aug 2001 | 0.06 |
| 15 Feb 2000 | 15 Feb 2002 | 0.065 |
| 15 Feb 2000 | 15 Aug 2002 | 0.075 |

To calculate the discount factors corresponding to these interest rates, call `rate2disc` using the syntax

```
Disc = rate2disc(Compounding, Rates, EndDates, StartDates,
ValuationDate)
```

where:

- `Compounding` represents the frequency at which the zero rates are compounded when annualized. For this example, assume this value to be 2.
- `Rates` is a vector of annualized percentage rates representing the interest rate applicable to each time interval.
- `EndDates` is a vector of dates representing the end of each interest-rate term (period).
- `StartDates` is a vector of dates representing the beginning of each interest-rate term.
- `ValuationDate` is the date of observation for which the discount factors are calculated. In this particular example, use February 15, 2000 as the beginning date for all interest-rate terms.

Next, set the variables in MATLAB.

```
StartDates = ['15-Feb-2000'];
EndDates  = ['15-Aug-2000'; '15-Feb-2001'; '15-Aug-2001';...
'15-Feb-2002'; '15-Aug-2002'];
Compounding = 2;
ValuationDate = ['15-Feb-2000'];
```

```
Rates = [0.05; 0.056; 0.06; 0.065; 0.075];
```

Finally, compute the discount factors.

```
Disc = rate2disc(Compounding, Rates, EndDates, StartDates,...
ValuationDate)

Disc =

    0.9756
    0.9463
    0.9151
    0.8799
    0.8319
```

By adding a fourth column to the rates table (see "Calculating Discount Factors from Rates" on page 2-23) to include the corresponding discounts, you can see the evolution of the discount factors.

| **From** | **To** | **Rate** | **Discount** |
|---|---|---|---|
| 15 Feb 2000 | 15 Aug 2000 | 0.05 | 0.9756 |
| 15 Feb 2000 | 15 Feb 2001 | 0.056 | 0.9463 |
| 15 Feb 2000 | 15 Aug 2001 | 0.06 | 0.9151 |
| 15 Feb 2000 | 15 Feb 2002 | 0.065 | 0.8799 |
| 15 Feb 2000 | 15 Aug 2002 | 0.075 | 0.8319 |

## Optional Time Factor Outputs

The function `rate2disc` optionally returns two additional output arguments: `EndTimes` and `StartTimes`. These vectors of time factors represent the start dates and end dates in discount periodic units. The scale of these units is determined by the value of the input variable `Compounding`.

To examine the time factor outputs, find the corresponding values in the previous example.

```
[Disc, EndTimes, StartTimes] = rate2disc(Compounding, Rates,...
```

```
EndDates, StartDates, ValuationDate);
```

Arrange the two vectors into a single array for easier visualization.

```
Times = [StartTimes, EndTimes]

Times =

     0     1
     0     2
     0     3
     0     4
     0     5
```

Because the valuation date is equal to the start date for all periods, the `StartTimes` vector is composed of 0s. Also, since the value of `Compounding` is 2, the rates are compounded semiannually, which sets the units of periodic discount to 6 months. The vector `EndDates` is composed of dates separated by intervals of 6 months from the valuation date. This explains why the `EndTimes` vector is a progression of integers from 1 to 5.

### Alternative Syntax (rate2disc)

The function `rate2disc` also accommodates an alternative syntax that uses periodic discount units instead of dates. Since the relationship between discount factors and interest rates is based on time periods and not on absolute dates, this form of `rate2disc` allows you to work directly with time periods. In this mode, the valuation date corresponds to 0, and the vectors `StartTimes` and `EndTimes` are used as input arguments instead of their date equivalents, `StartDates` and `EndDates`. This syntax for `rate2disc` is:

```
Disc = rate2disc(Compounding, Rates, EndTimes, StartTimes)
```

Using as input the `StartTimes` and `EndTimes` vectors computed previously, you should obtain the previous results for the discount factors.

```
Disc = rate2disc(Compounding, Rates, EndTimes, StartTimes)

Disc =

    0.9756
```

```
    0.9463
    0.9151
    0.8799
    0.8319
```

### Calculating Rates from Discounts

The function `disc2rate` is the complement to `rate2disc`. It finds the rates applicable to a set of compounding periods, given the discount factor in those periods. The syntax for calling this function is:

```
Rates = disc2rate(Compounding, Disc, EndDates, StartDates,
ValuationDate)
```

Each argument to this function has the same meaning as in `rate2disc`. Use the results found in the previous example to return the rate values you started with.

```
Rates = disc2rate(Compounding, Disc, EndDates, StartDates,...
ValuationDate)

Rates =

    0.0500
    0.0560
    0.0600
    0.0650
    0.0750
```

### Alternative Syntax (disc2rate)

As in the case of `rate2disc`, `disc2rate` optionally returns `StartTimes` and `EndTimes` vectors representing the start and end times measured in discount periodic units. Again, working with the same values as before, you should obtain the same numbers.

```
[Rates, EndTimes, StartTimes] = disc2rate(Compounding, Disc,...
EndDates, StartDates, ValuationDate);
```

Arrange the results in a matrix convenient to display.

```
Result = [StartTimes, EndTimes, Rates]

Result =

         0    1.0000    0.0500
         0    2.0000    0.0560
         0    3.0000    0.0600
         0    4.0000    0.0650
         0    5.0000    0.0750
```

As with `rate2disc`, the relationship between rates and discount factors is determined by time periods and not by absolute dates. Consequently, the alternate syntax for `disc2rate` uses time vectors instead of dates, and it assumes that the valuation date corresponds to time = 0. The time-based calling syntax is:

```
Rates = disc2rate(Compounding, Disc, EndTimes, StartTimes);
```

Using this syntax, you again obtain the original values for the interest rates.

```
Rates = disc2rate(Compounding, Disc, EndTimes, StartTimes)

Rates =

    0.0500
    0.0560
    0.0600
    0.0650
    0.0750
```

## Interest-Rate Term Conversions

Interest rate evolution is typically represented by a set of interest rates, including the beginning and end of the periods the rates apply to. For zero rates, the start dates are typically at the valuation date, with the rates extending from that valuation date until their respective maturity dates.

### Spot Curve to Forward Curve Conversion

Frequently, given a set of rates including their start and end dates, you may be interested in finding the rates applicable to different terms (periods). This problem is addressed by the function `ratetimes`. This function interpolates the interest rates given a change in the original terms. The syntax for calling `ratetimes` is

```
[Rates, EndTimes, StartTimes] = ratetimes(Compounding, RefRates,
RefEndDates, RefStartDates, EndDates, StartDates, ValuationDate);
```

where:

- `Compounding` represents the frequency at which the zero rates are compounded when annualized.
- `RefRates` is a vector of initial interest rates representing the interest rates applicable to the initial time intervals.
- `RefEndDates` is a vector of dates representing the end of the interest rate terms (period) applicable to `RefRates`.
- `RefStartDates` is a vector of dates representing the beginning of the interest rate terms applicable to `RefRates`.
- `EndDates` represent the maturity dates for which the interest rates are interpolated.
- `StartDates` represent the starting dates for which the interest rates are interpolated.
- `ValuationDate` is the date of observation, from which the `StartTimes` and `EndTimes` are calculated. This date represents time = 0.

The input arguments to this function can be separated into two groups:

- The initial or reference interest rates, including the terms for which they are valid
- Terms for which the new interest rates are calculated

As an example, consider the rate table specified in "Calculating Discount Factors from Rates" on page 2-23.

| **From** | **To** | **Rate** |
|---|---|---|
| 15 Feb 2000 | 15 Aug 2000 | 0.05 |
| 15 Feb 2000 | 15 Feb 2001 | 0.056 |

| From | To | Rate |
|---|---|---|
| 15 Feb 2000 | 15 Aug 2001 | 0.06 |
| 15 Feb 2000 | 15 Feb 2002 | 0.065 |
| 15 Feb 2000 | 15 Aug 2002 | 0.075 |

Assuming that the valuation date is February 15, 2000, these rates represent zero-coupon bond rates with maturities specified in the second column. Use the function `ratetimes` to calculate the forward rates at the beginning of all periods implied in the table. Assume a compounding value of 2.

```
% Reference Rates.
RefStartDates = ['15-Feb-2000'];
RefEndDates  = ['15-Aug-2000'; '15-Feb-2001'; '15-Aug-2001';...
'15-Feb-2002'; '15-Aug-2002'];
Compounding = 2;
ValuationDate = ['15-Feb-2000'];
RefRates = [0.05; 0.056; 0.06; 0.065; 0.075];

% New Terms.
StartDates = ['15-Feb-2000'; '15-Aug-2000'; '15-Feb-2001';...
'15-Aug-2001'; '15-Feb-2002'];
EndDates =   ['15-Aug-2000'; '15-Feb-2001'; '15-Aug-2001';...
'15-Feb-2002'; '15-Aug-2002'];
% Find the new rates.
Rates = ratetimes(Compounding, RefRates, RefEndDates,...
RefStartDates, EndDates, StartDates, ValuationDate)

Rates =

    0.0500
    0.0620
    0.0680
    0.0801
    0.1155
```

Place these values in a table like the previous one. Observe the evolution of the forward rates based on the initial zero-coupon rates.

| From | To | Rate |
|---|---|---|
| 15 Feb 2000 | 15 Aug 2000 | 0.0500 |
| 15 Aug 2000 | 15 Feb 2001 | 0.0620 |
| 15 Feb 2001 | 15 Aug 2001 | 0.0680 |
| 15 Aug 2001 | 15 Feb 2002 | 0.0801 |
| 15 Feb 2002 | 15 Aug 2002 | 0.1155 |

### Alternative Syntax (ratetimes)

The `ratetimes` function can provide the additional output arguments `StartTimes` and `EndTimes`, which represent the time factor equivalents to the `StartDates` and `EndDates` vectors. The `ratetimes` function uses time factors for interpolating the rates. These time factors are calculated from the start and end dates, and the valuation date, which are passed as input arguments. `ratetimes` can also use time factors directly, assuming time = 0 as the valuation date. This alternate syntax is:

```
[Rates, EndTimes, StartTimes] = ratetimes(Compounding, RefRates,
RefEndTimes, RefStartTimes, EndTimes, StartTimes);
```

Use this alternate version of `ratetimes` to find the forward rates again. In this case, you must first find the time factors of the reference curve. Use `date2time` for this.

```
RefEndTimes = date2time(ValuationDate, RefEndDates, Compounding)

RefEndTimes =

     1
     2
     3
     4
     5

RefStartTimes = date2time(ValuationDate, RefStartDates,...
Compounding)
```

```
RefStartTimes =

     0
```

These are the expected values, given semiannual discounts (as denoted by a value of 2 in the variable `Compounding`), end dates separated by 6-month periods, and the valuation date equal to the date marking beginning of the first period (time factor = 0).

Now call `ratetimes` with the alternate syntax.

```
[Rates, EndTimes, StartTimes] = ratetimes(Compounding,...
RefRates, RefEndTimes, RefStartTimes, EndTimes, StartTimes);
Rates =

    0.0500
    0.0620
    0.0680
    0.0801
    0.1155
```

`EndTimes` and `StartTimes` have, as expected, the same values they had as input arguments.

```
Times = [StartTimes, EndTimes]

Times =

     0      1
     1      2
     2      3
     3      4
     4      5
```

## Modeling the Interest-Rate Term Structure

Financial Instruments Toolbox software includes a set of functions to encapsulate interest-rate term information into a single structure. These functions present a convenient way to package all information related to interest-rate terms into a common format, and to resolve interdependencies when one or more of the parameters is modified. For information, see:

- "Creating or Modifying (intenvset)" on page 2-33 for a discussion of how to create or modify an interest-rate term structure (`RateSpec`) using the `intenvset` function
- "Obtaining Specific Properties (intenvget)" on page 2-35 for a discussion of how to extract specific properties from a `RateSpec`

### Creating or Modifying (intenvset)

The main function to create or modify an interest-rate term structure `RateSpec` (rates specification) is `intenvset`. If the first argument to this function is a previously created `RateSpec`, the function modifies the existing rate specification and returns a new one. Otherwise, it creates a `RateSpec`.

When using `RateSpec` to specify the rate term structure to price instruments based on yields (zero coupon rates) or forward rates, specify zero rates or forward rates as the input argument. However, the `RateSpec` structure is not limited or specific to this problem domain. `RateSpec` is an encapsulation of rates-times relationships; `intenvset` acts as either a constructor or a modifier, and `intenvget` as an accessor. The interest rate models supported by the Financial Instruments Toolbox software work either with zero coupon rates or forward rates.

The other `intenvset` arguments are property-value pairs, indicating the new value for these properties. The properties that can be specified or modified are:

- `Basis`
- `Compounding`
- `Disc`
- `EndDates`
- `EndMonthRule`
- `Rates`
- `StartDates`
- `ValuationDate`

To learn about the properties `EndMonthRule` and `Basis`, type `help ftbEndMonthRule` and `help ftbBasis` or see the Financial Toolbox™ documentation.

Consider again the original table of interest rates (see "Calculating Discount Factors from Rates" on page 2-23).

| From | To | Rate |
|---|---|---|
| 15 Feb 2000 | 15 Aug 2000 | 0.05 |
| 15 Feb 2000 | 15 Feb 2001 | 0.056 |
| 15 Feb 2000 | 15 Aug 2001 | 0.06 |
| 15 Feb 2000 | 15 Feb 2002 | 0.065 |
| 15 Feb 2000 | 15 Aug 2002 | 0.075 |

Use the information in this table to populate the `RateSpec` structure.

```
StartDates = ['15-Feb-2000'];
EndDates =   ['15-Aug-2000';
              '15-Feb-2001';
              '15-Aug-2001';
              '15-Feb-2002';
              '15-Aug-2002'];
Compounding = 2;
ValuationDate = ['15-Feb-2000'];
Rates = [0.05; 0.056; 0.06; 0.065; 0.075];

rs = intenvset('Compounding',Compounding,'StartDates',...
StartDates, 'EndDates', EndDates, 'Rates', Rates,...
'ValuationDate', ValuationDate)

rs =

       FinObj: 'RateSpec'
  Compounding: 2
         Disc: [5x1 double]
        Rates: [5x1 double]
```

```
     EndTimes: [5x1 double]
   StartTimes: [5x1 double]
     EndDates: [5x1 double]
   StartDates: 730531
ValuationDate: 730531
        Basis: 0
 EndMonthRule: 1
```

Some of the properties filled in the structure were not passed explicitly in the call to `RateSpec`. The values of the automatically completed properties depend on the properties that are explicitly passed. Consider for example the `StartTimes` and `EndTimes` vectors. Since the `StartDates` and `EndDates` vectors are passed in, and the `ValuationDate`, `intenvset` has all the information required to calculate `StartTimes` and `EndTimes`. Hence, these two properties are read-only.

## Obtaining Specific Properties (intenvget)

The complementary function to `intenvset` is `intenvget`, which gets function specific properties from the interest-rate term structure. Its syntax is:

```
ParameterValue = intenvget(RateSpec, 'ParameterName')
```

To obtain the vector `EndTimes` from the `RateSpec` structure, enter:

```
EndTimes = intenvget(rs, 'EndTimes')

EndTimes =

     1
     2
     3
     4
     5
```

To obtain `Disc`, the values for the discount factors that were calculated automatically by `intenvset`, type:

```
Disc = intenvget(rs, 'Disc')

Disc =
```

```
    0.9756
    0.9463
    0.9151
    0.8799
    0.8319
```

These discount factors correspond to the periods starting from `StartDates` and ending in `EndDates`.

---

**Caution** Although you can directly access these fields within the structure instead of using `intenvget`, it is advised not to do so. The format of the interest-rate term structure could change in future versions of the toolbox. Should that happen, any code accessing the `RateSpec` fields directly would stop working.

---

Now use the `RateSpec` structure with its functions to examine how changes in specific properties of the interest-rate term structure affect those depending on it. As an exercise, change the value of `Compounding` from 2 (semiannual) to 1 (annual).

```
rs = intenvset(rs, 'Compounding', 1);
```

Since `StartTimes` and `EndTimes` are measured in units of periodic discount, a change in `Compounding` from 2 to 1 redefines the basic unit from semiannual to annual. This means that a period of 6 months is represented with a value of 0.5, and a period of 1 year is represented by 1. To obtain the vectors `StartTimes` and `EndTimes`, enter:

```
StartTimes = intenvget(rs, 'StartTimes');
EndTimes = intenvget(rs, 'EndTimes');
Times = [StartTimes, EndTimes]

Times =

         0    0.5000
         0    1.0000
         0    1.5000
```

```
         0    2.0000
         0    2.5000
```

Since all the values in `StartDates` are the same as the valuation date, all `StartTimes` values are 0. On the other hand, the values in the `EndDates` vector are dates separated by 6-month periods. Since the redefined value of compounding is 1, `EndTimes` becomes a sequence of numbers separated by increments of 0.5.

# Pricing Using Interest-Rate Term Structure

**In this section...**

## Introduction

The instruments can be presented to the functions as a portfolio of different types of instruments or as groups of instruments of the same type. The current version of the toolbox can compute price and sensitivities for four instrument types using interest-rate curves:

- Bonds
- Fixed-rate notes
- Floating-rate notes
- Swaps
- OAS for callable and puttable bonds
- Agency OAS

In addition to these instruments, the toolbox also supports the calculation of price and sensitivities of arbitrary sets of cash flows.

Note that options and interest-rate floors and caps are absent from the above list of supported instruments. These instruments are not supported because their pricing and sensitivity function require a stochastic model for the evolution of interest rates. The interest-rate term structure used for pricing is treated as deterministic, and as such is not adequate for pricing these instruments.

Financial Instruments Toolbox software also contains functions that use the Heath-Jarrow-Morton (HJM) and Black-Derman-Toy (BDT) models to compute prices and sensitivities for financial instruments. These models support computations involving options and interest-rate floors and caps. See "Pricing Using Interest-Rate Tree Models" on page 2-67 for information on computing price and sensitivities of financial instruments using the HJM and BDT models.

## Computing Instrument Prices

The main function used for pricing portfolios of instruments is `intenvprice`. This function works with the family of functions that calculate the prices of individual types of instruments. When called, `intenvprice` classifies the portfolio contained in `InstSet` by instrument type, and calls the appropriate pricing functions. The map between instrument types and the pricing function `intenvprice` calls is

| | |
|---|---|
| `bondbyzero`: | Price a bond by a set of zero curves |
| `fixedbyzero`: | Price a fixed-rate note by a set of zero curves |
| `floatbyzero`: | Price a floating-rate note by a set of zero curves |
| `swapbyzero`: | Price a swap by a set of zero curves |

You can use each of these functions individually to price an instrument. Consult the reference pages for specific information on using these functions.

`intenvprice` takes as input an interest-rate term structure created with `intenvset`, and a portfolio of interest-rate contingent derivatives instruments created with `instadd`.

The syntax for using `intenvprice` to price an entire portfolio is

```
Price = intenvprice(RateSpec, InstSet)
```

where:

- `RateSpec` is the interest-rate term structure.
- `InstSet` is the name of the portfolio.

## Example: Pricing a Portfolio of Instruments

Consider this example of using the `intenvprice` function to price a portfolio of instruments supplied with Financial Instruments Toolbox software.

The provided MAT-file `deriv.mat` stores a portfolio as an instrument set variable `ZeroInstSet`. The MAT-file also contains the interest-rate term structure `ZeroRateSpec`. You can display the instruments with the function `instdisp`.

```
load deriv.mat;
instdisp(ZeroInstSet)

Index Type CouponRate Settle         Maturity       Period Basis...
1     Bond 0.04       01-Jan-2000    01-Jan-2003    1      NaN...
2     Bond 0.04       01-Jan-2000    01-Jan-2004    2      NaN...

Index Type  CouponRate Settle      Maturity    FixedReset Basis...
3     Fixed 0.04       01-Jan-2000 01-Jan-2003 1          NaN...

Index Type  Spread Settle         Maturity       FloatReset Basis...
4     Float 20     01-Jan-2000    01-Jan-2003    1          NaN...

Index Type LegRate    Settle         Maturity       LegReset Basis...
5     Swap [0.06 20]  01-Jan-2000    01-Jan-2003    [1  1]   NaN...
```

Use `intenvprice` to calculate the prices for the instruments contained in the portfolio `ZeroInstSet`.

```
format bank
Prices = intenvprice(ZeroRateSpec, ZeroInstSet)
Prices =

          98.72
          97.53
          98.72
         100.55
           3.69
```

The output `Prices` is a vector containing the prices of all the instruments in the portfolio in the order indicated by the `Index` column displayed by `instdisp`. Consequently, the first two elements in `Prices` correspond to the first two bonds; the third element corresponds to the fixed-rate note; the fourth to the floating-rate note; and the fifth element corresponds to the price of the swap.

## Computing Instrument Sensitivities

In general, you can compute sensitivities either as dollar price changes or as percentage price changes. The toolbox reports all sensitivities as dollar sensitivities.

Using the interest-rate term structure, you can calculate two types of derivative price sensitivities, delta and gamma. *Delta* represents the dollar sensitivity of prices to shifts in the observed forward yield curve. *Gamma* represents the dollar sensitivity of delta to shifts in the observed forward yield curve.

The `intenvsens` function computes instrument sensitivities and instrument prices. If you need both the prices and sensitivity measures, use `intenvsens`. A separate call to `intenvprice` is not required.

Here is the syntax

```
[Delta, Gamma, Price] = intenvsens(RateSpec, InstSet)
```

where, as before:

- `RateSpec` is the interest-rate term structure.
- `InstSet` is the name of the portfolio.

### Example: Sensitivities and Prices

Here is an example that uses `intenvsens` to calculate both sensitivities and prices.

```
format bank
load deriv.mat;
[Delta, Gamma, Price] = intenvsens(ZeroRateSpec, ZeroInstSet);
```

Display the results in a single matrix in bank format.

```
All = [Delta Gamma Price]

All =

       -272.64      1029.84       98.72
       -347.44      1622.65       97.53
       -272.64      1029.84       98.72
         -1.04         3.31      100.55
       -282.04      1059.62        3.69
```

To view the per-dollar sensitivity, divide the first two columns by the last one.

```
[Delta./Price, Gamma./Price, Price]

ans =

         -2.76        10.43         98.72
         -3.56        16.64         97.53
         -2.76        10.43         98.72
         -0.01         0.03        100.55
        -76.39       286.98          3.69
```

## OAS for Callable and Puttable Bonds

Option Adjusted Spread (OAS) is a useful way to value and compare securities with embedded options, like callable or puttable bonds. Basically, when the constant or flat spread is added to the interest-rate curve/rates in the tree, the pricing model value equals the market price. Financial Instruments Toolbox supports pricing American, European and Bermuda callable and puttable bonds using different interest rate models. The pricing for a bond with embedded options is:

- For a callable bond, where the holder has bought a bond and sold a call option to the issuer:

  `Price callable bond = Price Option free bond   Price call option`

- For a puttable bond, where the holder has bought a bond and a put option:

  `Price puttable bond = Price Option free bond + Price put option`

There are two additional sensitivities related to OAS for bonds with embedded options: Option Adjusted Duration and Option Adjusted Convexity. These are similar to the concepts of modified duration and convexity for option-free bonds. The measure Duration is a general term that describes how sensitive a bond's price is to a parallel shift in the yield curve. Modified Duration and Modified Convexity assume that the bond's cash flows do not change when the yield curve shifts. This is not true for OA Duration or OA Convexity because the cash flows may change due to the option risk component of the bond.

| **Function** | **Purpose** |
|---|---|
| `oasbybdt` | Compute OAS using a BDT model. |
| `oasbybk` | Compute OAS using a BK model. |
| `oasbyhjm` | Compute OAS using an HJM model. |
| `oasbyhw` | Compute OAS using an HW model. |

## Agency OAS

Often bonds are issued with embedded options, which then makes standard price/yield or spread measures irrelevant. For example, a municipality concerned about the chance that interest rates may fall in the future might issue bonds with a provision that allows the bond to be repaid before the bond's maturity. This is a call option on the bond and must be incorporated into the valuation of the bond. Option-adjusted spread (OAS), which adjusts a bond spread for the value of the option, is the standard measure for valuing bonds with embedded options. Financial Instruments Toolbox software supports computing option-adjusted spreads for bonds with single embedded options using the agency model.

The Securities Industry and Financial Markets Association (SIFMA) has a simplified approach to compute OAS for agency issues (Government Sponsored Entities like Fannie Mae and Freddie Mac) termed "Agency OAS". In this approach, the bond has only one call date (European call) and uses Black's model (a variation on Black Scholes, `http://en.wikipedia.org/wiki/Black_model`) to value the bond option. The price of the bond is computed as follows:

$$\mathtt{Price}_{\text{Callable}} = \mathtt{Price}_{\text{NonCallable}} - \mathtt{Price}_{\text{Option}}$$

where

$\texttt{Price}_{\text{Callable}}$ is the price of the callable bond.

$\texttt{Price}_{\text{NonCallable}}$ is the price of the noncallable bond, i.e., price of the bond using `bndspread`.

$\texttt{Price}_{\text{Option}}$ is the price of the option, i.e., price of the option using Black's model.

The Agency OAS is the spread, when used in the previous formula, yields the market price. Financial Instruments Toolbox software supports these functions:

| Agency OAS Functions | Purpose |
|---|---|
| `agencyoas` | Compute the OAS of the callable bond using the Agency OAS model. |
| `agencyprice` | Price the callable bond OAS using Agency using the OAS model. |

For more information on agency OAS, see "Agency Option-Adjusted Spreads" on page 6-2.

# Understanding Interest-Rate Tree Models

**In this section...**

## Introduction

Financial Instruments Toolbox software supports the Black-Derman-Toy (BDT), Black-Karasinski (BK), Heath-Jarrow-Morton (HJM), and Hull-White (HW) interest-rate models. The Heath-Jarrow-Morton model is one of the most widely used models for pricing interest-rate derivatives. The model considers a given initial term structure of interest rates and a specification of the volatility of forward rates to build a tree representing the evolution of the interest rates, based on a statistical process. For further explanation, see the book *Modelling Fixed Income Securities and Interest Rate Options* by Robert A. Jarrow.

The Black-Derman-Toy model is another analytical model commonly used for pricing interest-rate derivatives. The model considers a given initial zero rate term structure of interest rates and a specification of the yield volatilities of long rates to build a tree representing the evolution of the interest rates. For further explanation, see the paper "A One Factor Model of Interest Rates and its Application to Treasury Bond Options" by Fischer Black, Emanuel Derman, and William Toy.

The Hull-White model incorporates the initial term structure of interest rates and the volatility term structure to build a trinomial recombining tree of short rates. The resulting tree is used to value interest rate dependent securities. The implementation of the Hull-White model in Financial Instruments Toolbox software is limited to one factor.

The Black-Karasinski model is a single factor, log-normal version of the Hull-White model.

For further information on the Hull-White and Black-Karasinski models, see the book *Options, Futures, and Other Derivatives* by John C. Hull.

## Building a Tree of Forward Rates

The tree of forward rates is the fundamental unit representing the evolution of interest rates in a given period of time. This section explains how to create a forward-rate tree using Financial Instruments Toolbox software.

---

**Note** To avoid needless repetition, this document uses the HJM and BDT models to illustrate the creation and use of interest-rate trees. The HW and BK models are similar to the BDT model. Where specific differences exist, they are documented in "HW and BK Tree Structures" on page 2-62.

---

The MATLAB functions that create rate trees are `hjmtree` and `bdttree`. The `hjmtree` function creates the structure, `HJMTree`, containing time and forward-rate information for a bushy tree. The `bdttree` function creates a similar structure, `BDTTree`, for a recombining tree.

This structure is a self-contained unit that includes the tree of rates (found in the `FwdTree` field of the structure) and the volatility, rate, and time specifications used in building this tree.

These functions take three structures as input arguments:

- The volatility model `VolSpec`. (See "Specifying the Volatility Model (VolSpec)" on page 2-48.)
- The interest-rate term structure `RateSpec`. (See "Specifying the Interest-Rate Term Structure (RateSpec)" on page 2-51.)
- The tree time layout `TimeSpec`. (See "Specifying the Time Structure (TimeSpec)" on page 2-52.)

An easy way to visualize any trees you create is with the `treeviewer` function, which displays trees in a graphical manner. See "Graphical Representation of Trees" on page 2-91 for information about `treeviewer`.

### Calling Sequence

The calling syntax for `hjmtree` is

```
HJMTree = hjmtree(VolSpec, RateSpec, TimeSpec)
```

Similarly, the calling syntax for `bdttree` is

```
BDTTree = bdttree(VolSpec, RateSpec, TimeSpec)
```

Each of these functions requires `VolSpec`, `RateSpec`, and `TimeSpec` input arguments:

- `VolSpec` is a structure that specifies the forward-rate volatility process. You create `VolSpec` using either of the functions `hjmvolspec` or `bdtvolspec`.

  The `hjmvolspec` function supports the specification of up to three factors. It handles these models for the volatility of the interest-rate term structure:

  - Constant
  - Stationary
  - Exponential
  - Vasicek
  - Proportional

  A one-factor model assumes that the interest term structure is affected by a single source of uncertainty. Incorporating multiple factors allows you to specify different types of shifts in the shape and location of the interest-rate structure. See `hjmvolspec` for details.

  The `bdtvolspec` function supports only a single volatility factor. The volatility remains constant between pairs of nodes on the tree. You supply the input volatility values in a vector of decimal values. See `bdtvolspec` for details.

- `RateSpec` is the interest-rate specification of the initial rate curve. You create this structure with the function `intenvset`. (See "Modeling the Interest-Rate Term Structure" on page 2-32.)
- `TimeSpec` is the tree time layout specification. You create this variable with the functions `hjmtimespec` or `bdttimespec`. It represents the mapping between level times and level dates for rate quoting. This structure indirectly determines the number of levels in the tree.

## Specifying the Volatility Model (VolSpec)

Because HJM supports multifactor (up to 3) volatility models while BDT (also, BK and HW) supports only a single volatility factor, the `hjmvolspec` and `bdtvolspec` functions require different inputs and generate slightly different outputs. For examples, see "Creating an HJM Volatility Model" on page 2-48. For BDT examples see "Creating a BDT Volatility Model" on page 2-50.

### Creating an HJM Volatility Model

The function `hjmvolspec` generates the structure `VolSpec`, which specifies the volatility process $\sigma(t,T)$ used in the creation of the forward-rate trees. In this context capital $T$ represents the starting time of the forward rate, and $t$ represents the observation time. The volatility process can be constructed from a combination of factors specified sequentially in the call to function that creates it. Each factor specification starts with a string specifying the name of the factor, followed by the pertinent parameters.

**HJM Volatility Specification Example.** Consider an example that uses a single factor, specifically, a constant-sigma factor. The constant factor specification requires only one parameter, the value of $\sigma$. In this case, the value corresponds to 0.10.

```
HJMVolSpec = hjmvolspec('Constant', 0.10)

HJMVolSpec =

      FinObj: 'HJMVolSpec'
FactorModels: {'Constant'}
  FactorArgs: {{1x1 cell}}
  SigmaShift: 0
  NumFactors: 1
```

```
   NumBranch: 2
     PBranch: [0.5000 0.5000]
 Fact2Branch: [-1 1]
```

The `NumFactors` field of the `VolSpec` structure, `VolSpec.NumFactors = 1`, reveals that the number of factors used to generate `VolSpec` was one. The `FactorModels` field indicates that it is a `Constant` factor, and the `NumBranches` field indicates the number of branches. As a consequence, each node of the resulting tree has two branches, one going up, and the other going down.

Consider now a two-factor volatility process made from a proportional factor and an exponential factor.

```
% Exponential factor
Sigma_0 = 0.1;
Lambda = 1;
% Proportional factor
CurveProp = [0.11765; 0.08825; 0.06865];
CurveTerm = [   1   ;    2   ;    3   ];
% Build VolSpec
HJMVolSpec = hjmvolspec('Proportional', CurveProp, CurveTerm,...
1e6,'Exponential', Sigma_0, Lambda)

HJMVolSpec =

      FinObj: 'HJMVolSpec'
FactorModels: {'Proportional'  'Exponential'}
  FactorArgs: {{1x3 cell}  {1x2 cell}}
  SigmaShift: 0
  NumFactors: 2
   NumBranch: 3
     PBranch: [0.2500 0.2500 0.5000]
 Fact2Branch: [2x3 double]
```

The output shows that the volatility specification was generated using two factors. The tree has 3 branches per node. Each branch has probabilities of 0.25, 0.25, and 0.5, going from top to bottom.

### Creating a BDT Volatility Model

The function `bdtvolspec` generates the structure `VolSpec`, which specifies the volatility process. The function requires three input arguments:

- The valuation date `ValuationDate`
- The yield volatility end dates `VolDates`
- The yield volatility values `VolCurve`

An optional fourth argument `InterpMethod`, specifying the interpolation method, can be included.

The syntax used for calling `bdtvolspec` is:

```
BDTVolSpec = bdtvolspec(ValuationDate, VolDates, VolCurve,...
InterpMethod)
```

where:

- `ValuationDate` is the first observation date in the tree.
- `VolDates` is a vector of dates representing yield volatility end dates.
- `VolCurve` is a vector of yield volatility values.
- `InterpMethod` is the method of interpolation to use. The default is `linear`.

**BDT Volatility Specification Example.** Consider the following example:

```
ValuationDate = datenum('01-01-2000');
EndDates = datenum(['01-01-2001'; '01-01-2002'; '01-01-2003';
'01-01-2004'; '01-01-2005']);
Volatility = [.2; .19; .18; .17; .16];
```

Use `bdtvolspec` to create a volatility specification. Because no interpolation method is explicitly specified, the function uses the `linear` default.

```
BDTVolSpec = bdtvolspec(ValuationDate, EndDates, Volatility)

BDTVolSpec =
             FinObj: 'BDTVolSpec'
      ValuationDate: 730486
```

```
         VolDates: [5x1 double]
         VolCurve: [5x1 double]
  VolInterpMethod: 'linear'
```

## Specifying the Interest-Rate Term Structure (RateSpec)

The structure `RateSpec` is an interest term structure that defines the initial forward-rate specification from which the tree rates are derived. "Modeling the Interest-Rate Term Structure" on page 2-32 explains how to create these structures using the function `intenvset`, given the interest rates, the starting and ending dates for each rate, and the compounding value.

### Rate Specification Creation Example

Consider the following example:

```
Compounding = 1;
Rates = [0.02; 0.02; 0.02; 0.02];
StartDates = ['01-Jan-2000';
              '01-Jan-2001';
              '01-Jan-2002';
              '01-Jan-2003'];
EndDates =   ['01-Jan-2001';
              '01-Jan-2002';
              '01-Jan-2003';
              '01-Jan-2004'];
ValuationDate = '01-Jan-2000';

RateSpec = intenvset('Compounding',1,'Rates', Rates,...
'StartDates', StartDates, 'EndDates', EndDates,...
'ValuationDate', ValuationDate)

RateSpec =

        FinObj: 'RateSpec'
   Compounding: 1
          Disc: [4x1 double]
         Rates: [4x1 double]
      EndTimes: [4x1 double]
    StartTimes: [4x1 double]
      EndDates: [4x1 double]
```

```
    StartDates: [4x1 double]
 ValuationDate: 730486
         Basis: 0
  EndMonthRule: 1
```

Use the function `datedisp` to examine the dates defined in the variable `RateSpec`. For example:

```
datedisp(RateSpec.ValuationDate)
01-Jan-2000
```

## Specifying the Time Structure (TimeSpec)

The structure `TimeSpec` specifies the time structure for an interest-rate tree. This structure defines the mapping between the observation times at each level of the tree and the corresponding dates.

`TimeSpec` is built using either the `hjmtimespec` or `bdttimespec` function. These functions require three input arguments:

- The valuation date `ValuationDate`
- The maturity date `Maturity`
- The compounding rate `Compounding`

For example, the syntax used for calling `hjmtimespec` is

```
TimeSpec = hjmtimespec(ValuationDate, Maturity, Compounding)
```

where:

- `ValuationDate` is the first observation date in the tree.
- `Maturity` is a vector of dates representing the cash flow dates of the tree. Any instrument cash flows with these maturities fall on tree nodes.
- `Compounding` is the frequency at which the rates are compounded when annualized.

### Creating a Time Specification

Calling the time specification creation functions with the same data used to create the interest-rate term structure, `RateSpec` builds the structure that specifies the time layout for the tree.

**HJM Time Specification Example.** Consider the following example:

```
Maturity = EndDates;
HJMTimeSpec = hjmtimespec(ValuationDate, Maturity, Compounding)

HJMTimeSpec =

       FinObj: 'HJMTimeSpec'
ValuationDate: 730486
     Maturity: [4x1 double]
  Compounding: 1
        Basis: 0
 EndMonthRule: 1
```

Note that maturities specified when building `TimeSpec` need not coincide with the `EndDates` of the rate intervals in `RateSpec`. Since `TimeSpec` defines the time-date mapping of the tree, the rates in `RateSpec` are interpolated to obtain the initial rates with maturities equal to those in `TimeSpec`.

**Creating a BDT Time Specification.** Consider the following example:

```
Maturity = EndDates;
BDTTimeSpec = bdttimespec(ValuationDate, Maturity, Compounding)

BDTTimeSpec =

           FinObj: 'BDTTimeSpec'
    ValuationDate: 730486
         Maturity: [4x1 double]
      Compounding: 1
            Basis: 0
     EndMonthRule: 1
```

## Creating Trees

Use the `VolSpec`, `RateSpec`, and `TimeSpec` you have previously created as inputs to the functions used to create HJM and BDT trees.

### Creating an HJM Tree

```
% Reset the volatility factor to the Constant case
HJMVolSpec = hjmvolspec('Constant', 0.10);

HJMTree = hjmtree(HJMVolSpec, RateSpec, HJMTimeSpec)

HJMTree =

  FinObj: 'HJMFwdTree'
 VolSpec: [1x1 struct]
TimeSpec: [1x1 struct]
RateSpec: [1x1 struct]
    tObs: [0 1 2 3]
    TFwd: {[4x1 double] [3x1 double] [2x1 double] [3]}
  CFlowT: {[4x1 double] [3x1 double] [2x1 double] [4]}
 FwdTree:{[4x1 double][3x1x2 double][2x2x2 double][1x4x2 double]}
```

### Creating a BDT Tree

Now use the previously computed values for `VolSpec`, `RateSpec`, and `TimeSpec` as input to the function `bdttree` to create a BDT tree.

```
BDTTree = bdttree(BDTVolSpec, RateSpec, BDTTimeSpec)

BDTTree =

  FinObj: 'BDTFwdTree'
 VolSpec: [1x1 struct]
TimeSpec: [1x1 struct]
RateSpec: [1x1 struct]
    tObs: [0 1.00 2.00 3.00]
    TFwd: {[4x1 double]  [3x1 double]  [2x1 double]  [3.00]}
  CFlowT: {[4x1 double]  [3x1 double]  [2x1 double]  [4.00]}
 FwdTree: {[1.02] [1.02 1.02] [1.01 1.02 1.03] [1.01 1.02 1.02 1.03]}
```

## Examining Trees

When working with the models, Financial Instruments Toolbox software uses trees to represent forward rates, prices, and so on. At the highest level, these trees have structures wrapped around them. The structures encapsulate information required to interpret completely the information contained in a tree.

Consider this example, which uses the interest rate and portfolio data in the MAT-file `deriv.mat` included in the toolbox.

Load the data into the MATLAB workspace.

```
load deriv.mat
```

Display the list of the variables loaded from the MAT-file.

```
whos

 Name                Size             Bytes  Class     Attributes

  BDTInstSet         1x1              15956  struct
  BDTTree            1x1               5138  struct
  BKInstSet          1x1              15946  struct
  BKTree             1x1               5904  struct
  CRRInstSet         1x1              12434  struct
  CRRTree            1x1               5058  struct
  EQPInstSet         1x1              12434  struct
  EQPTree            1x1               5058  struct
  HJMInstSet         1x1              15948  struct
  HJMTree            1x1               5838  struct
  HWInstSet          1x1              15946  struct
  HWTree             1x1               5904  struct
  ITTInstSet         1x1              12438  struct
  ITTTree            1x1               8862  struct
  ZeroInstSet        1x1              10282  struct
  ZeroRateSpec       1x1               1580  struct
```

### HJM Tree Structure

You can now examine in some detail the contents of the `HJMTree` structure contained in this file.

```
HJMTree

HJMTree =

  FinObj: 'HJMFwdTree'
 VolSpec: [1x1 struct]
TimeSpec: [1x1 struct]
RateSpec: [1x1 struct]
    tObs: [0 1 2 3]
    TFwd: {[4x1 double]  [3x1 double]  [2x1 double]  [3]}
  CFlowT: {[4x1 double]  [3x1 double]  [2x1 double] [4]}
 FwdTree:{[4x1 double][3x1x2 double][2x2x2 double][1x4x2 double]}
```

`FwdTree` contains the actual forward-rate tree. MATLAB software represents it as a cell array with each cell array element containing a tree level.

The other fields contain other information relevant to interpreting the values in `FwdTree`. The most important are `VolSpec`, `TimeSpec`, and `RateSpec`, which contain the volatility, time structure, and rate structure information respectively.

**First Node.** Observe the forward rates in `FwdTree`. The first node represents the valuation date, `tObs = 0`.

```
HJMTree.FwdTree{1}

ans =

    1.0356
    1.0468
    1.0523
    1.0563
```

---

**Note** Financial Instruments Toolbox software uses *inverse discount* notation for forward rates in the tree. An inverse discount represents a factor by which the current value of an asset is multiplied to find its future value. In general, these forward factors are reciprocals of the discount factors.

---

Look closely at the `RateSpec` structure used in generating this tree to see where these values originate. Arrange the values in a single array.

```
[HJMTree.RateSpec.StartTimes HJMTree.RateSpec.EndTimes...
HJMTree.RateSpec.Rates]

ans =

         0    1.0000    0.0356
    1.0000    2.0000    0.0468
    2.0000    3.0000    0.0523
    3.0000    4.0000    0.0563
```

If you find the corresponding inverse discounts of the interest rates in the third column, you have the values at the first node of the tree. You can turn interest rates into inverse discounts using the function `rate2disc`.

```
Disc = rate2disc(HJMTree.TimeSpec.Compounding,...
HJMTree.RateSpec.Rates, HJMTree.RateSpec.EndTimes,...
HJMTree.RateSpec.StartTimes);
FRates = 1./Disc

FRates =
    1.0356
    1.0468
    1.0523
    1.0563
```

**Second Node.** The second node represents the first-rate observation time, `tObs = 1`. This node displays two states: one representing the branch going up and the other representing the branch going down.

Note that `HJMTree.VolSpec.NumBranch = 2`.

```
HJMTree.VolSpec

ans =

          FinObj: 'HJMVolSpec'
    FactorModels: {'Constant'}
      FactorArgs: {{1x1 cell}}
      SigmaShift: 0
      NumFactors: 1
       NumBranch: 2
         PBranch: [0.5000 0.5000]
     Fact2Branch: [-1 1]
```

Examine the rates of the node corresponding to the up branch.

```
HJMTree.FwdTree{2}(:,:,1)

ans =

    1.0364
    1.0420
    1.0461
```

Now examine the corresponding down branch.

```
HJMTree.FwdTree{2}(:,:,2)

ans =

    1.0574
    1.0631
    1.0672
```

**Third Node.** The third node represents the second observation time, `tObs = 2`. This node contains a total of four states, two representing the branches going up and the other two representing the branches going down. Examine the rates of the node corresponding to the up states.

```
HJMTree.FwdTree{3}(:,:,1)

ans =
```

```
    1.0317    1.0526
    1.0358    1.0568
```

Next examine the corresponding down states.

```
HJMTree.FwdTree{3}(:,:,2)

ans =

    1.0526    1.0738
    1.0568    1.0781
```

**Isolating a Specific Node.** Starting at the third level, indexing within the tree cell array becomes complex, and isolating a specific node can be difficult. The function `bushpath` isolates a specific node by specifying the path to the node as a vector of branches taken to reach that node. As an example, consider the node reached by starting from the root node, taking the branch up, then the branch down, and then another branch down. Given that the tree has only two branches per node, branches going up correspond to a 1, and branches going down correspond to a 2. The path up-down-down becomes the vector `[1 2 2]`.

```
FRates = bushpath(HJMTree.FwdTree, [1 2 2])

FRates =

    1.0356
    1.0364
    1.0526
    1.0674
```

`bushpath` returns the spot rates for all the nodes touched by the path specified in the input argument, the first one corresponding to the root node, and the last one corresponding to the target node.

Isolating the same node using direct indexing obtains

```
HJMTree.FwdTree{4}(:, 3, 2)
```

```
ans =

    1.0674
```

As expected, this single value corresponds to the last element of the rates returned by `bushpath`.

You can use these techniques with any type of tree generated with Financial Instruments Toolbox software, such as forward-rate trees or price trees.

### BDT Tree Structure

You can now examine in some detail the contents of the `BDTTree` structure.

```
BDTTree

BDTTree =

      FinObj: 'BDTFwdTree'
     VolSpec: [1x1 struct]
    TimeSpec: [1x1 struct]
    RateSpec: [1x1 struct]
        tObs: [0 1 2 3]
        TFwd: {[4x1 double]  [3x1 double]  [2x1 double]  [3]}
      CFlowT: {[4x1 double]  [3x1 double]  [2x1 double]  [4]}
     FwdTree: {1x4 cell}
```

`FwdTree` contains the actual rate tree. MATLAB software represents it as a cell array with each cell array element containing a tree level.

The other fields contain other information relevant to interpreting the values in `FwdTree`. The most important are `VolSpec`, `TimeSpec`, and `RateSpec`, which contain the volatility, time structure, and rate structure information respectively.

Look at the `RateSpec` structure used in generating this tree to see where these values originate. Arrange the values in a single array.

```
[BDTTree.RateSpec.StartTimes BDTTree.RateSpec.EndTimes...
BDTTree.RateSpec.Rates]
```

```
ans =

         0    1.0000    0.1000
         0    2.0000    0.1100
         0    3.0000    0.1200
         0    4.0000    0.1250
```

Look at the rates in `FwdTree`. The first node represents the valuation date, `tObs = 0`. The second node represents `tObs = 1`. Examine the rates at the second, third, and fourth nodes.

```
BDTTree.FwdTree{2}

ans =

      1.0979    1.1432
```

The second node represents the first observation time, `tObs = 1`. This node contains a total of two states, one representing the branch going up (`1.0979`) and the other representing the branch going down (`1.1432`).

---

**Note** The convention in this document is to display *prices* going up on the upper branch. Consequently, when displaying *rates*, rates are falling on the upper branch and increasing on the lower branch.

---

```
BDTTree.FwdTree{3}

ans =

    1.0976    1.1377    1.1942
```

The third node represents the second observation time, `tObs = 2`. This node contains a total of three states, one representing the branch going up (`1.0976`), one representing the branch in the middle (`1.1377`) and the other representing the branch going down (`1.1942`).

```
BDTTree.FwdTree{4}
```

```
ans =

    1.0872    1.1183    1.1606    1.2179
```

The fourth node represents the third observation time, `tObs = 3`. This node contains a total of four states, one representing the branch going up (`1.0872`), two representing the branches in the middle (`1.1183` and `1.1606`), and the other representing the branch going down (`1.2179`).

**Isolating a Specific Node.** The function `treepath` isolates a specific node by specifying the path to the node as a vector of branches taken to reach that node. As an example, consider the node reached by starting from the root node, taking the branch up, then the branch down, and finally another branch down. Given that the tree has only two branches per node, branches going up correspond to a 1, and branches going down correspond to a 2. The path up-down-down becomes the vector `[1 2 2]`.

```
FRates = treepath(BDTTree.FwdTree, [1 2 2])

FRates =

    1.1000
    1.0979
    1.1377
    1.1606
```

`treepath` returns the short rates for all the nodes touched by the path specified in the input argument, the first one corresponding to the root node, and the last one corresponding to the target node.

### HW and BK Tree Structures

The HW and BK tree structures are similar to the BDT tree structure. You can see this if you examine the sample HW tree contained in the file `deriv.mat`.

```
load deriv.mat:

HWTree

FinObj: 'HWFwdTree'
VolSpec: [1x1 struct]
```

```
TimeSpec: [1x1 struct]
RateSpec: [1x1 struct]
tObs: [0 1 2 3]
dObs: [731947 732313 732678 733043]
CFlowT: {[4x1 double]  [3x1 double]  [2x1 double]  [4]}
Probs: {[3x1 double]  [3x3 double]  [3x5 double]}
Connect: {[2]  [2 3 4]  [2 2 3 4 4]}
FwdTree: {1x4 cell}
```

All fields of this structure are similar to their BDT counterparts. There are two additional fields not present in BDT: `Probs` and `Connect`. The `Probs` field represents the occurrence probabilities at each branch of each node in the tree. The `Connect` field describes the connectivity of the nodes of a given tree level to nodes to the next tree level.

**Probs Field.** While BDT and one-factor HJM models have equal probabilities for each branch at a node, HW and BK do not. For HW and BK trees, the `Probs` field indicates the likelihood that a particular branch will be taken in moving from one node to another node on the next level.

The `Probs` field consists of a cell array with 1 cell per tree level. Each cell is a `3`-by-`NUMNODES` array with the top row representing the probability of an up movement, the middle row representing the probability of a middle movement, and the last row the probability of a down movement.

As an illustration, consider the first two elements of the `Probs` field of the structure, corresponding to the first (root) and second levels of the tree.

```
HWTree.Probs{1}

0.16666666666667
0.66666666666667
0.16666666666667

HWTree.Probs{2}

0.12361333418768   0.16666666666667   0.21877591615172
0.65761074966060   0.66666666666667   0.65761074966060
0.21877591615172   0.16666666666667   0.12361333418768
```

Reading from top to bottom, the values in `HWTree.Probs{1}` correspond to the up, middle, and down probabilities at the root node.

`HWTree.Probs{2}` is a `3`-by-`3` matrix of values. The first column represents the top node, the second column represents the middle node, and the last column represents the bottom node. As with the root node, the first, second, and third rows hold the values for up, middle, and down branching off each node.

As expected, the sum of all the probabilities at any node equals 1.

```
sum(HWTree.Probs{2})

1.0000    1.0000    1.0000
```

**Connect Field.** The other field that distinguishes HW and BK tree structures from the BDT tree structure is `Connect`. This field describes how each node in a given level connects to the nodes of the next level. The need for this field arises from the possibility of nonstandard branching in a tree.

The `Connect` field of the HW tree structure consists of a cell array with 1 cell per tree level.

```
HWTree.Connect

ans =

    [2]    [1x3 double]    [1x5 double]
```

Each cell contains a `1`-by-`NUMNODES` vector. Each value in the vector relates to a node in the corresponding tree level and represents the index of the node in the next tree level that the middle branch of the node connects to.

If you subtract 1 from the values contained in `Connect`, you reveal the index of the nodes in the next level that the up branch connects to. If you add 1 to the values, you reveal the index of the corresponding down branch.

As an illustration, consider `HWTree.Connect{1}`:

```
HWTree.Connect{1}

ans =
```

```
     2
```

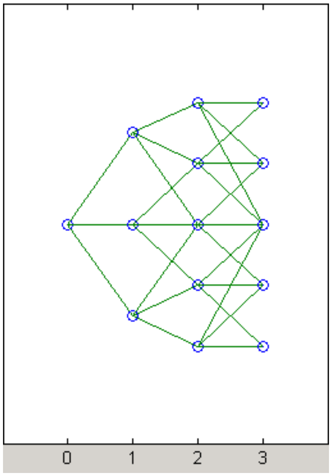
This indicates that the middle branch of the root node connects to the second (from the top) node of the next level, as expected. If you subtract 1 from this value, you obtain 1, which tells you that the up branch goes to the top node. If you add 1, you obtain 3, which points to the last node of the second level of the tree.

Now consider level 3 in this example:

```
HWTree.Connect{3}

2        2        3        4        4
```

On this level, there is nonstandard branching. This can be easily recognized because the middle branch of two nodes is connected to the same node on the next level.

To visualize this, consider the following illustration of the tree.



Here it becomes apparent that there is nonstandard branching at the third level of the tree, on the top and bottom nodes. The first and second nodes connect to the same trio of nodes on the next level. Similar branching occurs at the bottom and next-to-bottom nodes of the tree.

# Pricing Using Interest-Rate Tree Models

**In this section...**

## Introduction

For purposes of illustration, this section relies on the HJM and BDT models. The HW and BK functions that perform price and sensitivity computations are not explicitly shown here. Functions that use the HW and BK models operate similarly to the BDT model.

## Computing Instrument Prices

The portfolio pricing functions `hjmprice` and `bdtprice` calculate the price of any set of supported instruments, based on an interest-rate tree. The functions are capable of pricing these instrument types:

- Bonds
- Bond options
- Bond with embedded options
- Arbitrary cash flows
- Fixed-rate notes
- Floating-rate notes
- Caps
- Floors
- Range Notes
- Swaps
- Swaptions

For example, the syntax for calling `hjmprice` is:

```
[Price, PriceTree] = hjmprice(HJMTree, InstSet, Options)
```

Similarly, the calling syntax for `bdtprice` is:

```
[Price, PriceTree] = bdtprice(BDTTree, InstSet, Options)
```

Each function requires two input arguments: the interest-rate tree and the set of instruments, `InstSet`. An optional argument `Options` further controls the pricing and the output displayed. (See Appendix B, "Derivatives Pricing Options" for information about the `Options` argument.)

`HJMTree` is the Heath-Jarrow-Morton tree sampling of a forward-rate process, created using `hjmtree`. `BDTTree` is the Black-Derman-Toy tree sampling of an interest-rate process, created using `bdttree`. See "Building a Tree of Forward Rates" on page 2-46 to learn how to create these structures.

`InstSet` is the set of instruments to be priced. This structure represents the set of instruments to be priced independently using the model.

`Options` is an options structure created with the function `derivset`. This structure defines how the tree is used to find the price of instruments in the portfolio, and how much additional information is displayed in the command window when calling the pricing function. If this input argument is not specified in the call to the pricing function, a default Options structure is used. The pricing options structure is described in "Pricing Options Structure" on page B-2.

The portfolio pricing functions classify the instruments and call the appropriate instrument-specific pricing function for each of the instrument types. The HJM instrument-specific pricing functions are `bondbyhjm`, `cfbyhjm`, `fixedbyhjm`, `floatbyhjm`, `optbndbyhjm`, `rangefloatbyhjm`, `swapbyhjm`, and `swaptionbyhjm`. A similarly named set of functions exists for BDT models. You can also use these functions directly to calculate the price of sets of instruments of the same type.

## HJM Pricing Example

Consider the following example, which uses the portfolio and interest-rate data in the MAT-file `deriv.mat` included in the toolbox. Load the data into the MATLAB workspace.

```
load deriv.mat
```

Use the MATLAB `whos` command to display a list of the variables loaded from the MAT-file.

```
whos

Name                 Size             Bytes  Class     Attributes

  BDTInstSet         1x1              15956  struct
  BDTTree            1x1               5138  struct
  BKInstSet          1x1              15946  struct
  BKTree             1x1               5904  struct
  CRRInstSet         1x1              12434  struct
  CRRTree            1x1               5058  struct
  EQPInstSet         1x1              12434  struct
  EQPTree            1x1               5058  struct
  HJMInstSet         1x1              15948  struct
  HJMTree            1x1               5838  struct
  HWInstSet          1x1              15946  struct
  HWTree             1x1               5904  struct
  ITTInstSet         1x1              12438  struct
  ITTTree            1x1               8862  struct
  ZeroInstSet        1x1              10282  struct
  ZeroRateSpec       1x1               1580  struct
```

`HJMTree` and `HJMInstSet` are the input arguments required to call the function `hjmprice`.

Use the function `instdisp` to examine the set of instruments contained in the variable `HJMInstSet`.

```
instdisp(HJMInstSet)


Index Type CouponRate Settle       Maturity     Period Basis  ...  Name      Quantity
1     Bond 0.04       01-Jan-2000  01-Jan-2003  1      NaN    ...  4% bond   100
2     Bond 0.04       01-Jan-2000  01-Jan-2004  2      NaN    ...  4% bond    50

Index Type    UnderInd OptSpec Strike ExerciseDates  AmericanOpt Name        Quantity
3     OptBond 2        call    101    01-Jan-2003    NaN         Option 101    -50

Index Type  CouponRate Settle      Maturity    FixedReset Basis Principal Name     Quantity
4     Fixed 0.04       01-Jan-2000 01-Jan-2003 1          NaN   NaN       4% Fixed    80

Index Type  Spread Settle      Maturity    FloatReset  Basis Principal Name       Quantity
5     Float 20     01-Jan-2000 01-Jan-2003 1           NaN   NaN       20BP Float   8

Index Type Strike Settle         Maturity       CapReset Basis Principal Name    Quantity
6     Cap  0.03   01-Jan-2000    01-Jan-2004    1        NaN   NaN       3% Cap     30

Index Type  Strike Settle      Maturity       FloorReset Basis Principal Name     Quantity
7     Floor 0.03   01-Jan-2000 01-Jan-2004    1          NaN   NaN       3% Floor    40

Index Type LegRate   Settle   Maturity       LegReset Basis Principal LegType   Name Quantity
8     Swap [0.06 20] 01-Jan-2000 01-Jan-2003  [1  1]   NaN   NaN      [NaN]   6%/20BP Swap  10
```

Note that there are eight instruments in this portfolio set: two bonds, one bond option, one fixed-rate note, one floating-rate note, one cap, one floor, and one swap. Each instrument has a corresponding index that identifies the instrument prices in the price vector returned by `hjmprice`.

Now use `hjmprice` to calculate the price of each instrument in the instrument set.

```
Price = hjmprice(HJMTree, HJMInstSet)
Warning: Not all cash flows are aligned with the tree. Result will
be approximated.

Price =
```

```
  98.7159
  97.5280
   0.0486
  98.7159
 100.5529
   6.2831
   0.0486
   3.6923
```

---

**Note** The warning shown above appears because some of the cash flows for the second bond do not fall exactly on a tree node.

---

### BDT Pricing Example

Load the MAT-file `deriv.mat` into the MATLAB workspace.

```
load deriv.mat
```

Use the MATLAB `whos` command to display a list of the variables loaded from the MAT-file.

```
whos

 Name                 Size              Bytes  Class     Attributes

  BDTInstSet          1x1               15956  struct
  BDTTree             1x1                5138  struct
  BKInstSet           1x1               15946  struct
  BKTree              1x1                5904  struct
  CRRInstSet          1x1               12434  struct
  CRRTree             1x1                5058  struct
  EQPInstSet          1x1               12434  struct
  EQPTree             1x1                5058  struct
  HJMInstSet          1x1               15948  struct
  HJMTree             1x1                5838  struct
  HWInstSet           1x1               15946  struct
  HWTree              1x1                5904  struct
  ITTInstSet          1x1               12438  struct
```

```
  ITTTree          1x1               8862  struct
  ZeroInstSet      1x1              10282  struct
  ZeroRateSpec     1x1               1580  struct
```

`BDTTree` and `BDTInstSet` are the input arguments required to call the function `bdtprice`.

Use the function `instdisp` to examine the set of instruments contained in the variable `BDTInstSet`.

```
instdisp(BDTInstSet)



Index Type CouponRate Settle       Maturity     Period Basis ........  Name    Quantity
1     Bond 0.1        01-Jan-2000  01-Jan-2003  1      NaN.........    10% bond  100
2     Bond 0.1        01-Jan-2000  01-Jan-2004  2      NaN.........    10% bond  50


Index Type    UnderInd OptSpec Strike ExerciseDates  AmericanOpt Name      Quantity
3     OptBond 1        call    9501   Jan-2002       NaN         Option 95   -50

Index Type  CouponRate Settle      Maturity       FixedReset Basis Principal Name  Quantity
4     Fixed 0.10       01-Jan-2000 01-Jan-2003    1          NaN   NaN      10% Fixed 80

Index Type  Spread Settle      Maturity    FloatReset  Basis Principal Name       Quantity
5     Float 20     01-Jan-2000 01-Jan-2003 1           NaN   NaN       20BP Float 8

Index Type Strike Settle         Maturity       CapReset Basis Principal Name    Quantity
6     Cap  0.15   01-Jan-2000    01-Jan-2004    1        NaN   NaN       15% Cap 30

Index Type  Strike Settle      Maturity       FloorReset Basis Principal Name    Quantity
7     Floor 0.09   01-Jan-2000 01-Jan-2004    1          NaN   NaN       9% Floor 40

Index Type LegRate   Settle   Maturity  LegReset Basis Principal LegType Name   Quantity
8     Swap [0.15 10] 01-Jan-2000  01-Jan-2003  [1  1]  NaN   NaN  [NaN] 15%/10BP Swap 10
```

Note that there are eight instruments in this portfolio set: two bonds, one bond option, one fixed-rate note, one floating-rate note, one cap, one floor, and one swap. Each instrument has a corresponding index that identifies the instrument prices in the price vector returned by `bdtprice`.

Now use `bdtprice` to calculate the price of each instrument in the instrument set.

```
Price = bdtprice(BDTTree, BDTInstSet)
Warning: Not all cash flows are aligned with the tree. Result will
be approximated.

Price =

   95.5030
   93.9079
    1.7657
   95.5030
  100.4865
    1.4863
    0.0245
    7.4222
```

## Price Vector Output

The prices in the output vector `Price` correspond to the prices at observation time zero (`tObs = 0`), which is defined as the valuation date of the interest-rate tree. The instrument indexing within `Price` is the same as the indexing within `InstSet`.

In the HJM example, the prices in the `Price` vector correspond to the instruments in this order.

```
InstNames = instget(HJMInstSet, 'FieldName','Name')

InstNames =

4% bond
4% bond
Option 101
4% Fixed
```

```
20BP Float
3% Cap
3% Floor
6%/20BP Swap
```

Consequently, in the `Price` vector, the fourth element, 98.7159, represents the price of the fourth instrument (4% fixed-rate note); the sixth element, 6.2831, represents the price of the sixth instrument (3% cap).

In the BDT example, the prices in the `Price` vector correspond to the instruments in this order.

```
InstNames = instget(BDTInstSet, 'FieldName','Name')

InstNames =

10% Bond
10% Bond
Option 95
10% Fixed
20BP Float
15% Cap
9% Floor
15%/10BP Swap
```

Consequently, in the `Price` vector, the fourth element, 95.5030, represents the price of the fourth instrument (10% fixed-rate note); the sixth element, 1.4863, represents the price of the sixth instrument (15% cap).

### Price Tree Structure Output

If you call a pricing function with two output arguments, for example,

```
[Price, PriceTree] = hjmprice(HJMTree, HJMInstSet)
```

you generate a price tree along with the price information.

The optional output price tree structure `PriceTree` holds all the pricing information.

**HJM Price Tree.** In the HJM example, the first field of this structure, `FinObj`, indicates that this structure represents a price tree. The second field, `PBush`, is the tree holding the price of the instruments in each node of the tree. The third field, `AIBush`, is the tree holding the accrued interest of the instruments in each node of the tree. Finally, the fourth field, `tObs`, represents the observation time of each level of `PBush` and `AIBush`, with units in terms of compounding periods.

In this example, the price tree looks like

```
PriceTree =

FinObj: 'HJMPriceTree'
 PBush: {[8x1 double]  [8x1x2 double]  ...[8x8 double]}
AIBush: {[8x1 double]  [8x1x2 double] ... [8x8 double]}
  tObs: [0 1 2 3 4]
```

Both `PBush` and `AIBush` are `1`-by-`5` cell arrays, consistent with the five observation times of `tObs`. The data display has been shortened here to fit on a single line.

Using the command line interface, you can directly examine `PriceTree.PBush`, the field within the `PriceTree` structure that contains the price tree with the price vectors at every state. The first node represents `tObs = 0`, corresponding to the valuation date.

```
PriceTree.PBush{1}

ans =

   98.7159
   97.5280
    0.0486
   98.7159
  100.5529
    6.2831
    0.0486
    3.6923
```

With this interface, you can observe the prices for *all* instruments in the portfolio at *a specific time*.

**BDT Price Tree.** The BDT output price tree structure `PriceTree` holds all the pricing information. The first field of this structure, `FinObj`, indicates that this structure represents a price tree. The second field, `PTree`, is the tree holding the price of the instruments in each node of the tree. The third field, `AITree`, is the tree holding the accrued interest of the instruments in each node of the tree. The fourth field, `tObs`, represents the observation time of each level of `PTree` and `AITree`, with units in terms of compounding periods.

You can directly examine the field within the `PriceTree` structure, which contains the price tree with the price vectors at every state. The first node represents `tObs = 0`, corresponding to the valuation date.

```
[Price, PriceTree] = bdtprice(BDTTree, BDTInstSet)

PriceTree.PTree{1}

ans =

   95.5030
   93.9079
    1.7657
   95.5030
  100.4865
    1.4863
    0.0245
    7.4222
```

## Computing Instrument Sensitivities

Sensitivities can be reported either as dollar price changes or percentage price changes. The delta, gamma, and vega sensitivities that the toolbox computes are dollar sensitivities.

The functions `hjmsens` and `bdtsens` compute the delta, gamma, and vega sensitivities of instruments using an interest-rate tree. They also optionally return the calculated price for each instrument. The sensitivity functions require the same two input arguments used by the pricing functions (`HJMTree` and `HJMInstSet` for HJM; `BDTTree` and `BDTInstSet` for BDT).

Sensitivity functions calculate the dollar value of delta and gamma by shifting the observed forward yield curve by 100 basis points in each direction, and the dollar value of vega by shifting the volatility process by 1%. To obtain the per-dollar value of the sensitivities, divide the dollar sensitivity by the price of the corresponding instrument.

### HJM Sensitivities Example

The calling syntax for the function is:

```
[Delta, Gamma, Vega, Price] = hjmsens(HJMTree, HJMInstSet)
```

Use the previous example data to calculate the price of instruments.

```
load deriv.mat
[Delta, Gamma, Vega, Price] = hjmsens(HJMTree, HJMInstSet);
Warning: Not all cash flows are aligned with the tree. Result will
be approximated.
```

---

**Note** The warning appears because some of the cash flows for the second bond do not fall exactly on a tree node.

---

You can conveniently examine the sensitivities and the prices by arranging them into a single matrix.

```
All = [Delta, Gamma, Vega, Price]

All =

       -272.65       1029.90          0.00         98.72
       -347.43       1622.69         -0.04         97.53
         -8.08        643.40         34.07          0.05
       -272.65       1029.90          0.00         98.72
         -1.04          3.31             0        100.55
        294.97       6852.56         93.69          6.28
        -47.16       8459.99         93.69          0.05
       -282.05       1059.68          0.00          3.69
```

As with the prices, each row of the sensitivity vectors corresponds to the similarly indexed instrument in `HJMInstSet`. To view the *per-dollar sensitivities*, divide each dollar sensitivity by the corresponding instrument price.

### BDT Sensitivities Example

The calling syntax for the function is:

```
[Delta, Gamma, Vega, Price] = bdtsens(BDTTree, BDTInstSet);
```

Arrange the sensitivities and prices into a single matrix.

```
All = [Delta, Gamma, Vega, Price]

All =

    -232.67       803.71      -0.00       95.50
    -281.05      1181.93      -0.01       93.91
     -50.54       246.02       5.31        1.77
    -232.67       803.71          0       95.50
       0.84         2.45          0      100.49
      78.38       748.98      13.54        1.49
      -4.36       382.06       2.50        0.02
    -253.23       863.81          0        7.42
```

To view the *per-dollar sensitivities*, divide each dollar sensitivity by the corresponding instrument price.

```
All = [Delta ./ Price, Gamma ./ Price, Vega ./ Price, Price]

All =

      -2.44         8.42      -0.00       95.50
      -2.99        12.59      -0.00       93.91
     -28.63       139.34       3.01        1.77
      -2.44         8.42          0       95.50
       0.01         0.02          0      100.49
      52.73       503.92       9.11        1.49
    -177.89     15577.42     101.87        0.02
     -34.12       116.38          0        7.42
```

## Calibrating Hull-White Model Using Market Data

The pricing of interest rate derivative securities relies on models that describe the underlying process. These interest rate models depend on one or more parameters that you must determine by matching the model predictions to the existing data available in the market. In the Hull-White model, there are two parameters related to the short rate process: mean reversion and volatility. Calibration is used to determine these parameters, such that the model can reproduce, as close as possible, the prices of caps or floors observed in the market. The calibration routines find the parameters that minimize the difference between the model price predictions and the market prices for caps and floors.

For a Hull-White model, the minimization is two dimensional, with respect to mean reversion (α) and volatility (σ). That is, calibrating the Hull-White model minimizes the difference between the model prices and market prices for caps and floors:

$$\frac{(\text{ModelPrice}(\alpha\ ,\sigma)\text{ - MarketPrice})}{(\text{MarketPrice})}$$

### Hull-White Model Calibration Example

Use market data to identify the implied volatility (σ) and mean reversion (α) coefficients needed to build a Hull-White tree to price an instrument. The ideal case is to use the volatilities of the caps or floors used to calculate `Alpha` (α) and `Sigma` (σ). This will most likely not be the case, so market data must be interpolated to obtain the required values.

Consider a cap with these parameters:

```
Settle = ' Jan-21-2008';
Maturity = 'Mar-21-2011';
Strike = 0.0690;
Reset = 4;
Principal = 1000;
Basis = 0;
```

The caplets for this example would fall in:

```
capletDates = cfdates(Settle, Maturity, Reset, Basis);
```

```
datestr(capletDates')

ans =

21-Mar-2008
21-Jun-2008
21-Sep-2008
21-Dec-2008
21-Mar-2009
21-Jun-2009
21-Sep-2009
21-Dec-2009
21-Mar-2010
21-Jun-2010
21-Sep-2010
21-Dec-2010
21-Mar-2011
```

In the best case, look up the market volatilities for caplets with a `Strike` = 0.0690, and maturities in each reset date listed, but the likelihood of finding these exact instruments is low. As a consequence, use data that is available in the market and interpolate to find appropriate values for the caplets.

Based on the market data, you have the cap information for different dates and strikes. Assume that instead of having the data for `Strike` = 0.0690, you have the data for `Strike1` = 0.0590 and `Strike2` = 0.0790

| Maturity | Strike1 = 0.0590 | Strike2 = 0.0790 |
|---|---|---|
| 21-Mar-2008 | 0.1533 | 0. 1526 |
| 21-Jun-2008 | 0.1731 | 0. 1730 |
| 21-Sep-2008 | 0. 1727 | 0. 1726 |
| 21-Dec-2008 | 0. 1752 | 0. 1747 |
| 21-Mar-2009 | 0. 1809 | 0. 1808 |
| 21-Jun-2009 | 0. 1809 | 0. 1792 |
| 21-Sep-2009 | 0. 1805 | 0. 1797 |
| 21-Dec-2009 | 0. 1802 | 0. 1794 |

| **Maturity** | **Strike1 = 0.0590** | **Strike2 = 0.0790** |
|---|---|---|
| 21-Mar-2010 | 0. 1802 | 0. 1733 |
| 21-Jun-2010 | 0. 1757 | 0. 1751 |
| 21-Sep-2010 | 0. 1755 | 0. 1750 |
| 21-Dec-2010 | 0. 1755 | 0. 1745 |
| 21-Mar-2011 | 0. 1726 | 0. 1719 |

The nature of this data lends itself to matrix nomenclature, which is perfect for MATLAB. `hwcalbycap` requires that the dates, the strikes, and the actual volatility be separated into three variables: `MarketStrike`, `MarketMat`, and `MarketVol`.

```
MarketStrike = [0.0590; 0.0790];
MarketMat = {'21-Mar-2008';
'21-Jun-2008';
'21-Sep-2008';
'21-Dec-2008';
'21-Mar-2009';
'21-Jun-2009';
'21-Sep-2009';
'21-Dec-2009';
'21-Mar-2010';
'21-Jun-2010';
'21-Sep-2010';
'21-Dec-2010';
'21-Mar-2011'};


MarketVol = [0.1533 0.1731 0.1727 0.1752 0.1809 0.1800 0.1805 0.1802 0.1735 0.1757 ...
            0.1755 0.1755 0.1726;   % First column in table corresponding to Strike1
            0.1526 0.1730 0.1726 0.1747 0.1808 0.1792 0.1797 0.1794 0.1733 0.1751 ...
            0.1750 0.1745 0.1719];  % Second column in table corresponding to Strike2
```

Complete the input arguments using this data for `RateSpec`:

```
Rates= [0.0627;
0.0657;
0.0691;
```

```
0.0717;
0.0739;
0.0755;
0.0765;
0.0772;
0.0779;
0.0783;
0.0786;
0.0789;
0.0792;
0.0793];

ValuationDate = '21-Jan-2008';
EndDates = {'21-Mar-2008';'21-Jun-2008';'21-Sep-2008';'21-Dec-2008';...
            '21-Mar-2009';'21-Jun-2009';'21-Sep-2009';'21-Dec-2009';....
            '21-Mar-2010';'21-Jun-2010';'21-Sep-2010';'21-Dec-2010';....
            '21-Mar-2011';'21-Jun-2011'};
Compounding = 4;
Basis = 0;

RateSpec = intenvset('ValuationDate', ValuationDate, ...
'StartDates', ValuationDate, 'EndDates', EndDates, ...
'Rates', Rates, 'Compounding', Compounding, 'Basis', Basis);

RateSpec =

           FinObj: 'RateSpec'
      Compounding: 4
             Disc: [14x1 double]
            Rates: [14x1 double]
         EndTimes: [14x1 double]
       StartTimes: [14x1 double]
         EndDates: [14x1 double]
       StartDates: 733428
    ValuationDate: 733428
            Basis: 0
     EndMonthRule: 1
```

**Call the calibration routine to find values for volatility parameters Alpha and Sigma.** Use `hwcalbycap` to calculate the values of `Alpha` and `Sigma` based on market data. Internally, `hwcalbycap` calls the Optimization Toolbox™ function `lsqnonlin`. You can customize `lsqnonlin` by passing an optimization options structure created by `optimset` and then this can be passed to `hwcalbycap` using the name-value pair argument for `OptimOptions`. For example, `optimset` defines the target objective function tolerance as `100*eps` and then calls `hwcalbycap`:

```
o=optimset('TolFun',100*eps);

[Alpha, Sigma] = hwcalbycap(RateSpec, MarketStrike, MarketMat, MarketVol,...
Strike, Settle, Maturity, 'Reset', Reset, 'Principal', Principal, 'Basis',...
Basis, 'OptimOptions', o)
Local minimum possible.

Local minimum possible.

lsqnonlin stopped because the size of the current step is less than
the default value of the step size tolerance. Warning: LSQNONLIN did not converge to an optimal
solution. It exited with exitflag = 2.

> In hwcalbycapfloor at 97
  In hwcalbycap at 77

Alpha =

  1.0000e-006


Sigma =

    0.0127
```

The previous warning indicates that the conversion was not optimal. The search algorithm used by the Optimization Toolbox™ function `lsqnonlin` did not find a solution that conforms to all the constraints. To discern whether the solution is acceptable, look at the results of the optimization by specifying a third output (`OptimOut`) for `hwcalbycap`:

```
[Alpha, Sigma, OptimOut] = hwcalbycap(RateSpec, MarketStrike, MarketMat,...
```

```
MarketVol, Strike, Settle, Maturity, 'Reset', Reset, 'Principal', Principal,...
'Basis', Basis, 'OptimOptions', o);
```

The `OptimOut.residual` field of the `OptimOut` structure is the optimization residual. This value contains the difference between the Black caplets and those calculated during the optimization. You can use the `OptimOut.residual` value to calculate the percentual difference (error) compared to Black caplet prices and then decide whether the residual is acceptable. There is almost always some residual, so decide if parametrizing the market with a single value of `Alpha` and `Sigma` is acceptable.

**Price caplets using market data and Black's formula to obtain reference caplet values.** To determine the effectiveness of the optimization, calculate reference caplet values using Black's formula and the market data. Note, you must first interpolate the market data to obtain the caplets for calculation:

```
MarketMatNum = datenum(MarketMat);
[Mats, Strikes] = meshgrid(MarketMatNum, MarketStrike);
FlatVol = interp2(Mats, Strikes, MarketVol, datenum(Maturity), Strike, 'spline');
```

Compute the price of the cap using the Black model:

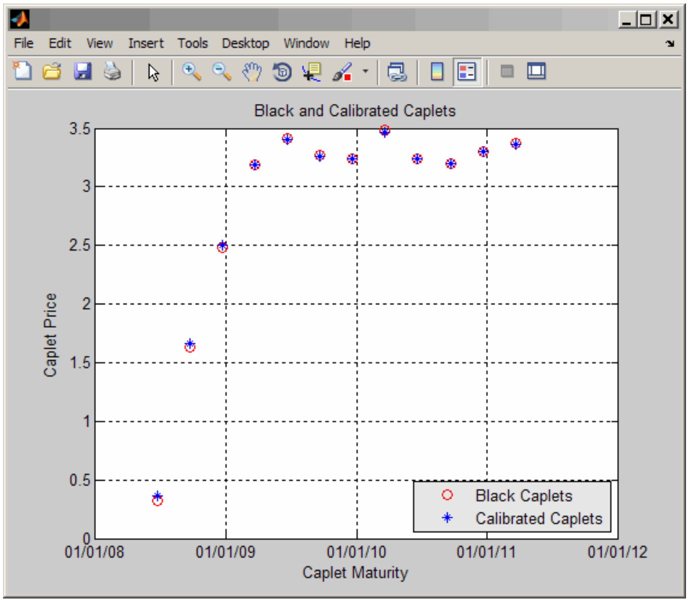
```
[CapPrice, Caplets] = capbyblk(RateSpec, Strike, Settle, Maturity, FlatVol,...
'Reset', Reset, 'Basis', Basis, 'Principal', Principal);
Caplets = Caplets(2:end)';
Caplets =

    0.3210
    1.6355
    2.4863
    3.1903
    3.4110
    3.2685
    3.2385
    3.4803
    3.2419
    3.1949
    3.2991
    3.3750
```

### Compare optimized values and Black values and display graphically.

After calculating the reference values for the caplets, compare the values, analytically and graphically, to determine whether the calculated single values of `Alpha` and `Sigma` provide an adequate approximation:

```
OptimCaplets = Caplets+OptimOut.residual;

disp('   ');
disp('    Black76   Calibrated Caplets');
disp([Caplets                   OptimCaplets])

plot(MarketMatNum(2:end), Caplets, 'or', MarketMatNum(2:end), OptimCaplets, '*b');
datetick('x', 2)
xlabel('Caplet Maturity');
ylabel('Caplet Price');
title('Black and Calibrated Caplets');
h = legend('Black Caplets', 'Calibrated Caplets');
set(h, 'color', [0.9 0.9 0.9]);
set(h, 'Location', 'SouthEast');
set(gcf, 'NumberTitle', 'off')
grid on
```

```
 Black76   Calibrated Caplets
    0.3210    0.3636
    1.6355    1.6603
    2.4863    2.4974
    3.1903    3.1874
    3.4110    3.4040
    3.2685    3.2639
    3.2385    3.2364
    3.4803    3.4683
    3.2419    3.2408
    3.1949    3.1957
    3.2991    3.2960
    3.3750    3.3663
```



**Compare cap prices using the Black, HW analytical, and HW tree models.** Using the calculated caplet values, compare the prices of the corresponding cap using the Black model, Hull-White analytical, and Hull-White tree models. To calculate a Hull-White tree based on `Alpha` and `Sigma`, use these calibration routines:

- Black model:

  ```
  CapPriceBLK = CapPrice;
  ```

- HW analytical model:

  ```
  CapPriceHWAnalytical = sum(OptimCaplets);
  ```

- HW tree model to price the cap derived from the calibration process:

  1. Create `VolSpec` from the calibration parameters `Alpha` and `Sigma`:

     ```
     VolDates     = EndDates;
     VolCurve     = Sigma*ones(14,1);
     AlphaDates   = EndDates;
     AlphaCurve   = Alpha*ones(14,1);
     ```

```
HWVolSpec = hwvolspec(ValuationDate, VolDates, VolCurve,AlphaDates, AlphaCurve);
```

**2** Create the `TimeSpec`:

```
HWTimeSpec  = hwtimespec(ValuationDate, EndDates, Compounding);
```

**3** Build the HW tree using the `HW2000` method:

```
HWTree = hwtree(HWVolSpec, RateSpec, HWTimeSpec, 'Method', 'HW2000');
```

**4** Price the cap:

```
Price = capbyhw(HWTree, Strike, Settle, Maturity, Reset, Basis, Principal);

disp('   ');
disp(['  CapPrice Black76 ..................:  ', num2str(CapPriceBLK,'%15.5f')]);
disp(['  CapPrice HW analytical..........:  ', num2str(CapPriceHWAnalytical,'%15.5f')]);
disp(['  CapPrice HW from capbyhw ..:  ', num2str(Price,'%15.5f')]);
disp('   ');

CapPrice Black76 ..........: 34.14220
CapPrice HW analytical.....: 34.18008
CapPrice HW from capbyhw ..: 34.14192
```

**Price a portfolio of instruments using the calibrated HW tree.** After building a Hull-White tree, based on parameters calibrated from market data, use `HWTree` to price a portfolio of these instruments:

- Two bonds

```
CouponRate = [0.07; 0.09];
Settle= ' Jan-21-2008';
Maturity = {'Mar-21-2010';'Mar-21-2011'};
Period = 1;
Face = 1000;
Basis = 0;
```

- Bond with an embedded American call option

```
CouponRateOEB = 0.08;
SettleOEB = ' Jan-21-2008';
MaturityOEB = 'Mar-21-2011';
```

```
OptSpec = 'call';
StrikeOEB = 950;
ExerciseDatesOEB = 'Mar-21-2011';
AmericanOpt= 1;
Period =1;
Face = 1000;
Basis =0;
```

To price this portfolio of instruments using the calibrated `HWTree`:

1. Use `instadd` to create the portfolio `InstSet`:

```
InstSet = instadd('Bond', CouponRate, Settle,  Maturity, Period, Basis, [], [], [], [], [], Face);
InstSet = instadd(InstSet,'OptEmBond',  CouponRateOEB, SettleOEB, MaturityOEB, OptSpec,...
StrikeOEB,   ExerciseDatesOEB, 'AmericanOpt', AmericanOpt, 'Period', Period,...
'Face',Face,  'Basis', Basis);
```

2. Add the cap instrument used in the calibration:

```
SettleCap = ' Jan-21-2008';
MaturityCap = 'Mar-21-2011';
StrikeCap = 0.0690;
Reset = 4;
Principal = 1000;

InstSet = instadd(InstSet,'Cap', StrikeCap, SettleCap, MaturityCap, Reset, Basis, Principal);
```

3. Assign names to the portfolio instruments:

```
Names = {'7% Bond'; '8% Bond'; 'BondEmbCall'; '6.9% Cap'};
InstSet = instsetfield(InstSet, 'Index',1:4, 'FieldName', {'Name'}, 'Data', Names );
```

4. Examine the set of instruments contained in `InstSet`:

```
instdisp(InstSet)

IdxType CoupRate Settle Mature Period Basis EOMRule IssueDate 1stCoupDate LastCoupDate StartDate Face Name

1 Bond 0.07       21-Jan-2008    21-Mar-2010    1  0  NaN  NaN     NaN   NaN  NaN  1000    7% Bond
2 Bond 0.09       21-Jan-2008    21-Mar-2011    1  0  NaN  NaN     NaN   NaN  NaN  1000    8% Bond

IdxType CoupRate Settle Mature OptSpec Stke ExDate Per Basis EOMRule IssDate 1stCoupDate LstCoupDate StrtDate Face AmerOpt Name
3 OptEmBond 0.08 21-Jan-2008 21-Mar-2011 call 950  21-Jan-2008  21-Mar-2011  1  0  1  NaN  NaN NaN  NaN  1000 1 BondEmbCall

Index Type Strike Settle     Maturity   CapReset Basis Principal Name
4 Cap  0.069  21-Jan-2008    21-Mar-2011    4      0     1000    6.9% Cap
```

**5** Use `hwprice` to price the portfolio using the calibrated `HWTree`:

```
format bank
PricePortfolio = hwprice(HWTree, InstSet)

PricePortfolio =
        980.45
       1023.05
        945.73
         34.14
```

# Interest-Rate Derivatives Using Closed-Form Solutions

## Pricing Caps and Floors Using the Black Option Model

Caps and floors are contracts that allow the holder to be protected if interest rates rise or decrease. The Black model uses a forward price as an underlier in place of a spot price. The assumption is that the forward price at maturity of the option is log-normally distributed.

Closed-form solutions for pricing caps and floors using the Black model support the following tasks:

| Task | Function |
|---|---|
| Price the interest rate caps using the Black option pricing model. | `capbyblk` |
| Price the interest rate floors using the Black option pricing model. | `floorbyblk` |

# Graphical Representation of Trees

**In this section...**

## Introduction

You can use the function `treeviewer` to display a graphical representation of a tree, allowing you to examine interactively the prices and rates on the nodes of the tree until maturity. To get started with this process, first load the data file `deriv.mat` included in this toolbox.

```
load deriv.mat
```

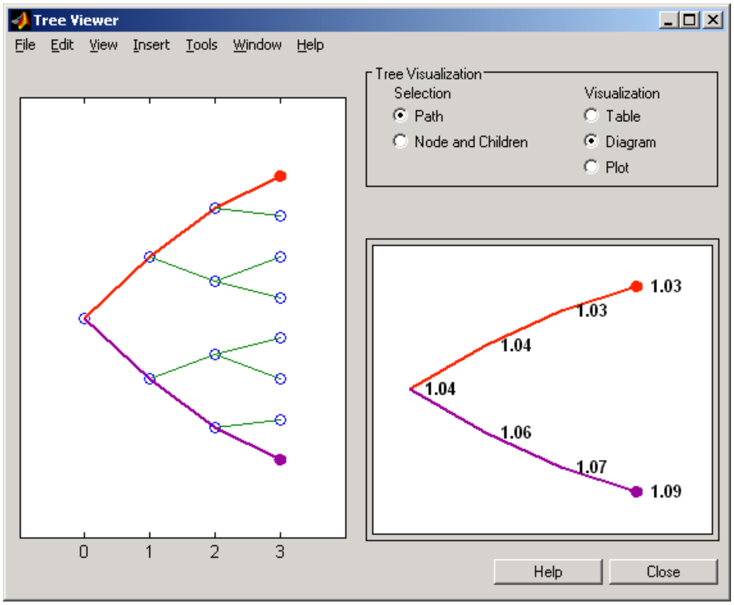
**Note** `treeviewer` price tree diagrams follow the convention that increasing prices appear on the upper branch of a tree and, consequently, decreasing prices appear on the lower branch. Conversely, for interest rate displays, *decreasing* interest rates appear on the upper branch (prices are rising) and *increasing* interest rates on the lower branch (prices are falling).

For information on the use of `treeviewer` to observe interest rate movement, see "Observing Interest Rates" on page 2-91. For information on using `treeviewer` to observe the movement of prices, see "Observing Instrument Prices" on page 2-95.

## Observing Interest Rates

If you provide the name of an interest rate tree to the `treeviewer` function, it displays a graphical view of the path of interest rates. For example, here is the `treeviewer` representation of all the rates along both the up and down branches of `HJMTree`.

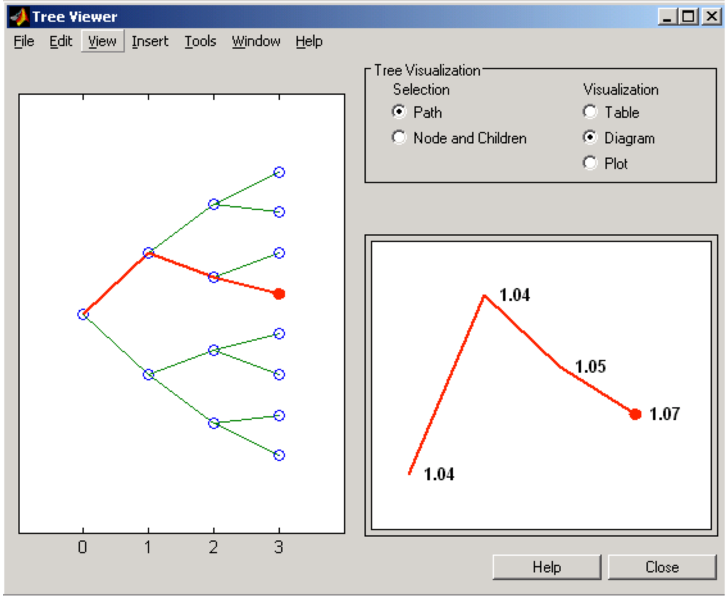
```
treeviewer(HJMTree)
```



The example in “Isolating a Specific Node for a CRRTree” on page 3-19 used `bushpath` to find the path of forward rates along an HJM tree by taking the first branch up and then two branches down the rate tree.

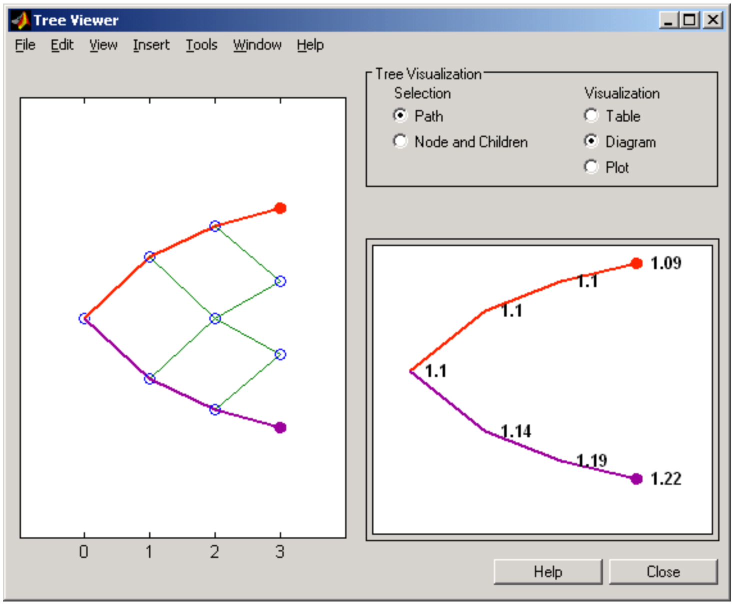
```
FRates = bushpath(HJMTree.FwdTree, [1 2 2])

FRates =

    1.0356
    1.0364
    1.0526
    1.0674
```

With the `treeviewer` function you can display the identical information by clicking along the same sequence of nodes, as shown next.



Next is a `treeviewer` representation of interest rates along several branches of `BDTTree`.

```
treeviewer(BDTTree)
```



---

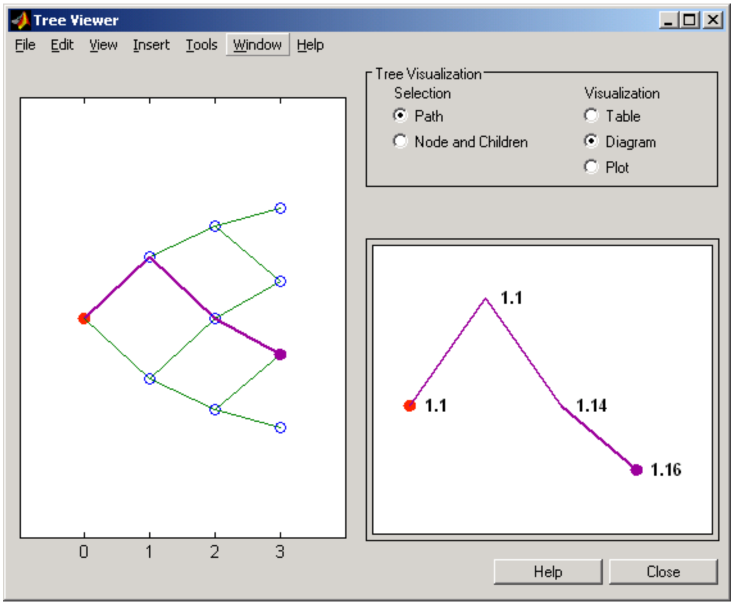
**Note** When using `treeviewer` with recombining trees, such as BDT, BK, and HW, you must click each node in succession from the beginning to the end. Because these trees can recombine, `treeviewer` is unable to complete the path automatically.

---

The example in "Isolating a Specific Node for a CRRTree" on page 3-19 used `treepath` to find the path of interest rates taking the first branch up and then two branches down the rate tree.

```
FRates = treepath(BDTTree.FwdTree, [1 2 2])

FRates =

    1.1000
    1.0979
    1.1377
    1.1606
```

You can display the identical information by clicking along the same sequence of nodes, as shown next.



Tree Visualization

Selection
● Path
○ Node and Children

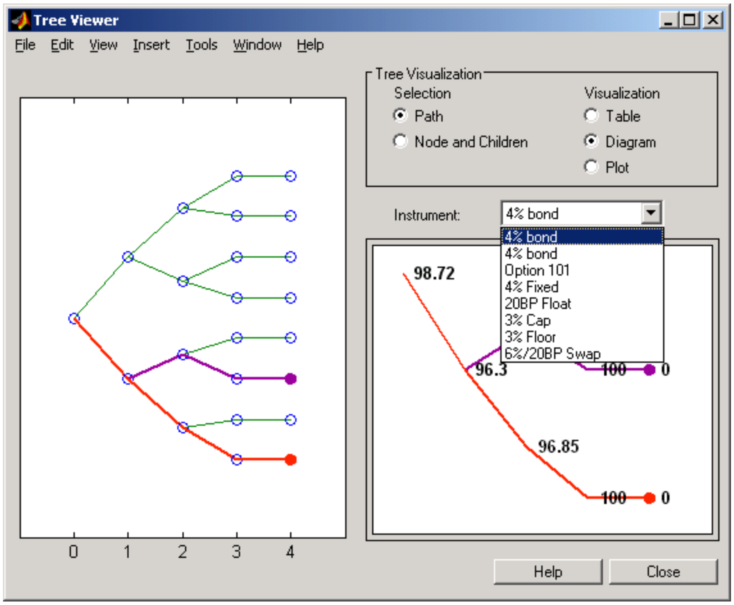
Visualization
○ Table
● Diagram
○ Plot

## Observing Instrument Prices

To use `treeviewer` to display a tree of instrument prices, provide the name of an instrument set along with the name of a price tree in your call to `treeviewer`, for example:

```
load deriv.mat
[Price, PriceTree] = hjmprice(HJMTree, HJMInstSet);
treeviewer(PriceTree, HJMInstSet)
```

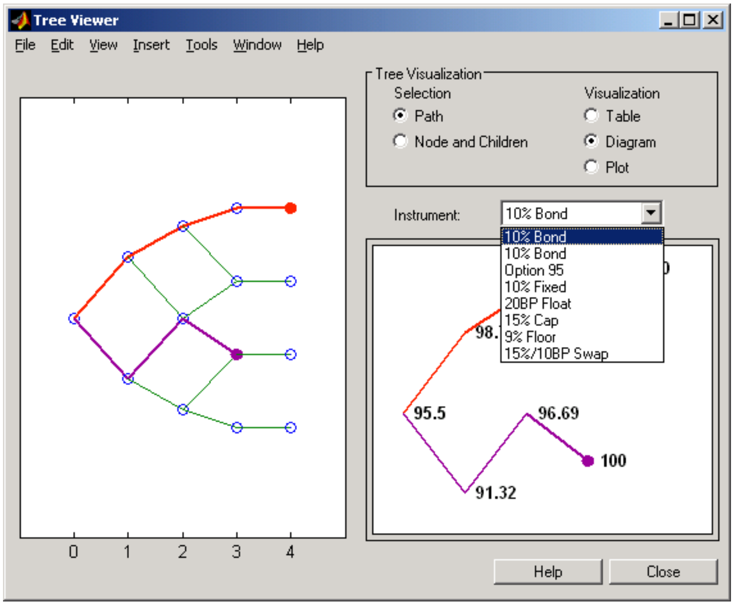
With `treeviewer` you select *each instrument individually* in the instrument portfolio for display.



Selection: ● Path
Visualization: ● Diagram
Instrument: 4% bond

You can use an analogous process to view instrument prices based on the BDT interest rate tree included in `deriv.mat`.

```
load deriv.mat
[BDTPrice, BDTPriceTree] = bdtprice(BDTTree, BDTInstSet);
treeviewer(BDTPriceTree, BDTInstSet)
```



Tree Visualization

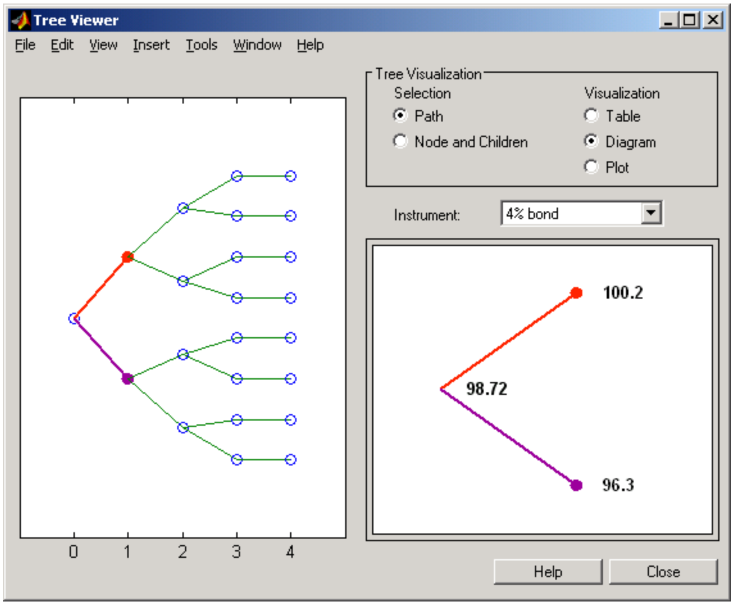
Selection
● Path
○ Node and Children

Visualization
○ Table
● Diagram
○ Plot

Instrument: 10% Bond

## Valuation Date Prices

You can use `treeviewer` instrument-by-instrument to observe instrument prices through time. For the first 4% bond in the HJM instrument portfolio, `treeviewer` indicates a valuation date price of 98.72, the same value obtained by accessing the `PriceTree` structure directly.



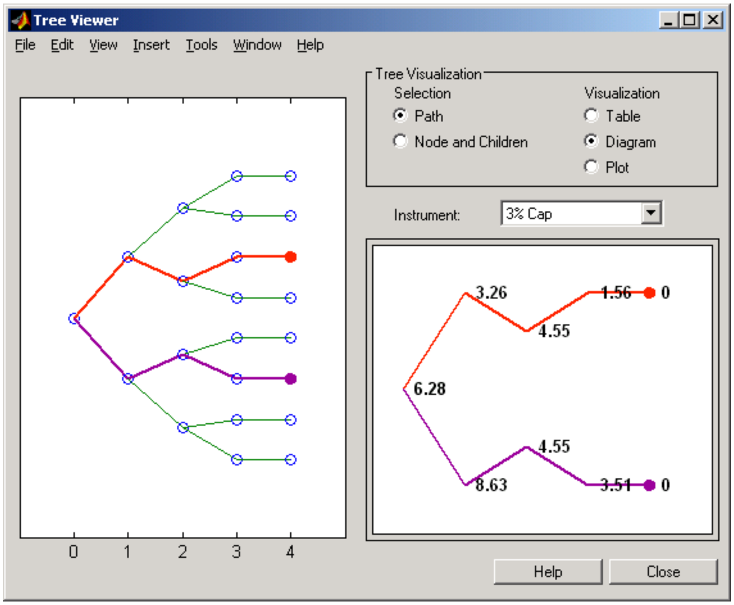
Tree Visualization

Selection: ● Path ○ Node and Children

Visualization: ○ Table ● Diagram ○ Plot

Instrument: 4% bond

As a further example, look at the sixth instrument in the price vector, the 3% cap. At the valuation date, its value obtained directly from the structure is 6.2831. Use `treeviewer` on this instrument to confirm this price.



Tree Visualization

Selection
● Path
○ Node and Children

Visualization
○ Table
● Diagram
○ Plot

Instrument: 3% Cap

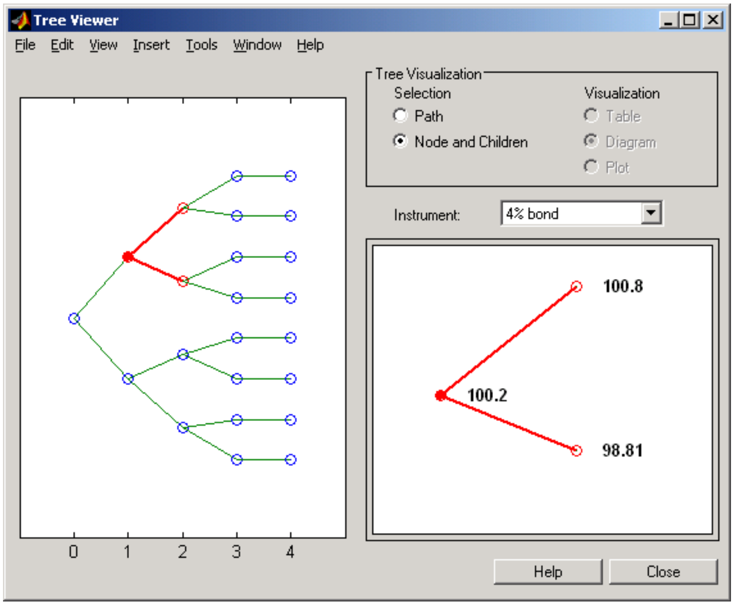
## Additional Observation Times

The second node represents the first-rate observation time, `tObs = 1`. This node displays two states, one representing the branch going up and the other one representing the branch going down.

Examine the prices of the node corresponding to the up branch.

```
PriceTree.PBush{2}(:,:,1)

ans =

  100.1563
   99.7309
    0.1007
  100.1563
  100.3782
    3.2594
    0.1007
    3.5597
```

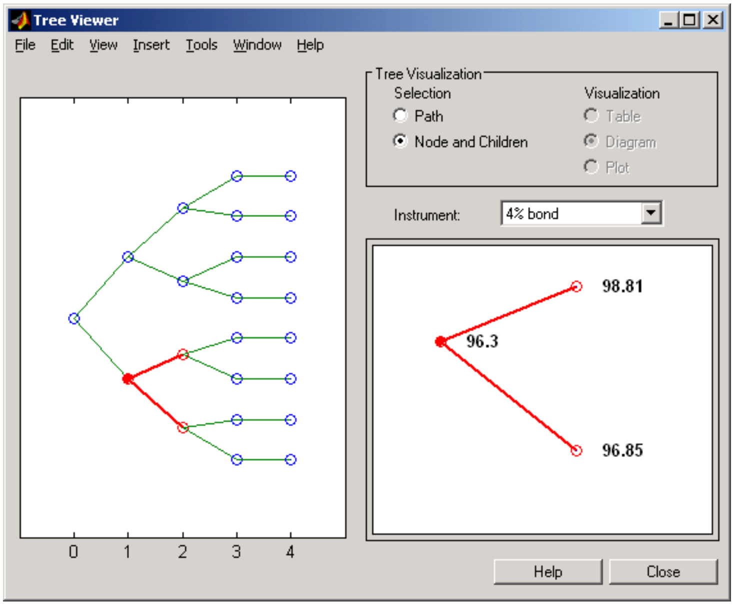
As before, you can use `treeviewer`, this time to examine the price for the 4% bond on the up branch. `treeviewer` displays a price of 100.2 for the first node of the up branch, as expected.



Now examine the corresponding down branch.

```
PriceTree.PBush{2}(:,:,2)

ans =

   96.3041
   94.1986
         0
   96.3041
  100.3671
    8.6342
         0
   -0.3923
```

Use `treeviewer` once again, now to observe the price of the 4% bond on the down branch. The displayed price of 96.3 conforms to the price obtained from direct access of the `PriceTree` structure. You may continue this process as far along the price tree as you want.



Tree Visualization

Selection
○ Path
● Node and Children

Visualization
○ Table
● Diagram
○ Plot

Instrument: 4% bond

# 3

# Equity Derivatives

# Understanding Equity Trees

**In this section...**

## Introduction

Financial Instruments Toolbox software supports three types of recombining tree models to represent the evolution of stock prices:

- Cox-Ross-Rubinstein (CRR) model
- Equal probabilities (EQP) model
- Leisen-Reimer (LR) model
- Implied trinomial tree (ITT) model

For a discussion of recombining trees, see "Rate and Price Trees" on page 2-20.

The CRR, EQP, LR, and ITT models are examples of discrete time models. A discrete time model divides time into discrete bits; prices can only be computed at these specific times.

The CRR model is one of the most common methods used to model the evolution of stock processes. The strength of the CRR model lies in its simplicity. It is a good model when dealing with many tree levels. The CRR model yields the correct expected value for each node of the tree and provides a good approximation for the corresponding local volatility. The approximation becomes better as the number of time steps represented in the tree is increased.

The EQP model is another discrete time model. It has the advantage of building a tree with the exact volatility in each tree node, even with small numbers of time steps. It also provides better results than CRR in some given trading environments, for example, when stock volatility is low and interest rates are high. However, this additional precision causes increased complexity, which is reflected in the number of calculations required to build a tree.

The LR model is another discrete time model. It has the advantage of producing estimates close to the Black-Scholes model using only a small number of steps, while also minimizing the oscillation.

The ITT model is a CRR-style implied trinomial tree which takes advantage of prices quoted from liquid options in the market with varying strikes and maturities to build a tree that more accurately represents the market. An ITT model is commonly used to price exotic options in such a way that they are consistent with the market prices of standard options.

## Building Equity Binary Trees

The tree of stock prices is the fundamental unit representing the evolution of the price of a stock over a given period of time. The MATLAB functions `crrtree`, `eqptree`, and `lrtree` create CRR trees, EQP trees, and LR trees, respectively. These functions create an output tree structure along with information about the parameters used for creating the tree.

The functions `crrtree`, `eqptree`, and `lrtree` take three structures as input arguments:

- The stock parameter structure `StockSpec`
- The interest-rate term structure `RateSpec`
- The tree time layout structure `TimeSpec`

### Calling Sequence for Equity Binary Trees

The calling syntax for `crrtree` is:

```
CRRTree = crrtree (StockSpec, RateSpec, TimeSpec)
```

Similarly, the calling syntax for `eqptree` is:

```
EQPTree = eqptree (StockSpec, RateSpec, TimeSpec)
```

And, the calling syntax for `lrtree` is:

```
LRTree = lrtree(StockSpec, RateSpec, TimeSpec, Strike)
```

All three functions require the structures `StockSpec`, `RateSpec`, and `TimeSpec` as input arguments:

- `StockSpec` is a structure that specifies parameters of the stock whose price evolution is represented by the tree. This structure, created using the function `stockspec`, contains information such as the stock's original price, its volatility, and its dividend payment information.
- `RateSpec` is the interest-rate specification of the initial rate curve. Create this structure with the function `intenvset`.
- `TimeSpec` is the tree time layout specification. Create these structures with the functions `crrtimespec`, `eqptimespec`, and `lrtimespec`. The structures contain information regarding the mapping of relevant dates into the tree structure, plus the number of time steps used for building the tree.

## Specifying the Stock Structure for Equity Binary Trees

The structure `StockSpec` encapsulates the stock-specific information required for building the binary tree of an individual stock's price movement.

You generate `StockSpec` with the function `stockspec`. This function requires two input arguments and accepts up to three additional input arguments that depend on the existence and type of dividend payments.

The syntax for calling `stockspec` is:

```
StockSpec = stockspec(Sigma, AssetPrice, DividendType, ...
DividendAmounts, ExDividendDates)
```

where:

- `Sigma` is the decimal annual volatility of the underlying security.
- `AssetPrice` is the price of the stock at the valuation date.
- `DividendType` is a string specifying the type of dividend paid by the stock. Allowed values are `cash`, `constant`, or `continuous`.

- `DividendAmounts` has a value that depends on the specification of `DividendType`. For `DividendType` `cash`, `DividendAmounts` is a vector of cash dividends. For `DividendType` `constant`, it is a vector of constant annualized dividend yields. For `DividendType` `continuous`, it is a scalar representing a continuously annualized dividend yield.
- `ExDividendDates` also has a value that depends on the nature of `DividendType`. For `DividendType` `cash` or `constant`, `ExDividendDates` is vector of dividend dates. For `DividendType` `continuous`, `ExDividendDates` is ignored.

### Stock Structure Example Using a Binary Tree

Consider a stock with a price of $100 and an annual volatility of 15%. Assume that the stock pays three cash $5.00 dividends on dates January 01, 2003; July 01, 2003; and January 01, 2004. You specify these parameters in MATLAB as:

```
Sigma = 0.15;
AssetPrice = 100;
DividendType = 'cash';
DividendAmounts = [5; 5; 5];
ExDividendDates = {'jan-01-2004', 'july-01-2005', 'jan-01-2006'};

StockSpec = stockspec(Sigma, AssetPrice, DividendType, ...
DividendAmounts, ExDividendDates)

StockSpec =

              FinObj: 'StockSpec'
               Sigma: 0.1500
          AssetPrice: 100
        DividendType: 'cash'
     DividendAmounts: [3x1 double]
     ExDividendDates: [3x1 double]
```

### Specifying the Interest-Rate Term Structure for Equity Binary Trees

The `RateSpec` structure defines the interest rate environment used when building the stock price binary tree. “Modeling the Interest-Rate Term Structure” on page 2-32 explains how to create these structures using the function `intenvset`, given the interest rates, the starting and ending dates for each rate, and the compounding value.

### Specifying the Tree-Time Term Structure for Equity Binary Trees

The `TimeSpec` structure defines the tree layout of the binary tree:

- It maps the valuation and maturity dates to their corresponding times.
- It defines the time of the levels of the tree by dividing the time span between valuation and maturity into equally spaced intervals. By specifying the number of intervals, you define the granularity of the tree time structure.

The syntax for building a `TimeSpec` structure is:

```
TimeSpec = crrtimespec(ValuationDate, Maturity, NumPeriods)
TimeSpec = eqptimespec(ValuationDate, Maturity, NumPeriods)

TimeSpec = lrtimespec(ValuationDate, Maturity, NumPeriods)
```

where:

- `ValuationDate` is a scalar date marking the pricing date and first observation in the tree (location of the root node). You enter `ValuationDate` either as a serial date number (generated with `datenum`) or a date string.
- `Maturity` is a scalar date marking the maturity of the tree, entered as a serial date number or a date string.
- `NumPeriods` is a scalar defining the number of time steps in the tree; for example, `NumPeriods = 10` implies 10 time steps and 11 tree levels (0, 1, 2, ..., 9, 10).

### TimeSpec Example Using a Binary Tree

Consider building a CRR tree, with a valuation date of January 1, 2003, a maturity date of January 1, 2008, and 20 time steps. You specify these parameters in MATLAB as:

```
ValuationDate = 'Jan-1-2003';
Maturity = 'Jan-1-2008';
NumPeriods = 20;
```

```
TimeSpec = crrtimespec(ValuationDate, Maturity, NumPeriods)
TimeSpec =

           FinObj: 'BinTimeSpec'
    ValuationDate: 731582
         Maturity: 733408
       NumPeriods: 20
            Basis: 0
     EndMonthRule: 1
             tObs: [1x21 double]
             dObs: [1x21 double]
```

Two vector fields in the `TimeSpec` structure are of particular interest: `dObs` and `tObs`. These two fields represent the observation times and corresponding dates of all tree levels, with `dObs(1)` and `tObs(1)`, respectively, representing the root node (`ValuationDate`), and `dObs(end)` and `tObs(end)` representing the last tree level (`Maturity`).

**Note** There is no relationship between the dates specified for the tree and the implied tree level times, and the maturities specified in the interest rate term structure. The rates in `RateSpec` are interpolated or extrapolated as required to meet the time distribution of the tree.

### Examples of Binary Tree Creation

You can now use the `StockSpec` and `TimeSpec` structures described previously to build an equal probability tree (`EQPTree`), a CRR tree (`CRRTree`), or a LR tree (`LRTree`). First, you must define the interest rate term structure. For this example, assume that the interest rate is fixed at 10% annually between the valuation date of the tree (January 1, 2003) until its maturity.

```
ValuationDate = 'Jan-1-2003';
Maturity = 'Jan-1-2008';
Rate = 0.1;
RateSpec = intenvset('Rates', Rate, 'StartDates', ...
ValuationDate, 'EndDates', Maturity, 'Compounding', -1);
```

To build a `CRRTree`, enter:

```
CRRTree = crrtree(StockSpec, RateSpec, TimeSpec)

CRRTree =

       FinObj: 'BinStockTree'
       Method: 'CRR'
    StockSpec: [1x1 struct]
     TimeSpec: [1x1 struct]
     RateSpec: [1x1 struct]
         tObs: [1x21 double]
         dObs: [1x21 double]
        STree: {1x21 cell}
      UpProbs: [1x20 double]
```

To build an `EQPTree`, enter:

```
EQPTree = eqptree(StockSpec, RateSpec, TimeSpec)

EQPTree =

       FinObj: 'BinStockTree'
       Method: 'EQP'
    StockSpec: [1x1 struct]
     TimeSpec: [1x1 struct]
     RateSpec: [1x1 struct]
         tObs: [1x21 double]
         dObs: [1x21 double]
        STree: {1x21 cell}
      UpProbs: [1x20 double]
```

## Building Implied Trinomial Trees

The tree of stock prices is the fundamental unit representing the evolution of the price of a stock over a given period of time. The MATLAB function `itttree` creates an output tree structure along with the information about the parameters used to create the tree.

The function `itttree` takes four structures as input arguments:

- The stock parameter structure `StockSpec`

- The interest-rate term structure `RateSpec`
- The tree time layout structure `TimeSpec`
- The stock option specification structure `StockOptSpec`

### Calling Sequence for Implied Trinomial Trees

The calling syntax for `itttree` is:

```
ITTTree = itttree (StockSpec,RateSpec,TimeSpec,StockOptSpec)
```

- `StockSpec` is a structure that specifies parameters of the stock whose price evolution is represented by the tree. This structure, created using the function `stockspec`, contains information such as the stock's original price, its volatility, and its dividend payment information.
- `RateSpec` is the interest-rate specification of the initial rate curve. Create this structure with the function `intenvset`.
- `TimeSpec` is the tree time layout specification. Create these structures with the function `itttimespec`. This structure contains information regarding the mapping of relevant dates into the tree structure, plus the number of time steps used for building the tree.
- `StockOptSpec` is a structure containing parameters of European stock options instruments. Create this structure with the function `stockoptspec`.

### Specifying the Stock Structure for Implied Trinomial Trees

The structure `StockSpec` encapsulates the stock-specific information required for building the trinomial tree of an individual stock's price movement.

You generate `StockSpec` with the function `stockspec`. This function requires two input arguments and accepts up to three additional input arguments that depend on the existence and type of dividend payments.

The syntax for calling `stockspec` is:

```
StockSpec = stockspec(Sigma, AssetPrice, DividendType, ...
DividendAmounts, ExDividendDates)
```

where:

- `Sigma` is the decimal annual volatility of the underlying security.
- `AssetPrice` is the price of the stock at the valuation date.
- `DividendType` is a string specifying the type of dividend paid by the stock. Allowed values are `cash`, `constant`, or `continuous`.
- `DividendAmounts` has a value that depends on the specification of `DividendType`. For `DividendType` `cash`, `DividendAmounts` is a vector of cash dividends. For `DividendType` `constant`, it is a vector of constant annualized dividend yields. For `DividendType` `continuous`, it is a scalar representing a continuously annualized dividend yield.
- `ExDividendDates` also has a value that depends on the nature of `DividendType`. For `DividendType` `cash` or `constant`, `ExDividendDates` is vector of dividend dates. For `DividendType` `continuous`, `ExDividendDates` is ignored.

### Stock Structure Example Using an Implied Trinomial Tree

Consider a stock with a price of $100 and an annual volatility of 12%. Assume that the stock is expected to pay a dividend yield of 6%. You specify these parameters in MATLAB as:

```
So=100;
DividendYield = 0.06;
Sigma=.12;

StockSpec = stockspec(Sigma, So, 'continuous', DividendYield)

StockSpec =

             FinObj: 'StockSpec'
              Sigma: 0.1200
         AssetPrice: 100
       DividendType: 'continuous'
    DividendAmounts: 0.0600
    ExDividendDates: []
```

### Specifying the Interest-Rate Term Structure for Implied Trinomial Trees

The structure `RateSpec` defines the interest rate environment used when building the stock price binary tree. "Modeling the Interest-Rate Term Structure" on page 2-32 explains how to create these structures using the function `intenvset`, given the interest rates, the starting and ending dates for each rate, and the compounding value.

### Specifying the Tree-Time Term Structure for Implied Trinomial Trees

The `TimeSpec` structure defines the tree layout of the trinomial tree:

- It maps the valuation and maturity dates to their corresponding times.
- It defines the time of the levels of the tree by dividing the time span between valuation and maturity into equally spaced intervals. By specifying the number of intervals, you define the granularity of the tree time structure.

The syntax for building a `TimeSpec` structure is:

```
TimeSpec = itttimespec(ValuationDate, Maturity, NumPeriods)
```

where:

- `ValuationDate` is a scalar date marking the pricing date and first observation in the tree (location of the root node). You enter `ValuationDate` either as a serial date number (generated with `datenum`) or a date string.
- `Maturity` is a scalar date marking the maturity of the tree, entered as a serial date number or a date string.
- `NumPeriods` is a scalar defining the number of time steps in the tree; for example, `NumPeriods = 10` implies 10 time steps and 11 tree levels (0, 1, 2, ..., 9, 10).

### TimeSpec Example Using an Implied Trinomial Tree

Consider building an ITT tree, with a valuation date of January 1, 2006, a maturity date of January 1, 2008, and four time steps. You specify these parameters in MATLAB as:

```
ValuationDate = '01-01-2006';
EndDate = '01-01-2008';
NumPeriods = 4;

TimeSpec = itttimespec(ValuationDate, EndDate, NumPeriods)

TimeSpec =

           FinObj: 'ITTTimeSpec'
    ValuationDate: 732678
         Maturity: 733408
       NumPeriods: 4
            Basis: 0
     EndMonthRule: 1
             tObs: [0 0.5000 1 1.5000 2]
             dObs: [732678 732860 733043 733225 733408]
```

Two vector fields in the `TimeSpec` structure are of particular interest: `dObs` and `tObs`. These two fields represent the observation times and corresponding dates of all tree levels, with `dObs(1)` and `tObs(1)`, respectively, representing the root node (`ValuationDate`), and `dObs(end)` and `tObs(end)` representing the last tree level (`Maturity`).

## Specifying the Option Stock Structure for Implied Trinomial Trees

The `StockOptSpec` structure encapsulates the option-stock-specific information required for building the implied trinomial tree. You generate `StockOptSpec` with the function `stockoptspec`. This function requires five input arguments. An optional sixth argument `InterpMethod`, specifying the interpolation method, can be included. The syntax for calling `stockoptspec` is:

```
[StockOptSpec] = stockoptspec(OptPrice, Strike, Settle, Maturity, OptSpec)
```

where:

- `Optprice` is a `NINST`-by-`1` vector of European option prices.
- `Strike` is a `NINST`-by-`1` vector of strike prices.
- `Settle` is a scalar date marking the settlement date.

- `Maturity` is a `NINST`-by-`1` vector of maturity dates.
- `OptSpec` is a `NINST`-by-`1` cell array of strings `'call'` or `'put'`.

### Option Stock Structure Example Using an Implied Trinomial Tree

Consider the following data quoted from liquid options in the market with varying strikes and maturity. You specify these parameters in MATLAB as:

```
Settle =   '01/01/06';

Maturity =    ['07/01/06';
    '07/01/06';
    '07/01/06';
    '07/01/06';
    '01/01/07';
    '01/01/07';
    '01/01/07';
    '01/01/07';
    '07/01/07';
    '07/01/07';
    '07/01/07';
    '07/01/07';
    '01/01/08';
    '01/01/08';
    '01/01/08';
    '01/01/08'];

Strike = [113;
   101;
   100;
    88;
   128;
   112;
   100;
    78;
   144;
   112;
   100;
    69;
```

```
   162;
   112;
   100;
    61];

OptPrice =[                 0;
   4.807905472659144;
   1.306321897011867;
   0.048039195057173;
                   0;
   2.310953054191461;
   1.421950392866235;
   0.020414826276740;
                   0;
   5.091986935627730;
   1.346534812295291;
   0.005101325584140;
                   0;
   8.047628153217246;
   1.219653432150932;
   0.001041436654748];


OptSpec = { 'call';
    'call';
    'put';
    'put';
    'call';
    'call';
    'put';
    'put';
    'call';
    'call';
    'put';
    'put';
    'call';
    'call';
    'put';
    'put'};
```

```
StockOptSpec = stockoptspec(OptPrice, Strike, Settle, Maturity, OptSpec)

StockOptSpec =

         FinObj: 'StockOptSpec'
       OptPrice: [16x1 double]
         Strike: [16x1 double]
         Settle: 732678
       Maturity: [16x1 double]
        OptSpec: {16x1 cell}
   InterpMethod: 'price'
```

---

**Note** The algorithm for building the ITT tree requires specifying option prices for all tree nodes. The maturities of those options correspond to those of the tree levels, and the strike to the prices on the tree nodes. The types of option are `Calls` for the nodes above the central nodes, and `Puts` for those below and including the central nodes.

Clearly, all these options will not be available in the market, hence making interpolation and extrapolation necessary to obtain the node option prices. The degree to which the tree reflects the market will unavoidably be tied to the results of these interpolations and extrapolations. Keeping in mind that extrapolation is less accurate than interpolation, and more so the further away the extrapolated points are from the data points, the function `itttree` issues a warning with a list of the options for which extrapolation was necessary.

In some cases, it may be desirable to view a list of ideal option prices to form an idea of the ranges needed. This can be achieved by calling the function `itttree` specifying only the first three input arguments. The second output argument is a structure array containing the list of ideal options needed.

---

### Creating an Implied Trinomial Tree

You can now use the `StockSpec`, `TimeSpec`, and `StockOptSpec` structures described in "Stock Structure Example Using an Implied Trinomial Tree" on page 3-10, "TimeSpec Example Using an Implied Trinomial Tree" on page 3-11, and "Option Stock Structure Example Using an Implied Trinomial Tree" on page 3-13 to build an implied trinomial tree (ITT). First, you must define the interest rate term structure. For this example, assume that the interest rate is fixed at 8% annually between the valuation date of the tree (January 1, 2006) until its maturity.

```
Rate = 0.08;
ValuationDate = '01-01-2006';
EndDate = '01-01-2008';

RateSpec = intenvset('StartDates', ValuationDate, 'EndDates', EndDate, ...
    'ValuationDate', ValuationDate, 'Rates', Rate, 'Compounding', -1);
```

To build an `ITTTree`, enter:

```
ITTTree = itttree(StockSpec, RateSpec, TimeSpec, StockOptSpec)

ITTTree =

          FinObj: 'ITStockTree'
       StockSpec: [1x1 struct]
    StockOptSpec: [1x1 struct]
        TimeSpec: [1x1 struct]
        RateSpec: [1x1 struct]
            tObs: [0 0.500000000000000 1 1.500000000000000 2]
            dObs: [732678 732860 733043 733225 733408]
           STree: {1x5 cell}
           Probs: {[3x1 double]  [3x3 double]  [3x5 double]  [3x7 double]}
```

## Examining Equity Trees

Financial Instruments Toolbox software uses equity binary and implied trinomial trees to represent prices of equity options and of underlying stocks. At the highest level, these trees have structures wrapped around them. The structures encapsulate information required to interpret information in the tree.

To examine an equity binary or trinomial tree, load the data in the MAT-file `deriv.mat` into the MATLAB workspace.

```
load deriv.mat
```

Display the list of variables loaded from the MAT-file with the `whos` command.

```
  Name               Size            Bytes  Class     Attributes

  BDTInstSet         1x1             15956  struct
  BDTTree            1x1              5138  struct
  BKInstSet          1x1             15946  struct
  BKTree             1x1              5904  struct
  CRRInstSet         1x1             12434  struct
  CRRTree            1x1              5058  struct
  EQPInstSet         1x1             12434  struct
  EQPTree            1x1              5058  struct
  HJMInstSet         1x1             15948  struct
  HJMTree            1x1              5838  struct
  HWInstSet          1x1             15946  struct
  HWTree             1x1              5904  struct
  ITTInstSet         1x1             12438  struct
  ITTTree            1x1              8862  struct
  ZeroInstSet        1x1             10282  struct
  ZeroRateSpec       1x1              1580  struct
```

## Examining a CRRTree

You can examine in some detail the contents of the `CRRTree` structure contained in this file.

```
CRRTree

       FinObj: 'BinStockTree'
       Method: 'CRR'
    StockSpec: [1x1 struct]
     TimeSpec: [1x1 struct]
     RateSpec: [1x1 struct]
         tObs: [0 1 2 3 4]
         dObs: [731582 731947 732313 732678 733043]
        STree: {[100]  [110.5171 90.4837]  [122.1403 100 81.8731]
               [1x4 double]  [1x5 double]}
      UpProbs: [0.7309 0.7309 0.7309 0.7309]
```

The `Method` field of the structure indicates that this is a CRR tree, not an EQP tree.

The fields `StockSpec`, `TimeSpec` and `RateSpec` hold the original structures passed into the function `crrtree`. They contain all the context information required to interpret the tree data.

The fields `tObs` and `dObs` are vectors containing the observation times and dates, the times and dates of the levels of the tree. In this particular case, `tObs` reveals that the tree has a maturity of 4 years (`tObs(end) = 4`) and that it has four time steps (the length of `tObs` is five).

The field `dObs` shows the specific dates for the tree levels, with a granularity of 1 day. This means that all values in `tObs` that correspond to a given day from 00:00 hours to 24:00 hours are mapped to the corresponding value in `dObs`. You can use the function `datestr` to convert these MATLAB serial dates into their string representations.

The field `UpProbs` is a vector representing the probabilities for up movements from any node in each level. This vector has 1 element per tree level. All nodes for a given level have the same probability of an up movement. In the specific case being examined, the probability of an up movement is 0.7309 for all levels, and the probability for a down movement is 0.2691 (1 - 0.7309).

Finally, the field `STree` contains the actual stock tree. It is represented in MATLAB as a cell array with each cell array element containing a vector of prices corresponding to a tree level. The prices are in descending order, that is, `CRRTree.STree{3}(1)` represents the topmost element of the third level of the tree, and `CRRTree.STree{3}(end)` represents the bottom element of the same level of the tree.

## Examining an ITTTree

You can examine in some detail the contents of the `ITTTree` structure contained in this file.

```
TTTree =


          FinObj: 'ITStockTree'
       StockSpec: [1x1 struct]
    StockOptSpec: [1x1 struct]
        TimeSpec: [1x1 struct]
        RateSpec: [1x1 struct]
```

```
  tObs: [0 1 2 3 4]
  dObs: [732678 733043 733408 733773 734139]
 STree: {1x5 cell}
 Probs: {[3x1 double]  [3x3 double]  [3x5 double]  [3x7 double]}
```

The fields `StockSpec`, `StockOptSpec`, `TimeSpec`, and `RateSpec` hold the original structures passed into the function `itttree`. They contain all the context information required to interpret the tree data.

The fields `tObs` and `dObs` are vectors containing the observation times and dates, the times and dates of the levels of the tree. In this particular case, `tObs` reveals that the tree has a maturity of 4 years (`tObs(end) = 4`) and that it has four time steps (the length of `tObs` is five).

The field `dObs` shows the specific dates for the tree levels, with a granularity of 1 day. This means that all values in `tObs` that correspond to a given day from 00:00 hours to 24:00 hours are mapped to the corresponding value in `dObs`. You can use the function `datestr` to convert these MATLAB serial dates into their string representations.

The field `Probs` is a vector representing the probabilities for movements from any node in each level. This vector has three elements per tree node. In the specific case being examined, at `tObs= 1`, the probability for an up movement is 0.4675, and the probability for a down movement is 0.1934.

Finally, the field `STree` contains the actual stock tree. It is represented in MATLAB as a cell array with each cell array element containing a vector of prices corresponding to a tree level. The prices are in descending order, that is, `ITTTree.STree{4}(1)` represents the top element of the fourth level of the tree, and `ITTTree.STree{4}(end)` represents the bottom element of the same level of the tree.

## Isolating a Specific Node for a CRRTree

The function `treepath` can isolate a specific set of nodes of a binary tree by specifying the path used to reach the final node. As an example, consider the nodes touched by starting from the root node, then following a down movement, then an up movement, and finally a down movement. You use a vector to specify the path, with `1` corresponding to an up movement and `2` corresponding to a down movement. An up-down-up path is then represented as `[2 1 2]`. To obtain the values of all nodes touched by this path, enter:

```
SVals = treepath(CRRTree.STree, [2 1 2])

SVals =

  100.0000
   90.4837
  100.0000
   90.4837
```

The first value in the vector `SVals` corresponds to the root node, and the last value corresponds to the final node reached by following the path indicated.

### Isolating a Specific Node for an ITTTree

The function `trintreepath` can isolate a specific set of nodes of a trinomial tree by specifying the path used to reach the final node. As an example, consider the nodes touched by starting from the root node, then following an up movement, then a middle movement, and finally a down movement. You use a vector to specify the path, with `1` corresponding to an up movement, `2` corresponding to a middle movement, and `3` corresponding to a down movement. An up-down-middle-down path is then represented as `[1 3 2 3]`. To obtain the values of all nodes touched by this path, enter:

```
pathSVals = trintreepath(ITTTree, [1 3 2 3])

pathSVals =

   50.0000
   66.3448
   50.0000
   50.0000
   37.6819
```

The first value in the vector `pathSVals` corresponds to the root node, and the last value corresponds to the final node reached by following the path indicated.

## Differences Between CRR and EQP Tree Structures

In essence, the structures representing CRR trees and EQP trees are similar. If you create a CRR or an EQP tree using identical input arguments, only a few of the tree structure fields differ:

- The `Method` field has a value of `'CRR'` or `'EQP'` indicating the method used to build the structure.
- The prices in the `STree` cell array have the same structure, but the prices within the cell array are different.
- For EQP, the structure field `UpProb` always holds a vector with all elements set to 0.5, while for CRR, these probabilities are calculated based on the input arguments passed when building the tree.

# Supported Equity Derivatives

**In this section…**

## Asian Option

An *Asian* option is a path-dependent option with a payoff linked to the average value of the underlying asset during the life (or some part of the life) of the option. They are similar to lookback options in that there are two types of Asian options: fixed (average price option) and floating (average strike option). Fixed Asian options have a specified strike, while floating Asian options have a strike equal to the average value of the underlying asset over the life of the option.

There are four Asian option types, each with its own characteristic payoff formula:

- Fixed call: $\max(0, S_{av} - X)$
- Fixed put: $\max(0, X - S_{av})$
- Floating call: $\max(0, S - S_{av})$
- Floating put: $\max(0, S_{av} - S)$

where:

$S_{av}$ is the average price of underlying stock found along the particular path followed to the node.

$S$ is the price of the underlying stock on the node.

$X$ is the strike price (applicable only to fixed Asian options).

$S_{av}$ is defined using either a geometric or an arithmetic average.

The following functions support Asian options.

| Function | Purpose |
|---|---|
| `asianbycrr` | Price Asian options from a CRR binomial tree. |
| `asianbyeqp` | Price Asian options from an EQP binomial tree. |
| `asianbyitt` | Price Asian options using an implied trinomial tree (ITT). |
| `instasian` | Construct an Asian option. |

## Barrier Option

A *barrier* option is similar to a vanilla put or call option, but its life either begins or ends when the price of the underlying stock passes a predetermined barrier value. There are four types of barrier options.

### Up Knock-In

This option becomes effective when the price of the underlying stock passes above a barrier that is above the initial stock price. Once the barrier has knocked in, it will not knock out even if the price of the underlying instrument moves below the barrier again.

### Up Knock-Out

This option terminates when the price of the underlying stock passes above a barrier that is above the initial stock price. Once the barrier has knocked out, it will not knock in even if the price of the underlying instrument moves below the barrier again.

### Down Knock-In

This option becomes effective when the price of the underlying stock passes below a barrier that is below the initial stock price. Once the barrier has knocked in, it will not knock out even if the price of the underlying instrument moves above the barrier again.

### Down Knock-Out

This option terminates when the price of the underlying stock passes below a barrier that is below the initial stock price. Once the barrier has knocked out, it will not knock in even if the price of the underlying instrument moves above the barrier again.

### Rebates

If a barrier option fails to exercise, the seller may pay a rebate to the buyer of the option. Knock-outs may pay a rebate when they are knocked out, and knock-ins may pay a rebate if they expire without ever knocking in.

The following functions support barrier options.

| Function | Purpose |
|---|---|
| `barrierbycrr` | Price barrier options from a CRR binomial tree. |
| `barrierbyeqp` | Price barrier options from an EQP binomial tree. |
| `barrierbyitt` | Price barrier options using an implied trinomial tree (ITT). |
| `instbarrier` | Construct a barrier option. |

## Basket Option

A *basket* option is an option on a portfolio of several underlying equity assets. Payout for a basket option depends on the cumulative performance of the collection of the individual assets. A basket option tends to be cheaper than the corresponding portfolio of plain vanilla options for these reasons:

- If the basket components correlate negatively, movements in the value of one component neutralize opposite movements of another component.

Unless all the components correlate perfectly, the basket option is cheaper than a series of individual options on each of the assets in the basket.

- A basket option minimizes transaction costs because an investor has to purchase only one option instead of several individual options.

The payoff for a basket option is as follows:

- For a call: $\max(\sum Wi * Si - K; 0)$
- For a put: $\max(\sum K - Wi * Si; 0)$

where:

$Si$ is the price of asset $i$ in the basket.

$Wi$ is the quantity of asset $i$ in the basket.

$K$ is the strike price.

Financial Instruments Toolbox software supports Longstaff-Schwartz and Nengiu Ju models for pricing basket options. The Longstaff-Schwartz model supports both European, Bermuda, and American basket options. The Nengiu Ju model only supports European basket options. If you want to price either an American or Bermuda basket option, use the functions for the Longstaff-Schwartz model. To price a European basket option, use either the functions for the Longstaff-Schwartz model or the Nengiu Ju model.

| Function | Purpose |
|---|---|
| `basketbyls` | Price basket options using the Longstaff-Schwartz model. |
| `basketsensbyls` | Calculate price and sensitivities for basket options using the Longstaff-Schwartz model. |
| `basketbyju` | Price European basket options using the Nengjiu Ju approximation model. |

| Function | Purpose |
|---|---|
| `basketsensbyju` | Calculate European basket options price and sensitivity using the Nengjiu Ju approximation model. |
| `basketstockspec` | Specify a basket stock structure. |

## Compound Option

A *compound* option is basically an option on an option; it gives the holder the right to buy or sell another option. With a compound option, a vanilla stock option serves as the underlying instrument. Compound options thus have two strike prices and two exercise dates.

There are four types of compound options:

- Call on a call
- Put on a put
- Call on a put
- Put on a call

**Note** The payoff formulas for compound options are too complex for this discussion. If you are interested in the details, consult the paper by Mark Rubinstein entitled "Double Trouble," published in *Risk 5* (1991).

Consider the third type, a call on a put. It gives the holder the right to buy a put option. In this case, on the first exercise date, the holder of the compound option pay the first strike price and receives a put option. The put option gives the holder the right to sell the underlying asset for the second strike price on the second exercise date.

The following functions support compound options.

| Function | Purpose |
|---|---|
| `compoundbycrr` | Price compound options from a CRR binomial tree. |
| `compoundbyeqp` | Price compound options from an EQP binomial tree. |
| `compoundbyitt` | Price compound options using an implied trinomial tree (ITT). |
| `instcompound` | Construct a compound option. |

## Lookback Option

A *lookback* option is a path-dependent option based on the maximum or minimum value the underlying asset achieves during the entire life of the option.

Financial Instruments Toolbox software supports two types of lookback options: fixed and floating. Fixed lookback options have a specified strike price, while floating lookback options have a strike price determined by the asset path. Consequently, there are a total of four lookback option types, each with its own characteristic payoff formula:

- Fixed call: $\max(0, S_{\max} - X)$
- Fixed put: $\max(0, X - S_{\min})$
- Floating call: $\max(0, S - S_{\min})$
- Floating put: $\max(0, S_{\max} - S)$

where:

$S_{\max}$ is the maximum price of underlying stock found along the particular path followed to the node.

$S_{\min}$ is the minimum price of underlying stock found along the particular path followed to the node.

$S$ is the price of the underlying stock on the node.

$X$ is the strike price (applicable only to fixed lookback options).

The following functions support lookback options.

| **Function** | **Purpose** |
|---|---|
| `lookbackbycrr` | Price lookback options from a CRR binomial tree. |
| `lookbackbyeqp` | Price lookback options from an EQP binomial tree. |
| `lookbackbyitt` | Price lookback options using an implied trinomial tree (ITT). |
| `instlookback` | Construct a lookback option. |

## Digital Option

A *digital* option is an option whose payoff is characterized as having only two potential values: a fixed payout, when the option is in the money or a zero payout otherwise. This is the case irrespective of how far the asset price at maturity is above (call) or below (put) the strike.

Digital options are attractive to sellers because they guarantee a known maximum loss in the event that the option is exercised. This overcomes a fundamental problem with the vanilla options, where the potential loss is unlimited. Digital options are attractive to buyers because the option payoff is a known constant amount, and this amount can be adjusted to provide the exact quantity of protection required.

Financial Instruments Toolbox supports four types of digital options:

- Cash-or-nothing option — Pays some fixed amount of cash if the option expires in the money.
- Asset-or-nothing option — Pays the value of the underlying security if the option expires in the money.
- Gap option — One strike decides if the option is in or out of money; another strike decides the size the size of the payoff.
- Supershare — Pays out a proportion of the assets underlying a portfolio if the asset lies between a lower and an upper bound at the expiry of the option.

The following functions calculate pricing and sensitivity for digital options.

| Function | Purpose |
|---|---|
| `cashbybls` | Calculate the price of cash-or-nothing digital options using the Black-Scholes model. |
| `assetbybls` | Calculate the price of asset-or-nothing digital options using the Black-Scholes model. |
| `gapbybls` | Calculate the price of gap digital options using the Black-Scholes model. |
| `supersharebybls` | Calculate the price of supershare digital options using the Black-Scholes model. |
| `cashsensbybls` | Calculate the price and sensitivities of cash-or-nothing digital options using the Black-Scholes model. |
| `assetsensbybls` | Calculate the price and sensitivities of asset-or-nothing digital options using the Black-Scholes model. |
| `gapsensbybls` | Calculate the price and sensitivities of gap digital options using the Black-Scholes model. |
| `supersharesensbybls` | Calculate the price and sensitivities of supershare digital options using the Black-Scholes model. |

## Rainbow Option

A rainbow option payoff depends on the relative price performance of two or more assets. A *rainbow* option gives the holder the right to buy or sell the best or worst of two securities, or options that pay the best or worst of two assets.

Rainbow options are popular because of the lower premium cost of the structure relative to the purchase of two separate options. The lower cost reflects the fact that the payoff is generally lower than the payoff of the two separate options.

Financial Instruments Toolbox supports two types of rainbow options:

- Minimum of two assets — The option holder has the right to buy(sell) one of two risky assets, whichever one is worth less.
- Maximum of two assets — The option holder has the right to buy(sell) one of two risky assets, whichever one is worth more.

The following rainbow options speculate/hedge on two equity assets.

| **Function** | **Purpose** |
|---|---|
| `minassetbystulz` | Calculate the European rainbow option price on minimum of two risky assets using the Stulz option pricing model. |
| `minassetsensbystulz` | Calculate the European rainbow option prices and sensitivities on minimum of two risky assets using the Stulz pricing model. |
| `maxassetbystulz` | Calculate the European rainbow option price on maximum of two risky assets using the Stulz option pricing model. |
| `maxassetsensbystulz` | Calculate the European rainbow option prices and sensitivities on maximum of two risky assets using the Stulz pricing model. |

## Vanilla Option

A *vanilla option* is a category of options that includes only the most standard components. A vanilla option has an expiration date and straightforward strike price. American-style options and European-style options are both categorized as vanilla options.

The payoff for a vanilla option is as follows:

- For a call: $\max(St - K, 0)$
- For a put: $\max(K - St, 0)$

where:

$St$ is the price of the underlying stock at time $t$.

*K* is the strike price.

The following functions support specifying or pricing a vanilla option.

| Function | Purpose |
|---|---|
| `optstockbycrr` | Calculate the price of a European, Bermuda, or American stock option using a CRR tree. |
| `optstockbyeqp` | Calculate the price of a European, Bermuda, or American stock option using an EQP tree. |
| `optstockbyitt` | Calculate the price of a European, Bermuda, or American stock option using an ITT tree. |
| `optstockbylr` | Calculate the price of a European, Bermuda, or American stock option using the Leisen-Reimer (LR) binomial tree model. |
| `optstockbybls` | Price options using the Black-Scholes option pricing model. |
| `optstocksensbybls` | Calculate option prices and sensitivities using the Black-Scholes option pricing model. |
| `optstockbyrgw` | Calculate American call option prices using the Roll-Geske-Whaley option pricing model. |
| `optstocksensbyrgw` | Calculate American call option prices and sensitivities using the Roll-Geske-Whaley option pricing model. |
| `optstockbybjs` | Price American options using the Bjerksund-Stensland 2002 option pricing model. |
| `optstocksensbybjs` | Calculate American option prices and sensitivities using the Bjerksund-Stensland 2002 option pricing model. |
| `instoptstock` | Specify a European or Bermuda option. |

## Bermuda Put and Call Schedule

A Bermuda option resembles a hybrid of American and European options. You exercise it on predetermined dates only, usually monthly. In Financial

Instruments Toolbox software, you indicate the relevant information for a Bermuda option in two input matrices:

- `Strike` — Contains the strike price values for the option.
- `ExerciseDates` — Contains the schedule when you can exercise the option.

# Pricing Equity Derivatives Using Trees

**In this section...**

## Computing Instrument Prices

The portfolio pricing functions `crrprice`, `eqpprice`, and `ittprice` calculate the price of any set of supported instruments based on a binary equity price tree or an implied trinomial price tree. These functions are capable of pricing the following instrument types:

- Vanilla stock options
  - American and European puts and calls
- Exotic options
  - Asian
  - Barrier
  - Compound
  - Lookback
  - Stock options (Bermuda put and call schedules)

The syntax for calling the function `crrprice` is:

```
[Price, PriceTree] = crrprice(CRRTree, InstSet, Options)
```

The syntax for `eqpprice` is:

```
[Price, PriceTree] = eqpprice(EQPTree, InstSet, Options)
```

The syntax for `ittprice` is:

```
Price = ittprice(ITTTree, ITTInstSet, Options)
```

These functions require two input arguments: the equity price tree and the set of instruments, `InstSet`, and allow a third optional argument.

### Required Arguments

`CRRTree` is a CRR equity price tree created using `crrtree`. `EQPTree` is an equal probability equity price tree created using `eqptree`. `ITTTree` is an ITT equity price tree created using `itttree`. See “Building Equity Binary Trees” on page 3-3 and “Building Implied Trinomial Trees” on page 3-8 to learn how to create these structures.

`InstSet` is a structure that represents the set of instruments to be priced independently using the model.

### Optional Argument

You can enter a third optional argument, `Options`, used when pricing barrier options. For more specific information, see Appendix B, “Derivatives Pricing Options”.

These pricing functions internally classify the instruments and call the appropriate individual instrument pricing function for each of the instrument types. The CRR pricing functions are `asianbycrr`, `barrierbycrr`, `compoundbycrr`, `lookbackbycrr`, and `optstockbycrr`. A similar set of functions exists for EQP and ITT pricing. You can also use these functions directly to calculate the price of sets of instruments of the same type. See the reference pages for these individual functions for further information.

## Computing Prices Using CRR

Consider the following example, which uses the portfolio and stock price data in the MAT-file `deriv.mat` included in the toolbox. Load the data into the MATLAB workspace.

```
load deriv.mat
```

Use the MATLAB `whos` command to display a list of the variables loaded from the MAT-file.

```
 Name                 Size                Bytes  Class     Attributes

  BDTInstSet          1x1                 15956  struct
  BDTTree             1x1                  5138  struct
  BKInstSet           1x1                 15946  struct
  BKTree              1x1                  5904  struct
  CRRInstSet          1x1                 12434  struct
  CRRTree             1x1                  5058  struct
  EQPInstSet          1x1                 12434  struct
  EQPTree             1x1                  5058  struct
  HJMInstSet          1x1                 15948  struct
  HJMTree             1x1                  5838  struct
  HWInstSet           1x1                 15946  struct
  HWTree              1x1                  5904  struct
  ITTInstSet          1x1                 12438  struct
  ITTTree             1x1                  8862  struct
  ZeroInstSet         1x1                 10282  struct
  ZeroRateSpec        1x1                  1580  struct
```

`CRRTree` and `CRRInstSet` are the required input arguments to call the function `crrprice`.

Use `instdisp` to examine the set of instruments contained in the variable `CRRInstSet`.

```
instdisp(CRRInstSet)
```

```
Index Type     OptSpec Strike Settle         ExerciseDates  AmericanOpt Name  Quantity
1     OptStock call    105    01-Jan-2003    01-Jan-2005    1           Call1 10
2     OptStock put     105    01-Jan-2003    01-Jan-2006    0           Put1   5

Index Type    OptSpec Strike Settle      ExerciseDates AmericanOpt BarrierSpec Barrier Rebate Name  Quantity
3     Barrier call    105    01-Jan-2003 01-Jan-2006   1           ui          102     0      Barrier1 1

Index Type     UOptSpec ....COptSpec CStrike CSettle        CExerciseDates CAmericanOpt Name      Quantity
4     Compound call      ....put      5       01-Jan-2003    01-Jan-2005    1            Compound1 3

Index Type     OptSpec Strike Settle         ExerciseDates  AmericanOpt Name      Quantity
5     Lookback call    115    01-Jan-2003    01-Jan-2006    0           Lookback1 7
6     Lookback call    115    01-Jan-2003    01-Jan-2007    0           Lookback2 9

Index Type  OptSpec Strike Settle      ExerciseDates AmericanOpt AvgType    AvgPrice AvgDate Name   Quantity
7     Asian put     110    01-Jan-2003  01-Jan-2006   0           arithmetic NaN      NaN     Asian1 4
8     Asian put     110    01-Jan-2003  01-Jan-2007   0           arithmetic NaN      NaN     Asian2 6
```

---

**Note** Because of space considerations, the compound option above (`Index 4`) has been condensed to fit the page. The `instdisp` command displays all compound option fields on your computer screen.

---

The instrument set contains eight instruments:

- Two vanilla options (`Call1`, `Put1`)
- One barrier option (`Barrier1`)
- One compound option (`Compound1`)
- Two lookback options (`Lookback1`, `Lookback2`)
- Two Asian options (`Asian1`, `Asian2`)

Each instrument has a corresponding index that identifies the instrument prices in the price vector returned by `crrprice`.

Now use `crrprice` to calculate the price of each instrument in the instrument set.

```
Price = crrprice(CRRTree, CRRInstSet)

Price =
```

```
  8.2863
  2.5016
 12.1272
  3.3241
  7.6015
 11.7772
  4.1797
  3.4219
```

## Computing Prices Using EQP

Load the data into the MATLAB workspace.

```
load deriv.mat
```

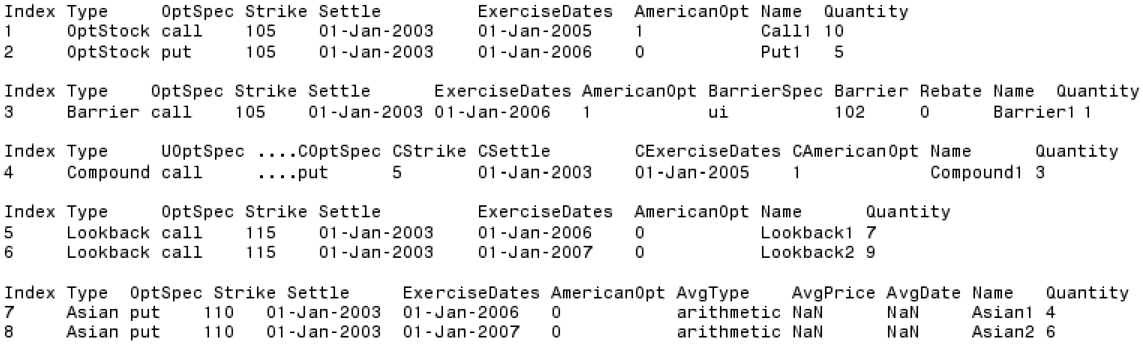
Use the MATLAB `whos` command to display a list of the variables loaded from the MAT-file.

```
 Name                 Size              Bytes  Class     Attributes

  BDTInstSet          1x1               15956  struct
  BDTTree             1x1                5138  struct
  BKInstSet           1x1               15946  struct
  BKTree              1x1                5904  struct
  CRRInstSet          1x1               12434  struct
  CRRTree             1x1                5058  struct
  EQPInstSet          1x1               12434  struct
  EQPTree             1x1                5058  struct
  HJMInstSet          1x1               15948  struct
  HJMTree             1x1                5838  struct
  HWInstSet           1x1               15946  struct
  HWTree              1x1                5904  struct
  ITTInstSet          1x1               12438  struct
  ITTTree             1x1                8862  struct
  ZeroInstSet         1x1               10282  struct
  ZeroRateSpec        1x1                1580  struct
```

`EQPTree` and `EQPInstSet` are the input arguments required to call the function `eqpprice`.

Use the command `instdisp` to examine the set of instruments contained in the variable `EQPInstSet`.

```
instdisp(EQPInstSet)
```



```
Index Type     OptSpec Strike Settle         ExerciseDates  AmericanOpt Name  Quantity
1     OptStock call    105    01-Jan-2003    01-Jan-2005    1           Call1 10
2     OptStock put     105    01-Jan-2003    01-Jan-2006    0           Put1   5

Index Type    OptSpec Strike Settle      ExerciseDates AmericanOpt BarrierSpec Barrier Rebate Name  Quantity
3     Barrier call    105    01-Jan-2003 01-Jan-2006   1           ui          102     0      Barrier1 1

Index Type     UOptSpec ....COptSpec CStrike CSettle        CExerciseDates CAmericanOpt Name      Quantity
4     Compound call     ....put      5       01-Jan-2003    01-Jan-2005    1            Compound1 3

Index Type     OptSpec Strike Settle         ExerciseDates  AmericanOpt Name      Quantity
5     Lookback call    115    01-Jan-2003    01-Jan-2006    0           Lookback1 7
6     Lookback call    115    01-Jan-2003    01-Jan-2007    0           Lookback2 9

Index Type  OptSpec Strike Settle     ExerciseDates AmericanOpt AvgType    AvgPrice AvgDate Name   Quantity
7     Asian put    110   01-Jan-2003  01-Jan-2006   0           arithmetic NaN      NaN     Asian1 4
8     Asian put    110   01-Jan-2003  01-Jan-2007   0           arithmetic NaN      NaN     Asian2 6
```

---

**Note** Because of space considerations, the compound option above (`Index 4`) has been condensed to fit the page. The `instdisp` command displays all compound option fields on your computer screen.

---

The instrument set contains eight instruments:

- Two vanilla options (`Call1`, `Put1`)
- One barrier option (`Barrier1`)
- One compound option (`Compound1`)
- Two lookback options (`Lookback1`, `Lookback2`)
- Two Asian options (`Asian1`, `Asian2`)

Each instrument has a corresponding index that identifies the instrument prices in the price vector returned by `eqpprice`.

Now use `eqpprice` to calculate the price of each instrument in the instrument set.

```
Price = eqpprice(EQPTree, EQPInstSet)

Price =

    8.4791
    2.6375
   12.2632
    3.5091
    8.7941
   12.9577
    4.7444
    3.9178
```

## Computing Prices Using ITT

Consider the following example, which uses the portfolio and stock price data in the MAT-file `deriv.mat` included in the toolbox. Load the data into the MATLAB workspace.

```
load deriv.mat
```

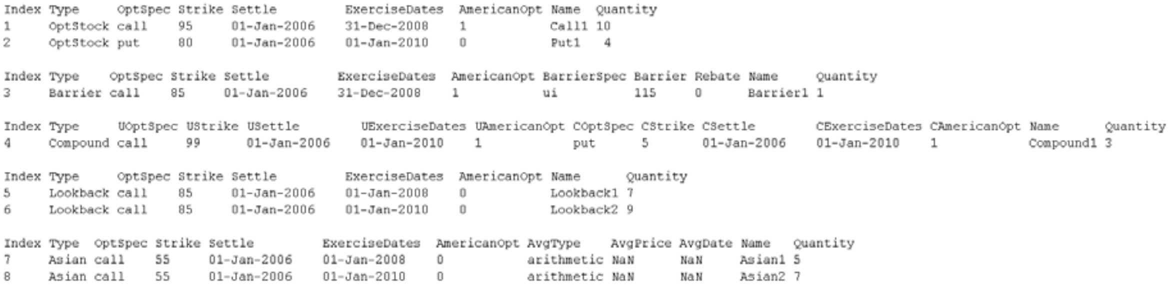
Use the MATLAB `whos` command to display a list of the variables loaded from the MAT-file.

```
  Name                  Size              Bytes  Class     Attributes

  BDTInstSet            1x1               15956  struct
  BDTTree               1x1                5138  struct
  BKInstSet             1x1               15946  struct
  BKTree                1x1                5904  struct
  CRRInstSet            1x1               12434  struct
  CRRTree               1x1                5058  struct
  EQPInstSet            1x1               12434  struct
  EQPTree               1x1                5058  struct
  HJMInstSet            1x1               15948  struct
  HJMTree               1x1                5838  struct
  HWInstSet             1x1               15946  struct
  HWTree                1x1                5904  struct
  ITTInstSet            1x1               12438  struct
  ITTTree               1x1                8812  struct
  ZeroInstSet           1x1               10282  struct
```

```
  ZeroRateSpec      1x1                1580  struct
```

`ITTTree` and `ITTInstSet` are the input arguments required to call the function `ittprice`. Use the command `instdisp` to examine the set of instruments contained in the variable `ITTInstSet`.

```
instdisp(ITTInstSet)
```



```
Index Type     OptSpec Strike Settle        ExerciseDates  AmericanOpt Name  Quantity
1     OptStock call    95     01-Jan-2006   31-Dec-2008    1           Call1 10
2     OptStock put     80     01-Jan-2006   01-Jan-2010    0           Put1   4

Index Type    OptSpec Strike Settle        ExerciseDates  AmericanOpt BarrierSpec Barrier Rebate Name     Quantity
3     Barrier call    85     01-Jan-2006   31-Dec-2008    1           ui          115     0      Barrier1 1

Index Type     UOptSpec UStrike USettle       UExerciseDates UAmericanOpt COptSpec CStrike CSettle       CExerciseDates CAmericanOpt Name      Quantity
4     Compound call     99      01-Jan-2006   01-Jan-2010    1            put      5       01-Jan-2006   01-Jan-2010    1            Compound1 3

Index Type     OptSpec Strike Settle        ExerciseDates  AmericanOpt Name      Quantity
5     Lookback call    85     01-Jan-2006   01-Jan-2008    0           Lookback1 7
6     Lookback call    85     01-Jan-2006   01-Jan-2010    0           Lookback2 9

Index Type  OptSpec Strike Settle        ExerciseDates  AmericanOpt AvgType    AvgPrice AvgDate Name   Quantity
7     Asian call    55     01-Jan-2006   01-Jan-2008    0           arithmetic NaN      NaN     Asian1 5
8     Asian call    55     01-Jan-2006   01-Jan-2010    0           arithmetic NaN      NaN     Asian2 7
```

The instrument set contains eight instruments:

- Two vanilla options (`Call1`, `Put1`)
- One barrier option (`Barrier1`)
- One compound option (`Compound1`)
- Two lookback options (`Lookback1`, `Lookback2`)
- Two Asian options (`Asian1`, `Asian2`)

Each instrument has a corresponding index that identifies the instrument prices in the price vector returned by `ittprice`.

Now use `ittprice` to calculate the price of each instrument in the instrument set.

```
Price = ittprice(ITTTree, ITTInstSet)

Price =

    1.650
```

```
   10.68
    2.407
    3.229
    0.542
    6.184
    3.205
    6.607
```

## Examining Output from the Pricing Functions

The prices in the output vector `Price` correspond to the prices at observation time zero (`tObs = 0`), which is defined as the valuation date of the equity tree. The instrument indexing within `Price` is the same as the indexing within `InstSet`.

In the CRR example, the prices in the `Price` vector correspond to the instruments in this order.

```
InstNames = instget(CRRInstSet, 'FieldName','Name')

InstNames =

Call1
Put1
Barrier1
Compound1
Lookback1
Lookback2
Asian1
Asian2
```

Consequently, in the `Price` vector, the fourth element, `3.3241`, represents the price of the fourth instrument (`Compound1`), and the sixth element, `11.7772`, represents the price of the sixth instrument (`Lookback2`).

In the ITT example, the prices in the `Price` vector correspond to the instruments in this order.

```
InstNames = instget(ITTInstSet, 'FieldName','Name')

InstNames =
```

```
Call1
Put1
Barrier1
Compound1
Lookback1
Lookback2
Asian1
Asian2
```

Consequently, in the `Price` vector, the first element, 1.650, represents the price of the first instrument (`Call1`), and the eight element, 6.607, represents the price of the eighth instrument (`Asian2`).

## Price Tree Output for CRR

If you call a pricing function with two output arguments, for example:

```
[Price, PriceTree] = crrprice(CRRTree, CRRInstSet)
```

you generate a price tree structure along with the price information.

This price tree structure `PriceTree` holds all pricing information.

```
PriceTree =
FinObj: 'BinPriceTree'
PTree: {[8x1 double] [8x2 double] [8x3 double] [8x4 double] [8x5 double]}
tObs: [0 1 2 3 4]
dObs: [731582 731947 732313 732678 733043]
```

The first field of this structure, `FinObj`, indicates that this structure represents a price tree. The second field, `PTree`, is the tree holding the prices of the instruments in each node of the tree. Finally, the third and fourth fields, `tObs` and `dObs`, represent the observation time and date of each level of `PTree`, with `tObs` using units in terms of compounding periods.

Using the command-line interface, you can directly examine `PriceTree.PTree`, the field within the `PriceTree` structure that contains the price tree with the price vectors at every state. The first node represents `tObs = 0`, corresponding to the valuation date.

```
PriceTree.PTree{1}
ans =
8.2863
2.5016
12.1272
3.3241
7.6015
11.7772
4.1797
3.4219
```

With this interface, you can observe the prices for all instruments in the portfolio at a specific time.

The function `eqpprice` also returns a price tree that you can examine in the same way.

## Price Tree Output for ITT

If you call a pricing function with two output arguments, for example:

```
[Price, PriceTree] = ittprice(ITTTree, ITTInstSet)
```

you generate a price tree structure along with the price information.

This price tree structure `PriceTree` holds all pricing information.

```
PriceTree =


   FinObj: 'TrinPriceTree'
    PTree: {[8x1 double]  [8x3 double]  [8x5 double]  [8x7 double]  [8x9 double]}
     tObs: [0 1 2 3 4]
     dObs: [732678 733043 733408 733773 734139]
```

The first field of this structure, `FinObj`, indicates that this structure represents a trinomial price tree. The second field, `PTree` is the tree holding the prices of the instruments in each node of the tree. Finally, the third and fourth fields, `tObs` and `dObs`, represent the observation time and date of each level of `PTree`, with `tObs` using units in terms of compounding periods.

Using the command-line interface, you can directly examine `PriceTree.PTree`, the field within the `PriceTree` structure that contains the price tree with the price vectors at every state. The first node represents `tObs = 0`, corresponding to the valuation date.

```
PriceTree.PTree{1}

    1.6506
   10.6832
    2.4074
    3.2294
    0.5426
    6.1845
    3.2052
    6.6074
```

With this interface, you can observe the prices for all instruments in the portfolio at a specific time.

## Prices for Lookback and Asian Options for Equity Trees

Lookback options and Asian options are path dependent, and, as such, there are no unique prices for any node except the root node. Consequently, the corresponding values for lookback and Asian options in the price tree are set to `NaN`, the only exception being the root node. This becomes apparent if you examine the prices in the second node (`tobs = 1`) of the CRR price tree:

```
PriceTree.PTree{2}

ans =

   11.9176         0
    0.9508    7.1914
   16.4600    2.6672
    2.5896    5.0000
       NaN       NaN
       NaN       NaN
       NaN       NaN
       NaN       NaN
```

Examining the prices in the second node (`tobs = 1`) of the ITT price tree displays:

```
PriceTree.PTree{2}

   ans =

    3.9022         0         0
    6.3736   13.3743   22.1915
    5.6914         0         0
    2.7663    3.8594    5.0000
       NaN       NaN       NaN
       NaN       NaN       NaN
       NaN       NaN       NaN
       NaN       NaN       NaN
```

## Computing Instrument Sensitivities

Sensitivities can be reported either as dollar price changes or percentage price changes. The delta, gamma, and vega sensitivities that the toolbox computes are dollar sensitivities.

The functions `crrsens`, `eqpsens`, and `ittsens` compute the delta, gamma, and vega sensitivities of instruments using a stock tree. They also optionally return the calculated price for each instrument. The sensitivity functions require the same two input arguments used by the pricing functions (`CRRTree` and `CRRInstSet` for CRR, `EQPTree` and `EQPInstSet` for EQP, and`ITTTree` and `ITTInstSet` for ITT).

As with the instrument pricing functions, the optional input argument `Options` is also allowed. You would include this argument if you want a sensitivity function to generate a price for a barrier option as one of its outputs and want to control the method that the toolbox uses to perform the pricing operation. See Appendix B, “Derivatives Pricing Options” or the `derivset` function for more information.

For path-dependent options (lookback and Asian), delta and gamma are computed by finite differences in calls to `crrprice`, `eqpprice`, and `ittprice`. For the other options (stock option, barrier, and compound), delta and gamma are computed from the CRR, EQP, and ITT trees and the corresponding option price tree. (See Chriss, Neil, *Black-Scholes and Beyond*, pp. 308-312.)

### CRR Sensitivities Example

The calling syntax for the sensitivity function is:

```
[Delta, Gamma, Vega, Price] = crrsens(CRRTree, InstSet, Options)
```

Using the example data in `deriv.mat`, calculate the sensitivity of the instruments.

```
load deriv.mat
[Delta, Gamma, Vega, Price] = crrsens(CRRTree, CRRInstSet);
```

You can conveniently examine the sensitivities and the prices by arranging them into a single matrix.

```
format bank
All = [Delta, Gamma, Vega, Price]

All =

       0.59             0.04         53.45            8.29
      -0.31             0.03         67.00            2.50
       0.69             0.03         67.00           12.13
      -0.12            -0.01        -98.08            3.32
      -0.40        -45926.32         88.18            7.60
      -0.42       -112143.15        119.19           11.78
       0.60         45926.32         49.21            4.18
       0.82        112143.15         41.71            3.42
```

As with the prices, each row of the sensitivity vectors corresponds to the similarly indexed instrument in `CRRInstSet`. To view the per-dollar sensitivities, divide each dollar sensitivity by the corresponding instrument price.

```
All = [Delta ./ Price, Gamma ./ Price, Vega ./ Price, Price]
All =

        0.07           0.00          6.45          8.29
       -0.12           0.01         26.78          2.50
        0.06           0.00          5.53         12.13
       -0.04          -0.00        -29.51          3.32
```

```
      -0.05      -6041.77        11.60         7.60
      -0.04      -9522.02        10.12        11.78
       0.14      10987.98        11.77         4.18
       0.24      32771.92        12.19         3.42
```

### ITT Sensitivities Example

The calling syntax for the sensitivity function is:

```
[Delta, Gamma, Vega, Price] = ittsens(ITTTree, ITTInstSet,
Options)
```

Using the example data in `deriv.mat`, calculate the sensitivity of the instruments.

```
load deriv.mat
warning('off', 'fininst:itttree:Extrapolation');
[Delta, Gamma, Vega, Price] = ittsens(ITTTree, ITTInstSet);
```

You can conveniently examine the sensitivities and the prices by arranging them into a single matrix.

```
format bank
All = [Delta, Gamma, Vega, Price]

All =

           0.24          0.03         19.35          1.65
          -0.43          0.02         49.69         10.68
           0.35          0.04         12.29          2.41
          -0.07          0.00          6.73          3.23
           0.63     142945.66         38.90          0.54
           0.60      22703.21         68.92          6.18
           0.32    -142945.66         18.48          3.21
           0.67     -22703.21         17.75          6.61
```

As with the prices, each row of the sensitivity vectors corresponds to the similarly indexed instrument in `ITTInstSet`.

---

**Note** In this example, the extrapolation warnings are turned off before calculating the sensitivities to avoid displaying many warnings on the Command Window as the sensitivities are calculated.

---

If the extrapolation warnings are turned on

```
warning('on', 'fininst:itttree:Extrapolation');
```

and `ittsens` is rerun, the extrapolation warnings scroll as the command executes:

```
[Delta, Gamma, Vega, Price] = ittsens(ITTTree, ITTInstSet)

Warning: The option set specified in StockOptSpec was too narrow for the generated tree.
This makes extrapolation necessary. The list of options outside of the
range of those specified in StockOptSpec are:

Option Type: 'call'   Maturity: 01-Jan-2007  Strike=66.3529
Option Type: 'put'   Maturity: 01-Jan-2007  Strike=50.0061
Option Type: 'put'   Maturity: 01-Jan-2008  Strike=50.0061
Option Type: 'put'   Maturity: 31-Dec-2008  Strike=50.0061
Option Type: 'call'   Maturity: 01-Jan-2010  Strike=155.0141
Option Type: 'put'   Maturity: 01-Jan-2010  Strike=50.006
> In itttree>InterpOptPrices at 675
  In itttree at 277
  In stocktreesens>stocktreedeltagamma_PD at 127
  In stocktreesens at 83
  In ittsens at 81

Warning: The option set specified in StockOptSpec was too narrow for the generated tree.
This made extrapolation necessary. Below is a list of the options that were outside of the
range of those specified in StockOptSpec.

Option Type: 'call'   Maturity: 01-Jan-2007  Strike=66.3367
Option Type: 'put'   Maturity: 01-Jan-2007  Strike=37.6773
Option Type: 'call'   Maturity: 01-Jan-2008  Strike=66.3367
Option Type: 'put'   Maturity: 01-Jan-2008  Strike=28.3951
Option Type: 'call'   Maturity: 31-Dec-2008  Strike=66.3367
Option Type: 'call'   Maturity: 01-Jan-2010  Strike=66.3367
```

```
Option Type: 'put'   Maturity: 01-Jan-2010  Strike=16.1276

> In itttree>InterpOptPrices at 675
  In itttree at 277
  In stocktreesens>stocktreedeltagamma_PD at 131
  In stocktreesens at 83
  In ittsens at 81

Warning: The option set specified in StockOptSpec was too narrow for the generated tree.
This made extrapolation necessary. Below is a list of the options that were outside of the
range of those specified in StockOptSpec.

Option Type: 'call'   Maturity: 01-Jan-2007  Strike=67.2897
Option Type: 'put'   Maturity: 01-Jan-2007  Strike=37.1528
Option Type: 'put'   Maturity: 01-Jan-2008  Strike=27.6066
Option Type: 'put'   Maturity: 31-Dec-2008  Strike=20.5132
Option Type: 'call'   Maturity: 01-Jan-2010  Strike=164.0157
Option Type: 'put'   Maturity: 01-Jan-2010  Strike=15.2424

> In itttree>InterpOptPrices at 675
  In itttree at 277
  In stocktreesens>stocktreevega at 191
  In stocktreesens at 92
  In ittsens at 81
```

These warnings are a consequence of having to extrapolate to find the option price of the tree nodes. In this example, the set of inputs options was too narrow for the shift in the tree nodes introduced by the disturbance used to calculate the sensitivities. As a consequence extrapolation for some of the nodes was needed. Since the input data is quite close the extrapolated data, the error introduced by extrapolation is fairly low.

## Graphical Representation of Equity Derivative Trees

You can use the function `treeviewer` to display a graphical representation of a tree, allowing you to examine interactively the prices and rates on the nodes of the tree until maturity. The graphical representations of CRR, EQP, and LR trees are equivalent to Black-Derman-Toy (BDT) trees, given that they are all binary recombining trees. The graphical representations of ITT trees are equivalent to Hull-White (HW) trees, given that they are all trinomial recombining trees. See “Graphical Representation of Trees” on page 2-91 for an overview on the use of `treeviewer` with CRR trees, EQP trees, LR trees, and ITT trees and their corresponding option price trees. Follow the instructions for BDT trees.

# Equity Derivatives Using Closed-Form Solutions

**In this section...**

## Introduction

Financial Instruments Toolbox software supports four types of closed-form solutions and analytical approximations to calculate price and sensitivities (greeks) of vanilla options:

- Black-Scholes model
- Black model
- Roll-Geske-Whaley model
- Bjerksund-Stensland 2002 model

## Black-Scholes Model

The Black-Scholes model is one of the most commonly used models to price European calls and puts. It serves as a basis for many closed-form solutions used for pricing options. The standard Black-Scholes model is based on the following assumptions:

- There are no dividends paid during the life of the option.
- The option can only be exercised at maturity.

- The markets operate under a Markov process in continuous time.
- No commissions are paid.
- The risk-free interest rate is known and constant.
- Returns on the underlying stocks are log-normally distributed.

---

**Note** The Black-Scholes model implemented in Financial Instruments Toolbox software allows dividends. The following three dividend methods are supported:

- Cash dividend
- Continuous dividend yield
- Constant dividend yield

However, not all Black-Scholes closed-form pricing functions support all three dividend methods. For more information on specifying the dividend methods, see `stockspec`.

---

Closed-form solutions based on a Black-Scholes model support the following tasks.

| Task | Function |
|---|---|
| Price European options with different dividends using the Black-Scholes option pricing model. | `optstockbybls` |
| Calculate European option prices and sensitivities using the Black-Scholes option pricing model. | `optstocksensbybls` |
| Calculate implied volatility on European options using the Black-Scholes option pricing model. | `impvbybls` |
| Price European simple chooser options using Black-Scholes model. | `chooserbybls` |

For an example using the Black-Scholes model, see "Pricing Using the Black-Scholes Model" on page 3-55.

## Black Model

Use the Black model for pricing European options on physical commodities, forwards or futures. The Black model supported by Financial Instruments Toolbox software is a special case of the Black-Scholes model. The Black model uses a forward price as an underlier in place of a spot price. The assumption is that the forward price at maturity of the option is log-normally distributed.

Closed-form solutions for a Black model support the following tasks.

| **Task** | **Function** |
|---|---|
| Price European options on futures using the Black option pricing model. | `optstockbyblk` |
| Calculate European option prices and sensitivities on futures using the Black option pricing model. | `optstocksensbyblk` |
| Calculate implied volatility for European options using the Black option pricing model. | `impvbyblk` |

For an example using the Black model, see "Pricing Using the Black Model" on page 3-57.

## Roll-Geske-Whaley Model

Use the Roll-Geske-Whaley approximation method to price American call options paying a single cash dividend. This model is based on the modification of the observed stock price for the present value of the dividend and also supports a compound option to account for the possibility of early exercise. The Roll-Geske-Whaley model has drawbacks due to an escrowed dividend price approach which may lead to arbitrage. For further explanation, see *Options, Futures, and Other Derivatives* by John Hull.

Closed-form solutions for a Roll-Geske-Whaley model support the following tasks.

| Task | Function |
|---|---|
| Price American call options with a single cash dividend using the Roll-Geske-Whaley option pricing model. | `optstockbyrgw` |
| Calculate American call prices and sensitivities using the Roll-Geske-Whaley option pricing model. | `optstocksensbyrgw` |
| Calculate implied volatility for American call options using the Roll-Geske-Whaley option pricing model. | `impvbyrgw` |

For an example using the Roll-Geske-Whaley model, see "Pricing Using the Roll-Geske-Whaley Model" on page 3-58.

## Bjerksund-Stensland 2002 Model

Use the Bjerksund-Stensland 2002 model for pricing American puts and calls with continuous dividend yield. This model works by dividing the time to maturity of the option in two separate parts, each with its own flat exercise boundary (trigger price). The Bjerksund-Stensland 2002 method is a generalization of the Bjerksund and Stensland 1993 method and is considered to be computationally efficient . For further explanation, see *Closed Form Valuation of American Options* by Bjerksund and Stensland.

Closed-form solutions for a Bjerksund-Stensland 2002 model support the following tasks.

| Task | Function |
|---|---|
| Price American options with continuous dividend yield using the | `optstockbybjs` |

| Task | Function |
|---|---|
| Bjerksund-Stensland 2002 option pricing model. | |
| Calculate American options prices and sensitivities using the Bjerksund-Stensland 2002 option pricing model. | `optstocksensbybjs` |
| Calculate implied volatility for American options using the Bjerksund-Stensland 2002 option pricing model. | `impvbybjs` |

For an example using the Bjerksund-Stensland 2002 model, see "Pricing Using the Bjerksund-Stensland Model" on page 3-59.

## Pricing Using the Black-Scholes Model

Consider a European stock option with an exercise price of $40 on January 1, 2008 that expires on July 1, 2008. Assume the underlying stock pays dividends of $0.50 on March 1 and June 1. The stock is trading at $40 and has a volatility of 30% per annum. The risk-free rate is 4% per annum. Using this data, calculate the price of a call and a put option on the stock using the Black-Scholes option pricing model:

```
Strike = 40;
AssetPrice = 40;
Sigma = .3;
Rates = 0.04;
Settle = 'Jan-01-08';
Maturity = 'Jul-01-08';

Div1 = 'March-01-2008';
Div2 = 'Jun-01-2008';
```

Create `RateSpec` and `StockSpec`:

```
RateSpec = intenvset('ValuationDate', Settle, 'StartDates', Settle, 'EndDates',...
Maturity, 'Rates', Rates, 'Compounding', -1);

StockSpec = stockspec(Sigma, AssetPrice, {'cash'}, 0.50,{Div1,Div2});
```

Define two options, one call and one put:

```
OptSpec = {'call'; 'put'};
```

Calculate the price of the European options:

```
Price = optstockbybls(RateSpec, StockSpec, Settle, Maturity, OptSpec, Strike)

Price =

    3.2063
    3.4027
```

The first element of the `Price` vector represents the price of the call ($3.21); the second is the price of the put ($3.40). Use the function `optstocksensbybls` to compute six sensitivities for the Black-Scholes model: `delta`, `gamma`, `vega`, `lambda`, `rho`, and `theta` and the `price` of the option.

The selection of output parameters and their order is determined by the optional input parameter `OutSpec`. This parameter is a cell array of strings, each one specifying a desired output parameter. The order in which these output parameters are returned by the function is the same as the order of the strings contained in `OutSpec`.

As an example, consider the same options as the previous example. To calculate their `Delta`, `Rho`, `Price`, and `Gamma`, build the cell array `OutSpec` as follows:

```
OutSpec = {'delta', 'rho', 'price', 'gamma'};

[Delta, Rho, Price, Gamma] =optstocksensbybls(RateSpec, StockSpec, Settle,...
Maturity, OptSpec, Strike, 'OutSpec', OutSpec)

Delta =

    0.5328
   -0.4672


Rho =
```

```
    8.7902
  -10.8138


Price =

    3.2063
    3.4027


Gamma =

    0.0480
    0.0480
```

## Pricing Using the Black Model

Consider two European call options on a futures contract with exercise prices of $20 and $25 that expire on September 1, 2008. Assume that on May 1, 2008 the contract is trading at $20 and has a volatility of 35% per annum. The risk-free rate is 4% per annum. Using this data, calculate the price of the call futures options using the Black model:

```
Strike = [20; 25];
AssetPrice = 20;
Sigma = .35;
Rates = 0.04;
Settle = 'May-01-08';
Maturity = 'Sep-01-08';
```

Create `RateSpec` and `StockSpec`:

```
RateSpec = intenvset('ValuationDate', Settle, 'StartDates', Settle,...
'EndDates', Maturity, 'Rates', Rates, 'Compounding', -1);

StockSpec = stockspec(Sigma, AssetPrice);
```

Define the call option:

```
OptSpec = {'call'};
```

Calculate price and all sensitivities of the European futures options:

```
OutSpec = {'All'}

[Delta, Gamma, Vega, Lambda, Rho, Theta, Price] = optstocksensbyblk(RateSpec,...
StockSpec, Settle, Maturity, OptSpec, Strike, 'OutSpec', OutSpec);

Price =

    1.5903
    0.3037
```

The first element of the `Price` vector represents the price of the call with an exercise price of $20 ($1.59); the second is the price of the call with an exercise price of $25 ($2.89).

The function `impvbyblk` is used to compute the implied volatility using the Black option pricing model. Assuming that the previous European call futures are trading at $1.5903 and $0.3037, you can calculate their implied volatility:

```
Volatility = impvbyblk(RateSpec, StockSpec, Settle, Maturity,...
OptSpec,  Strike, Price);
```

As expected, you get volatilities of 35%. If the call futures were trading at $1.50 and $0.50 in the market, the implied volatility would be 33% and 42%:

```
Volatility = impvbyblk(RateSpec, StockSpec, Settle, Maturity,...
OptSpec,  Strike, [1.50;0.5])

Volatility =

    0.3301
    0.4148
```

## Pricing Using the Roll-Geske-Whaley Model

Consider two American call options, with exercise prices of $110 and $100 on June 1, 2008, that expire on June 1, 2009. Assume the underlying stock pays dividends of $0.001 on December 1, 2008. The stock is trading at $80 and has a volatility of 20% per annum. The risk-free rate is 6% per annum. Using this data, calculate the price of the American calls using the Roll-Geske-Whaley option pricing model:

```
AssetPrice = 80;
Settle = 'Jun-01-2008';
Maturity = 'Jun-01-2009';
Strike = [110; 100];

Rate = 0.06;
Sigma  = 0.2;

DivAmount = 0.001;
DivDate = 'Dec-01-2008';
```

Create `RateSpec` and `StockSpec`:

```
StockSpec = stockspec(Sigma, AssetPrice, {'cash'}, DivAmount, DivDate);

RateSpec = intenvset('ValuationDate', Settle, 'StartDates', Settle,...
'EndDates', Maturity, 'Rates', Rate, 'Compounding', -1);
```

Calculate the call prices:

```
Price  = optstockbyrgw(RateSpec, StockSpec, Settle, Maturity, Strike)

Price =

    0.8398
    2.0236
```

The first element of the `Price` vector represents the price of the call with an exercise price of $110 ($0.84); the second is the price of the call with an exercise price of $100 ($2.02).

## Pricing Using the Bjerksund-Stensland Model

Consider four American stock options (two calls and two puts) with an exercise price of $100 that expire on July 1, 2008. Assume the underlying stock pays a continuous dividend yield of 4% as of January 1, 2008. The stock has a volatility of 20% per annum and the risk-free rate is 8% per annum. Using this data, calculate the price of the American calls and puts assuming the following current prices of the stock: \$80, \$90 (for the calls) and \$100 and \$110 (for the puts):

```
Settle = 'Jan-1-2008';
Maturity = 'Jul-1-2008';
Strike = 100;
AssetPrice = [80; 90; 100; 110];
DivYield = 0.04;

Rate = 0.08;
Sigma = 0.20;
```

Create `RateSpec` and `StockSpec`:

```
StockSpec = stockspec(Sigma, AssetPrice, {'continuous'}, DivYield);

RateSpec = intenvset('ValuationDate', Settle, 'StartDates', Settle,...
'EndDates', Maturity, 'Rates', Rate, 'Compounding', -1);
```

Define the option type:

```
OptSpec = {'call'; 'call'; 'put'; 'put'};
```

Compute the option prices:

```
Price = optstockbybjs(RateSpec, StockSpec, Settle, Maturity, OptSpec, Strike)

Price =

    0.4144
    2.1804
    4.7253
    1.7164
```

The first two elements of the `Price` vector represent the price of the calls (\$0.41 and \$2.18), the last two elements represent the price of the put options (\$4.72 and \$1.72). Use the function `optstocksensbybjs` to compute six sensitivities for the Bjerksund-Stensland model: `delta`, `gamma`, `vega`, `lambda`, `rho`, and `theta` and the `price` of the option. The selection of output parameters and their order is determined by the optional input parameter `OutSpec`. This parameter is a cell array of strings, each one specifying a desired output parameter. The order in which these output parameters are returned by the function is the same as the order of the strings contained in `OutSpec`. As an example, consider the same options as the previous example. To calculate their `delta`, `gamma`, and `price`, build the cell array `OutSpec` as follows:

```
OutSpec = {'delta', 'gamma', 'price'};
```

The outputs of `optstocksensbybjs` will be in the same order as in `OutSpec`.

```
[Delta ,Gamma, Price]= optstocksensbybjs(RateSpec, StockSpec, Settle,...
Maturity, OptSpec, Strike, 'OutSpec', OutSpec)

Delta =

    0.0843
    0.2912
    0.4803
    0.2261

Gamma =

    0.0136
    0.0267
    0.0304
    0.0217

Price =

    0.4144
    2.1804
    4.7253
    1.7164
```

For more information on the Bjerksund-Stensland model, see “Closed-Form Solutions Modeling” on page C-9.

# Pricing European Call Options Using Different Equity Models

This example illustrates how the Financial Instruments Toolbox™ can be used to price European vanilla call options using different equity models.

The example compares call option prices using the Cox-Ross-Rubinstein model, the Leisen-Reimer model and the Black-Scholes closed formula.

### Define the Call Instrument

Consider a European call option, with an exercise price of $30 on January 1, 2010. The option expires on Sep 1, 2010. Assume that the underlying stock provides no dividends. The stock is trading at $25 and has a volatility of 35% per annum. The annualized continuously compounded risk-free rate is 1.11% per annum.

```
% Option
Settle = 'Jan-01-2010';
Maturity = 'Sep-01-2010';
Strike = 30;
OptSpec ='call';

% Stock
AssetPrice = 25;
Sigma = .35;
```

### Create the Interest Rate Term Structure

```
StartDates = '01 Jan 2010';
EndDates =   '01 Jan 2013';
Rates = 0.0111;
ValuationDate ='01 Jan 2010';
Compounding = -1;

RateSpec = intenvset('Compounding',Compounding,'StartDates', StartDates,...
                     'EndDates', EndDates, 'Rates', Rates,'ValuationDate',
```

### Create the Stock Structure

Suppose we wish to create two scenarios. The first one assumes that AssetPrice is currently $25, the option is out of the money (OTM). The second scenario assumes that the option is at the money (ATM), and therefore AssetPriceATM = 30.

```
AssetPriceATM = 30;

StockSpec = stockspec(Sigma, AssetPrice);
StockSpecATM = stockspec(Sigma, AssetPriceATM);
```

### Price the Options Using the Black-Scholes Closed Formula

Use the function 'optstockbybls' in the Financial Instruments Toolbox to compute the price of the European call options.

```
% Price the option with AssetPrice = 25
PriceBLS = optstockbybls(RateSpec, StockSpec, Settle, Maturity, OptSpec, St

% Price the option with AssetPrice = 30
PriceBLSATM = optstockbybls(RateSpec, StockSpecATM, Settle, Maturity, OptSp
```

### Build the Cox-Ross-Rubinstein Tree

```
% Create the time specification of the tree
NumPeriods = 15;

CRRTimeSpec = crrtimespec(ValuationDate, Maturity, NumPeriods);

% Build the tree
CRRTree = crrtree(StockSpec, RateSpec, CRRTimeSpec);
CRRTreeATM = crrtree(StockSpecATM, RateSpec, CRRTimeSpec);
```

### Build the Leisen-Reimer Tree

```
% Create the time specification of the tree
LRTimeSpec = lrtimespec(ValuationDate, Maturity, NumPeriods);

% Use the default method 'PP1' (Peizer-Pratt method 1 inversion)to build
% the tree
LRTree = lrtree(StockSpec, RateSpec, LRTimeSpec, Strike);
LRTreeATM = lrtree(StockSpecATM, RateSpec, LRTimeSpec, Strike);
```

### Price the Options Using the Cox-Ross-Rubinstein (CRR) Model

```
PriceCRR = optstockbycrr(CRRTree, OptSpec, Strike, Settle, Maturity);
PriceCRRATM = optstockbycrr(CRRTreeATM, OptSpec, Strike, Settle, Maturity);
```

### Price the Options Using the Leisen-Reimer (LR) Model

```
PriceLR = optstockbylr(LRTree, OptSpec, Strike, Settle, Maturity);
PriceLRATM = optstockbylr(LRTreeATM, OptSpec, Strike, Settle, Maturity);
```

### Compare BLS, CRR and LR Results

```
sprintf('PriceBLS: \t%f\nPriceCRR: \t%f\nPriceLR:\t%f\n', PriceBLS, ...
    PriceCRR, PriceLR)

sprintf('\t== ATM ==\nPriceBLS ATM: \t%f\nPriceCRR ATM: \t%f\nPriceLR ATM:\
    PriceCRRATM,   PriceLRATM)
```

```
ans =

PriceBLS:  1.275075
PriceCRR:  1.294979
PriceLR: 1.275838



ans =

 == ATM ==
PriceBLS ATM:  3.497891
PriceCRR ATM:  3.553938
PriceLR ATM: 3.498571
```

### Convergence of CRR and LR Models to a BLS Solution

The following tables compare call option prices using the CRR and LR models against the results obtained with the Black-Scholes formula.

While the CRR binomial model and the Black-Scholes model converge as the number of time steps gets large and the length of each step gets small, this convergence, except for at the money options, is anything but smooth or uniform.

The tables below show that the Leisen-Reimer model reduces the size of the error with even as few steps of 45.

```
  Strike = 30,    Asset Price = 30
-------------------------------------
  #Steps         LR          CRR
   15          3.4986       3.5539
   25          3.4981       3.5314
   45          3.4980       3.5165
   65          3.4979       3.5108
   85          3.4979       3.5077
   105         3.4979       3.5058
   201         3.4979       3.5020
   501         3.4979       3.4996
   999         3.4979       3.4987

  Strike = 30,    Asset Price = 25
-------------------------------------
  #Steps         LR          CRR
   15          1.2758       1.2950
   25          1.2754       1.2627
   45          1.2751       1.2851
   65          1.2751       1.2692
   85          1.2751       1.2812
   105         1.2751       1.2766
   201         1.2751       1.2723
   501         1.2751       1.2759
   999         1.2751       1.2756
```

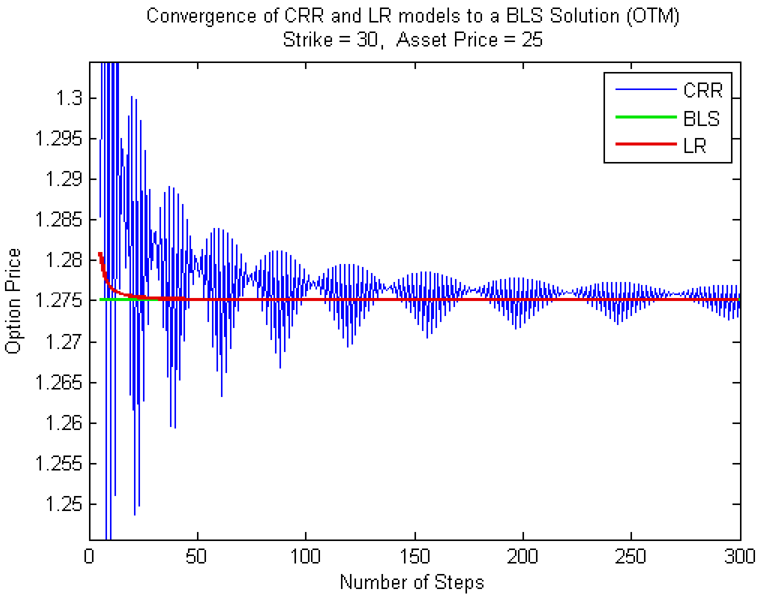
### Analyze the Effect of the Number of Periods on the Price of the Options

The following graphs show how convergence changes as the number of steps in the binomial calculation increases, as well as the impact on convergence to changes to the stock price. Observe that the Leisen-Reimer model removes the oscillation and produces estimates close to the Black-Scholes model using only a small number of steps.

```
NPoints = 300;

% Cox-Ross-Rubinstein
NumPeriodCRR  = 5 : 1 : NPoints;
NbStepCRR     = length(NumPeriodCRR);
PriceCRR = nan(NbStepCRR, 1);
PriceCRRATM = PriceCRR;

for i = 1 : NbStepCRR
    CRRTimeSpec = crrtimespec(ValuationDate, Maturity, NumPeriodCRR(i));
    CRRT = crrtree(StockSpec, RateSpec, CRRTimeSpec);
    PriceCRR(i) = optstockbycrr(CRRT, OptSpec, Strike,ValuationDate, Maturi

    CRRTATM = crrtree(StockSpecATM, RateSpec, CRRTimeSpec);
    PriceCRRATM(i) = optstockbycrr(CRRTATM, OptSpec, Strike,ValuationDate,
end;


% Now with Leisen-Reimer
NumPeriodLR  = 5 : 2 : NPoints;
NbStepLR     = length(NumPeriodLR);
PriceLR = nan(NbStepLR, 1);
PriceLRATM = PriceLR;

for i = 1 : NbStepLR
    LRTimeSpec = lrtimespec(ValuationDate, Maturity, NumPeriodLR(i));
    LRT = lrtree(StockSpec, RateSpec, LRTimeSpec, Strike);
    PriceLR(i) = optstockbylr(LRT, OptSpec, Strike,ValuationDate, Maturity)

    LRTATM = lrtree(StockSpecATM, RateSpec, LRTimeSpec, Strike);
    PriceLRATM(i) = optstockbylr(LRTATM, OptSpec, Strike,ValuationDate, Mat
end;
```

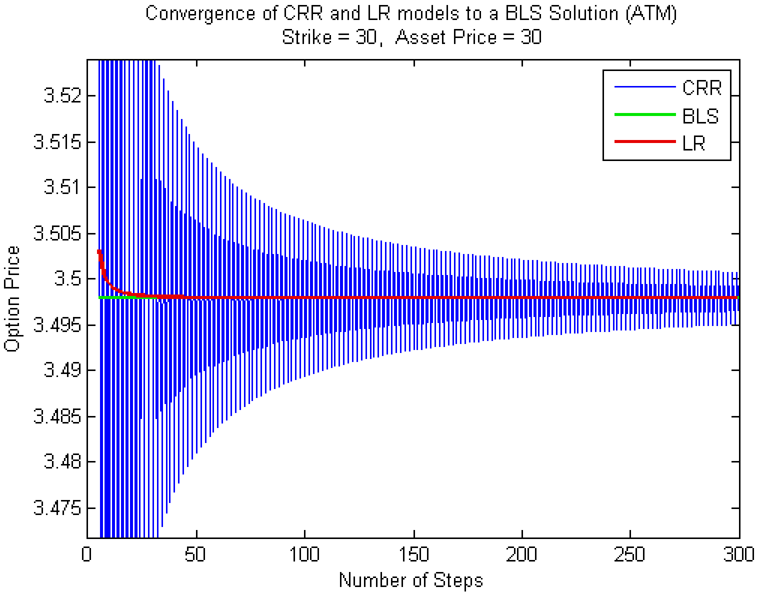
First scenario: Out of the Money call option

```
% For Cox-Ross-Rubinstein
plot(NumPeriodCRR, PriceCRR);hold on;
plot(NumPeriodCRR, PriceBLS*ones(NbStepCRR,1),'Color',[0 0.9 0], 'linewidth

% For Leisen-Reimer
plot(NumPeriodLR, PriceLR, 'Color',[0.9 0 0], 'linewidth', 1.5);

% Concentrate in the area of interest by clipping on the Y axis at 5x the
% LR Price:
YLimDelta = 5*abs(PriceLR(1) - PriceBLS);
set(gca, 'ylim', [PriceBLS-YLimDelta PriceBLS+YLimDelta]);

% Annotate Plot
titleString = sprintf('\nConvergence of CRR and LR models to a BLS Solution
title(titleString)
ylabel('Option Price')
xlabel('Number of Steps')
legend('CRR', 'BLS', 'LR', 'Location', 'NorthEast')
```



Second scenario: At the Money call option

```
% For Cox-Ross-Rubinstein
figure;
plot(NumPeriodCRR, PriceCRRATM);hold on;
plot(NumPeriodCRR, PriceBLSATM*ones(NbStepCRR,1),'Color',[0 0.9 0], 'linewi

% For Leisen-Reimer
plot(NumPeriodLR, PriceLRATM, 'Color',[0.9 0 0], 'linewidth', 1.5);

% Concentrate in the area of interest by clipping on the Y axis at 5x the
% LR Price:
YLimDelta = 5*abs(PriceLRATM(1) - PriceBLSATM);
set(gca, 'ylim', [PriceBLSATM-YLimDelta PriceBLSATM+YLimDelta]);

% Annotate Plot
titleString = sprintf('\nConvergence of CRR and LR models to a BLS Solution
title(titleString)
ylabel('Option Price')
xlabel('Number of Steps')
legend('CRR', 'BLS', 'LR', 'Location', 'NorthEast')
```


Convergence of CRR and LR models to a BLS Solution (ATM)
Strike = 30, Asset Price = 30
CRR
BLS
LR
Option Price
3.52
3.515
3.51
3.505
3.5
3.495
3.49
3.485
3.48
3.475
0
50
100
150
200
250
300
Number of Steps

# Hedging Portfolios

# Hedging

Hedging is an important consideration in modern finance. Whether or not to hedge, how much portfolio insurance is adequate, and how often to rebalance a portfolio are important considerations for traders, portfolio managers, and financial institutions alike.

If there were no transaction costs, financial professionals would prefer to rebalance portfolios continually, thereby minimizing exposure to market movements. However, in practice, the transaction costs associated with frequent portfolio rebalancing may be expensive. Therefore, traders and portfolio managers must carefully assess the cost required to achieve a particular portfolio sensitivity (for example, maintaining delta, gamma, and vega neutrality). Thus, the hedging problem involves the fundamental tradeoff between portfolio insurance and the cost of such insurance coverage.

# Hedging Functions

**In this section...**

## Introduction

Financial Instruments Toolbox software offers two functions for assessing the fundamental hedging tradeoff, `hedgeopt` and `hedgeslf`.

The first function, `hedgeopt`, addresses the most general hedging problem. It allocates an optimal hedge to satisfy either of two goals:

- Minimize the cost of hedging a portfolio given a set of target sensitivities.
- Minimize portfolio sensitivities for a given set of maximum target costs.

`hedgeopt` allows investors to modify portfolio allocations among instruments according to either of the goals. The problem is cast as a constrained linear least-squares problem. For additional information about `hedgeopt`, see "Hedging with hedgeopt" on page 4-4.

The second function, `hedgeslf`, attempts to allocate a self-financing hedge among a portfolio of instruments. In particular, `hedgeslf` attempts to maintain a constant portfolio value consistent with reduced portfolio sensitivities (that is, the rebalanced portfolio is hedged against market moves and is closest to being self-financing). If `hedgeslf` cannot find a self-financing hedge, it rebalances the portfolio to minimize overall portfolio sensitivities. For additional information on `hedgeslf`, see "Self-Financing Hedges with hedgeslf" on page 4-12.

The examples in this section consider the *delta*, *gamma*, and *vega* sensitivity measures. In this toolbox, when you work with *interest-rate derivatives*, delta is the price sensitivity measure of shifts in the forward yield curve, gamma is the delta sensitivity measure of shifts in the forward yield curve, and vega is the price sensitivity measure of shifts in the volatility process. See `bdtsens` or `hjmsens` for details on the computation of sensitivities for interest-rate derivatives.

For *equity exotic options*, the underlying instrument is the stock price instead of the forward yield curve. Consequently, delta now represents the price sensitivity measure of shifts in the stock price, gamma is the delta sensitivity measure of shifts in the stock price, and vega is the price sensitivity measure of shifts in the volatility of the stock. See `crrsens`, `eqpsens`, or `ittsens` for details on the computation of sensitivities for equity derivatives.

For examples showing the computation of sensitivities for interest-rate based derivatives, see "Computing Instrument Sensitivities" on page 2-41. Likewise, for examples showing the computation of sensitivities for equity exotic options, see "Computing Instrument Sensitivities" on page 3-45.

---

**Note** The delta, gamma, and vega sensitivities that the toolbox calculates are dollar sensitivities.

---

## Hedging with hedgeopt

---

**Note** The numerical results in this section are displayed in the MATLAB bank format. Although the calculations are performed in floating-point double precision, only two decimal places are displayed.

---

To illustrate the hedging facility, consider the portfolio `HJMInstSet` obtained from the example file `deriv.mat`. The portfolio consists of eight instruments: two bonds, one bond option, one fixed-rate note, one floating-rate note, one cap, one floor, and one swap.

Both hedging functions require some common inputs, including the current portfolio holdings (allocations), and a matrix of instrument sensitivities. To create these inputs, load the example portfolio into memory

```
load deriv.mat;
```

compute price and sensitivities

```
[Delta, Gamma, Vega, Price] = hjmsens(HJMTree, HJMInstSet);
Warning: Not all cash flows are aligned with the tree. Result will
be approximated.
```

and extract the current portfolio holdings.

```
Holdings = instget(HJMInstSet, 'FieldName', 'Quantity');
```

For convenience place the delta, gamma, and vega sensitivity measures into a matrix of sensitivities.

```
Sensitivities = [Delta Gamma Vega];
```

Each row of the `Sensitivities` matrix is associated with a different instrument in the portfolio, and each column with a different sensitivity measure.

To summarize the portfolio information

```
disp([Price  Holdings  Sensitivities])

 98.72       100.00       -272.65       1029.90        0.00
 97.53        50.00       -347.43       1622.69       -0.04
  0.05       -50.00         -8.08        643.40       34.07
 98.72        80.00       -272.65       1029.90        0.00
100.55         8.00         -1.04          3.31           0
  6.28        30.00        294.97       6852.56       93.69
  0.05        40.00        -47.16       8459.99       93.69
  3.69        10.00       -282.05       1059.68        0.00
```

The first column above is the dollar unit price of each instrument, the second is the holdings of each instrument (the quantity held or the number of contracts), and the third, fourth, and fifth columns are the dollar delta, gamma, and vega sensitivities, respectively.

The current portfolio sensitivities are a weighted average of the instruments in the portfolio.

```
TargetSens  = Holdings' * Sensitivities

TargetSens =
```

```
     -61910.22     788946.21       4852.91
```

## Maintaining Existing Allocations

To illustrate using `hedgeopt`, suppose that you want to maintain your existing portfolio. The first form of `hedgeopt` minimizes the cost of hedging a portfolio given a set of target sensitivities. If you want to maintain your existing portfolio composition and exposure, you should be able to do so without spending any money. To verify this, set the target sensitivities to the current sensitivities.

```
[Sens, Cost, Quantity] = hedgeopt(Sensitivities, Price,...
Holdings, [], [], [], TargetSens)

Sens =

     -61910.22     788946.21       4852.91

Cost =

             0

Quantity' =

        100.00
         50.00
        -50.00
         80.00
          8.00
         30.00
         40.00
         10.00
```

Portfolio composition and sensitivities are unchanged, and the cost associated with doing nothing is zero. The cost is defined as the change in portfolio value. This number cannot be less than zero because the rebalancing cost is defined as a nonnegative number.

If `Value0` and `Value1` represent the portfolio value before and after rebalancing, respectively, the zero cost can also be verified by comparing the portfolio values.

```
Value0 = Holdings' * Price

Value0 =

     23674.62

Value1 = Quantity * Price

Value1 =

     23674.62
```

### Partially Hedged Portfolio

Building on the example in "Maintaining Existing Allocations" on page 4-6, suppose you want to know the cost to achieve an overall portfolio dollar sensitivity of `[-23000 -3300 3000]`, while allowing trading only in instruments 2, 3, and 6 (holding the positions of instruments 1, 4, 5, 7, and 8 fixed). To find the cost, first set the target portfolio dollar sensitivity.

```
TargetSens = [-23000 -3300 3000];
```

Then, specify the instruments to be fixed.

```
FixedInd = [1 4 5 7 8];
```

Finally, call `hedgeopt`

```
[Sens, Cost, Quantity] = hedgeopt(Sensitivities, Price,...
Holdings, FixedInd, [], [], TargetSens);
```

and again examine the results.

```
Sens =

     -23000.00      -3300.00       3000.00
```

```
Cost =

      19174.02

Quantity' =

        100.00
       -141.03
        137.26
         80.00
          8.00
        -57.96
         40.00
         10.00
```

Recompute `Value1`, the portfolio value after rebalancing.

```
Value1 = Quantity * Price

Value1 =

      4500.60
```

As expected, the cost, $19174.02, is the difference between `Value0` and `Value1`, $23674.62 — $4500.60. Only the positions in instruments 2, 3, and 6 have been changed.

## Fully Hedged Portfolio

The example in “Partially Hedged Portfolio” on page 4-7 illustrates a partial hedge, but perhaps the most interesting case involves the cost associated with a fully hedged portfolio (simultaneous delta, gamma, and vega neutrality). In this case, set the target sensitivity to a row vector of 0s and call `hedgeopt` again. The following example uses data from “Hedging with hedgeopt” on page 4-4.

```
TargetSens = [0 0 0];
[Sens, Cost, Quantity] = hedgeopt(Sensitivities, Price, ...
Holdings, FixedInd, [], [], TargetSens);
```

Examining the outputs reveals that you have obtained a fully hedged portfolio

```
Sens =

         -0.00          -0.00          -0.00
```

but at an expense of over $20,000.

```
Cost =

      23055.90
```

The positions required to achieve a fully hedged portfolio

```
Quantity' =

        100.00
       -182.36
        -19.55
         80.00
          8.00
        -32.97
         40.00
         10.00
```

result in the new portfolio value

```
Value1 = Quantity * Price

Value1 =

      618.72
```

## Minimizing Portfolio Sensitivities

The examples in “Fully Hedged Portfolio” on page 4-8 illustrate how to use `hedgeopt` to determine the minimum cost of hedging a portfolio given a set of target sensitivities. In these examples, portfolio target sensitivities are treated as equality constraints during the optimization process. You tell `hedgeopt` what sensitivities you want, and it tells you what it will cost to get those sensitivities.

A related problem involves minimizing portfolio sensitivities for a given set of maximum target costs. For this goal, the target costs are treated as inequality constraints during the optimization process. You tell `hedgeopt` the most you are willing spend to insulate your portfolio, and it tells you the smallest portfolio sensitivities you can get for your money.

To illustrate this use of `hedgeopt`, compute the portfolio dollar sensitivities along the entire cost frontier. From the previous examples, you know that spending nothing replicates the existing portfolio, while spending $23,055.90 completely hedges the portfolio.

Assume, for example, you are willing to spend as much as $50,000, and want to see what portfolio sensitivities will result along the cost frontier. Assume that the same instruments are held fixed, and that the cost frontier is evaluated from $0 to $50,000 at increments of $1000.

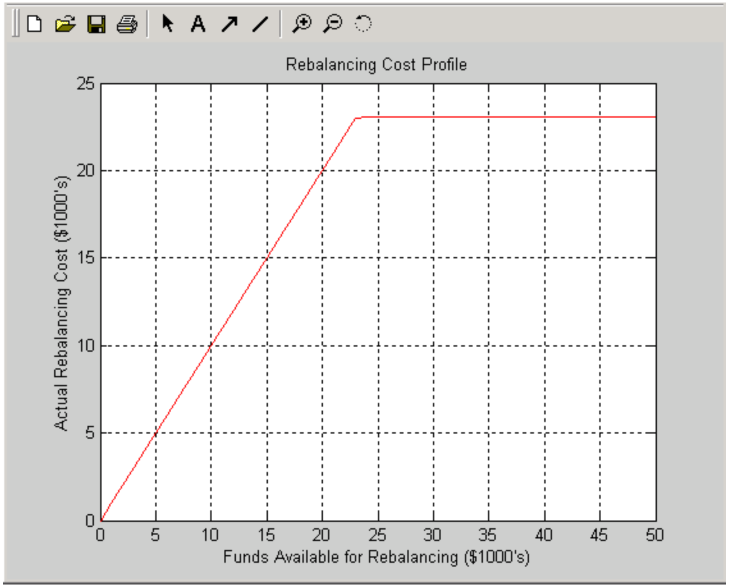
```
MaxCost = [0:1000:50000];
```

Now, call `hedgeopt`.

```
[Sens, Cost, Quantity] = hedgeopt(Sensitivities, Price, ...
Holdings, FixedInd, [], MaxCost);
```

With this data, you can plot the required hedging cost versus the funds available (the amount you are willing to spend)

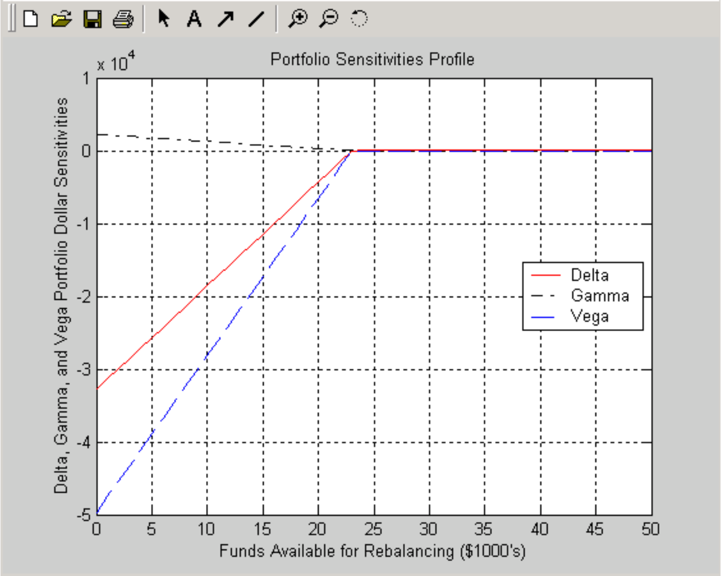
```
plot(MaxCost/1000, Cost/1000, 'red'), grid
xlabel('Funds Available for Rebalancing ($1000''s)')
ylabel('Actual Rebalancing Cost ($1000''s)')
title ('Rebalancing Cost Profile')
```



**Rebalancing Cost Profile**

and the portfolio dollar sensitivities versus the funds available.

```
figure
plot(MaxCost/1000, Sens(:,1), '-red')
hold('on')
plot(MaxCost/1000, Sens(:,2), '-.black')
plot(MaxCost/1000, Sens(:,3), '--blue')
grid
xlabel('Funds Available for Rebalancing ($1000''s)')
ylabel('Delta, Gamma, and Vega Portfolio Dollar Sensitivities')
title ('Portfolio Sensitivities Profile')
legend('Delta', 'Gamma', 'Vega', 0)
```



**Funds Available for Rebalancing**

## Self-Financing Hedges with hedgeslf

The figures Rebalancing Cost Profile on page 4-11 and Funds Available for Rebalancing on page 4-12 indicate that there is no benefit because the funds available for hedging exceed $23,055.90, the point of maximum expense required to obtain simultaneous delta, gamma, and vega neutrality. You can also find this point of delta, gamma, and vega neutrality using `hedgeslf`.

```
[Sens, Value1, Quantity] = hedgeslf(Sensitivities, Price,...
Holdings, FixedInd);

Sens =

         -0.00
         -0.00
         -0.00

Value1 =

      618.72
```

```
Quantity =

        100.00
       -182.36
        -19.55
         80.00
          8.00
        -32.97
         40.00
         10.00
```

Similar to `hedgeopt`, `hedgeslf` returns the portfolio dollar sensitivities and instrument quantities (the rebalanced holdings). However, in contrast, the second output parameter of `hedgeslf` is the value of the rebalanced portfolio, from which you can calculate the rebalancing cost by subtraction.

```
Value0 - Value1

ans =

      23055.90
```

In this example, the portfolio is clearly not self-financing, so `hedgeslf` finds the best possible solution required to obtain zero sensitivities.

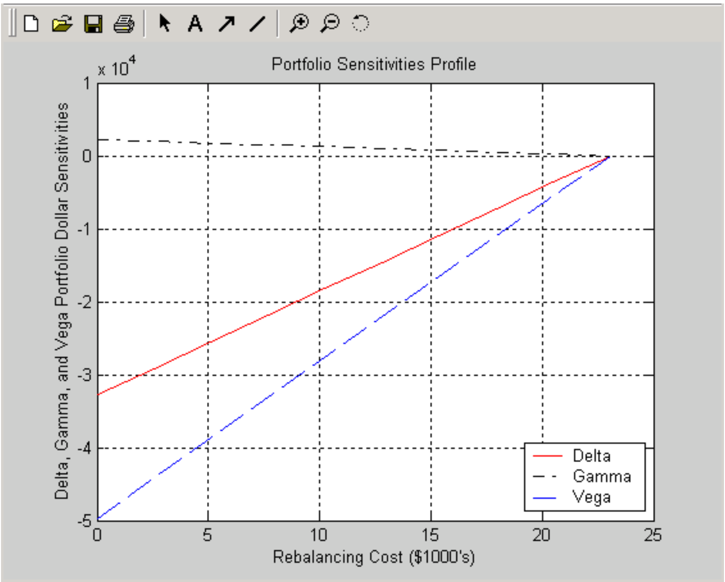
There is, in fact, a third calling syntax available for `hedgeopt` directly related to the results shown above for `hedgeslf`. Suppose, instead of directly specifying the funds available for rebalancing (the most money you are willing to spend), you want to simply specify the number of points along the cost frontier. This call to `hedgeopt` samples the cost frontier at 10 equally spaced points between the point of minimum cost (and potentially maximum exposure) and the point of minimum exposure (and maximum cost).

```
[Sens, Cost, Quantity] = hedgeopt(Sensitivities, Price,...
Holdings, FixedInd, 10);

Sens =
     -32784.46       2231.83     -49694.33
     -29141.74       1983.85     -44172.74
```

```
   -25499.02       1735.87     -38651.14
   -21856.30       1487.89     -33129.55
   -18213.59       1239.91     -27607.96
   -14570.87        991.93     -22086.37
   -10928.15        743.94     -16564.78
    -7285.43        495.96     -11043.18
    -3642.72        247.98      -5521.59
        0.00         -0.00          0.00

Cost =
          0.00
       2561.77
       5123.53
       7685.30
      10247.07
      12808.83
      15370.60
      17932.37
      20494.14
      23055.90
```

Now plot this data.

```
figure
plot(Cost/1000, Sens(:,1), '-red')
hold('on')
plot(Cost/1000, Sens(:,2), '-.black')
plot(Cost/1000, Sens(:,3), '--blue')
grid
xlabel('Rebalancing Cost ($1000''s)')
ylabel('Delta, Gamma, and Vega Portfolio Dollar Sensitivities')
title ('Portfolio Sensitivities Profile')
legend('Delta', 'Gamma', 'Vega', 0)
```



### Rebalancing Cost

In this calling form, `hedgeopt` calls `hedgeslf` internally to determine the maximum cost needed to minimize the portfolio sensitivities ($23,055.90), and evenly samples the cost frontier between $0 and $23,055.90.

Note that both `hedgeopt` and `hedgeslf` cast the optimization problem as a constrained linear least squares problem. Depending on the instruments and constraints, neither function is guaranteed to converge to a solution. In some cases, the problem space may be unbounded, and additional instrument equality constraints, or user-specified constraints, may be necessary for convergence. See "Hedging with Constrained Portfolios" on page 4-21 for additional information.

# Specifying Constraints with ConSet

**In this section...**

## Introduction

Both `hedgeopt` and `hedgeslf` accept an optional input argument, `ConSet`, that allows you to specify a set of linear inequality constraints for instruments in your portfolio. The examples in this section are brief. For additional information regarding portfolio constraint specifications, refer to "Analyzing Portfolios".

## Setting Constraints

For the first example of setting constraints, return to the fully hedged portfolio example that used `hedgeopt` to determine the minimum cost of obtaining simultaneous delta, gamma, and vega neutrality (target sensitivities all 0). Recall that when `hedgeopt` computes the cost of rebalancing a portfolio, the input target sensitivities you specify are treated as equality constraints during the optimization process. The situation is reproduced next for convenience.

```
TargetSens = [0 0 0];
[Sens, Cost, Quantity] = hedgeopt(Sensitivities, Price,...
Holdings, FixedInd, [], [], TargetSens);
```

The outputs provide a fully hedged portfolio

```
Sens =
          -0.00            -0.00            -0.00
```

at an expense of over $23,000.

```
Cost =
      23055.90
```

The positions required to achieve this fully hedged portfolio are

```
Quantity' =

        100.00
       -182.36
        -19.55
         80.00
          8.00
        -32.97
         40.00
         10.00
```

Suppose now that you want to place some upper and lower bounds on the individual instruments in your portfolio. You can specify these constraints, along with a variety of general linear inequality constraints, with Financial Toolbox function `portcons`.

As an example, assume that, in addition to holding instruments 1, 4, 5, 7, and 8 fixed as before, you want to bound the position of all instruments to within +/- 180 contracts (for each instrument, you cannot short or long more than 180 contracts). Applying these constraints disallows the current position in the second instrument (short 182.36). All other instruments are currently within the upper/lower bounds.

You can generate these constraints by first specifying the lower and upper bounds vectors and then calling `portcons`.

```
LowerBounds = [-180 -180 -180 -180 -180 -180 -180 -180];
UpperBounds = [ 180  180   180 180  180  180  180  180];
ConSet = portcons('AssetLims', LowerBounds, UpperBounds);
```

To impose these constraints, call `hedgeopt` with `ConSet` as the last input.

```
[Sens, Cost, Quantity] = hedgeopt(Sensitivities, Price,...
Holdings, FixedInd, [], [], TargetSens, ConSet);
```

Examine the outputs and see that they are all set to `NaN`, indicating that the problem, given the constraints, is not solvable. Intuitively, the results mean that you cannot obtain simultaneous delta, gamma, and vega neutrality with these constraints at any price.

To see how close you can get to portfolio neutrality with these constraints, call `hedgeslf`.

```
[Sens, Value1, Quantity] = hedgeslf(Sensitivities, Price,...
Holdings, FixedInd, ConSet);

Sens =

       -352.43
         21.99
       -498.77

Value1 =

      855.10

Quantity =

        100.00
       -180.00
        -37.22
         80.00
          8.00
        -31.86
         40.00
         10.00
```

`hedgeslf` enforces the lower bound for the second instrument, but the sensitivity is far from neutral. The cost to obtain this portfolio is

```
Value0 - Value1

ans =

      22819.52
```

## Portfolio Rebalancing

As a final example of user-specified constraints, rebalance the portfolio using the second hedging goal of `hedgeopt`. Assume that you are willing to spend as much as $20,000 to rebalance your portfolio, and you want to know what minimum portfolio sensitivities you can get for your money. In this form, recall that the target cost ($20,000) is treated as an inequality constraint during the optimization process.

For reference, startup `hedgeopt` without any user-specified linear inequality constraints.

```
[Sens, Cost, Quantity] = hedgeopt(Sensitivities, Price,...
Holdings, FixedInd, [], 20000);

Sens =

      -4345.36        295.81      -6586.64
Cost =

      20000.00

Quantity' =

        100.00
       -151.86
       -253.47
         80.00
          8.00
        -18.18
         40.00
         10.00
```

This result corresponds to the $20,000 point along the Portfolio Sensitivities Profile shown in the figure Rebalancing Cost on page 4-15.

Assume that, in addition to holding instruments 1, 4, 5, 7, and 8 fixed as before, you want to bound the position of all instruments to within +/- 150 contracts (for each instrument, you cannot short more than 150 contracts and you cannot long more than 150 contracts). These bounds disallow the current position in the second and third instruments (-151.86 and -253.47). All other instruments are currently within the upper/lower bounds.

As before, you can generate these constraints by first specifying the lower and upper bounds vectors and then calling `portcons`.

```
LowerBounds = [-150 -150 -150 -150 -150 -150 -150 -150];
UpperBounds = [ 150  150  150  150  150  150  150  150];
ConSet = portcons('AssetLims', LowerBounds, UpperBounds);
```

To impose these constraints, again call `hedgeopt` with `ConSet` as the last input.

```
[Sens, Cost, Quantity] = hedgeopt(Sensitivities, Price,...
Holdings,FixedInd, [], 20000, [], ConSet);

Sens =

      -8818.47        434.43      -4010.79

Cost =

      19876.89

Quantity' =

        100.00
       -150.00
       -150.00
         80.00
          8.00
        -28.32
         40.00
         10.00
```

With these constraints, `hedgeopt` enforces the lower bound for the second and third instruments. The cost incurred is $19,876.89.

# Hedging with Constrained Portfolios

**In this section...**

## Overview

Both hedging functions cast the optimization as a constrained linear least-squares problem. (See the function `lsqlin` in the Optimization Toolbox documentation for details.) In particular, `lsqlin` attempts to minimize the constrained linear least squares problem

$$\min_x \frac{1}{2}\|Cx-d\|_2^2 \quad \text{such that} \quad \begin{aligned} A\cdot x &\le b \\ Aeq\cdot x &= beq \\ lb \le x &\le ub \end{aligned}$$

where $C$, $A$, and $Aeq$ are matrices, and $d$, $b$, $beq$, $lb$, and $ub$ are vectors. For Financial Instruments Toolbox software, $x$ is a vector of asset holdings (contracts).

Depending on the constraint and the number of assets in the portfolio, a solution to a particular problem may or may not exist. Furthermore, if a solution is found, it may not be unique. For a unique solution to exist, the least squares problem must be sufficiently and appropriately constrained.

## Example: Fully Hedged Portfolio

Recall that `hedgeopt` allows you to allocate an optimal hedge by one of two goals:

- Minimize the cost of hedging a portfolio given a set of target sensitivities.

- Minimize portfolio sensitivities for a given set of maximum target costs.

As an example, reproduce the results for the fully hedged portfolio example.

```
TargetSens = [0 0 0];
FixedInd   = [1 4 5 7 8];
[Sens,Cost,Quantity] = hedgeopt(Sensitivities, Price,...
Holdings, FixedInd, [], [], TargetSens);

Sens =

         -0.00          -0.00          -0.00

Cost =

      23055.90

Quantity' =

         98.72
       -182.36
        -19.55
         80.00
          8.00
        -32.97
         40.00
         10.00
```

This example finds a unique solution at a cost of just over $23,000. The matrix `C` (formed internally by `hedgeopt` and passed to `lsqlin`) is the asset `Price` vector expressed as a row vector.

```
C = Price' = [98.72 97.53 0.05 98.72 100.55 6.28 0.05 3.69]
```

The vector `d` is the current portfolio value `Value0 = 23674.62`. The example maintains, as closely as possible, a constant portfolio value subject to the specified constraints.

### Additional Constraints

In the absence of any additional constraints, the least squares objective involves a single equation with eight unknowns. This is an under-determined system of equations. Because such systems generally have an infinite number of solutions, you need to specify additional constraints to achieve a solution with practical significance.

The additional constraints can come from two sources:

- User-specified equality constraints
- Target sensitivity equality constraints imposed by `hedgeopt`

The example in "Fully Hedged Portfolio" on page 4-8 specifies five equality constraints associated with holding assets 1, 4, 5, 7, and 8 fixed. This reduces the number of unknowns from eight to three, which is still an under-determined system. However, when combined with the first goal of `hedgeopt`, the equality constraints associated with the target sensitivities in `TargetSens` produce an additional system of three equations with three unknowns. This additional system guarantees that the weighted average of the delta, gamma, and vega of assets 2, 3, and 6, together with the remaining assets held fixed, satisfy the overall portfolio target sensitivity needs in `TargetSens`.

Combining the least-squares objective equation with the three portfolio sensitivity equations provides an overall system of four equations with three unknown asset holdings. This is no longer an under-determined system, and the solution is as shown.

If the assets held fixed are reduced, for example, `FixedInd = [1 4 5 7]`, `hedgeopt` returns a no cost, fully hedged portfolio (`Sens = [0 0 0]` and `Cost = 0`).

If you further reduce `FixedInd` (for example, `[1 4 5]`, `[1 4]`, or even `[]`), `hedgeopt` always returns a no cost, fully hedged portfolio. In these cases, insufficient constraints result in an under-determined system. Although `hedgeopt` identifies no cost, fully hedged portfolios, there is nothing unique about them. These portfolios have little practical significance.

Constraints must be *sufficient* and *appropriately defined*. Additional constraints having no effect on the optimization are called *dependent constraints*. As a simple example, assume that parameter $Z$ is constrained such that $Z \le 1$. Furthermore, assume you somehow add another constraint that effectively restricts $Z \le 0$. The constraint $Z \le 1$ now has no effect on the optimization.

## Example: Minimize Portfolio Sensitivities

To illustrate using `hedgeopt` to minimize portfolio sensitivities for a given maximum target cost, specify a target cost of $20,000 and determine the new portfolio sensitivities, holdings, and cost of the rebalanced portfolio.

```
MaxCost = 20000;
[Sens, Cost, Quantity] = hedgeopt(Sensitivities, Price,...
Holdings, [1 4 5 7 8], [], MaxCost);

Sens =

      -4345.36        295.81      -6586.64

Cost =

       20000.00

Quantity' =

         100.00
        -151.86
        -253.47
          80.00
           8.00
         -18.18
          40.00
          10.00
```

This example corresponds to the $20,000 point along the cost axis in the figures Rebalancing Cost Profile on page 4-11, Funds Available for Rebalancing on page 4-12, and Rebalancing Cost on page 4-15.

When minimizing sensitivities, the maximum target cost is treated as an inequality constraint; in this case, `MaxCost` is the most you are willing to spend to hedge a portfolio. The least-squares objective matrix `C` is the matrix transpose of the input asset sensitivities

```
C = Sensitivities'
```

a `3`-by-`8` matrix in this example, and `d` is a `3`-by-`1` column vector of zeros, `[0 0 0]'`.

Without any additional constraints, the least-squares objective results in an under-determined system of three equations with eight unknowns. By holding assets 1, 4, 5, 7, and 8 fixed, you reduce the number of unknowns from eight to three. Now, with a system of three equations with three unknowns, `hedgeopt` finds the solution shown.

## Example: Under-Determined System

Reducing the number of assets held fixed creates an under-determined system with meaningless solutions. For example, see what happens with only four assets constrained.

```
FixedInd = [1 4 5 7];
[Sens, Cost, Quantity] = hedgeopt(Sensitivities, Price,...
Holdings, FixedInd, [], MaxCost);

Sens =

         -0.00          -0.00          -0.00

Cost =

      20000.00

Quantity' =

        100.00
        -149.31
        -14.91
         80.00
```

```
         8.00
       -34.64
        40.00
       -32.60
```

You have spent $20,000 (all the funds available for rebalancing) to achieve a fully hedged portfolio.

With an increase in available funds to $50,000, you still spend all available funds to get another fully hedged portfolio.

```
MaxCost  = 50000;
[Sens, Cost, Quantity] = hedgeopt(Sensitivities, Price,...
Holdings, FixedInd, [],MaxCost);

Sens =

        -0.00          0.00          0.00
Cost =

      50000.00

Quantity' =

       100.00
      -473.78
       -60.51
        80.00
         8.00
       -18.20
        40.00
       385.60
```

All solutions to an under-determined system are meaningless. You buy and sell various assets to obtain zero sensitivities, spending all available funds every time. If you reduce the number of fixed assets any further, this problem is insufficiently constrained, and you find no solution (the outputs are all `NaN`).

Note also that no solution exists whenever constraints are *inconsistent*. Inconsistent constraints create an infeasible solution space; the outputs are all `NaN`.

## Example: Portfolio Constraints with hedgeslf

The other hedging function, `hedgeslf`, attempts to minimize portfolio sensitivities such that the rebalanced portfolio maintains a constant value (the rebalanced portfolio is hedged against market moves and is closest to being self-financing). If a self-financing hedge is not found, `hedgeslf` tries to rebalance a portfolio to minimize sensitivities.

From a least-squares systems approach, `hedgeslf` first attempts to minimize cost in the same way that `hedgeopt` does. If it cannot solve this problem (a no cost, self-financing hedge is not possible), `hedgeslf` proceeds to minimize sensitivities like `hedgeopt`. Thus, the discussion of constraints for `hedgeopt` is directly applicable to `hedgeslf` as well.

To illustrate this hedging facility using equity exotic options, consider the portfolio `CRRInstSet` obtained from the example MAT-file `deriv.mat`. The portfolio consists of eight option instruments: two stock options, one barrier, one compound, two lookback, and two Asian.

The hedging functions require inputs that include the current portfolio holdings (allocations) and a matrix of instrument sensitivities. To create these inputs, start by loading the example portfolio into memory

```
load deriv.mat;
```

Next, compute the prices and sensitivities of the instruments in this portfolio.

```
[Delta, Gamma, Vega, Price] = crrsens(CRRTree, CRRInstSet);
```

Extract the current portfolio holdings (the quantity held or the number of contracts).

```
Holdings = instget(CRRInstSet, 'FieldName', 'Quantity');
```

For convenience place the delta, gamma, and vega sensitivity measures into a matrix of sensitivities.

```
Sensitivities = [Delta Gamma Vega];
```

Each row of the `Sensitivities` matrix is associated with a different instrument in the portfolio and each column with a different sensitivity measure.

```
disp([Price  Holdings  Sensitivities])

         8.29        10.00         0.59         0.04        53.45
         2.50         5.00        -0.31         0.03        67.00
        12.13         1.00         0.69         0.03        67.00
         3.32         3.00        -0.12        -0.01       -98.08
         7.60         7.00        -0.40    -45926.32        88.18
        11.78         9.00        -0.42   -112143.15       119.19
         4.18         4.00         0.60     45926.32        49.21
         3.42         6.00         0.82    112143.15        41.71
```

The first column contains the dollar unit price of each instrument, the second contains the holdings of each instrument, and the third, fourth, and fifth columns contain the delta, gamma, and vega dollar sensitivities, respectively.

Suppose that you want to obtain a delta, gamma and vega neutral portfolio using `hedgeslf`.

```
[Sens, Value1, Quantity]= hedgeslf(Sensitivities, Price, ...
Holdings)

Sens =

          0.00
         -0.00
          0.00

Value1 =

        313.93

Quantity =

         10.00
```

```
         7.64
        -1.56
        26.13
         9.94
         3.73
        -0.75
         8.11
```

`hedgeslf` returns the portfolio dollar sensitivities (`Sens`), the value of the rebalanced portfolio (`Value1`) and the new allocation for each instrument (`Quantity`).

If `Value0` and `Value1` represent the portfolio value before and after rebalancing, respectively, you can verify the cost by comparing the portfolio values.

```
Value0= Holdings' * Price

Value0 =

       313.93
```

In this example, the portfolio is fully hedged (simultaneous delta, gamma, and vega neutrality) and self-financing (the values of the portfolio before and after balancing (`Value0` and `Value1`) are the same.

Suppose now that you want to place some upper and lower bounds on the individual instruments in your portfolio. By using Financial Toolbox function `portcons`, you can specify these constraints, along with a variety of general linear inequality constraints.

As an example, assume that, in addition to holding instrument 1 fixed as before, you want to bound the position of all instruments to within +/- 20 contracts (for each instrument, you cannot short or long more than 20 contracts). Applying these constraints disallows the current position in the fourth instrument (long 26.13). All other instruments are currently within the upper/lower bounds.

You can generate these constraints by first specifying the lower and upper bounds vectors and then calling `portcons`.

```
LowerBounds = [-20  -20  -20  -20  -20  -20  -20  -20];
UpperBounds = [20  20  20  20  20  20  20  20];
ConSet = portcons('AssetLims', LowerBounds, UpperBounds);
```

To impose these constraints, call `hedgeslf` with `ConSet` as the last input.

```
[Sens, Cost, Quantity1] = hedgeslf(Sensitivities, Price, ...
Holdings, 1, ConSet)

Sens =

         -0.00
          0.00
          0.00

Cost =

        313.93

Quantity1 =

         10.00
          5.28
         10.98
         20.00
         20.00
         -6.99
        -20.00
          9.39
```

Observe that `hedgeslf` enforces the upper bound on the fourth instrument, and the portfolio continues to be fully hedged and self-financing.

# 5

# Mortgage-Backed Securities

# What Are Mortgage-Backed Securities?

Mortgage-backed securities (MBSs) are a type of investment that represents ownership in a group of mortgages. Principal and interest from the individual mortgages are used to pay principal and interest on the MBS.

Ownership in a group of mortgages is typically represented by a *pass-through certificate* (PC). Most pass-through certificates are issued by the Government National Mortgage Agency, a branch of the United States government, or by one of two private corporations: Fannie Mae or Freddie Mac. With these certificates, homeowners' payments pass from the originating bank through the issuing agency to holders of the certificates. These agencies also frequently guarantee that the certificate holder receives timely payment of principal and interest from the PCs.

# Fixed-Rate Mortgage Pool

**In this section...**

## Introduction

Financial Instruments Toolbox software supports calculations involved with generic fixed-rate mortgage pools and balloon mortgages. Pass-through certificates typically have embedded call options in the form of prepayment. Prepayment is an excess payment applied to the principal of a PC. These accelerated payments reduce the effective life of a PC.

The toolbox comes with a standard Bond Market Association (PSA) prepayment model and can generate multiples of standard prepayment speeds. The Public Securities Association provides a set of uniform practices for calculating the characteristics of mortgage-backed securities when there is an assumed prepayment function.

Alternatively, aside from the standard PSA implementation in this toolbox, you can supply your own projected prepayment vectors. At this time, however, custom prepayment functionality that incorporates pool-specific information and interest rate forecasts are not available in this toolbox. If you plan to use custom prepayment vectors in your calculations, you presumably already own such a suite in MATLAB.

## Inputs to Functions

Because of the generic, all-purpose nature of the toolbox pass-through functions, you can fine-tune them to conform to a particular mortgage. Most functions require at least this set of inputs:

- Gross coupon rate
- Settlement date
- Issue (effective) date
- Maturity date

Typical optional inputs include standard prepayment speed (or customized vector), net coupon rate (if different from gross coupon rate), and payment delay in number of days.

All calculations are based on expected payment dates and actual cash flow to the investor. For example, when `GrossRate` and `CouponRate` differ as inputs to `mbsdurp`, the function returns a modified duration based on `CouponRate`. (A notable exception is `mbspassthrough`, which returns interest quantities based on the `GrossRate`.)

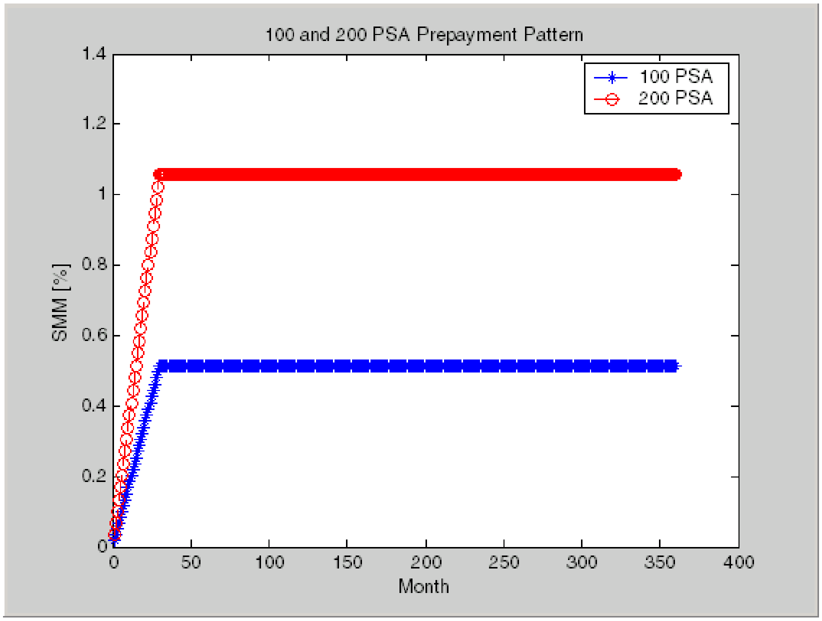
## Generating Prepayment Vectors

You can generate PSA multiple prepayment vectors quickly. To generate prepayment vectors of 100 and 200 PSA, type

```
PSASpeed = [100, 200];
[CPR, SMM] = psaspeed2rate(PSASpeed);
```

This function computes two prepayment values: conditional prepayment rate (CPR) and single monthly mortality (SMM) rate. CPR is the percentage of outstanding principal prepaid in 1 year. SMM is the percentage of outstanding principal prepaid in 1 month. In other words, CPR is an annual version of SMM.

Since the entire 360-by-2 array is too long to show in this document, observe the SMM (100 and 200 PSA) plots, spaced one month apart, instead.



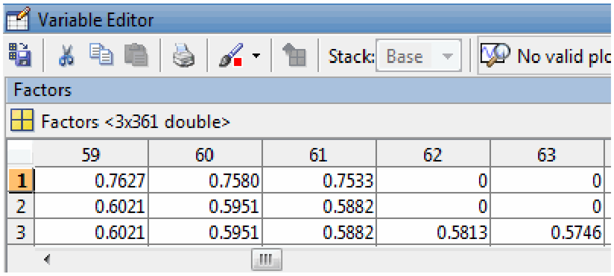
Prepayment assumptions form the basis upon which far more comprehensive MBS calculations are based. As an illustration observe the following example, which shows the use of the function `mbscfamounts` to generate cash flows and timings based on a set of standard prepayments.

Consider three mortgage pools that were sold on the issue date (which starts unamortized). The first two pools "balloon out" in 60 months, and the third is regularly amortized to the end. The prepayment speeds are assumed to be 100, 200, and 200 PSA, respectively.

```
Settle     = [datenum('1-Feb-2000');
              datenum('1-Feb-2000');
              datenum('1-Feb-2000')];

Maturity   = [datenum('1-Feb-2030')];

IssueDate  = datenum('1-Feb-2000');
GrossRate  = 0.08125;
CouponRate = 0.075;
Delay = 14;
```

```
PSASpeed = [100, 200];
[CPR, SMM] = psaspeed2rate(PSASpeed);

PrepayMatrix = ones(360,3);
PrepayMatrix(1:60,1:2) = SMM(1:60,1:2);
PrepayMatrix(:,3) = SMM(:,2);

[CFlowAmounts, CFlowDates, TFactors, Factors] = ...
mbscfamounts(Settle, Maturity, IssueDate, GrossRate, ...
CouponRate, Delay, [], PrepayMatrix);
```

The fourth output argument, `Factors`, indicates the fraction of the balance still outstanding at the beginning of each month. A snapshot of this argument in the MATLAB Variables editor illustrates the 60-month life of the first two of the mortgages with balloon payments and the continuation of the third mortgage until the end (360 months).



| | 59 | 60 | 61 | 62 | 63 |
|---|---|---|---|---|---|
| 1 | 0.7627 | 0.7580 | 0.7533 | 0 | 0 |
| 2 | 0.6021 | 0.5951 | 0.5882 | 0 | 0 |
| 3 | 0.6021 | 0.5951 | 0.5882 | 0.5813 | 0.5746 |

You can readily see that `mbscfamounts` is the building block of most fixed rate and balloon pool cash flows.

## Mortgage Prepayments

Prepayment is beneficial to the pass-through owner when a mortgage pool has been purchased at discount. The next example compares mortgage yields (compounded monthly) versus the purchase clean price with constant prepayment speed. The example illustrates that when you have purchased a pool at a discount, prepayment generates a higher yield with decreasing purchase price.

```
Price = [85; 90; 95];
Settle = datenum('15-Apr-2002');
Maturity = datenum('1 Jan 2030');
IssueDate = datenum('1-Jan-2000');
GrossRate = 0.08125;
CouponRate = 0.075;
Delay = 14;
Speed = 100;
```

Compute the mortgage and bond-equivalent yields.

```
[MYield, BEMBSYield] = mbsyield(Price, Settle, Maturity, ...
IssueDate, GrossRate, CouponRate, Delay, Speed)

MYield =

    0.1018
    0.0918
    0.0828

BEMBSYield =

    0.1040
    0.0936
    0.0842
```

If for this same pool of mortgages, there was no prepayment (`Speed = 0`), the yields would decline to

```
MYield =

    0.0926
    0.0861
    0.0802

BEMBSYield =

    0.0944
    0.0877
    0.0815
```

Likewise, if the rate of prepayment doubled (`Speed = 200`), the yields would increase to

```
MYield =

    0.1124
    0.0984
    0.0858

BEMBSYield =

    0.1151
    0.1004
    0.0873
```

For the same prepayment vector, deeper discount pools earn higher yields. For more information, see `mbsprice` and `mbsyield`.

## Risk Measurement

Financial Instruments Toolbox software provides the most basic risk measures of a pool portfolio:

- Modified duration
- Convexity
- Average life of pool

Consider the following example, which calculates the Macaulay and modified durations given the price of a mortgage pool.

```
Price = [95; 100; 105];
Settle = datenum('15-Apr-2002');
Maturity = datenum('1-Jan-2030');
IssueDate = datenum('1-Jan-2000');
GrossRate = 0.08125;
CouponRate = 0.075;
Delay = 14;
Speed = 100;
```

```
[YearDuration, ModDuration] = mbsdurp(Price, Settle, ...
Maturity, IssueDate, GrossRate, CouponRate, Delay, Speed)

YearDuration =

    6.1341
    6.3882
    6.6339

ModDuration =

    5.8863
    6.1552
    6.4159
```

Using Financial Instruments Toolbox functions, you can obtain modified duration and convexity from either price or yield, as long as you specify a prepayment vector or an assumed prepayment speed. The toolbox risk-measurement functions (`mbsdurp`, `mbsdury`, `mbsconvp`, `mbsconvy`, and `mbswal`) adhere to the guidelines listed in the *PSA Uniform Practices* manual.

## Mortgage Pool Valuation

For accurate valuation of a mortgage pool, you must generate interest rate paths and use them with mortgage pool characteristics to properly value the pool. A widely used methodology is the option-adjusted spread (OAS). OAS measures the yield spread that is not directly attributable to the characteristics of a fixed-income investment.

### Calculating OAS

Prepayment alters the cash flows of an otherwise regularly amortizing mortgage pool. A comprehensive option-adjusted spread calculation typically begins with the generation of a set of paths of spot rates to predict prepayment. A path is collection of $i$ spot-rate paths, with corresponding $j$ cash flows on each of those paths.

The effect of the OAS on pool pricing is shown mathematically in the following equation, where $K$ is the option-adjusted spread.

$$PoolPrice = \frac{1}{NumberofPaths} \times \sum_{i}^{NumberofPaths} \sum_{j}^{CF_{ij}} \frac{CF_{ij}}{(1 + zerorates_{ij} + K)^{T_{ij}}}$$

### Calculating Effective Duration

Alternatively, if you are more interested in the sensitivity of a mortgage pool to interest rate changes, use effective duration, which is a more appropriate measure. Effective duration is defined mathematically with the following equation.

$$Effective\ Duration = \frac{P(y + \Delta y) - P(y - \Delta y)}{2P(y)\Delta y}$$

### Calculating Market Price

The toolbox has all the components required to calculate OAS and effective duration if you supply prepayment vectors or assumptions. For OAS, given a prepayment vector, you can generate a set of cash flows with `mbscfamounts`. Discounting these cash flows with the reference curve and then adding OAS produces the market price. See “Computing Option-Adjusted Spread” on page 5-10 for a discussion on the computation of option-adjusted spread.

Effective duration is a more difficult issue. While modified duration changes the discounting process (by changing the yield used to discount cash flows), effective duration must account for the change in cash flow because of the change in yield. A possible solution is to recompute prices using `mbsprice` for a small change in yield, in both the upwards and downwards directions. In this case, you must recompute the prepayment input. Internally, this alters the cash flows of the mortgage pool. Assuming that the OAS stays constant in all yield environments, you can apply a set of discounting factors to the cash flows in up and down yield environments to find the effective duration.

## Computing Option-Adjusted Spread

The option-adjusted spread (OAS) is an amount of extra interest added above (or below if negative) the reference zero curve. To compute the OAS, you must provide the zero curve as an extra input. You can specify the zero curve in any intervals and with any compounding method. (To minimize any error due to interpolation, keep the intervals as regular and frequent as possible.)

You must supply a prepayment vector or specify a speed corresponding to a standard PSA prepayment vector.

One way to compute the appropriate zero curve for an agency is to look at its bond yields and bootstrap them from the shortest maturity onwards. You can do this with Financial Toolbox functions `zbtprice` and `zbtyield`.

The following example shows how to calculate an appropriate zero curve followed by computation of the pool's OAS. This example calculates the OAS of a 30-year fixed rate mortgage with about a 28-year weighted average maturity left, given an assumption of 0, 50, and 100 PSA prepayment speeds.

Create curve for `zerorates`.

```
Bonds = [datenum('11/21/2002')   0        100    0    2    1;
         datenum('02/20/2003')   0        100    0    2    1;
         datenum('07/31/2004')   0.03     100    2    3    1;
         datenum('08/15/2007')   0.035    100    2    3    1;
         datenum('08/15/2012')   0.04875  100    2    3    1;
         datenum('02/15/2031')   0.05375  100    2    3    1];

Yields = [0.0162;
          0.0163;
          0.0211;
          0.0328;
          0.0420;
          0.0501];
```

Since the above is Treasury data and not selected agency data, a term structure of spread is assumed. In this example, the spread declines proportionally from a maximum of 250 basis points at the shortest maturity.

```
Yields = Yields + 0.025 * (1./[1:6]');
```

Get parameters from `Bonds` matrix.

```
Settle = datenum('20-Aug-2002');
Maturity = Bonds(:,1);
CouponRate = Bonds(:,2);
Face = Bonds(:,3);
Period = Bonds(:,4);
```

```
Basis = Bonds(:,5);
EndMonthRule = Bonds(:,6);

[Prices, AccruedInterest] = bndprice(Yields, CouponRate, ...
Settle, Maturity, Period, Basis, EndMonthRule, [], [], [], [], ...
Face);
```

Use `zbtprice` to solve for zero rates.

```
[ZeroRatesP, CurveDatesP] = zbtprice(Bonds, Prices, Settle);
ZeroCompounding = 2*ones(size(ZeroRatesP));
ZeroMatrix = [CurveDatesP, ZeroRatesP, ZeroCompounding];
```

Use output from `zbtprice` to calculate the OAS.

```
Price = 95;
Settle = datenum('20-Aug-2002');
Maturity = datenum('2-Jan-2030');
IssueDate = datenum('2-Jan-2000');
GrossRate = 0.08125;
CouponRate = 0.075;
Delay = 14;
Interpolation = 1;
PrepaySpeed = [0; 50; 100];

OAS = mbsprice2oas(ZeroMatrix, Price, Settle, Maturity, ...
IssueDate, GrossRate, CouponRate, Delay, Interpolation, ...
PrepaySpeed)

OAS =

    26.0502
    28.6348
    31.2222
```

This example shows that one cash flow set is being discounted and solved for its OAS, as contrasted with the `NumberOfPaths` set of cash flows as shown in “Mortgage Pool Valuation” on page 5-9. Averaging the sets of cash flows resulting from all simulations into one average cash flow vector and solving for the OAS, discounts the averaged cash flows to have a present value of today’s (average) price.

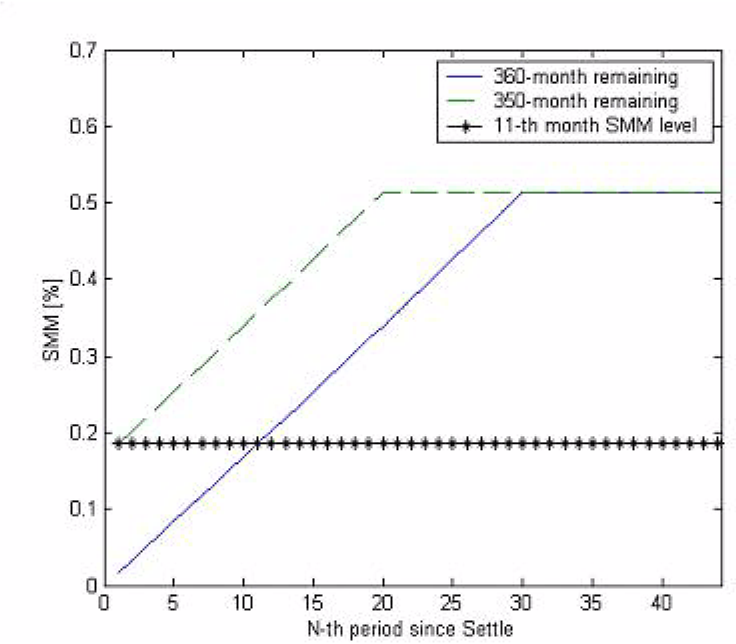
While this example uses the mortgage pool price (`mbsprice2oas`) to determine the OAS, you can also use yield to resolve it (`mbsyield2oas`). Also, there are reverse OAS functions that return prices and yields given OAS (`mbsoas2price` and `mbsoas2yield`).

The example also restates earlier examples that show discount securities benefit from higher level of prepayment, keeping everything else unchanged. The relation is reversed for premium securities.

## Prepayments with Fewer Than 360 Months Remaining

When fewer than 360 months remain in the pool, the applicable PSA prepayment vector is "seasoned" by the pool’s age. (Elements in the 360-element prepayment vector that represent past payments are skipped. For example, on a 30-year mortgage that is 10 months old, only the final 350 prepayments are applied.)

Assume, for example, that you have two 30-year loans, one new and another 10 months old. Both have the same PSA speed of 100 and prepay using the vectors plotted below.



Still within the scope of relative valuation, you could also solve for the percentage of the standard PSA prepayment vector given the pool's arbitrary, user-supplied prepayment vector, such that the PSA speed gives the same Macaulay duration as the user-supplied prepayment vector.

If you supply a custom prepayment vector, you must account for the number of months remaining.

```
Price = 101;
Settle = datenum('1-Jan-2001');
Maturity = datenum('1-Jan-2030');
IssueDate = datenum('1-Jan-2000');
GrossRate = 0.08125;
PrepayMatrix = 0.005*ones(348,1);
CouponRate   = 0.075;
Delay = 14;

ImpliedSpeed = mbsprice2speed(Price, Settle, Maturity, ...
IssueDate, GrossRate, PrepayMatrix, CouponRate, Delay)
```

```
ImpliedSpeed =

  104.2543
```

Examine the prepayment input. The remaining 29 years require 348 monthly elements in the prepayment vector. Suppose then, keeping everything the same, you change `Settle` to February 14, 2003.

```
Settle = datenum('14-Feb-2003');
```

You can use `cpncount` to count all incoming coupons received after `Settle` by invoking

```
NumCouponsRemaining = cpncount(Settle, Maturity, 12, 1, [], ...
IssueDate)

NumCouponsRemaining =
323
```

The input `12` defines the monthly payment frequency, `1` defines the 30/360 basis, and `IssueDate` defines aging and determination-of-holder date. Thus, you must supply a 323-element vector to account for a prepayment corresponding to each monthly payment.

## Pools with Different Numbers of Coupons Remaining

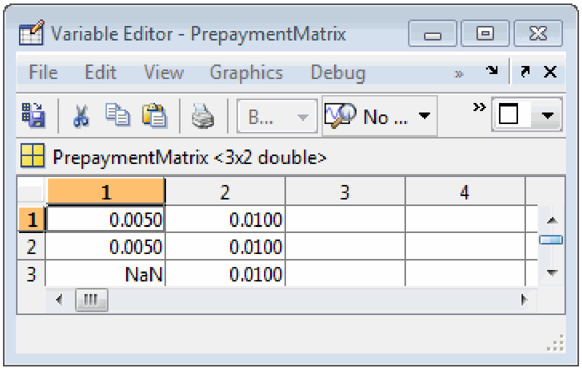
Suppose one pool has two remaining coupons, and the other has three. MATLAB software expects the prepayment matrix to be in the following format:

```
V11       V21
V12       V22
NaN       V23
```

$V_{ij}$ denotes the single monthly mortality (SMM) rate for pool `i` during the `j`th coupon period since `Settle`.

The use of `NaN` to pad the prepayment matrix is necessary because MATLAB cannot concatenate vectors of different lengths into a matrix. Also, it can serve as an error check against any unintended operation (any MATLAB operation that would return `NaN`).

For example, assume that the 2 month pool has a constant SMM of 0.5% and the 3 month has a constant SMM of 1% in every period. The prepayment matrix you would create is depicted below.



PrepaymentMatrix <3x2 double>

|   | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| 1 | 0.0050 | 0.0100 |  |  |
| 2 | 0.0050 | 0.0100 |  |  |
| 3 | NaN | 0.0100 |  |  |

Create this input in whatever manner is best for you.

## Summary of Prepayment Data Vector Representation

- When you specify a PSA prepayment speed, MATLAB "seasons" the pool according to its age.
- When you specify your own prepayment matrix, identify the maximum number of coupons remaining using `cpncount`. Then supply the matrix elements up to the point when cash flow ceases to exist.
- When different length pools must exist in the same matrix, pad the shorter one(s) with `NaN`. Each column of the prepayment matrix corresponds to a specific pool.

# Prepayment Modeling with a Two Factor Hull White Model and a LIBOR Market Model

This example shows how to model prepayment in MATLAB using the stochastic differential equation (SDE) functionality found in the Econometrics Toolbox™ and functionality from the Financial Instruments Toolbox™. Specifically, a variation of the Richard and Roll prepayment model is implemented using a two factor Hull-White interest-rate model and a LIBOR Market Model to simulate future interest-rate paths. A mortgage-backed security is priced with both the custom and default prepayment models.

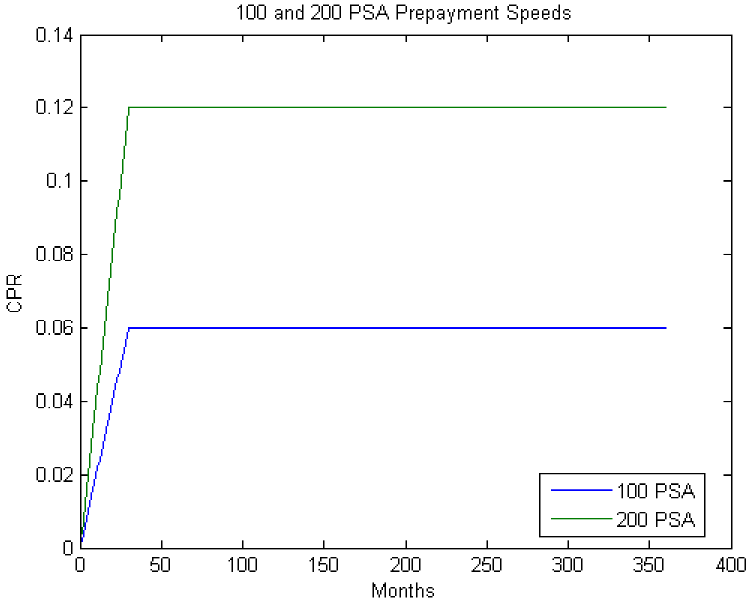
### Introduction

Prepayment modeling is crucial to the analysis of mortgage-backed securities (MBS). Prepayments by individual mortgage holders affect both the amount and timing of cash flows -- and for collateralized mortgage obligations (e.g.: interest-only securities), prepayment can greatly affect the value of the securities.

### PSA Model

The most basic prepayment model is the Public Securities Association (PSA) model, which assumes a ramp-up phase and then a constant conditional prepayment rate (CPR). The PSA model can be generated in MATLAB using the Financial Instruments Toolbox function PSASPEED2RATE

```
G2PP_CPR = psaspeed2rate([100 200]);
figure
plot(G2PP_CPR)
title('100 and 200 PSA Prepayment Speeds')
xlabel('Months')
ylabel('CPR')
ylim([0 .14])
legend({'100 PSA','200 PSA'}, 'Location', 'Best')
```



### Mortgage-Backed Security

The MBS analyzed in this example matures in 2020 and has the properties outlined in this section. Cash flows are generated for PSA prepayment speeds simply by entering the PSA speed as an input argument.

```
% Parameters for MBS passthrough to be priced
Settle = datenum('15-Dec-2007');
Maturity = datenum('15-Dec-2020');
IssueDate = datenum('15-Dec-2000');
GrossRate = .0475;
CouponRate = .045;
Delay = 14;
Period = 12;
Basis = 4;

% Generate cash flows and dates for baseline case using 100 PSA
[CFlowAmounts, CFlowDates] = mbscfamounts(Settle,Maturity, IssueDate,...
    GrossRate, CouponRate, Delay,100);
```

```
CFlowTimes = yearfrac(Settle,CFlowDates);
NumCouponsRemaining = cpncount(Settle, Maturity, Period,Basis, 1, IssueDate
```

### Richard and Roll Model

While prepayment modeling often involves quite complex and sophisticated modeling, often at the loan level, this example will use a slightly modified approach based on the model proposed by Richard and Roll in [6].

The Richard and Roll prepayment model involves the following factors:

- Refinancing incentive
- Seasonality (month of the year)
- Seasoning or age of the mortgage
- Burnout

Richard and Roll propose a multiplicative model of the following:

$$CPR = RefiIncentive * SeasoningMultiplier * SeasonalityMultiplier * BurnoutM$$

For the custom model in this example, the Burnout Multiplier, which describes the tendency of prepayment to slow when a significant number of homeowners have already refinanced, is ignored and the first three terms are used.

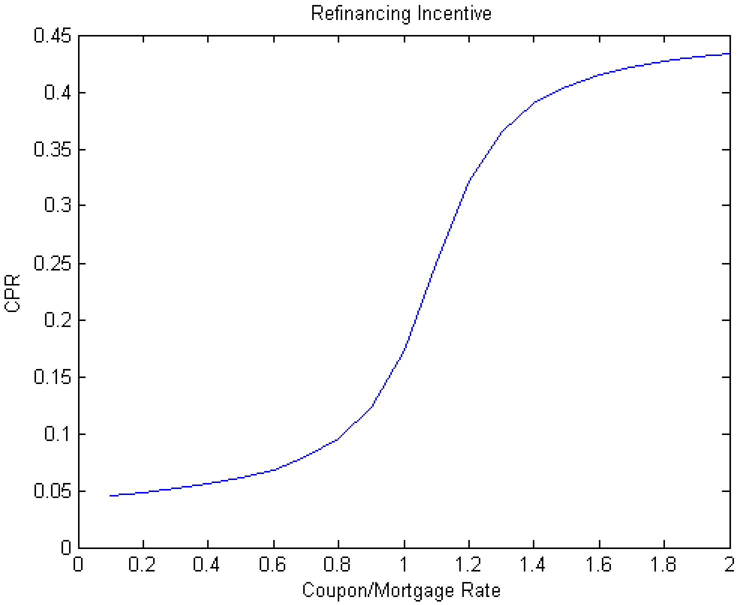
The refinancing incentive is a function of the ratio of the coupon-rate of the mortgage to the available mortgage rate at that particular point in time. For example, the Office of Thrift Supervision (OTS) proposes the following model:

$$Refi = .2406 - .1389 * arctan(5.952 * (1.089 - \frac{CouponRate}{MortgageRate}))$$

The refinancing incentive requires a simulation of future interest rates. This will be discussed later in this example.

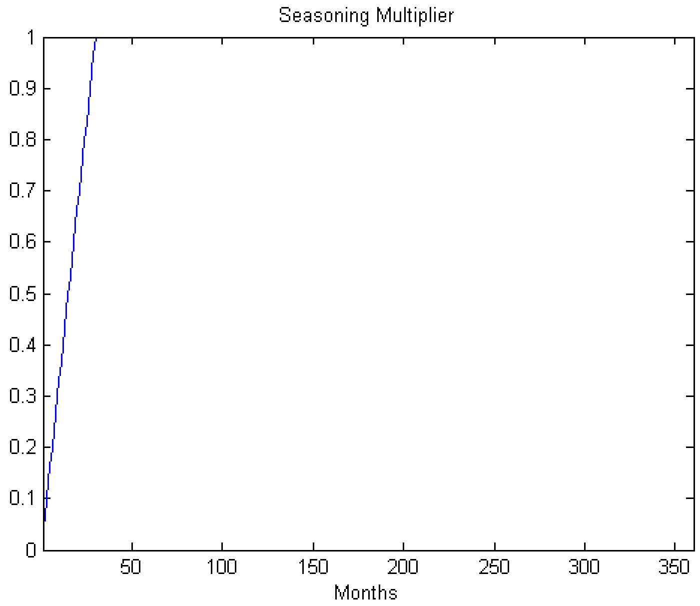
```
C_M = .1:.1:2;
G2PP_Refi = .2406 - .1389 * atan(5.952*(1.089 - C_M));
figure
plot(C_M,G2PP_Refi)
```

```
xlabel('Coupon/Mortgage Rate')
ylabel('CPR')
title('Refinancing Incentive')
```



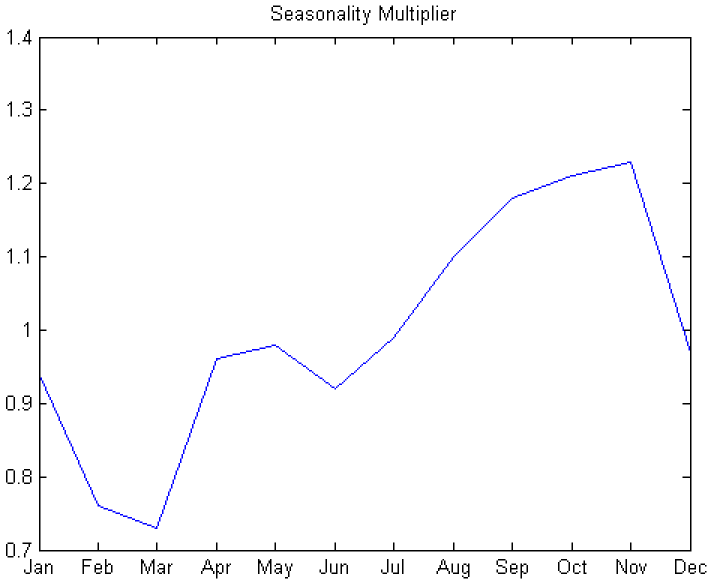
Seasoning captures the tendency of prepayment to ramp up at the beginning of a mortgage before leveling off. The OTS models the seasoning multiplier as follows:

```
Seasoning = ones(360,1);
Seasoning(1:29) = (1:29)/30;
figure
plot(Seasoning)
xlim([1 360])
title('Seasoning Multiplier')
xlabel('Months')
```



The seasonality multiplier simply models the seasonal behavior of prepayments -- this data is based on Figure 3 of [6], which applies to the behavior of Ginnie Mae 30-year, single-family MBSs.

```
Seasonality = [.94 .76 .73 .96 .98 .92 .99 1.1 1.18 1.21 1.23 .97];
figure
plot(Seasonality)
xlim([1 12])
set(gca,'xtick',1:12)
set(gca,'xtickLabel',{'Jan','Feb','Mar','Apr','May','Jun','Jul','Aug',...
    'Sep','Oct','Nov','Dec'})
title('Seasonality Multiplier')
```



### G2++ Interest-Rate Model

Since the refinancing incentive requires a simulation of future interest rates, an interest-rate model must be used. One choice is a two-factor additive Gaussian model, referred to as G2++ by Brigo and Mercurio [2].

The G2++ Interest Rate Model is:

$$r(t) = x(t) + y(t) + \varphi(t)$$

$$dx(t) = -ax(t)dt + \sigma dW_1(t)$$

$$dy(t) = -by(t)dt + \eta dW_2(t)$$

where $dW_1(t)dW_2(t)$ is a two-dimensional Brownian motion with correlation $\rho$

$$dW_1(t)dW_2(t) = \rho$$

$$\varphi(t) = f^M(0,T) + \frac{\sigma^2}{2a^2}(1 - e^{-aT})^2 + \frac{\eta^2}{2b^2}(1 - e^{-bT})^2 + \rho\frac{\sigma\eta}{ab}(1 - e^{-aT})(1 - e^{-bT})$$

and $r(t)$ is the short rate, $a$ and $b$ are mean reversion constants and $\sigma$ and $\eta$ are volatility constants, and $f^M(0,T)$ is the market forward rate, or the forward rate observed on the Settle date.

The entire discount curve can be recovered from two factors using the following formula:

$$P(t,T) = \frac{P^M(0,T)}{P^M(0,t)}e^{A(t,T)}$$

where $P^M$ is the market discount curve and

$$A(t,T) = \frac{1}{2}[V(t,T) - V(0,T) + V(0,t)] - \frac{1 - e^{-a(T-t)}}{a}x(t) - \frac{1 - e^{-b(T-t)}}{b}y(t)$$

$$V(t,T) = \frac{\sigma^2}{a^2}[T - t + \frac{2}{a}e^{-a(T-t)} - \frac{1}{2a}e^{-2a(T-t)} - \frac{3}{2a}]$$

$$+\frac{\eta^2}{b^2}[T - t + \frac{2}{b}e^{-b(T-t)} - \frac{1}{2b}e^{-2b(T-t)} - \frac{3}{2b}]$$

$$+2\rho\frac{\sigma\eta}{ab}[T - t + \frac{e^{-a(T-t)} - 1}{a} - \frac{e^{-b(T-t)} - 1}{b} - \frac{e^{-(a+b)(T-t)} - 1}{a+b}]$$

#### LIBOR Market Model

The LIBOR Market Model (LMM) differs from short-rate models in that it evolves a set of discrete forward rates. Specifically, the lognormal LMM specifies the following diffusion equation for each forward rate:

$$\frac{dF_i(t)}{F_i} = -\mu_i dt + \sigma_i(t)dW_i$$

where

dW is an N dimensional geometric Brownian motion with:

$$dW_i(t)dW_j(t) = \rho_{ij}$$

The LMM relates the drifts of the forward rates based on no-arbitrage arguments. Specifically, under the Spot LIBOR measure, the drifts are expressed as the following:

$$\mu_i(t) = -\sigma_i(t) \sum_{j=q(t)}^{i} \frac{\tau_j \rho_{i,j} \sigma_j(t) F_j(t)}{1 + \tau_j F_j(t)}$$

where

$\tau_i$ is the time fraction associated with the ith forward rate

$q(t)$ is an index function defined by the relation $T_{q(t)-1} < t < T_{q(t)}$

and the Spot LIBOR numeraire is defined as the following:

$$B(t) = P(t, T_{q(t)}) \prod_{n=0}^{q(t)-1} (1 + \tau_n F_n(T_n))$$

Given the above, the choice with the LMM is how to model volatility and correlation.

The volatility of the rates can be modeled with a stochastic volatility, but for this example a deterministic volatility is used, and so a functional form needs to be specified. One of the most popular functional forms in the literature is the following:

$$\sigma_i(t) = \phi_i(a(T_i - t) + b)e^{c(T_i - t)} + d$$

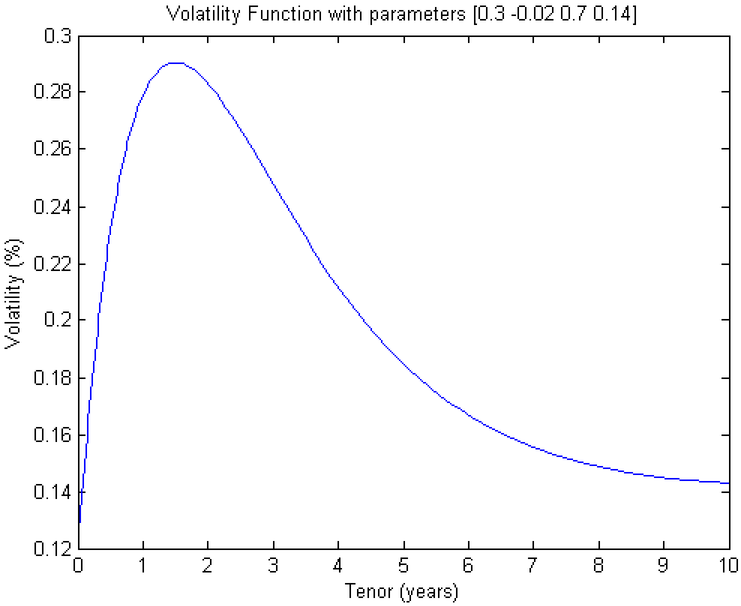
where $\phi$ adjusts the curve to match the volatility for the $i^{th}$ forward rate.

Similarly, the correlation between the forward rates needs to be specified. This can be estimated from historical data or fitted to option prices. For this example, the following functional form will be used:

$$\rho_{i,j} = e^{-\beta|i-j|}$$

Once the volatility and correlation are specified, the parameters need to be calibrated -- this can be done with historical or market data, typically swaptions or caps/floors. For this example, we simply use reasonable estimates for the correlation and volatility parameters.

```
% The volatility function to be used -- and one choice for the parameters
LMMVolFunc = @(a,t) (a(1)*t + a(2)).*exp(-a(3)*t) + a(4);
LMMVolParams = [.3 -.02 .7 .14];

% Volatility specification
fplot(@(t) LMMVolFunc(LMMVolParams,t),[0 10])
title(['Volatility Function with parameters ' mat2str(LMMVolParams)])
ylabel('Volatility (%)')
xlabel('Tenor (years)')
```

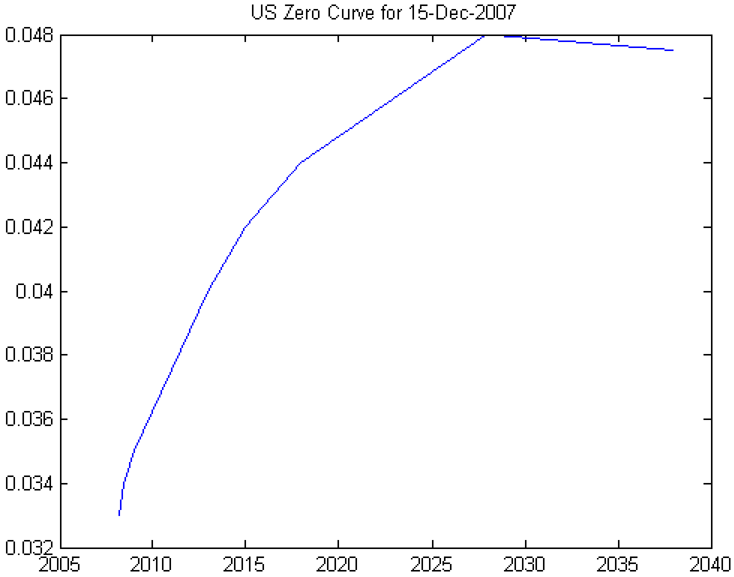


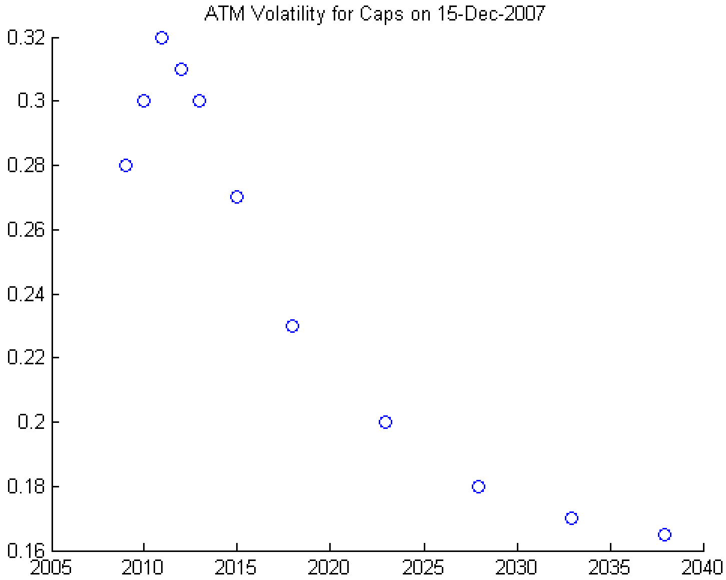
### Calibration to Market Data

The parameters in the G2++ model can be calibrated to market data. Typically, the parameters are calibrated to observed interest-rate cap, floor and/or swaption data. For now, market cap data is used for calibration.

This data is hardcoded but could be imported into MATLAB with the Database Toolbox™ or Datafeed Toolbox™.

```
% Zero Curve -- this data is hardcoded for now but could be bootstrapped
% using the bootstrap method of IRDataCurve
ZeroTimes = [3/12 6/12 1 5 7 10 20 30]';
ZeroRates = [0.033 0.034 0.035 0.040 0.042 0.044 0.048 0.0475]';
ZeroDates = daysadd(Settle,360*ZeroTimes,1);
DiscountRates = zero2disc(ZeroRates,ZeroDates,Settle);

figure
plot(ZeroDates,ZeroRates)
datetick
title(['US Zero Curve for ' datestr(Settle)])

% Cap Data
Reset = 2;
Notional = 100;
CapMaturity = daysadd(Settle,360*[1:5 7 10 15 20 25 30],1);
CapVolatility = [.28 .30 .32 .31 .30 .27 .23 .2 .18 .17 .165]';

% ATM strikes could be computed with SWAPBYZERO
Strike = [0.0353 0.0366 0.0378 0.0390 0.0402 0.0421 0.0439 ...
    0.0456 0.0471 0.0471 0.0471]';

% This could be computed with CAPBYBLK
BlackCapPrices = [0.1532 0.6416 1.3366 2.0290 2.7366 4.2960 6.5992 ...
    9.6787 12.2580 14.0969 15.7873]';

figure
scatter(CapMaturity,CapVolatility)
datetick
title(['ATM Volatility for Caps on ' datestr(Settle)])
```


US Zero Curve for 15-Dec-2007
0.048
0.046
0.044
0.042
0.04
0.038
0.036
0.034
0.032
2005
2010
2015
2020
2025
2030
2035
2040



Brigo and Mercurio derive analytic formulae for valuing Caps using the G2++ model.

Specifically:

$$Cap(t,T,\tau,N,X) = N\sum_{i=1}^{n}\Bigg[-(1+X\tau_i)P(t,T_i)\phi\left(\frac{ln\frac{P(t,T_{i-1})}{(1+X\tau_i)P(t,T_i))}}{\Sigma(t,T_{i-1},T_i)} - \frac{1}{2}\Sigma(t,T_{i-1},T_i)\right)$$

$$+P(t,T_{i-1})\phi\left(\frac{ln\frac{P(t,T_{i-1})}{(1+X\tau_i)P(t,T_i))}}{\Sigma(t,T_{i-1},T_i)} + \frac{1}{2}\Sigma(t,T_{i-1},T_i)\right)\Bigg]$$

where

$$\Sigma(t,T,S) = \frac{\sigma^2}{2a^3}[1-e^{-a(S-T)}]^2[1-e^{-2a(T-t)}]$$

$$+\frac{\eta^2}{2b^3}[1-e^{-b(S-T)}]^2[1-e^{-2b(T-t)}]$$

$$+2\rho\frac{\sigma\eta}{ab(a+b)}[1-e^{-a(S-T)}][1-e^{-b(S-T)}][1-e^{-(a+b)(T-t)}]$$

$N =$ Notional Value of Cap

$T_i =$ Payment dates for Cap

$\tau =$ Year Fraction between $T_{i-1}$ and $T_i$

$X =$ Cap Strike

To calibrate the model parameters, a parameter set will be found that minimizes the sum of the squared differences between the G2++ predicted Cap values and the observed Black Cap values. The Optimization Toolbox™ function LSQNONLIN is used in this example, although other approaches (e.g. Global Optimization) may also be applicable.

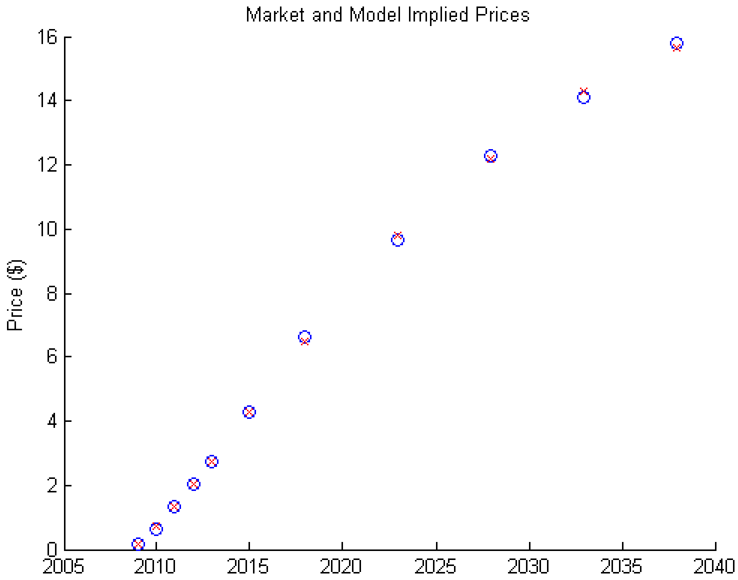
Upper and lower bounds for the model parameters are set to be relatively constrained. As Brigo and Mercurio discuss, the correlation parameter, $rho$ , can often be close to -1 when fitting a G2++ model to interest-rate cap prices. Therefore, $rho$ is constrained to be between -.7 and .7 to ensure that the parameters represent a truly two-factor model. The remaining mean reversion and volatility parameters are constrained to be between 0 and .5. Calibration remains a complex task, and while the plot below indicates that the best fit parameters seem to do a reasonably good job of reproducing the Cap prices, it should be noted that the procedure outlined here simply represents one approach.

```
% Generate the payment dates for the Caps
CapCFDates = cfdates(Settle,CapMaturity+1,Reset);
T = yearfrac(Settle,CapCFDates);
tau = diff(T,[],2);

% Discount factor function
PM = @(t) interp1(ZeroTimes,DiscountRates,t,'pchip','extrap');

% Sigma function
```

```
SF = @(t,T,S,a,b,sigma,eta,rho) sqrt(sigma^2/(2*a^3).*(1 - exp(-a*(S-T))).^
    eta^2/(2*b^3).*(1 - exp(-b*(S-T))).^2.*(1 - exp(-2*b*(T-t))) + ...
    2*rho*sigma*eta/(a*b*(a+b))*(1 - exp(-a*(S-T))).*(1 - exp(-b*(S-T))).*(

% Log Term in Cap equation above
logTerm = log(PM(T(:,1:end-1))./PM(T(:,2:end))./(1 + bsxfun(@times,Strike,t

CapByG2PP = @(a,b,sigma,eta,rho) Notional*nansum(PM(T(:,1:end-1)).* ...
    normcdf(logTerm./SF(0,T(:,1:end-1),T(:,2:end),a,b,sigma,eta,rho) + ...
    .5*SF(0,T(:,1:end-1),T(:,2:end),a,b,sigma,eta,rho)) - ...
    (1 + bsxfun(@times,Strike,tau)).*PM(T(:,2:end)) ...
    .*normcdf(logTerm./SF(0,T(:,1:end-1),T(:,2:end),a,b,sigma,eta,rho) - ..
    .5*SF(0,T(:,1:end-1),T(:,2:end),a,b,sigma,eta,rho)),2);

% Call to LSQNONLIN to calibrate parameters
objfun = @(x) BlackCapPrices - CapByG2PP(x(1),x(2),x(3),x(4),x(5));
x0 = [.5 .05 .1 .01 -.1];
lb = [0 0 0 0 -.7];
ub = [.5 .5 .5 .5 .7];

G2PP_Params = lsqnonlin(objfun,x0,lb,ub);

a = G2PP_Params(1);
b = G2PP_Params(2);
sigma = G2PP_Params(3);
eta = G2PP_Params(4);
rho = G2PP_Params(5);

% Compare the results
figure
scatter(CapMaturity,BlackCapPrices)
hold on
scatter(CapMaturity,CapByG2PP(a,b,sigma,eta,rho),'rx')
datetick
title('Market and Model Implied Prices')
ylabel('Price ($)')
```

```
Local minimum possible.
```

```
lsqnonlin stopped because the final change in the sum of squares relative t
its initial value is less than the default value of the function tolerance.
```



### G2++ Model Implementation

The Hull-White/Vasicek (HWV) object in the Econometrics Toolbox can be used to specify the G2++ model and simulate future paths of the two interest-rate factors. Then, using four function handles, the full interest-rate term structure can be recovered.

```
% G2++ model from Brigo and Mercurio with time homogeneous volatility
% parameters
G2PP = hwv(diag([a;b]),zeros(2,1), diag([sigma;eta]),'Correlation',...
    [1 rho;rho 1],'StartState',[0;0]);

% Formula for V -- used in A formula below
```

```
V = @(t,T) sigma^2*(T - t + 2/a*exp(-a*(T-t)) - 1/(2*a)*exp(-2*a*(T-t)) - 3
    eta^2*(T - t + 2/b*exp(-a*(T-t)) - 1/(2*b)*exp(-2*a*(T-t)) - 3/2/b) + .
    2*rho*sigma*eta/(a*b)*(T - t + (exp(-a*(T-t)) - 1)/a + (exp(-b*(T-t)) -
    (exp(-(a + b)*(T-t)) - 1)/(a + b));

% Formula for A -- used in P formula below
A = @(t,T,x,y) 1/2*(V(t,T) - V(0,T) + V(0,t)) - (1 - exp(-a*(T-t)))/a.*x -

% Formula for recovering the discount curve from two factors
P = @(t,T,x,y) PM(T)./PM(t).*exp(A(t,T,x,y));

% Formula for recovering the zero rate
ZR = @(t,T,x,y) -log(P(t,T,x,y))./(T-t);
```

**LIBOR Market Model Implementation**

After the volatility and correlation have been calibrated, Monte Carlo simulation is used to evolve the rates forward in time. The GBM object, from the Econometrics Toolbox, is used to simulate the forward rates.

While factor reduction is often used with the LMM to reduce computational complexity, there is no factor reduction in this example.

6M LIBOR rates are chosen to be evolved in this simulation. Since a monthly prepayment vector must be computed, interpolation is used to generate the intermediate rates. Simple linear interpolation is used.

```
% Get forward dates at 6M intervals
ForwardDates = datemnth(datenum('15-Jun-2008'),0:6:(12*23-1))';
ForwardTimes = yearfrac(Settle,ForwardDates);
ZeroRatesInterp = interp1(ZeroDates,ZeroRates,ForwardDates);
ForwardRates = zero2fwd(ZeroRatesInterp,ForwardDates,Settle,1,12);
tau = [ForwardTimes(1);diff(ForwardTimes)];
numForwardRates = length(ForwardRates);

LMMPeriod = 2; % Semi-annual rates
LMMNumPeriods = NumCouponsRemaining/12*LMMPeriod; % Number of semi-annual p
LMMSimTimes = 0:1/LMMPeriod:LMMNumPeriods/LMMPeriod;
LMMNTRIALS = 100;
```

```
% Instead of being fit, VolPhi is simply hard-coded  --
% representative of a declining volatility over time.
VolPhi = linspace(1.2,.8,numForwardRates)';
VolMat = VolPhi*LMMVolFunc(LMMVolParams,ForwardTimes)';

Beta = .08; % Hard-coded
CorrFunc = @(i,j,Beta) exp(-Beta*abs(i-j));
CorrMat = CorrFunc(meshgrid(1:numForwardRates)',meshgrid(1:numForwardRates)

% Construct a function handle for the LMM diffusion term. Note that this
% simply returns the volatility matrix entry appropriate to the simulation
% time
LMMDiffusion = @(t,L) diag([zeros(1,find(t==LMMSimTimes)-1) ...
    VolMat(max(1,find(t==LMMSimTimes)-1),1:(numForwardRates - find(t == LMM

% Call the GBM constructor -- note that we are calling hLMMDriftSpot, a loc
% function that implements the drift equation for the Spot LIBOR measure
LMM = gbm(@(t,L) hLMMDriftSpot(t,L,VolMat,CorrMat,tau,LMMSimTimes),LMMDiffu
    'Correlation',CorrMat,'StartState',ForwardRates);
```

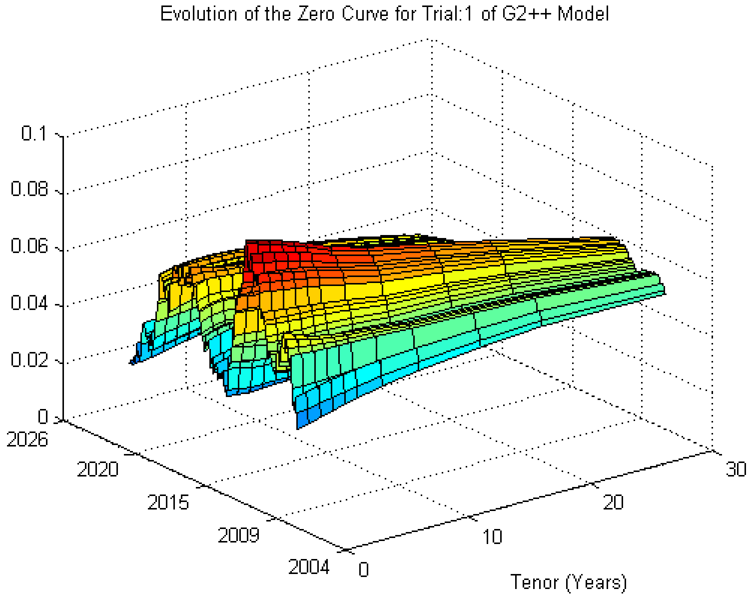
**G2++ Monte Carlo Simulation**

The various interest-rate paths can be simulated by calling the simulate method of the HWV object.

One limitation to two-factor Gaussian models like this one is that it does permit negative interest rates.  This is a concern, particularly in low interest-rate environments. To handle this possibility, any interest-rate paths with negative rates are simply rejected.

```
nPeriods = NumCouponsRemaining;
nTrials = 100;
stepSize = 1/12;

% Generate factors and short rates
[G2PPPaths, SimTimes] = G2PP.simulate(nPeriods,'NTRIALS',nTrials,'DeltaTime

SimDates = daysadd(Settle,round(360*SimTimes),1);

% Tenors that will be recovered for each simulation date. The stepsize is
```

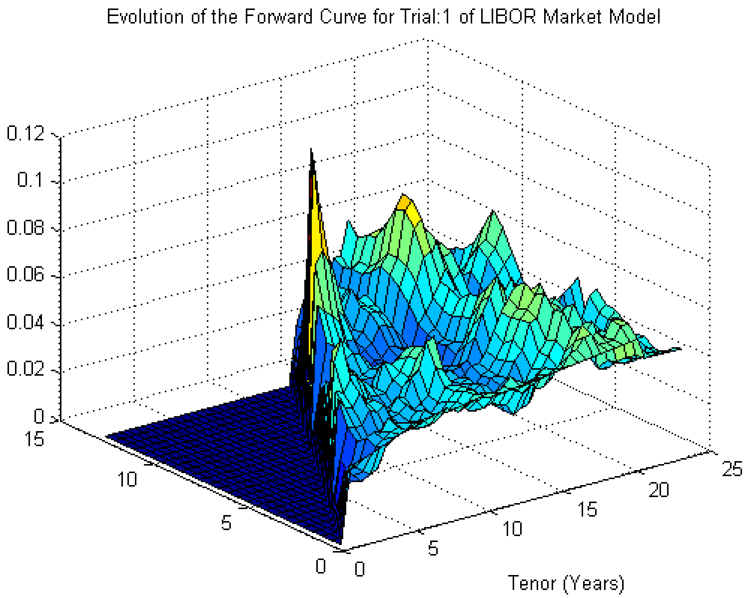
```
% included here to facilitate computing a discount factor for each
% simulation path.
Tenors = [1/12 1 2 3 4 5 7 10 15 20 30];
nTenors = length(Tenors);

G2PP_SimZeroRates = zeros(nPeriods+1,nTenors,nTrials);

% Compute full term structure from factors
for trialidx = 1:nTrials
    for stepidx=1:nPeriods+1
        G2PP_SimZeroRates(stepidx,:,trialidx) = ZR(stepSize*(stepidx-1),...
            Tenors+stepSize*(stepidx-1),G2PPPaths(stepidx,1,trialidx),G2PPP
    end
end

% Remove any paths that go negative
NegIdx = squeeze(any(any(G2PP_SimZeroRates < 0,1),2));
G2PP_SimZeroRates(:,:,NegIdx) = [];
nTrials = size(G2PP_SimZeroRates,3);

% Plot evolution of one sample path
trialIdx = 1;
figure
surf(Tenors,SimDates,G2PP_SimZeroRates(:,:,trialIdx))
datetick y keepticks keeplimits
title(['Evolution of the Zero Curve for Trial:' num2str(trialIdx) ' of G2++
xlabel('Tenor (Years)')
```



**LIBOR Market Model Simulation**

The various interest-rate paths can be simulated by calling the simulate method of the GBM object.

```
% Simulate
Paths = LMM.simBySolution(LMMNumPeriods,'NTRIALS',LMMNTRIALS,'DeltaTime',1/

% Kill off the rates once the simulation has moved past them
LMMRates = bsxfun(@times,Paths,triu(ones([size(Paths,1) size(Paths,2)])));

% Plot evolution of one sample path
trialIdx = 1;
figure
surf(ForwardTimes,LMMSimTimes,LMMRates(:,:,trialIdx))
title(['Evolution of the Forward Curve for Trial:' num2str(trialIdx) ' of L
xlabel('Tenor (Years)')
```



#### Compute Mortgage Rates from Simulation

Once the interest-rate paths have been simulated, the mortgage rate needs to be computed -- one approach, discussed by [7], is to compute the mortgage rate from a combination of the 2-year and 10-year rates.

For this example, the following is used:

$$MortgageRate = .024 + .2 * TwoYearRate + .6 * TenYearRate$$

```
% Compute mortgage rates from interest rate paths
TwoYearRates = squeeze(G2PP_SimZeroRates(:,Tenors == 2,:));
TenYearRates = squeeze(G2PP_SimZeroRates(:,Tenors == 7,:));
G2PP_MortgageRates = .024 + .2*TwoYearRates + .6*TenYearRates;

% Compute zero rates from simulated forward rates
LMMZeroRates = bsxfun(@mtimes,cumprod(1 + LMMRates*.5,2) - 1,repmat(1./Forw

LMMTwoYearRates = zeros(LMMNumPeriods,LMMNTRIALS);
```

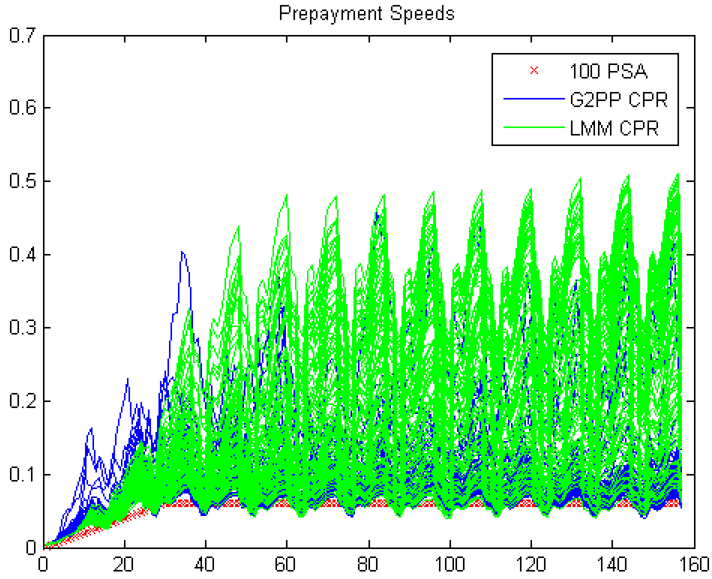
```
LMMTenYearRates = zeros(LMMNumPeriods,LMMNTRIALS);
LMMDiscountFactors = zeros(LMMNumPeriods,LMMNTRIALS);

% Compute 2 and 10 year rates
for trialidx=1:LMMNTRIALS
   LMMTwoYearRates(:,trialidx) = diag(LMMZeroRates(:,4:29,trialidx));
   LMMTenYearRates(:,trialidx) = diag(LMMZeroRates(:,20:45,trialidx));

   LMMDiscountFactors(:,trialidx) = cumprod(diag(1./(1 + LMMRates(:,1:(LMMN
end

% TenYearRates = squeeze(LMMRates(:,Tenors == 7,:));
LMMMortgageRates = .024 + .2*LMMTwoYearRates + .6*LMMTenYearRates;

% Interpolate to get monthly mortgage rates
MonthlySimTimes = 0:1/12:LMMNumPeriods/LMMPeriod;
LMMMonthlyMortgageRates = zeros(nPeriods+1,LMMNTRIALS);
LMMMonthlyDF = zeros(nPeriods+1,LMMNTRIALS);
for trialidx=1:LMMNTRIALS
   LMMMonthlyMortgageRates(:,trialidx) = interp1(LMMSimTimes(2:end),LMMMort
   LMMMonthlyDF(:,trialidx) = interp1(LMMSimTimes(2:end),LMMDiscountFactors
end
```

**Computing CPR and Generating and Valuing Cash Flows**

Once the Mortgage Rates have been simulated, the CPR can be computed from the multiplicative model for each interest-rate path.

```
% Compute Seasoning and Refinancing Multipliers
Seasoning = ones(nPeriods+1,1);
Seasoning(1:30) = 1/30*(1:30);
G2PP_Refi = .2406 - .1389 * atan(5.952*(1.089 - CouponRate./G2PP_MortgageRa
LMM_Refi = .2406 - .1389 * atan(5.952*(1.089 - CouponRate./LMMMonthlyMortga

% CPR is simply computed by evaluating the multiplicative model
G2PP_CPR = bsxfun(@times,G2PP_Refi,Seasoning.*(Seasonality(month(CFlowDates
LMM_CPR = bsxfun(@times,LMM_Refi,Seasoning.*(Seasonality(month(CFlowDates))

% Compute single monthly mortality (SMM) from CPR
G2PP_SMM = 1 - (1 - G2PP_CPR).^(1/12);
```

```
LMM_SMM = 1 - (1 - LMM_CPR).^(1/12);

% Plot CPR's against 100 PSA
CPR_PSA100 = psaspeed2rate(100);
figure
PSA_handle = plot(CPR_PSA100(1:nPeriods),'rx');
hold on
G2PP_handle = plot(G2PP_CPR,'b');
LMM_handle = plot(LMM_CPR,'g');
title('Prepayment Speeds')
legend([PSA_handle(1) G2PP_handle(1) LMM_handle(1)],{'100 PSA','G2PP CPR','
```



### Generate Cash Flows and Compute Present Value

With a vector of single monthly mortalities (SMM) computed for each interest-rate path, cash flows for the MBS can be computed and discounted.

```
% Compute the baseline zero rate at each cash flow time
CFlowZero = interp1(ZeroTimes,ZeroRates,CFlowTimes,'linear','extrap');
```

```
% Compute DF for each cash flow time
CFlowDF_Zero = zero2disc(CFlowZero,CFlowDates,Settle);

% Compute the price of the MBS using the zero curve
Price_Zero = CFlowAmounts*CFlowDF_Zero';

% Generate the cash flows for each IR Path
G2PP_CFlowAmounts = mbscfamounts(Settle, ...
    repmat(Maturity,1,nTrials), IssueDate, GrossRate, CouponRate, Delay, []

% Compute the DF for each IR path
G2PP_CFlowDFSim = cumprod(exp(squeeze(-G2PP_SimZeroRates(:,1,:).*stepSize))

% Present value the cash flows for each MBS
G2PP_Price_Ind = sum(G2PP_CFlowAmounts.*G2PP_CFlowDFSim',2);
G2PP_Price = mean(G2PP_Price_Ind);

% Repeat for LMM
LMM_CFlowAmounts = mbscfamounts(Settle, ...
    repmat(Maturity,1,LMMNTRIALS), IssueDate, GrossRate, CouponRate, Delay,

% Present value the cash flows for each MBS
LMM_Price_Ind = sum(LMM_CFlowAmounts.*LMMMonthlyDF',2);
LMM_Price = mean(LMM_Price_Ind);
```

The results from the different approaches can be compared. The number of trials for the G2++ model will typically be less than 100 due to the filtering out of any paths that produce negative interest rates.

Additionally, while the number of trials for the G2++ model in this example is set to be 100, it is often the case that a larger number of simulations need to be run to produce an accurate valuation.

```
fprintf('                     # of Monte Carlo Trials: %8d\n'     , nTrials)
fprintf('                     # of Time Periods/Trial: %8d\n\n'   , nPeriods
fprintf('                      MBS Price with PSA 100: %8.4f\n'   , Price_Ze
fprintf(' MBS Price with Custom G2PP Prepayment Model: %8.4f\n\n', G2PP_Pri
fprintf(' MBS Price with Custom LMM Prepayment Model: %8.4f\n\n', LMM_Price
```

```
                  # of Monte Carlo Trials:       84
                  # of Time Periods/Trial:      156

                    MBS Price with PSA 100:   1.0187
MBS Price with Custom G2PP Prepayment Model:   0.9913

 MBS Price with Custom LMM Prepayment Model:   1.0243
```

#### Conclusion

This example shows how to calibrate and simulate a G2++ interest-rate model and how to use the generated interest-rate paths in a prepayment model loosely based on the Richard and Roll model. This example also provides a useful starting point to using the G2++ and LMM interest-rate models in other financial applications.

#### Bibliography

This example is based on the following books, papers and journal articles:

1. Andersen, L. and V. Piterbarg (2010). Interest Rate Modeling, Atlantic Financial Press.

2. Brigo, D. and F. Mercurio (2001). Interest Rate Models - Theory and Practice with Smile, Inflation and Credit (2nd ed. 2006 ed.). Springer Verlag. ISBN 978-3-540-22149-4.

3. Hayre, L, ed., Salomon Smith Barney Guide to Mortgage-Backed and Asset-Backed Securities. New York: John Wiley & Sons, 2001b

4. Karpishpan, Y., O. Turel, and A. Hasha, Introducing the Citi LMM Term Structure Model for Mortgages, The Journal of Fixed Income, Volume 20 (2010) 44-58.

5. Rebonato, R., K. McKay, and R. White (2010). The Sabr/Libor Market Model: Pricing, Calibration and Hedging for Complex Interest-Rate Derivatives. John Wiley & Sons.

6. Richard, S. F., and R. Roll, 1989, "Prepayments on Fixed Rate Mortgage-Backed Securities" ,Journal of Portfolio Management.

**7** Office of Thrift Supervision, "Net Portfolio Value Model Manual", March 2000.

**8** Stein, H. J., Belikoff, A. L., Levin, K. and Tian, X., Analysis of Mortgage Backed Securities: Before and after the Credit Crisis (January 5, 2007). Credit Risk Frontiers: Subprime Crisis, Pricing and Hedging, CVA, MBS, Ratings, and Liquidity; Bielecki, Tomasz,; Damiano Brigo and Frederic Patras, eds., February 2011. Available at SSRN: http://ssrn.com/abstract=955358

# Pricing Mortgage Backed Securities Using the Black-Derman-Toy Model

This example illustrates how the Financial Toolbox™ and Financial Instruments Toolbox™ can be used to price a level mortgage backed security using the BDT model.

### Load the BDT Tree Stored in the Data File

```
 load mbsexample.mat
```

### Observe the Interest Rate Tree

Visualize the interest rate evolution along the tree by looking at the output structure BDTTree. BDTTree returns an inverse discount tree, which you can convert into an interest rate tree with the cvtree function.

```
BDTTreeR = cvtree(BDTTree);
```

Look at the upper branch and lower branch paths of the tree:

```
OldFormat = get(0, 'format');
format short

%Rate at root node:
RateRoot      = treepath(BDTTreeR.RateTree, [0])

%Rates along upper branch:
RatePathUp    = treepath(BDTTreeR.RateTree, [1 1 1 1 1])

%Rates along lower branch:
RatePathDown = treepath(BDTTreeR.RateTree, [2 2 2 2 2])
```

```
RateRoot =

    0.0399


RatePathUp =
```

```
    0.0399
    0.0397
    0.0391
    0.0383
    0.0373
    0.0360


RatePathDown =

    0.0399
    0.0470
    0.0550
    0.0638
    0.0734
    0.0841
```

**Compute the Price Tree for the Non-Prepayable Mortgage**

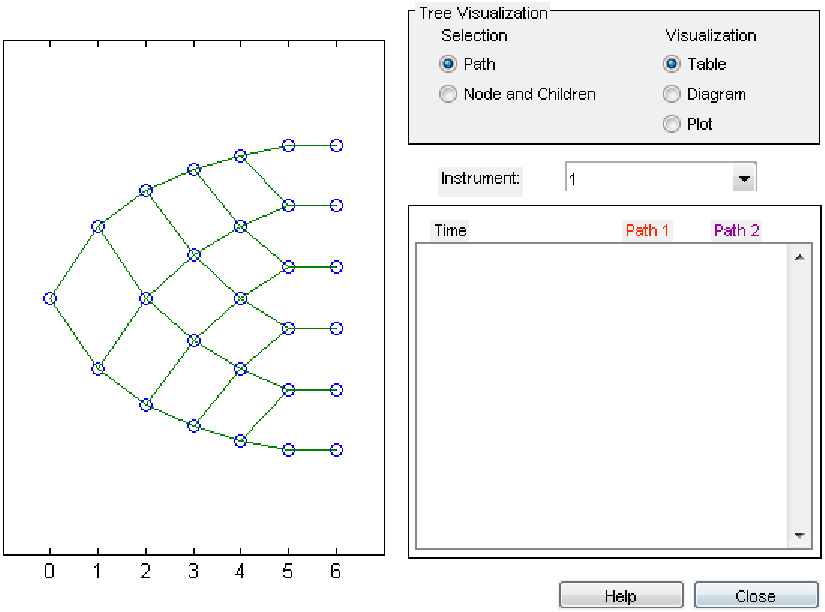
Let's say that we have a 3 year $10000 level prepayable loan, with a mortgage interest rate of 4.64% semi-annually compounded.

```
MortgageAmount = 10000;
CouponRate = 0.0464;
Period = 2;
Settle='01-Jan-2007';
Maturity='01-Jan-2010';
Compounding = BDTTree.TimeSpec.Compounding;

format bank

% Use the function 'amortize.m' in the Financial Toolbox to calculate the m
% payment of the loan (MP), the interest and principal components, and the
% outstanding principal balance.

NumPeriods = date2time(Settle,Maturity, Compounding)';

[Principal, InterestPayment, OutstandingBalance, MP] = amortize(CouponRate/
```

```
% Display Principal, Interest and Outstanding balances
PrincipalAmount = Principal'
InterestPaymentAmount = InterestPayment'
OutstandingBalanceAmount =OutstandingBalance'

CFlowAmounts = MP*ones(1,NumPeriods);
% The CFlowDates are the same as the tree level dates
CFlowDates= {'01-Jul-2007' ,'01-Jan-2008' ,'01-Jul-2008' , '01-Jan-2009' ,

% Calculate the price of the non-prepayable mortgage
[PriceNonPrepayableMortgage, PriceTreeNonPrepayableMortgage] = cfbybdt(BDTT
for iLevel = 2:length(PriceTreeNonPrepayableMortgage.PTree)
    PriceTreeNonPrepayableMortgage.PTree{iLevel}(:,:)= PriceTreeNonPrepayab
end

% Look at the price of the mortgage today (tObs = 0)
PriceNonPrepayableMortgage

% The value of the non-prepayable mortgage is $10017.47. This value exceeds
% the $10000 amount borrowed since the homeowner received not only $10000,
% also a prepayment option.

% Look at the value of the mortgage on the last date, right after the last
% mortgage payment, is zero:
PriceTreeNonPrepayableMortgage.PTree{end};

% Visualize the price tree for the non-prepayable mortgage.
treeviewer(PriceTreeNonPrepayableMortgage)
```

```
PrincipalAmount =

       1572.59
       1609.07
       1646.40
       1684.60
       1723.68
       1763.67
```

```
InterestPaymentAmount =

        232.00
        195.52
        158.19
        119.99
         80.91
         40.92


OutstandingBalanceAmount =

       8427.41
       6818.34
       5171.94
       3487.35
       1763.67
          0.00


PriceNonPrepayableMortgage =

      10017.47
```



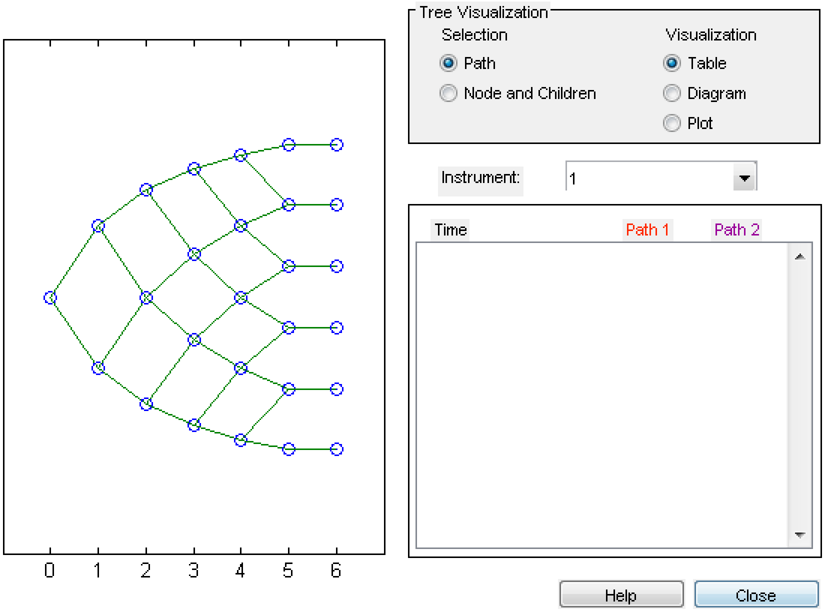
Tree Visualization
Selection
● Path
○ Node and Children
Visualization
● Table
○ Diagram
○ Plot
Instrument: 1

### Compute the Price Tree of the Prepayment Option

```
% The Prepayment option is like a call option on a bond.
%
% The exercise price or strike will be equal to the outstanding principal a
% which has been calculated using the function 'amortize.m'.

OptSpec = 'call';
Strike = [MortgageAmount OutstandingBalance];
ExerciseDates =[Settle CFlowDates];
AmericanOpt = 0;
Maturity = CFlowDates(end);

% Compute the price of the prepayment option:
[PricePrepaymentOption, PriceTreePrepaymentOption] = prepaymentbybdt(BDTTre
                            0, Settle, Maturity,[], [], [], ...
                           [], [], [],  [], 0, [], CFlowAmounts);
```

```
% Look at the price of the prepayment option today (tObs = 0)
PricePrepaymentOption

% The value of the prepayment option is $17.47 as expected.

% Visualize the price tree for the prepayment option
treeviewer(PriceTreePrepaymentOption)
```

```
PricePrepaymentOption =

         17.47
```



● Path
○ Node and Children
● Table
○ Diagram
○ Plot

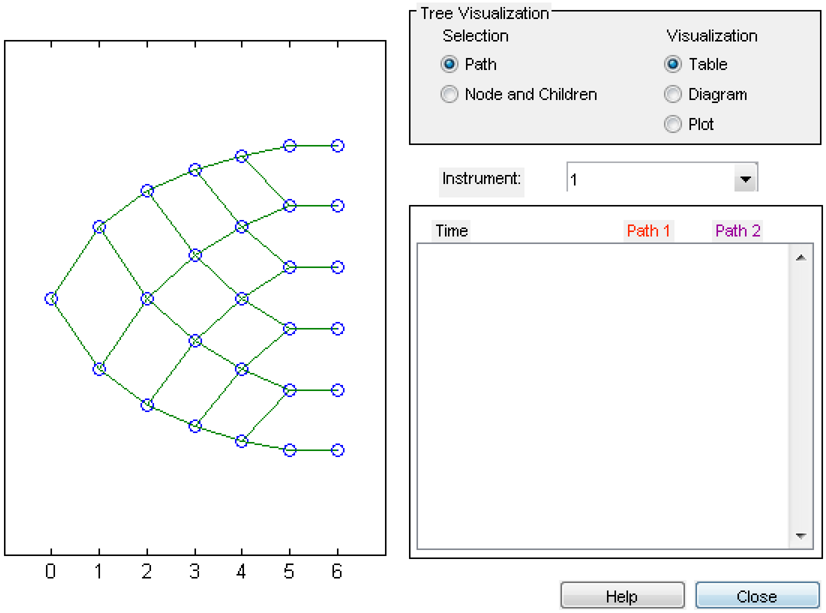
Instrument: 1

### Calculate the Price Tree of the Prepayable Mortgage.

```
% Compute the price of the prepayable mortgage.

PricePrepayableMortgage = PriceNonPrepayableMortgage - PricePrepaymentOptio
```

```
PriceTreePrepayableMortgage = PriceTreeNonPrepayableMortgage;

for iLevel = 1:length(PriceTreeNonPrepayableMortgage.PTree)
    PriceTreePrepayableMortgage.PTree{iLevel}(:,:)= PriceTreeNonPrepayableM
         PriceTreePrepaymentOption.PTree{iLevel}(:,:);
end

% Look at the price of the prepayable mortgage today (tObs = 0)
PricePrepayableMortgage

% The value of the prepayable mortgage is $10000 as expected.

% Visualize the price and price tree for the prepayable mortgage
treeviewer(PriceTreePrepayableMortgage)
set(0, 'format', OldFormat);
```

```
PricePrepayableMortgage =

      10000.00
```



Tree Visualization

Selection

● Path
○ Node and Children

Visualization

● Table
○ Diagram
○ Plot

Instrument: 1

| Time | Path 1 | Path 2 |
|---|---|---|

# Using Collateralized Mortgage Obligations (CMOs)

**In this section...**

## What Are CMOs?

Financial Instruments Toolbox supports collateralized mortgage obligations (CMOs) to provide investors with a greater range of risk and return characteristics than mortgage-backed securities (MBS). In contrast to an MBS, which simply redirects principal and interest cash flows to investors on a pro rata basis, a CMO structures cash flows to different tranches, or slices, to create securities that are better tailored to specific investors.

For example, banks might be primarily concerned with *extension risk*, or the risk that their investment lengthens in time due to increasing interest rates, given that they typically have short-term deposits as liabilities. Insurance companies and pension funds might be concerned primarily with *contraction risk*, or the risk that their investment will pay off too soon, with liabilities that have much longer lives. A CMO structure addresses the interest-rate risk of extension or contraction with a blend of short-term and long-term CMO securities, called tranches.

## Prepayment Risk

Prepayment risk is the risk that the term of the security varies according to differing rates of repayment of principal by borrowers (repayments from refinancings, sales, curtailments, or foreclosures). In a CMO, you can structure the principal (and associated coupon) stream from the underlying mortgage pool collateral to allocate prepayment risk. If principal is prepaid faster than expected (for example, if mortgage rates fall and borrowers refinance), then the overall term of the mortgage pool collateral shortens.

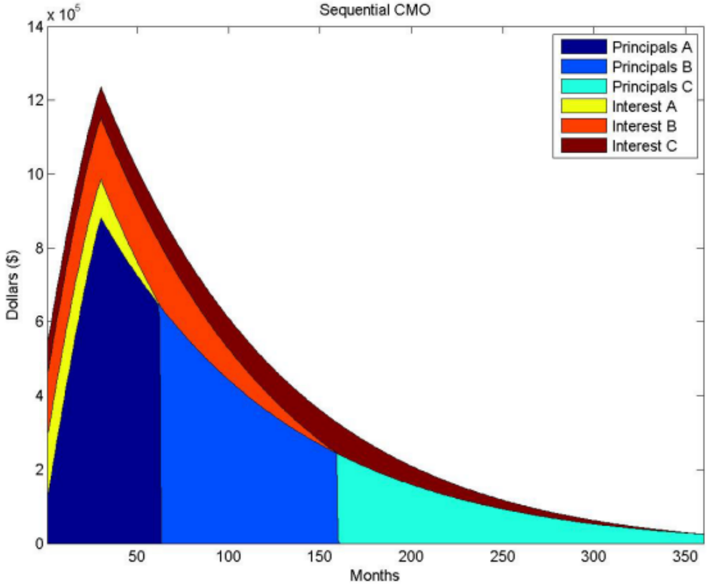
You cannot remove prepayment risk, but you can reallocate it among CMO tranches so that some tranches have some protection against this risk, and other tranches will absorb more of this risk. To facilitate this allocation of prepayment risk, CMOs are structured such that prepayments are allocated among tranches using a fixed set of rules. The most common schemes for prepayment tranching are:

- Sequential tranching, with or without, Z-bond tranching
- Schedule bond tranching
  - Planned amortization class (PAC) bonds
  - Target amortization class (TAC) bonds

Financial Instruments Toolbox supports these schemes for prepayment tranching for CMOs and tools for pricing and scheduling cash flows between the tranches, as well as analyzing the price and yield for CMOs. Financial Instruments Toolbox functionality for CMOs does not model credit risk. Therefore, this functionality is most appropriate for CMOs where credit risk is not an issue (for example, agency CMOs where the underlying mortgage pool collateral is insured for default by the agency Government-Sponsored Enterprises (GSEs), such as Fannie Mae and Freddie Mac).

### Sequential Tranches Without a Z-Bond

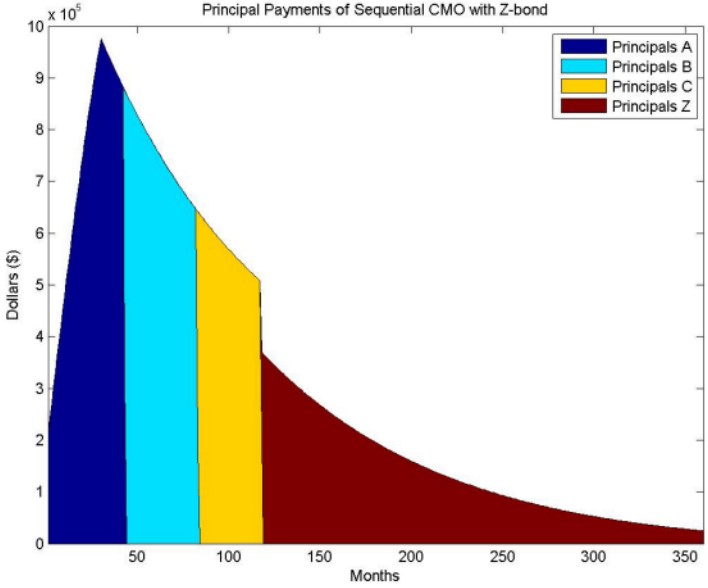
All available principal and interest payments go to the first sequential tranche, until its balance decrements to zero, then to the second, and so on. For example, consider the following example where all principal and interest from the underlying mortgage pool is repaid on tranche A first, then tranche B, then tranche C. Note that interest is paid on each tranche as long as the principal for the tranche has not been retired.



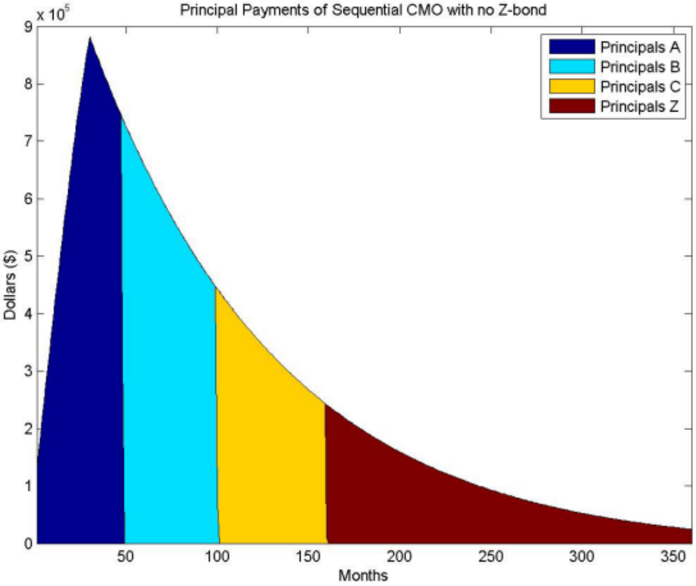
## Sequential Tranches With a Z-Bond

The Z-bond, also called an accural bond, is a type of interest and principal pay rule. The Z-bond tranche supports other sequential pay tranches by not receiving an interest payment. The interest payment that would have accrued to the Z-bond tranche pays off the principal of other bonds, and the principal of the Z-bond tranche increases. The Z-bond tranche starts receiving interest and principal payments only after the other tranches in the CMO have been fully paid. The Z-bond tranche is used in a sequential-pay structure to accelerate the principal repayments of the sequential-pay bonds.

A Z-bond differs from other CMO instruments because it is not tranching principal but interest. The Z-bond receives no cash flows until all other securities have been paid off. In the interim, the interest that is owed to the Z-bond is accrued to its principal. The following chart demonstrates the difference between a Z-bond and a normal sequential pay tranche. Note that the C tranche pays off sooner with the Z-bond, because the interest cash flows to the Z-bond are being used to pay down the principal of the C tranche.



For comparison, the following graphic is the same sequential CMO with no Z-bond.



## PAC Tranches

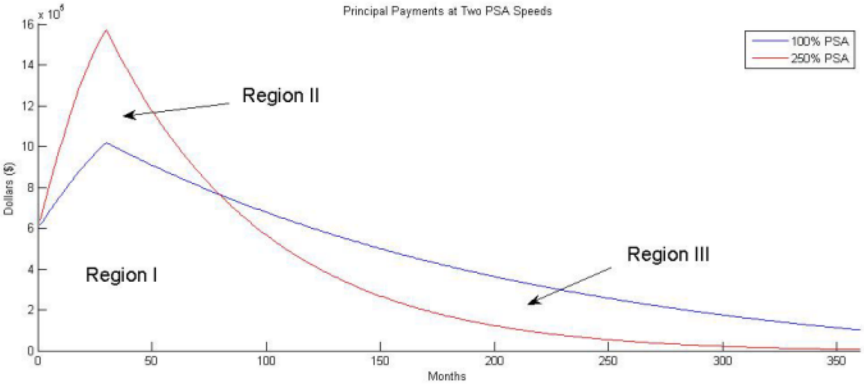
Planned amortization class (PAC) bonds help reduce the effects of prepayment risk. They are designed to produce more stable cash flows by redirecting prepayments from the underlying mortgage collateral to other classes (tranches) called companion or support classes. PAC bonds have a principal payment rate over a predetermined period of time. The PAC bond payment schedule is determined by two different prepayment rates, which together form a band (also called a collar). Early in the life of the CMO, the prepayment at the lower PSA yields a lower prepayment. Later in its life, the principal in the higher PSA declines enough that it yields a lower prepayment. The PAC tranche receives whichever rate is lower, so it will change prepayment at one PSA for the first part of its life, then switch to the other rate. The ability to stay on this schedule is maintained by a support bond, which absorbs excess prepayments, and receives less prepayments to prevent extension of average life.

However, the PAC is only protected from extension to the amount that prepayments are made on the underlying MBSs. If there is a sustained period of fast prepayments, then that might completely eliminate a PAC bond's outstanding support class. When the principal of the associated PAC bond is exhausted, the CMO is called a "busted PAC", or "busted collar". Alternatively, in times of slow prepayments, amortization of the support bonds is delayed if there is not enough principal for the currently paying PAC bond. This extends the average life of the class.

A PAC bond protects against both extension and contraction risk by:

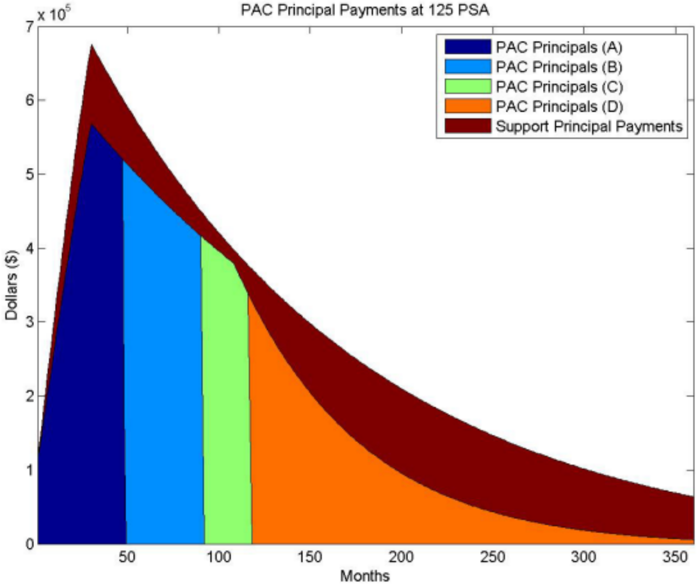
- Specifying a schedule of principal payments for the PAC bond
- Including support tranches that are allocated prepayments inside a specified prepayment band

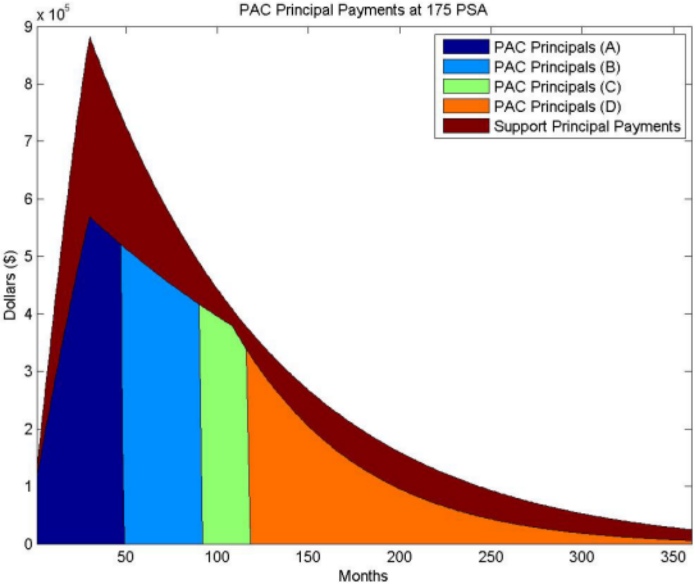
PAC bonds typically specify a band expressed using the PSA model. A PAC bond with a range of 100 to 250% has this principal schedule.

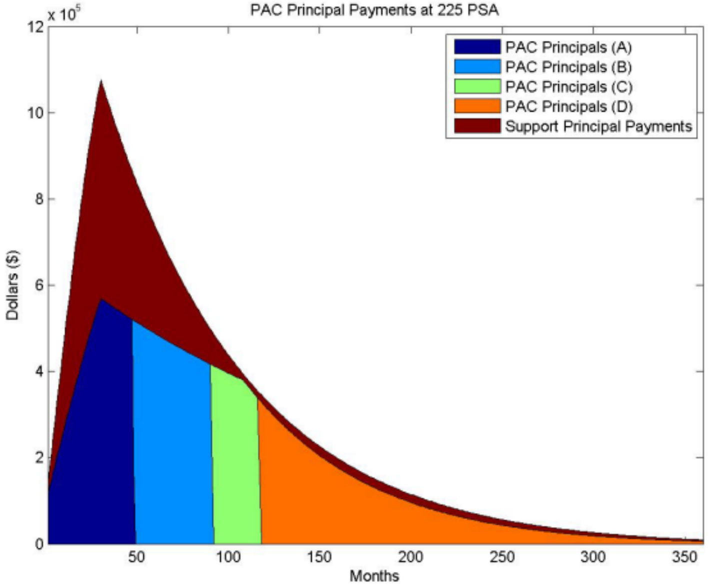


The principal repayment schedule is the minimum principal payment as Region 1 shows. Note that this is the principal payment schedule as long as the actual prepayment stays within the prepayment band of 100 to 250% PSA.

For example, for different prepayment speeds of 125%, 175%, and 225% PSA, the actual principal payments are shown in the following graphs. Note that at higher prepayment speeds, the support tranche is allocated principal earlier while the principal timing for the other tranches remains constant.


PAC Principal Payments at 125 PSA
x 10^5
7
6
5
4
3
2
1
0
Dollars ($)
50
100
150
200
250
300
350
Months
PAC Principals (A)
PAC Principals (B)
PAC Principals (C)
PAC Principals (D)
Support Principal Payments

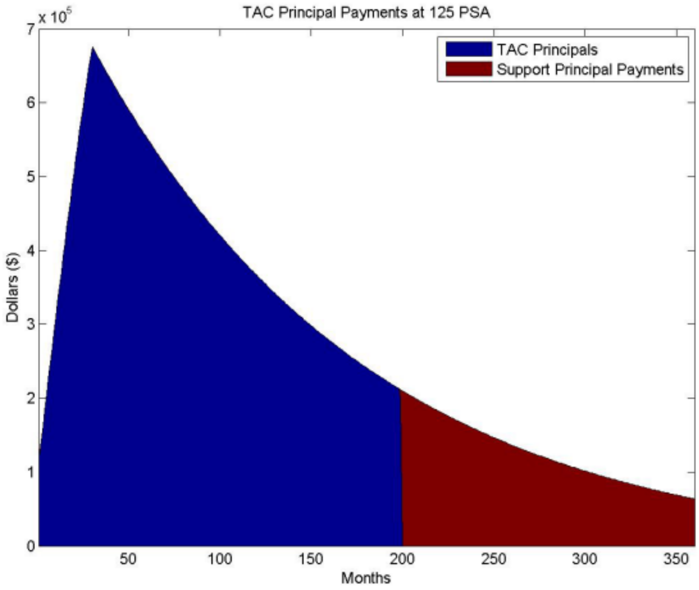

PAC Principal Payments at 175 PSA
x 10^5
9
8
7
6
5
4
3
2
1
0
Dollars ($)
50
100
150
200
250
300
350
Months
PAC Principals (A)
PAC Principals (B)
PAC Principals (C)
PAC Principals (D)
Support Principal Payments

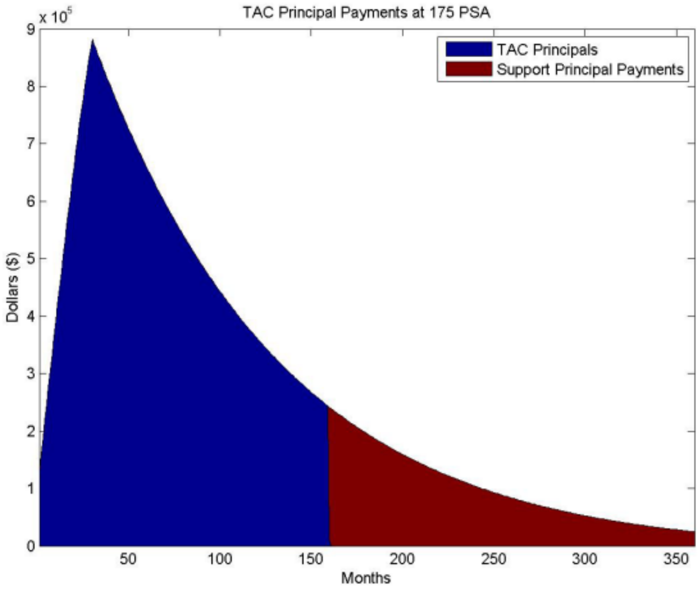


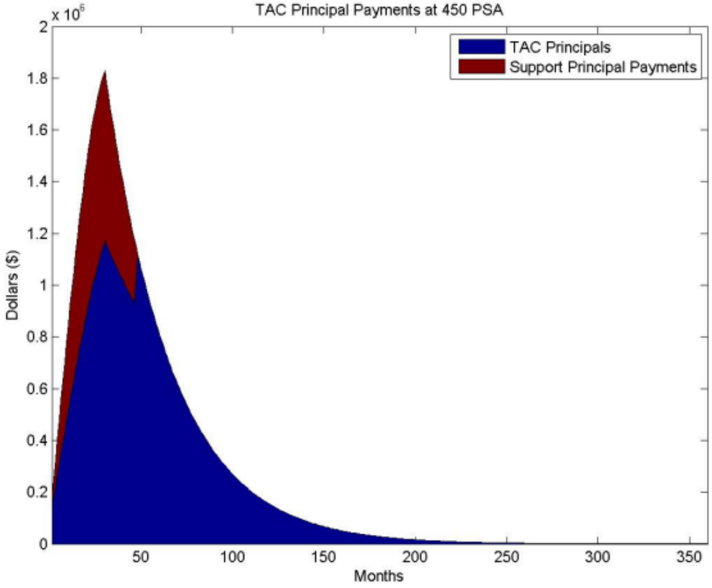
## TAC Tranches

Target amortization class (TAC) bonds are similar to PAC bonds, but they do not provide protection against extension of average life. Create the schedule of principal payments by using just a single PSA. TAC bonds pay a “targeted” principal payment schedule at a single, constant prepayment speed. As long as the underlying mortgage collateral does not prepay at a rate slower than this speed, the TAC bond payment schedule is met. TAC bonds can protect against increasing prepayments and early retirement of the TAC bond investment. If the principal cash flow from the mortgage collateral exceeds the TAC schedule, the excess is allocated to TAC companion (support) classes. Alternatively, if prepayments fall below the speed necessary to maintain the TAC schedule, the weighted average life of the TAC is extended. The TAC bond does not protect against low prepayment rates.

For example, here is a TAC structure rated for 125%, 175%, and 450% PSA.


TAC Principal Payments at 125 PSA
x 10^5
7
6
5
4
3
2
1
0
Dollars ($)
50
100
150
200
250
300
350
Months
TAC Principals
Support Principal Payments


TAC Principal Payments at 175 PSA
x 10^5
9
8
7
6
5
4
3
2
1
0
Dollars ($)
50
100
150
200
250
300
350
Months
TAC Principals
Support Principal Payments



Note that for prepayments below 175% PSA, the TAC bond extends like a normal sequential pay CMO. TAC bonds are appealing because they offer higher yields than comparable PAC bonds. The unaddressed risk from low prepayment rates generally does not concern investors as much as risk from high prepayment rates.

## CMO Workflow

In general, the CMO workflow is:

1. Calculate underlying mortgage cash flows.
2. Define CMO tranches
3. If using a PAC or TAC CMO, calculate the principal schedule.
4. Calculate cash flows for each tranche.
5. Analyze the CMO by computing price, yield, spread of CMO cash flows.

### Calculate Underlying Mortgage Cash Flows

Underlying mortgage pool pass-through cash flows are calculated by the existing function `mbspassthrough`. The CMO cash flow functions require the principal payments (including prepayments) calculated from existing functions `mbspassthrough` or `mbscfamounts`.

```
principal = 10000000;
coupon = 0.06;
terms = 360;
psa = 150;

[principal_balance, monthly_payments, sched_principal_payments,...
interest_payments, prepayments] = mbspassthrough(principal,...
coupon, terms, terms, psa, []);

principal_payments = sched_principal_payments.' + prepayments.';
```

After determining principal payments for the underlying mortgage collateral, you can generate cash flows for a sequential CMO, with or without a Z-bond, by using `cmoseqcf`. For a PAC or TAC CMO, the cash flows are generated using `cmoschedcf`

### Define CMO Tranches

Define CMO tranche; for example, define a CMO with 2 tranches:

```
TranchePrincipals = [500000; 500000];
TrancheCoupons = [0.06; 0.06];
```

### If Using a PAC or TAC CMO, Calculate Principal Schedule

Calculate the PAC/TAC principal balance schedule based on a band of PSA speeds. For scheduled CMOs (PAC/TAC), the CMO cash flow functions additionally take in the principal balance schedule calculated by the CMO schedule function `cmosched`.

```
speed = [100 300];
[balanceSchedule, initialBalance] = cmosched(principal, coupon,...
terms, terms, speed, TranchePrincipals(1));
```

## Calculate Cash Flows for Each Tranche

You can reuse the output from the cash flow generation functions to further divide the cash flows into tranches. For example, the output from `cmoschedcf` for a PAC tranche can be divided into sequential tranches by passing the principal cash flows of the PAC tranche into the `cmoschedcf` function. The output of the CMO cash flow functions are the principal and interest cash flows, as well as the principal balance.

```
[principal_balances, principal_cashflows, interest_cashflows] = cmoschedcf(principal_payments,...
TranchePrincipals, TrancheCoupons, balanceSchedule);
```

## Analyze CMO by Computing Price, Yield, and Spread of CMO Cash Flows

The outputs from the CMO functions (`cmoseqcf` and `cmoschedcf`) are cash flows. The functions used to analyze a CMO are based on these cash flows. To that end, you can use `cfbyzero`, `cfspread`, `cfyield`, and `cfprice` to compute prices, yield, and spreads for the CMO cash flows. In addition, using the following, you can calculate a weighted average life (WAL) for each tranche in the CMO:

$$WAL = \sum_{i=1}^{n} \frac{P_i}{P} t_i$$

where:

$P$ is the total principal.

$P_i$ is the principal repayment of the coupon $i$.

$\frac{P_i}{P}$ is the fraction of the principal repaid in coupon $i$.

$t_i$ is the time in years from the start to coupon $i$.

**See Also** `cmoseqcf` | `cmosched` | `cmoschedcf` | `mbscfamounts` | `mbspassthrough`

**Related Examples**

- "Create PAC and Sequential CMO" on page 5-65
- "Fixed-Rate Mortgage Pool" on page 5-3

**Concepts**

- "What Are Mortgage-Backed Securities?" on page 5-2

# Create PAC and Sequential CMO

This example shows how to use an underlying mortgage-backed security (MBS) pool for a 30-year fixed-rate mortgage of 6% to define a PAC bond, and then define a sequential CMO from the PAC bond. Analyze the CMO by comparing the CMO spread to a zero-rate curve for a 30-year Treasury bond and then calculate the weighted-average life (WAL) for the PAC bond.

**Step 1. Define the underlying mortgage pool.**

```
principal = 100000000;
grossrate = 0.06;
coupon = 0.05;
originalTerm = 360;
termRemaining = 360;
speed = 100;
delay = 14;

Settle      = datenum('1-Jan-2011');
IssueDate   = datenum('1-Jan-2011');
Maturity    = addtodate(IssueDate, 360, 'month');
```

**Step 2. Calculate underlying pool cash flow.**

```
[CFlowAmounts, CFlowDates, ~, ~, ~, UnitPrincipal, UnitInterest, ...
UnitPrepayment] = mbscfamounts(Settle, Maturity, IssueDate, grossrate, ...
coupon, delay, speed, []);
```

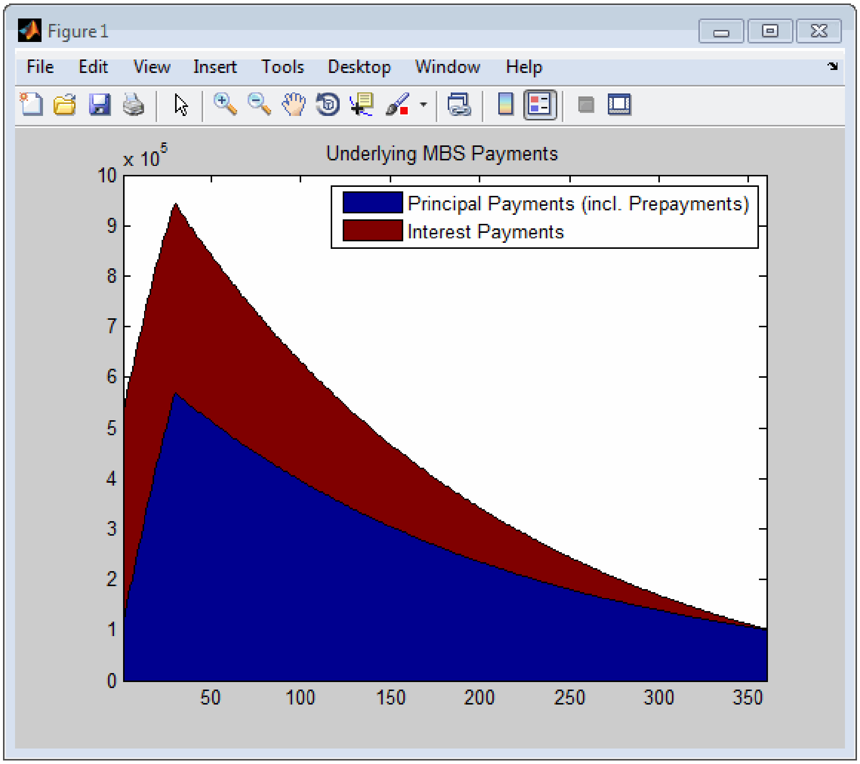
**Step 3. Calculate prepayments.**

```
principalPayments = UnitPrincipal * principal;
netInterest = UnitInterest * principal;
prepayments = UnitPrepayment * principal;
dates = CFlowDates' + delay;
```

Generate a plot for the underlying MBS payments:

```
area([principalPayments'+prepayments', netInterest'])
title('Underlying MBS Payments');
```

```
legend('Principal Payments (incl. Prepayments)', 'Interest Payments')
```

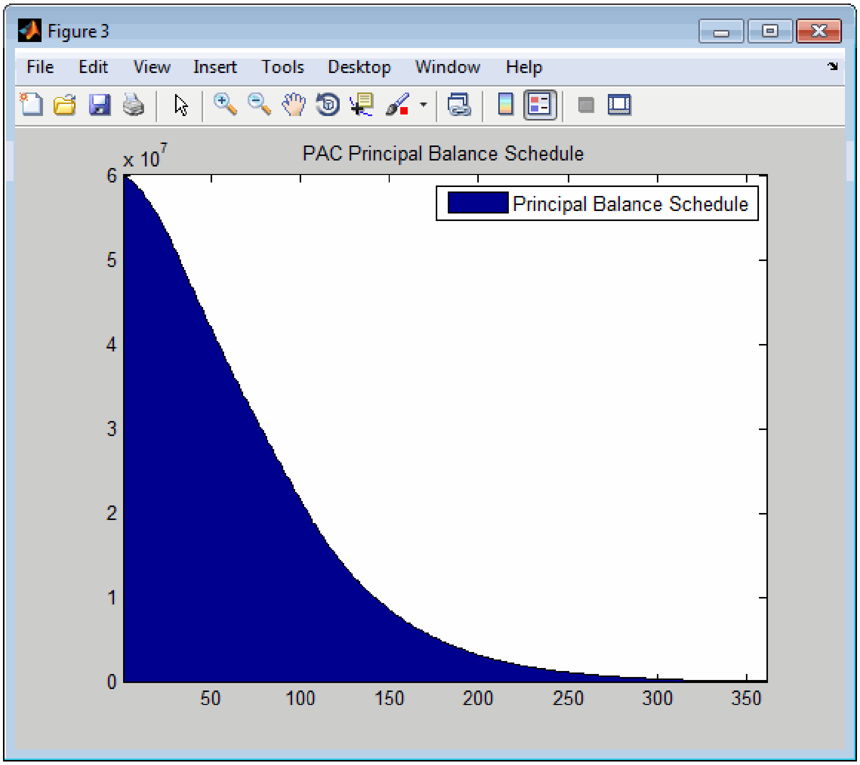


**Step 4. Calculate the PAC schedule.**

```
pacSpeed = [80 300];
[balanceSchedule, pacInitBalance] = ...
cmosched(principal, grossrate, originalTerm, termRemaining, ...
pacSpeed, []);
```

Generate a plot for the PAC principal balance schedule:

```
figure;
area([pacInitBalance'; balanceSchedule'])
title('PAC Principal Balance Schedule');
legend('Principal Balance Schedule');
```

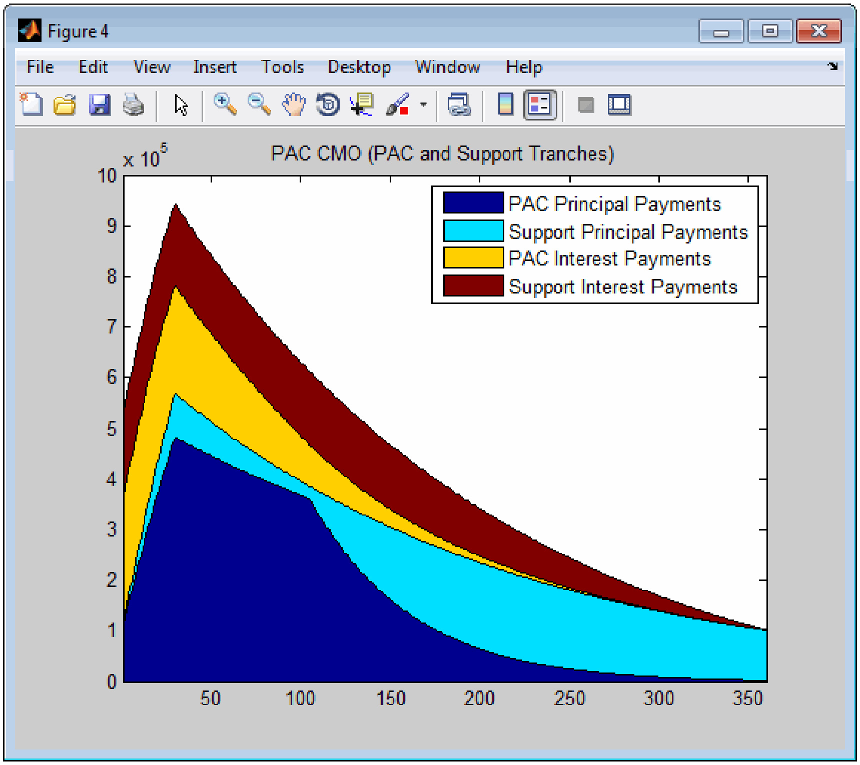


**Step 5. Calculate PAC cash flow.**

```
pacTranchePrincipals = [pacInitBalance; principal-pacInitBalance];
pacTrancheCoupons = [0.05; 0.05];
[pacBalances, pacPrincipals, pacInterests] = ...
cmoschedcf(principalPayments+prepayments, ...
pacTranchePrincipals, pacTrancheCoupons, balanceSchedule);
```

Generate a plot for the PAC CMO tranches:

```
figure;
area([pacPrincipals' pacInterests']);
title('PAC CMO (PAC and Support Tranches)');
legend('PAC Principal Payments', 'Support Principal Payments', ...
'PAC Interest Payments', 'Support Interest Payments');
```



**Step 6. Create sequential CMO from the PAC bond.**

```
% CMO tranches, A, B, C, and D
seqTranchePrincipals = ...
[20000000; 20000000; 10000000; pacInitBalance-50000000];
seqTrancheCoupons = [0.05; 0.05; 0.05; 0.05];
```

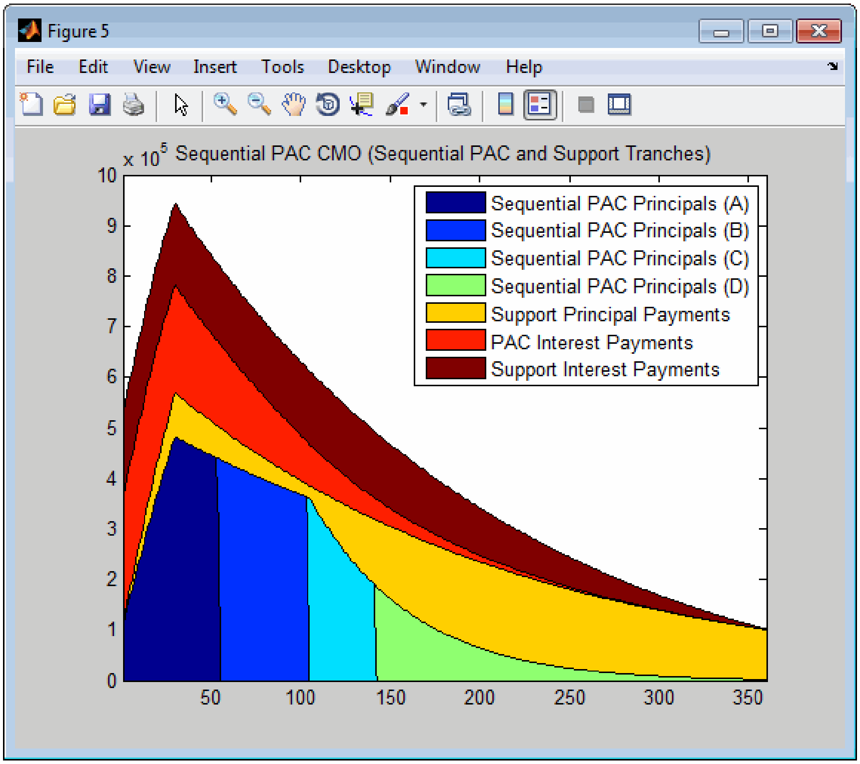
```
seqTrancheCoupons =

    0.0500
    0.0500
    0.0500
    0.0500
```

**Step 7. Calculate cash flows for each tranche.**

```
[seqBalances, seqPrincipals, seqInterests] = ...
cmoseqcf(pacPrincipals(1, :), seqTranchePrincipals, ...
seqTrancheCoupons, false);
```

Generate a plot for the sequential PAC CMO:

```
figure
area([seqPrincipals' pacPrincipals(2, :)' pacInterests']);
title('Sequential PAC CMO (Sequential PAC and Support Tranches)');
legend('Sequential PAC Principals (A)', 'Sequential PAC Principals (B)', ...
'Sequential PAC Principals (C)', 'Sequential PAC Principals (D)', ...
'Support Principal Payments', 'PAC Interest Payments', ...
'Support Interest Payments');
```



**Step 8. Create the discount curve.**

```
CurveSettle = datenum('1-Jan-2011');
ZeroRates = [0.01 0.03 0.10 0.19 0.45 0.81 1.76 2.50 3.18 4.09 4.38]'/100;
CurveTimes = [1/12 3/12 6/12 1 2 3 5 7 10 20 30]';
CurveDates = daysadd(CurveSettle, 360 * CurveTimes, 1);
zeroCurve = intenvset('Rates', ZeroRates, 'StartDates', CurveSettle, ...
'EndDates', CurveDates);
```

```
zeroCurve =

           FinObj: 'RateSpec'
      Compounding: 2
             Disc: [11x1 double]
            Rates: [11x1 double]
         EndTimes: [11x1 double]
       StartTimes: [11x1 double]
         EndDates: [11x1 double]
       StartDates: 734504
    ValuationDate: 734504
            Basis: 0
     EndMonthRule: 1
```

### Step 9. Price the CMO cash flows.

The cash flow for the sequential PAC principal A tranche is calculated using the cash flow functions `cfbyzero`, `cfyield`, `cfprice`, and `cfspread`.

```
cflows = seqPrincipals(1, :)+seqInterests(1, :);
cfdates = dates(2:end)';
price1 = cfbyzero(zeroCurve, cflows, cfdates, Settle, 4)
yield = cfyield(cflows, cfdates, price1, Settle, 'Basis', 4)
price2 = cfprice(cflows, cfdates, yield, Settle, 'Basis', 4)
spread = cfspread(zeroCurve, price2, cflows, cfdates, Settle, 'Basis', 4)
WAL = sum(cflows .* yearfrac(Settle, cfdates, 4)) / sum(cflows)
```

```
price1 =

   2.2109e+07


yield =
```

```
    0.0090


price2 =

   2.2109e+07


spread =

  -3.4093e-13


WAL =

    2.5408
```

The weighted average life (WAL) for the sequential PAC principal A tranche is 2.54 years.

## See Also

`cmoseqcf | cmosched | cmoschedcf | mbscfamounts`

## Related Examples

- “Fixed-Rate Mortgage Pool” on page 5-3

## Concepts

- “Using Collateralized Mortgage Obligations (CMOs)” on page 5-50

# Debt Instruments

# Agency Option-Adjusted Spreads

Often bonds are issued with embedded options, which then makes standard price/yield or spread measures irrelevant. For example, a municipality concerned about the chance that interest rates may fall in the future might issue bonds with a provision that allows the bond to be repaid before the bond's maturity. This is a call option on the bond and must be incorporated into the valuation of the bond. Option-adjusted spread (OAS), which adjusts a bond spread for the value of the option, is the standard measure for valuing bonds with embedded options. Financial Instruments Toolbox software supports computing option-adjusted spreads for bonds with single embedded options using the agency model.

The Securities Industry and Financial Markets Association (SIFMA) has a simplified approach to compute OAS for agency issues (Government Sponsored Entities like Fannie Mae and Freddie Mac) termed "Agency OAS". In this approach, the bond has only one call date (European call) and uses Black's model (a variation on Black Scholes, `http://en.wikipedia.org/wiki/Black_model`) to value the bond option. The price of the bond is computed as follows:

$$\mathtt{Price}_{\text{Callable}} = \mathtt{Price}_{\text{NonCallable}} - \mathtt{Price}_{\text{Option}}$$

where

$\mathtt{Price}_{\text{Callable}}$ is the price of the callable bond.

$\mathtt{Price}_{\text{NonCallable}}$ is the price of the noncallable bond, i.e., price of the bond using `bndspread`.

$\mathtt{Price}_{\text{Option}}$ is the price of the option, i.e., price of the option using Black's model.

The Agency OAS is the spread, when used in the previous formula, yields the market price. Financial Instruments Toolbox software supports these functions:

**(Continued)**

| Agency OAS Functions | Purpose |
|---|---|
| `agencyoas` | Compute the OAS of the callable bond using the Agency OAS model. |
| `agencyprice` | Price the callable bond OAS using Agency using the OAS model. |

## Computing the Agency OAS for Bonds

To compute the Agency OAS using `agencyoas`, you must provide the zero curve as the input `ZeroData`. You can specify the zero curve in any intervals and with any compounding method. You can do this using Financial Toolbox™ functions `zbtprice` and `zbtyield`. Or, you can use `IRDataCurve` to construct an `IRDataCurve` object, and then use the `getZeroRates` to convert to dates and data for use in the `ZeroData` input.

After creating the `ZeroData` input for `agencyoas`, you can then:

1. Assign parameters for `CouponRate`, `Settle`, `Maturity`, `Vol`, `CallDate`, and `Price`.

2. Compute the option-adjusted spread using `agencyoas` to derive the `OAS` output.

   If you have the Agency OAS for the callable bond, you can use the `OAS` value as an input to `agencyprice` to determine the price for a callable bond.

In the following example, the Agency OAS is computed using `agencyoas` for a range of bond prices and the spread of an identically priced noncallable bond is calculated using `bndspread`.

```
%% Data
% Bond data -- note that there is only 1 call date
Settle = datenum('20-Jan-2010');
Maturity = datenum('30-Dec-2013');
Coupon = .022;
Vol = .5117;
```

```
CallDate = datenum('30-Dec-2010');
Period = 2;
Basis = 1;
Face = 100;

% Zero Curve data
ZeroTime = [.25 .5 1 2 3 4 5 7 10 20 30]';
ZeroDates = daysadd(Settle,360*ZeroTime,1);
ZeroRates = [.0008 .0017 .0045 .0102 .0169 .0224 .0274 .0347 .0414 .0530 .0740]';
ZeroData = [ZeroDates ZeroRates];
CurveCompounding = 2;
CurveBasis = 1;

Price = 94:104;
OAS = agencyoas(ZeroData, Price', Coupon, Settle,Maturity, Vol, CallDate,'Basis',Basis)
Spread = bndspread(ZeroData, Price', Coupon, Settle, Maturity)
plot(OAS,Price)
hold on
plot(Spread,Price,'r')
xlabel('Spread (bp)')
ylabel('Price')
title('AOAS and Spread for an Agency and Equivalent Noncallable Bond')
legend({'Callable Issue','Noncallable Issue'})

OAS =

  163.4942
  133.7306
  103.8735
   73.7505
   43.1094
   11.5608
  -21.5412
  -57.3869
  -98.5675
 -152.5226
 -239.6462


Spread =
```

```
 168.1412
 139.7047
 111.6123
  83.8561
  56.4286
  29.3227
   2.5314
 -23.9523
 -50.1348
 -76.0226
-101.6218
```

The following plot demonstrates as the price increases, the value of the embedded option in the Agency issue increases, and the value of the issue itself does not increase as much as it would for a noncallable bond, illustrating the negative convexity of this issue:

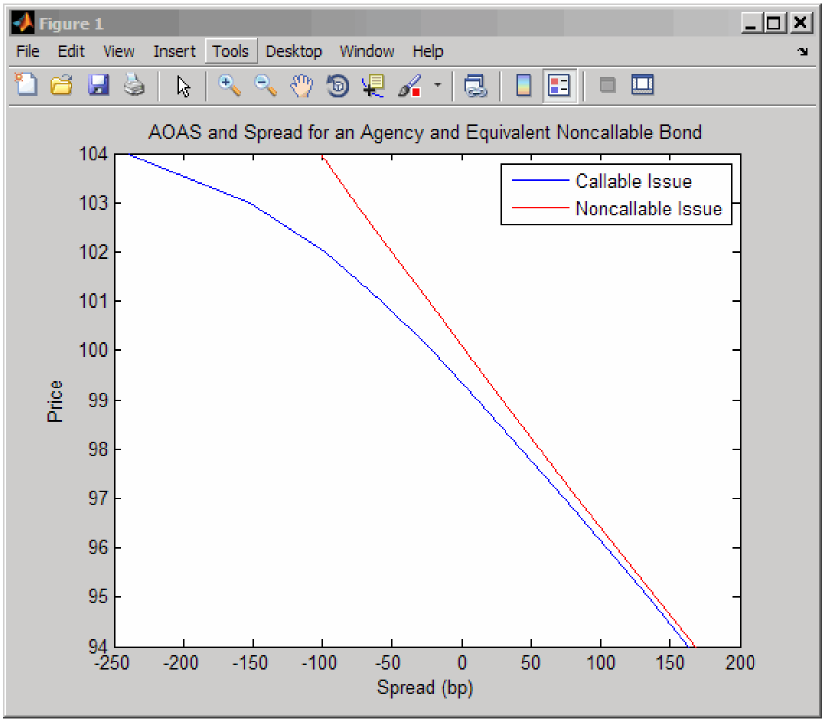


Figure 1
File Edit View Insert Tools Desktop Window Help
AOAS and Spread for an Agency and Equivalent Noncallable Bond
Callable Issue
Noncallable Issue
Price
104
103
102
101
100
99
98
97
96
95
94
-250
-200
-150
-100
-50
0
50
100
150
200
Spread (bp)

# Treasury Bills Defined

Treasury bills are short-term securities (issued with maturities of 1 year or less) sold by the United States Treasury. Sales of these securities are frequent, usually weekly. From time to time, the Treasury also offers longer duration securities called Treasury notes and Treasury bonds.

A Treasury bill is a discount security. The holder of the Treasury bill does not receive periodic interest payments. Instead, at the time of sale, a percentage discount is applied to the face value. At maturity, the holder redeems the bill for full face value.

The basis for Treasury bill interest calculation is actual/360. Under this system, interest accrues on the actual number of elapsed days between purchase and maturity, and each year contains 360 days.

# Computing Treasury Bill Price and Yield

| In this section... |
|---|
| "Introduction" on page 6-8 |
| "Treasury Bill Repurchase Agreements" on page 6-8 |
| "Treasury Bill Yields" on page 6-10 |

## Introduction

Financial Instruments Toolbox software provides the following suite of functions for computing price and yield on Treasury bills.

### Treasury Bill Functions

| Function | Purpose |
|---|---|
| `tbilldisc2yield` | Convert discount rate to yield. |
| `tbillprice` | Price Treasury bill given its yield or discount rate. |
| `tbillrepo` | Break-even discount of repurchase agreement. |
| `tbillyield` | Yield and discount of Treasury bill given its price. |
| `tbillyield2disc` | Convert yield to discount rate. |
| `tbillval01` | The value of 1 basis point given the characteristics of the Treasury bill, as represented by its settlement and maturity dates. You can relate the basis point to discount, money-market, or bond-equivalent yield. |

For all functions with yield in the computation, you can specify yield as money-market or bond-equivalent yield. The functions all assume a face value of $100 for each Treasury bill.

## Treasury Bill Repurchase Agreements

The following example shows how to compute the break-even discount rate. This is the rate that correctly prices the Treasury bill such that the profit from selling the tail equals 0.

```
Maturity = '26-Dec-2002';
InitialDiscount = 0.0161;
PurchaseDate = '26-Sep-2002';
SaleDate = '26-Oct-2002';
RepoRate = 0.0149;

BreakevenDiscount = tbillrepo(RepoRate, InitialDiscount, ...
PurchaseDate, SaleDate, Maturity)

BreakevenDiscount =

    0.0167
```

You can check the result of this computation by examining the cash flows in and out from the repurchase transaction. First compute the price of the Treasury bill on the purchase date (September 26).

```
PriceOnPurchaseDate = tbillprice(InitialDiscount, ...
PurchaseDate, Maturity, 3)

PriceOnPurchaseDate =

   99.5930
```

Next compute the interest due on the repurchase agreement.

```
RepoInterest = ...
RepoRate*PriceOnPurchaseDate*days360(PurchaseDate,SaleDate)/360

RepoInterest =

    0.1237
```

`RepoInterest` for a 1.49% 30-day term repurchase agreement (30/360 basis) is 0.1237.

Finally, compute the price of the Treasury bill on the sale date (October 26).

```
PriceOnSaleDate = tbillprice(BreakevenDiscount, SaleDate, ...
Maturity, 3)
```

```
PriceOnSaleDate =

   99.7167
```

Examining the cash flows, observe that the break-even discount causes the sum of the price on the purchase date plus the accrued 30-day interest to be equal to the price on sale date. The next table shows the cash flows.

**Cash Flows from Repurchase Agreement**

| Date | Cash Out Flow | | Cash In Flow | |
|---|---|---|---|---|
| 9/26/2002 | Purchase T-bill | 99.593 | Repo money | 99.593 |
| 10/26/2002 | Payment of repo | 99.593 | Sell T-bill | 99.7168 |
| | Repo interest | 0.1238 | | |
| | Total | 199.3098 | | 199.3098 |

## Treasury Bill Yields

Using the same data as before, you can examine the money-market and bond-equivalent yields of the Treasury bill at the time of purchase and sale. The function `tbilldisc2yield` can perform both computations at one time.

```
Maturity = '26-Dec-2002';
InitialDiscount = 0.0161;
PurchaseDate = '26-Sep-2002';
SaleDate = '26-Oct-2002';
RepoRate = 0.0149;
BreakevenDiscount = tbillrepo(RepoRate, InitialDiscount, ...
PurchaseDate, SaleDate, Maturity)

[BEYield, MMYield] = ...
tbilldisc2yield([InitialDiscount; BreakevenDiscount], ...
[PurchaseDate; SaleDate], Maturity)

BreakevenDiscount =
```

```
          0.0167


      BEYield =

          0.0164
          0.0170


      MMYield =

          0.0162
          0.0168
```

For the short Treasury bill (fewer than 182 days to maturity), the money-market yield is 360/365 of the bond-equivalent yield, as this example shows.

# Using Zero-Coupon Bonds

**In this section...**

## Introduction

A zero-coupon bond is a corporate, Treasury, or municipal debt instrument that pays no periodic interest. Typically, the bond is redeemed at maturity for its full face value. It will be a security issued at a discount from its face value, or it may be a coupon bond stripped of its coupons and repackaged as a zero-coupon bond.

Financial Instruments Toolbox software provides functions for valuing zero-coupon debt instruments. These functions supplement existing coupon bond functions such as `bndprice` and `bndyield` that are available in Financial Toolbox software.

## Measuring Zero-Coupon Bond Function Quality

Zero-coupon function quality is measured by how consistent the results are with coupon-bearing bonds. Because the zero coupon's yield is bond-equivalent, comparisons with coupon-bearing bonds are possible.

In the textbook case, where time ($t$) is measured continuously and the rate ($r$) is continuously compounded, the value of a zero bond is the principal multiplied by $e^{-rt}$. In reality, the rate quoted is continuous and the basis can be variable, requiring a more consistent approach to meet the stricter demands of accurate pricing.

The following two examples

- "Pricing Treasury Notes" on page 6-13
- "Pricing Corporate Bonds" on page 6-15

show how the zero functions are consistent with supported coupon bond functions.

## Pricing Treasury Notes

A Treasury note can be considered to be a package of zeros. The toolbox functions that price zeros require a coupon bond equivalent yield. That yield can originate from any type of coupon paying bond, with any periodic payment, or any accrual basis. The next example shows the use of the toolbox to price a Treasury note and compares the calculated price with the actual price quotation for that day.

```
Settle = datenum('02-03-2003');
MaturityCpn = datenum('05-15-2009');
Period = 2;
Basis = 0;

% Quoted yield.
QYield = 0.03342;

% Quoted price.
QPriceACT = 112.127;

CouponRate = 0.055;
```

Extract the cash flow and compute price from the sum of zeros discounted.

```
[CFlows, CDates] = cfamounts(CouponRate, Settle, MaturityCpn, ...
Period, Basis);
MaturityofZeros = CDates;
```

Compute the price of the coupon bond identically as a collection of zeros by multiplying the discount factors to the corresponding cash flows.

```
PriceofZeros = CFlows * zeroprice(QYield, Settle, ...
MaturityofZeros, Period, Basis)/100;
```

The following table shows the intermediate calculations.

| Cash Flows | Discount Factors | Discounted Cash Flows |
|---|---|---|
| -1.2155 | 1.0000 | -1.2155 |
| 2.7500 | 0.9908 | 2.7246 |
| 2.7500 | 0.9745 | 2.6799 |
| 2.7500 | 0.9585 | 2.6359 |
| 2.7500 | 0.9427 | 2.5925 |
| 2.7500 | 0.9272 | 2.5499 |
| 2.7500 | 0.9120 | 2.5080 |
| 2.7500 | 0.8970 | 2.4668 |
| 2.7500 | 0.8823 | 2.4263 |
| 2.7500 | 0.8678 | 2.3864 |
| 2.7500 | 0.8535 | 2.3472 |
| 2.7500 | 0.8395 | 2.3086 |
| 2.7500 | 0.8257 | 2.2706 |
| 102.7500 | 0.8121 | 83.4451 |
|  | Total | 112.1263 |

Compare the quoted price and the calculated price based on zeros.

```
[QPriceACT PriceofZeros]

ans =

112.1270   112.1263
```

This example shows that `zeroprice` can satisfactorily price a Treasury note, a semiannual actual/actual basis bond, as if it were a composed of a series of zero-coupon bonds.

## Pricing Corporate Bonds

You can similarly price a corporate bond, for which there is no corresponding zero-coupon bond, as opposed to a Treasury note, for which corresponding zeros exist. You can create a synthetic zero-coupon bond and arrive at the quoted coupon-bond price when you later sum the zeros.

```
Settle = datenum('02-05-2003');
MaturityCpn = datenum('01-14-2009');
Period = 2;
Basis = 1;
% Quoted yield.
QYield = 0.05974;
% Quoted price.
QPrice30 = 99.382;
CouponRate = 0.05850;
```

Extract cash flow and compute price from the sum of zeros.

```
[CFlows, CDates] = cfamounts(CouponRate, Settle, MaturityCpn, ...
Period, Basis);

Maturity = CDates;
```

Compute the price of the coupon bond identically as a collection of zeros by multiplying the discount factors to the corresponding cash flows.

```
Price30 = CFlows * zeroprice(QYield, Settle, Maturity, Period, ...
Basis)/100;
```

Compare quoted price and calculated price based on zeros.

```
[QPrice30 Price30]

ans =

99.3820   99.3828
```

As a test of fidelity, intentionally giving the wrong basis, say actual/actual (`Basis = 0`) instead of 30/360, gives a price of 99.3972. Such a systematic error, if recurring in a more complex pricing routine, quickly adds up to large inaccuracies.

In summary, the zero functions in MATLAB software facilitate extraction of present value from virtually any fixed-coupon instrument, up to any period in time.

# Stepped-Coupon Bonds

**In this section...**

## Introduction

A stepped-coupon bond has a fixed schedule of changing coupon amounts. Like fixed coupon bonds, stepped-coupon bonds could have different periodic payments and accrual bases.

The functions `stepcpnprice` and `stepcpnyield` compute prices and yields of such bonds. An accompanying function `stepcpncfamounts` produces the cash flow schedules pertaining to these bonds.

## Cash Flows from Stepped-Coupon Bonds

Consider a bond that has a schedule of two coupons. Suppose the bond starts out with a 2% coupon that steps up to 4% in 2 years and onward to maturity. Assume that the issue and settlement dates are both March 15, 2003. The bond has a 5 year maturity. Use `stepcpncfamounts` to generate the cash flow schedule and times.

```
Settle      = datenum('15-Mar-2003');
Maturity    = datenum('15-Mar-2008');
ConvDates   = [datenum('15-Mar-2005')];
CouponRates = [0.02, 0.04];

[CFlows, CDates, CTimes] = stepcpncfamounts(Settle, Maturity, ...
ConvDates, CouponRates)
```

Notably, `ConvDates` has 1 less element than `CouponRates` because MATLAB software assumes that the first element of `CouponRates` indicates the coupon schedule between `Settle` (March 15, 2003) and the first element of `ConvDates` (March 15, 2005), shown diagrammatically below.

| | | | |
|---|---|---|---|
| | Pay 2% from March 15, 2003 | | Pay 4% from March 15, 2003 |
| Effective 2% on March 15, 2003 | | Effective 4% on March 15, 2005 | |

| Coupon Dates | Semiannual Coupon Payment |
|---|---|
| 15-Mar-03 | 0 |
| 15-Sep-03 | 1 |
| 15-Mar-04 | 1 |
| 15-Sep-04 | 1 |
| 15-Mar-05 | 1 |
| 15-Sep-05 | 2 |
| 15-Mar-06 | 2 |
| 15-Sep-06 | 2 |
| 15-Mar-07 | 2 |
| 15-Sep-07 | 2 |
| 15-Mar-08 | 102 |

The payment on March 15, 2005 is still a 2% coupon. Payment of the 4% coupon starts with the next payment, September 15, 2005. March 15, 2005 is the end of first coupon schedule, not to be confused with the beginning of the second.

In summary, MATLAB takes user input as the end dates of coupon schedules and computes the next coupon dates automatically.

The payment due on settlement (zero in this case) represents the accrued interest due on that day. It is negative if such amount is nonzero. Comparison with `cfamounts` in Financial Toolbox shows that the two functions operate identically.

## Price and Yield of Stepped-Coupon Bonds

The toolbox provides two basic analytical functions to compute price and yield for stepped-coupon bonds. Using the above bond as an example, you can compute the price when the yield is known.

You can estimate the yield to maturity as a number-of-year weighted average of coupon rates. For this bond, the estimated yield is:

$$\frac{(2\times2)+(4\times3)}{5}$$

.

or 3.33%. While definitely not exact (due to nonlinear relation of price and yield), this estimate suggests close to par valuation and serves as a quick first check on the function.

```
Yield = 0.0333;

[Price, AccruedInterest] = stepcpnprice(Yield, Settle, ...
Maturity, ConvDates, CouponRates)
```

The price returned is 99.2237 (per $100 notional), and the accrued interest is zero, consistent with our earlier assertions.

To validate that there is consistency among the stepped-coupon functions, you can use the above price and see if indeed it implies a 3.33% yield by using `stepcpnyield`.

```
YTM = stepcpnyield(Price, Settle, Maturity, ConvDates, ...
CouponRates)

YTM =

    0.0333
```

# Term Structure Calculations

**In this section…**

## Introduction

So far, a more formal definition of "yield" and its application has not been developed. In many situations when cash flow is available, discounting factors to the cash flows may not be immediately apparent. In other cases, what is relevant is often a *spread*, the difference between curves (also known as the term structure of spread).

All these calculations require one main ingredient, the Treasury spot, par-yield, or forward curve. Typically, the generation of these curves starts with a series of on-the-run and selected off-the-run issues as inputs.

MATLAB software uses these bonds to find spot rates one at a time, from the shortest maturity onwards, using bootstrap techniques. All cash flows are used to construct the spot curve, and rates between maturities (for these coupons) are interpolated linearly.

## Computing Spot and Forward Curves

For an illustration of how this works, observe the use of `zbtyield` (or equivalently `zbtprice`) on a portfolio of six Treasury bills and bonds.

| Bills | Maturity Date | Current Yield |
|---|---|---|
| 3 month | 4/17/03 | 1.15 |
| 6 month | 7/17/03 | 1.18 |

| Notes/Bonds | Coupon | Maturity Date | Current Yield |
|---|---|---|---|
| 2 year | 1.750 | 12/31/04 | 1.68 |
| 5 year | 3.000 | 11/15/07 | 2.97 |
| 10 year | 4.000 | 11/15/12 | 4.01 |
| 30 year | 5.375 | 2/15/31 | 4.92 |

You can specify prices or yields to the bonds above to infer the spot curve. The function `zbtyield` accepts yields (bond-equivalent yield, to be exact).

To proceed, first assemble the above table into a variable called `Bonds`. The first column contains maturities, the second contains coupons, and the third contains notionals or face values of the bonds. (Note that bills have zero coupons.)

```
Bonds = [datenum('04/17/2003')    0        100;
         datenum('07/17/2003')    0        100;
         datenum('12/31/2004')    0.0175   100;
         datenum('11/15/2007')    0.03     100;
         datenum('11/15/2012')    0.04     100;
         datenum('02/15/2031')    0.05375  100];
```

Then specify the corresponding yields.

```
Yields  = [0.0115;
           0.0118;
           0.0168;
           0.0297;
           0.0401;
           0.0492];
```

You are now ready to compute the spot curve for each of these six maturities. The spot curve is based upon a settlement date of January 17, 2003.

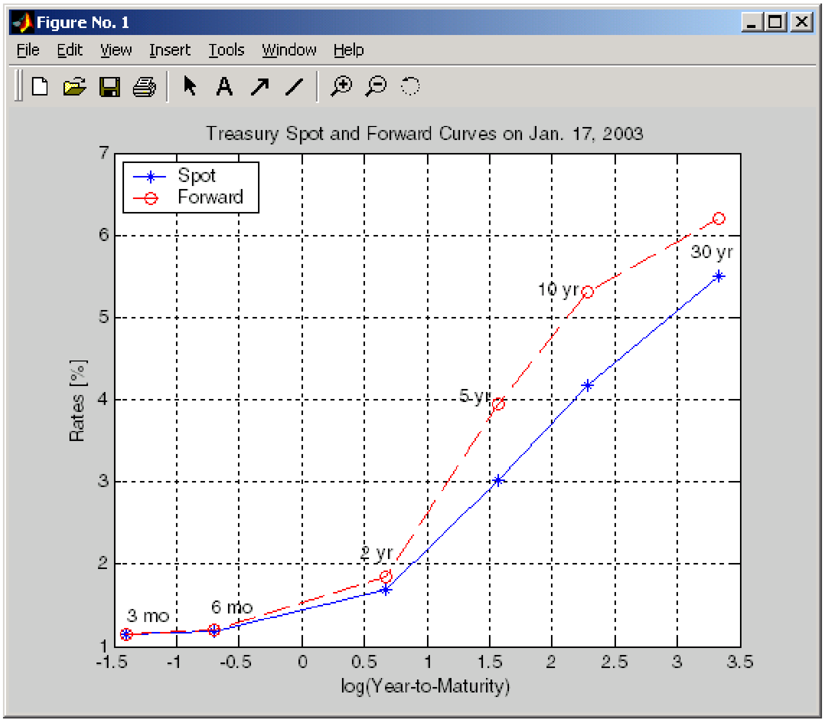
```
Settle = datenum('17-Jan-2003');
[ZeroRates, CurveDates] = zbtyield(Bonds, Yields, Settle)
```

This gets you the Treasury spot curve for the day.

You can compute the forward curve from this spot curve with `zero2fwd`.

```
[ForwardRates, CurveDates] = zero2fwd(ZeroRates, CurveDates, ...
Settle)
```

Here the notion of forward rates refers to rates between the maturity dates shown above, not to a certain period (forward 3 month rates, for example).



## Computing Spreads

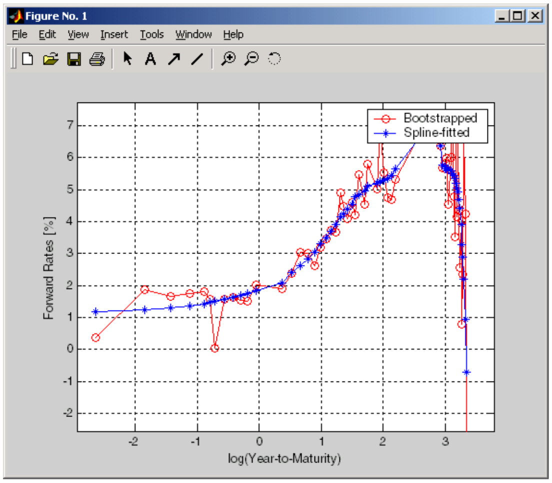
Calculating the spread between specific, fixed forward periods (such as the Treasury-Eurodollar spread) requires an extra step. Interpolate the zero rates (or zero prices, instead) for the corresponding maturities on the interval dates. Then use the interpolated zero rates to deduce the forward rates, and thus the spread of Eurodollar forward curve segments versus the relevant forward segments from Treasury bills.

Additionally, the variety of curve functions (including `zero2fwd`) helps to standardize such calculations. For instance, by making both rates quoted with quarterly compounding and on an actual/360 basis, the resulting spread structure is fully comparable. This avoids the small inconsistency that occurs when directly comparing the bond-equivalent yield of a Treasury bill to the quarterly forward rates implied by Eurodollar futures.

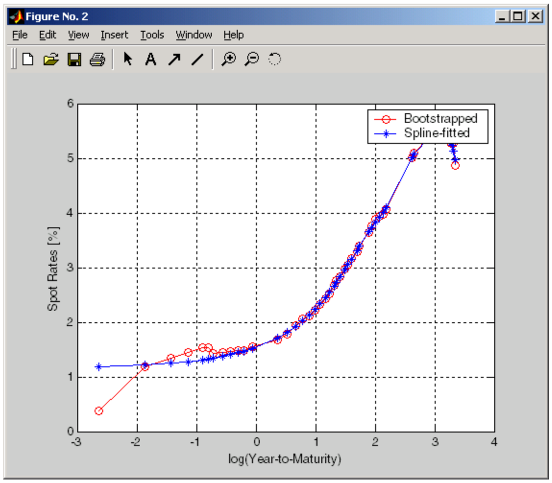
### Noise in Curve Computations

When introducing more bonds in constructing curves, noise may become a factor and may need some "smoothing" (with splines, for example); this helps obtain a smoother forward curve.

The following spot and forward curves are constructed from 67 Treasury bonds. The fitted and bootstrapped spot curve (bottom right figure) displays comparable stability. The forward curve (upper-left figure) contains significant noise and shows an improbable forward rate structure. The noise is not necessarily bad; it could uncover trading opportunities for a relative-value approach. Yet, a more balanced approach is desired when the bootstrapped forward curve oscillates this much and contains a negative rate as large as -10% (not shown in the plot because it is outside the limits).



**Implied Forward Curves. The jagged curve comes from direct bootstrapping. The smooth curve shows the effect of smoothing with splines.**



**Implied Spot Rate Curves. These curves correspond to the forward curve above.**

This example uses `termfit`, an example function from Financial Toolbox software that also requires the use of Curve Fitting Toolbox™ software.

7

# Derivative Securities

# Interest Rate Swaps

**In this section...**

## Swap Pricing Assumptions

Financial Instruments Toolbox software contains the function `liborfloat2fixed`, which computes a fixed-rate par yield that equates the floating-rate side of a swap to the fixed-rate side. The solver sets the present value of the fixed side to the present value of the floating side without having to line up and compare fixed and floating periods.

### Assumptions on Floating-Rate Input

- Rates are quarterly, for example, that of Eurodollar futures.
- Effective date is the first third Wednesday after the settlement date.
- All delivery dates are spaced 3 months apart.
- All periods start on the third Wednesday of delivery months.
- All periods end on the same dates of delivery months, 3 months after the start dates.
- Accrual basis of floating rates is actual/360.
- Applicable forward rates are estimated by interpolation in months when forward-rate data is not available.

### Assumptions on Fixed-Rate Output

- Design allows you to create a bond of any coupon, basis, or frequency, based upon the floating-rate input.
- The start date is a valuation date, that is, a date when an agreement to enter into a contract by the settlement date is made.

- Settlement can be on or after the start date. If it is after, a forward fixed-rate contract results.
- Effective date is assumed to be the first third Wednesday after settlement, the same date as that of the floating rate.
- The end date of the bond is a designated number of years away, on the same day and month as the effective date.
- Coupon payments occur on anniversary dates. The frequency is determined by the period of the bond.
- Fixed rates are not interpolated. A fixed-rate bond of the same present value as that of the floating-rate payments is created.

## Swap Pricing Example

This example shows the use of the functions in computing the fixed rate applicable to a series of 2-, 5-, and 10-year swaps based on Eurodollar market data. According to the Chicago Mercantile Exchange (`http://www.cme.com`), Eurodollar data on Friday, October 11, 2002, was as shown in the following table.

---

**Note** This example illustrates swap calculations in MATLAB software. Timing of the data set used was not rigorously examined and was assumed to be the proxy for the swap rate reported on October 11, 2002.

---

### Eurodollar Data on Friday, October 11, 2002

| Month | Year | Settle |
|---|---|---|
| 10 | 2002 | 98.21 |
| 11 | 2002 | 98.26 |
| 12 | 2002 | 98.3 |
| 1 | 2003 | 98.3 |
| 2 | 2003 | 98.31 |
| 3 | 2003 | 98.275 |
| 6 | 2003 | 98.12 |

**Eurodollar Data on Friday, October 11, 2002 (Continued)**

| Month | Year | Settle |
|---|---|---|
| 9 | 2003 | 97.87 |
| 12 | 2003 | 97.575 |
| 3 | 2004 | 97.26 |
| 6 | 2004 | 96.98 |
| 9 | 2004 | 96.745 |
| 12 | 2004 | 96.515 |
| 3 | 2005 | 96.33 |
| 6 | 2005 | 96.135 |
| 9 | 2005 | 95.955 |
| 12 | 2005 | 95.78 |
| 3 | 2006 | 95.63 |
| 6 | 2006 | 95.465 |
| 9 | 2006 | 95.315 |
| 12 | 2006 | 95.16 |
| 3 | 2007 | 95.025 |
| 6 | 2007 | 94.88 |
| 9 | 2007 | 94.74 |
| 12 | 2007 | 94.595 |
| 3 | 2008 | 94.48 |
| 6 | 2008 | 94.375 |
| 9 | 2008 | 94.28 |
| 12 | 2008 | 94.185 |
| 3 | 2009 | 94.1 |
| 6 | 2009 | 94.005 |
| 9 | 2009 | 93.925 |
| 12 | 2009 | 93.865 |

### Eurodollar Data on Friday, October 11, 2002 (Continued)

| Month | Year | Settle |
|---|---|---|
| 3 | 2010 | 93.82 |
| 6 | 2010 | 93.755 |
| 9 | 2010 | 93.7 |
| 12 | 2010 | 93.645 |
| 3 | 2011 | 93.61 |
| 6 | 2011 | 93.56 |
| 9 | 2011 | 93.515 |
| 12 | 2011 | 93.47 |
| 3 | 2012 | 93.445 |
| 6 | 2012 | 93.41 |
| 9 | 2012 | 93.39 |

Using this data, you can compute 1-, 2-, 3-, 4-, 5-, 7-, and 10-year swap rates with the toolbox function `liborfloat2fixed`. The function requires you to input only Eurodollar data, the settlement date, and tenor of the swap. MATLAB software then performs the required computations.

To illustrate how this function works, first load the data contained in the supplied Excel® worksheet `EDdata.xls`.

```
[EDRawData, textdata] = xlsread('EDdata.xls');
```

Extract the month from the first column and the year from the second column. The rate used as proxy is the arithmetic average of rates on opening and closing.

```
Month = EDRawData(:,1);
Year  = EDRawData(:,2);
IMMData = (EDRawData(:,4)+EDRawData(:,6))/2;
EDFutData = [Month, Year, IMMData];
```

Next, input the current date.

```
Settle = datenum('11-Oct-2002');
```

To compute for the 2 year swap rate, set the tenor to `2`.

```
Tenor = 2;
```

Finally, compute the swap rate with `liborfloat2fixed`.

```
[FixedSpec, ForwardDates, ForwardRates] = ...
liborfloat2fixed(EDFutData, Settle, Tenor)
```

MATLAB returns a par-swap rate of 2.23% using the default setting (quarterly compounding and 30/360 accrual), and forward dates and rates data (quarterly compounded), comparable to 2.17% of Friday's average broker data in Table H15 of *Federal Reserve Statistical Release* (`http://www.federalreserve.gov/releases/h15/update/`).

```
FixedSpec =

       Coupon: 0.0223
       Settle: '16-Oct-2002'
     Maturity: '16-Oct-2004'
       Period: 4
        Basis: 1

ForwardDates =

      731505
      731596
      731687
      731778
      731869
      731967
      732058
      732149

ForwardRates =

    0.0178
    0.0168
```

```
    0.0171
    0.0189
    0.0216
    0.0250
    0.0280
    0.0306
```

In the `FixedSpec` output, note that the swap rate actually goes forward from the third Wednesday of October 2002 (October 16, 2002), 5 days after the original `Settle` input (October 11, 2002). This, however, is still the best proxy for the swap rate on `Settle`, as the assumption merely starts the swap's effective period and does not affect its valuation method or its length.

The correction suggested by Hull and White improves the result by turning on convexity adjustment as part of the input to `liborfloat2fixed`. (See Hull, J., *Options, Futures, and Other Derivatives*, 4th Edition, Prentice-Hall, 2000.) For a long swap, for example, 5 years or more, this correction could prove to be large.

The adjustment requires additional parameters:

- `StartDate`, which you make the same as `Settle` (the default) by providing an empty matrix `[]` as input.
- `ConvexAdj` to tell `liborfloat2fixed` to perform the adjustment.
- `RateParam`, which provides the parameters `a` and `S` as input to the Hull-White short rate process.
- Optional parameters `InArrears` and `Sigma`, for which you can use empty matrices `[]` to accept the MATLAB defaults.
- `FixedCompound`, with which you can facilitate comparison with values cited in Table H15 of *Federal Reserve Statistical Release* by turning the default quarterly compounding into semiannual compounding, with the (default) basis of 30/360.

  ```
  StartDate = [];
  Interpolation = [];
  ConvexAdj = 1;
  RateParam = [0.03; 0.017];
  FixedCompound = 2;
  ```

```
[FixedSpec, ForwardDaates, ForwardRates] = ...
liborfloat2fixed(EDFutData, Settle, Tenor, StartDate, ...
Interpolation, ConvexAdj, RateParam, [], [], FixedCompound)
```

This returns 2.21% as the 2-year swap rate, quite close to the reported swap rate for that date.

Analogously, the following table summarizes the solutions for 1-, 3-, 5-, 7-, and 10-year swap rates (convexity-adjusted and unadjusted).

**Calculated and Market Average Data of Swap Rates on Friday, October 11, 2002**

| Swap Length (Years) | Unadjusted | Adjusted | Table H15 | Adjusted Error (Basis Points) |
|---|---|---|---|---|
| 1 | 1.80% | 1.79% | 1.80% | -1 |
| 2 | 2.24% | 2.21% | 2.22% | -1 |
| 3 | 2.70% | 2.66% | 2.66% | 0 |
| 4 | 3.12% | 3.03% | 3.04% | -1 |
| 5 | 3.50% | 3.37% | 3.36% | +1 |
| 7 | 4.16% | 3.92% | 3.89% | +3 |
| 10 | 4.87% | 4.42% | 4.39% | +3 |

## Portfolio Hedging

You can use these results further, such as for hedging a portfolio. The `liborduration` function provides a duration-hedging capability. You can isolate assets (or liabilities) from interest-rate risk exposure with a swap arrangement.

Suppose you own a bond with these characteristics:

- $100 million face value
- 7% coupon paid semiannually

- 5% yield to maturity
- Settlement on October 11, 2002
- Maturity on January 15, 2010
- Interest accruing on an actual/365 basis

Use of the `bnddury` function from Financial Toolbox software shows a modified duration of 5.6806 years.

To immunize this asset, you can enter into a pay-fixed swap, specifically a swap in the amount of notional principal (*Ns*) such that *Ns***SwapDuration* + $100M*5.6806 = 0 (or *Ns* = -100*5.6806/*SwapDuration*).

Suppose again, you choose to use a 5-, 7-, or 10-year swap (3.37%, 3.92%, and 4.42% from the previous table) as your hedging tool.

```
SwapFixRate = [0.0337; 0.0392; 0.0442];
Tenor = [5; 7; 10];
Settle = '11-Oct-2002';
PayFixDuration = liborduration(SwapFixRate, Tenor, Settle)
```

This gives a duration of -3.6835, -4.7307, and -6.0661 years for 5-, 7-, and 10-year swaps. The corresponding notional amount is computed by

```
Ns = -100*5.6806./PayFixDuration

Ns =

  154.2163
  120.0786
   93.6443
```

The notional amount entered in pay-fixed side of the swap instantaneously immunizes the portfolio.

## Convertible Bond Valuation

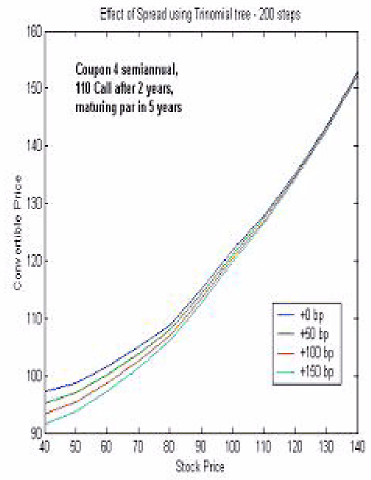
A convertible bond (CB) is a debt instrument that can be converted into a predetermined amount of a company's equity at certain times before the bond's maturity.

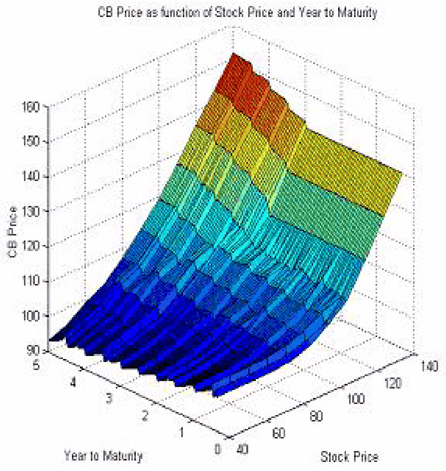
Financial Instruments Toolbox software uses a binomial and trinomial tree approach (`cbprice`) to value convertible bonds. The value of a convertible bond is determined by the uncertainty of the related stock. Once the stock evolution is modeled, backward discounting is computed.

The last column of such trees provides the data to decide which is more profitable: the debt notional (plus interest, if any) or the equivalent number of shares per the notional.

Where debt prevails, the toolbox discounts backward with the risk-free rate plus the spread reflecting the credit risk of the issuer. Where stock prevails, the toolbox discounts with the risk-free rate. The intrinsic value of a convertible bond is the sum of the (probability-adjusted) debt and stock portions from the last node. This is compared to current stock price, to see if voluntary or forced conversion may take place. Otherwise, its value is the intrinsic value. From here, the same discounting process is repeated after adjusting debt portion to be equal to 0 if any conversion takes place. Dividends and coupons are handled discretely, at the date they occur.

The approach is equivalent to solving a one-dimensional partial differential equation such as one described by Tsiveriotis and Fernandes. (See Tsiveriotis, K. and C. Fernandes (1998), "Valuing Convertible Bonds with Credit Risk," *The Journal of Fixed Income*, 8 (3), 95-102.) Using the same example of bond specifications that they use (4% annual coupon, payable twice a year, callable after 2 years at 110, and redeemable at par in 5 years), the toolbox gives results like theirs.





The figure on the left shows the bond "floor" of the convertible (a 5% yield, given a 4% coupon at about 97% par) when share prices are low.

The change of curvature located at the end of the second year is due to the activation of the embedded (soft) call feature (visible on the surface plot in the right figure).

Finally, there is the flat section when time is nearing expiration and share prices are high, indicating a delta of unity, a characteristic of in-the-money equity options embedded in a bond.

# Bond Futures

**In this section...**

## Supported Bond Futures

Bond futures are futures contracts where the commodity for delivery is a government bond. There are established global markets for government bond futures. Bond futures provide a liquid alternative for managing interest-rate risk.

In the U.S. market, the Chicago Mercantile Exchange (CME) offers futures on Treasury bonds and notes with maturities of 2, 5, 10, and 30 years. Typically, the following bond future contracts from the CME have maturities of 3, 6, 9, and 12 months:

- 30-year U.S. Treasury bond
- 10-year U.S. Treasury bond
- 5-year U.S. Treasury bond
- 2-year U.S. Treasury bond

The short position in a Treasury bond or note future contract must deliver to the long position in one of many possible existing Treasury bonds. For example, in a 30-year Treasury bond future, the short position must deliver a Treasury bond with at least 15 years to maturity. Because these bonds have different values, the bond future contract is standardized by computing a conversion factor. The conversion factor normalizes the price of a bond to a theoretical bond with a coupon of 6%. The price of a bond future contract is represented as:

$$InvoicePrice = FutPrice \times CF + AI$$

where:

*FutPrice* is the price of the bond future.

*CF* is the conversion factor for a bond to deliver in a futures contract.

*AI* is the accrued interest.

You can reference these conversion factors at U.S. Treasury Bond Futures Contract. The short position in a futures contract has the option of which bond to deliver and, in the U.S. bond market, when in the delivery month to deliver the bond. The short position typically chooses to deliver the bond known as the Cheapest to Deliver (CTD). The CTD bond most often delivers on the last delivery day of the month.

Financial Instruments Toolbox software supports the following bond futures:

- U.S. Treasury bonds and notes
- German Bobl, Bund, Buxl, and Schatz
- UK gilts
- Japanese government bonds (JGBs)

The functions supporting all bond futures are:

| Function | Purpose |
|---|---|
| `convfactor` | Calculates bond conversion factors for U.S. Treasury bonds, German Bobl, Bund, Buxl, and Schatz, U.K. gilts, and JGBs. |
| `bndfutprice` | Prices bond future given repo rates. |
| `bndfutimprepo` | Calculates implied repo rates for a bond future given price. |

The functions supporting U.S. Treasury bond futures are:

| Function | Purpose |
|---|---|
| `tfutbyprice` | Calculates future prices of Treasury bonds given the spot price. |
| `tfutbyyield` | Calculates future prices of Treasury bonds given current yield. |
| `tfutimprepo` | Calculates implied repo rates for the Treasury bond future given price. |
| `tfutpricebyrepo` | Calculates implied repo rates given the Treasury bond future price. |
| `tfutyieldbyrepo` | Calculates implied repo rates given the Treasury bond future yield. |

## Analysis of Bond Futures

The following example demonstrates analyzing German Euro-Bund futures traded on Eurex. However, `convfactor`, `bndfutprice`, and `bndfutimprepo` apply to bond futures in the U.S., U.K., Germany, and Japan. The workflow for this analysis is:

1. Calculate bond conversion factors.
2. Calculate implied repo rates to find the CTD bond.
3. Price the bond future using the term implied repo rate.

### Calculating Bond Conversion Factors

Use conversion factors to normalize the price of a particular bond for delivery in a futures contract. When using conversion factors, the assumption is that a bond for delivery has a 6% coupon. Use `convfactor` to calculate conversion factors for all bond futures from the U.S., Germany, Japan, and U.K.

For example, conversion factors for Euro-Bund futures on Eurex are listed at `www.eurexchange.com`. The delivery date for Euro-Bund futures is the 10th day of the month, as opposed to bond futures in the U.S., where the short position has the option of choosing when to deliver the bond.

For the 4% bond, compute the conversion factor with:

```
CF1 = convfactor('10-Sep-2009','04-Jul-2018', .04,.06,3)
CF1 =

     0.8659
```

This syntax for `convfactor` works fine for bonds with standard coupon periods. However, some deliverable bonds have long or short first coupon periods. Compute the conversion factors for such bonds using the optional input parameters `StartDate` and `FirstCouponDate`. Specify all optional input arguments for `convfactor` as parameter/value pairs:

```
CF2 = convfactor('10-Sep-2009','04-Jan-2019', .0375,'Convention',3,'startdate',...
datenum('14-Nov-2008'))
CF2 =

    0.8426
```

## Calculating Implied Repo Rates to Find the CTD Bond

To determine the availability of the cheapest bond for deliverable bonds against a futures contract, compute the implied repo rate for each bond. The bond with the highest repo rate is the cheapest because it has the lowest initial value, thus yielding a higher return, provided you deliver it with the stated futures price. Use `bndfutimprepo` to calculate repo rates:

```
% Bond Properties
CouponRate = [.0425;.0375;.035];
Maturity = [datenum('04-Jul-2018');datenum('04-Jan-2019');datenum('04-Jul-2019')];
CF = [0.882668;0.842556;0.818193];
Price = [105.00;100.89;98.69];

% Futures Properties
FutSettle = '09-Jun-2009';
FutPrice = 118.54;
Delivery = '10-Sep-2009';

% Note that the default for BNDFUTIMPREPO is for the bonds to be
% semi-annual with a day count basis of 0.  Since these are German
% bonds, we need to have a Basis of 8 and a Period of 1
ImpRepo = bndfutimprepo(Price, FutPrice, FutSettle, Delivery, CF, ...
CouponRate, Maturity,'Basis',8,'Period',1)
```

```
ImpRepo =

    0.0261
   -0.0022
   -0.0315
```

### Pricing Bond Futures Using the Term Implied Repo Rate

Use `bndfutprice` to perform price calculations for all bond futures from the U.S., Germany, Japan, and U.K. To price the bond, given a term repo rate:

```
% Assume a term repo rate of .0091;
RepoRate = .0091;
[FutPrice,AccrInt] = bndfutprice(RepoRate, Price(1), FutSettle,...
Delivery, CF(1), CouponRate(1), Maturity(1),...
'Basis',8,'Period',1)

FutPrice =

  118.0126


AccrInt =

    0.7918
```

## Managing Present Value with Bond Futures

The Present Value of a Basis Point (PVBP) is used to manage interest-rate risk. PVBP is a measure that quantifies the change in price of a bond given a one-basis point shift in interest rates. The PVBP of a bond is computed with the following:

$$PVBP_{Bond} = \frac{Duration \times MarketValue}{100}$$

The PVBP of a bond futures contract can be computed with the following:

$$PVBP_{Futures} = \frac{PVBP_{CTDBond}}{CTDConversionFactor}$$

Use `bnddurp` and `bnddury` from Financial Toolbox software to compute the modified durations of CTD bonds. For more information, see “Managing Interest Rate Risk with Bond Futures” on page 7-18 and “Fitting the Diebold Li Model” on page 7-27.

# Managing Interest Rate Risk with Bond Futures

This example shows how to hedge the interest rate risk of a portfolio using bond futures.

### Modifying the Duration of a Portfolio with Bond Futures

In managing a bond portfolio, you can use a benchmark portfolio to evaluate performance. Sometimes a manager is constrained to keep the portfolio's duration within a particular band of the duration of the benchmark. One way to modify the duration of the portfolio is to buy and sell bonds, however, there may be reasons why a portfolio manager wishes to maintain the existing composition of the portfolio (e.g., the current holdings reflect fundamental research/views about future returns). Therefore, another option for modifying the duration is to buy and sell bond futures.

Bond futures are futures contracts where the commodity to be delivered is a government bond that meets the standard outlined in the futures contract (e.g., the bond has a specified remaining time to maturity).

Since often many bonds are available, and each bond may have a different coupon, you can use a conversion factor to normalize the payment by the long to the short.

There exist well developed markets for government bond futures. Specifically, the Chicago Board of Trade offers futures on the following:

- 2 Year Note
- 3 Year Note
- 5 Year Note
- 10 Year Note
- 30 Year Bond

http://www.cmegroup.com/trading/interest-rates/

Eurex offers futures on the following:

- Euro-Schatz Futures 1.75 to 2.25

- Euro-Bobl Futures 4.5 to 5.5
- Euro-Bund Futures 8.5 to 10.5
- Euro-Buxl Futures 24.0 to 35

http://www.eurexchange.com

Bond futures can be used to modify the duration of a portfolio. Since bond futures derive their value from the underlying instrument, the duration of a bond futures contract is related to the duration of the underlying bond.

There are two challenges in computing this duration:

- Since there are many available bonds for delivery, the short in the contract has a choice in which bond to deliver.
- Some contracts allow the short flexibility in choosing the delivery date.

Typically, the bond used for analysis is the bond that is cheapest for the short to deliver (CTD).

One approach is to compute duration measures using the CTD's duration and the conversion factor.

For example, the Present Value of a Basis Point (PVBP) can be computed from the following:

$$PVBP_{Futures} = \frac{PVBP_{CTD}}{ConversionFactor_{CTD}}$$

$$PVBP_{CTD} = \frac{Duration_{CTD} * Price_{CTD}}{100}$$

Note that these definitions of duration for the futures contract are approximate, and do not account for the value of the delivery options for the short.

If the goal is to modify the duration of a portfolio, use the following:

$$NumContracts = \frac{(Dur_{Target} - Dur_{Initial}) * Value_{Portfolio}}{Dur_{CTD} * Price_{CTD} * ContractSize} * ConvFactor_{CTD}$$

Note that the contract size is typically for 100,000 face value of a bond -- so the contract size is typically 1000, as the bond face value is 100.

The following example assumes an initial duration, portfolio value, and target duration for a portfolio with exposure to the Euro interest rate. The June Euro-Bund Futures contract is used to modify the duration of the portfolio.

Note that typically futures contracts are offered for March, June, September and December.

```
% Assume the following for the portfolio and target
PortfolioDuration = 6.4;
PortfolioValue = 100000000;
BenchmarkDuration = 4.8;

% Deliverable Bunds -- note that these conversion factors may also be
% computed with the MATLAB(R) function CONVFACTOR
BondPrice = [106.46;108.67;104.30];
BondMaturity = datenum({'04-Jan-2018','04-Jul-2018','04-Jan-2019'});
BondCoupon = [.04;.0425;.0375];
ConversionFactor = [.868688;.880218;.839275];

% Futures data -- found from http://www.eurexchange.com
FuturesPrice = 122.17;
FuturesSettle = '23-Apr-2009';
FuturesDelivery = '10-Jun-2009';

% To find the CTD bond we can compute the implied repo rate
ImpliedRepo = bndfutimprepo(BondPrice,FuturesPrice,FuturesSettle,...
    FuturesDelivery,ConversionFactor,BondCoupon,BondMaturity);

% Note that the bond with the highest implied repo rate is the CTD
[CTDImpRepo,CTDIndex] = max(ImpliedRepo);

% Compute the CTD's Duration -- note the period and basis for German Bunds
Duration = bnddurp(BondPrice,BondCoupon,FuturesSettle,BondMaturity,1,8);
```

```
ContractSize = 1000;

% Use the formula above to compute the number of contracts to sell
NumContracts = (BenchmarkDuration - PortfolioDuration)*PortfolioValue./...
    (BondPrice(CTDIndex)*ContractSize*Duration(CTDIndex))*ConversionFactor(

disp(['To achieve the target duration, ' num2str(abs(round(NumContracts)))
    ' Euro-Bund Futures must be sold.'])
```

```
To achieve the target duration, 180 Euro-Bund Futures must be sold.
```

### Modifying the Key Rate Durations of a Portfolio with Bond Futures

One of the shortcomings of using duration as a risk measure is that it assumes parallel shifts in the yield curve. While many studies have shown that this explains roughly 85% of the movement in the yield curve, changes in the slope or shape of the yield curve are not captured by duration -- and therefore, hedging strategies are not successful at addressing these dynamics.

One approach is to use key rate duration -- this is particularly relevant when using bond futures with multiple maturities, like Treasury futures.

The following example uses 2, 5, 10 and 30 year Treasury Bond futures to hedge the key rate duration of a portfolio.

Computing key rate durations requires a zero curve. This example uses the zero curve published by the Treasury and found at the following location:

http://www.ustreas.gov/offices/domestic-finance/debt-management/interest-rate/yield.shtm

Note that this zero curve could also be derived using the Interest Rate Curve functionality found in IRDataCurve and IRFunctionCurve.

```
% Assume the following for the portfolio and target, where the duration
% vectors are key rate durations at 2, 5, 10, and 30 years.
PortfolioDuration = [.5 1 2 6];
PortfolioValue = 100000000;
BenchmarkDuration = [.4 .8 1.6 5];

% The following are the CTD Bonds for the 30, 10, 5 and 2 year futures
```

```
% contracts -- these were determined using the procedure outlined in the
% previous section.
CTDCoupon = [4.75 3.125 5.125 7.5]'/100;
CTDMaturity = datenum({'3/31/2011','08/31/2013','05/15/2016','11/15/2024'})
CTDConversion = [0.9794 0.8953 0.9519 1.1484]';
CTDPrice = [107.34 105.91 117.00 144.18]';

ZeroRates = [0.07 0.10 0.31 0.50 0.99 1.38 1.96 2.56 3.03 3.99 3.89]'/100;
ZeroDates = daysadd(FuturesSettle,[30 360 360*2 360*3 360*5 ...
    360*7 360*10 360*15 360*20 360*25 360*30],1);

% Compute the key rate durations for each of the CTD bonds.
CTDKRD = bndkrdur([ZeroDates ZeroRates], CTDCoupon,FuturesSettle,...
    CTDMaturity,'KeyRates',[2 5 10 30]);

% Note that the contract size for the 2 Year Note Future is $200,000
ContractSize = [2000;1000;1000;1000];

NumContracts = (bsxfun(@times,CTDPrice.*ContractSize./CTDConversion,CTDKRD)
    (BenchmarkDuration - PortfolioDuration)'*PortfolioValue;

sprintf(['To achieve the target duration, \n' ...
    num2str(-round(NumContracts(1))) ' 2 Year Treasury Note Futures must be
    num2str(-round(NumContracts(2))) ' 5 Year Treasury Note Futures must be
    num2str(-round(NumContracts(3))) ' 10 Year Treasury Note Futures must b
    num2str(-round(NumContracts(4))) ' Treasury Bond Futures must be sold,
```

```
ans =

To achieve the target duration,
24 2 Year Treasury Note Futures must be sold,
47 5 Year Treasury Note Futures must be sold,
68 10 Year Treasury Note Futures must be sold,
120 Treasury Bond Futures must be sold,
```

### Improving the Performance of a Hedge with Regression

An additional component to consider in hedging interest rate risk with bond futures, again related to movements in the yield curve, is that typically the yield curve moves more at the short end than at the long end.

Therefore, if a position is hedged with a future where the CTD bond has a maturity that is different than the portfolio this could lead to a situation where the hedge under- or over-compensates for the actual interest rate risk of the portfolio.

One approach is to perform a regression on historical yields at different maturities to determine a Yield Beta, which is a value that represents how much more the yield changes for different maturities.

This example shows how to use this approach with UK Long Gilt futures and historical data on Gilt Yields.

Market data on Gilt futures is found at the following:

http://www.euronext.com

Historical data on gilts is found at the following;

http://www.dmo.gov.uk

Note that while this approach does offer the possibility of improving the performance of a hedge, any analysis using historical data depends on historical relationships remaining consistent.

Also note that an additional enhancement takes into consideration the correlation between different maturities. While this approach is outside the scope of this example, you can use this to implement a minimum variance hedge.

```
% Assume the following for the portfolio and target
PortfolioDuration = 6.4;
PortfolioValue = 100000000;
BenchmarkDuration = 4.8;

% This is the CTD Bond for the Long Gilt Futures contract
CTDBondPrice = 113.40;
```

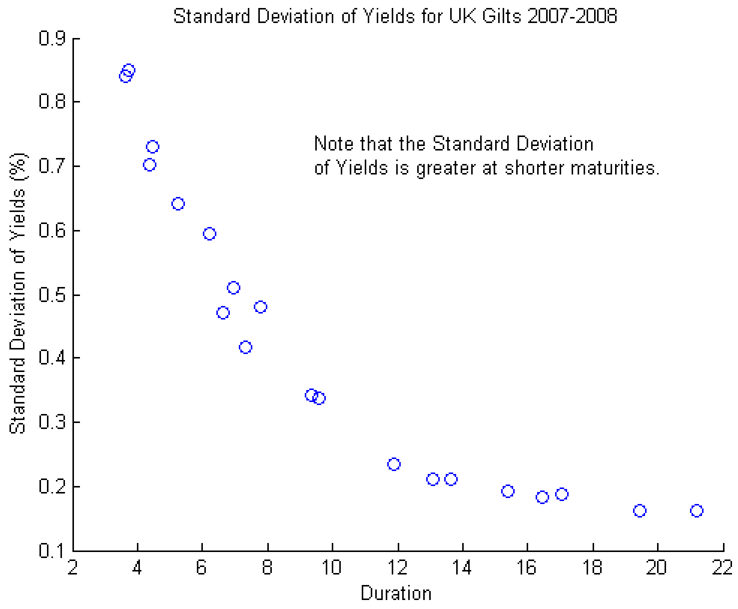
```
CTDBondMaturity = datenum('7-Mar-2018');
CTDBondCoupon = .05;
CTDConversionFactor = 0.9325024;

% Market data for the Long Gilt Futures contract
FuturesPrice = 120.80;
FuturesSettle = '23-Apr-2009';
FuturesDelivery = '10-Jun-2009';

CTDDuration = bnddurp(CTDBondPrice,CTDBondCoupon,FuturesSettle,CTDBondMatur

ContractSize = 1000;

NumContracts = (BenchmarkDuration - PortfolioDuration)*PortfolioValue./...
    (CTDBondPrice*ContractSize*CTDDuration)*CTDConversionFactor;

disp(['To achieve the target duration with a conventional hedge ' ...
    num2str(-round(NumContracts)) ...
    ' Long Gilt Futures must be sold.'])
```

```
To achieve the target duration with a conventional hedge 182 Long Gilt Futu
```

To improve the accuracy of this hedge, historical data is used to determine a relationship between the standard deviation of the yields. Specifically, standard deviation of yields is plotted and regressed vs bond duration. This relationship is then used to compute a Yield Beta for the hedge.

```
% Load data from XLS spreadsheet
load ukbonddata_20072008

Duration = bnddury(Yield(1,:)',Coupon,Dates(1,:),Maturity);

scatter(Duration,100*std(Yield))
title('Standard Deviation of Yields for UK Gilts 2007-2008')
ylabel('Standard Deviation of Yields (%)')
xlabel('Duration')
annotation(gcf,'textbox',[0.4067 0.685 0.4801 0.0989],...
    'String',{'Note that the Standard Deviation',...
    'of Yields is greater at shorter maturities.'},...
    'FitBoxToText','off',...
```

```
    'EdgeColor','none');

stats = regstats(std(Yield),Duration);
YieldBeta = (stats.beta'*[1 PortfolioDuration]')./(stats.beta'*[1 CTDDurati
```


Standard Deviation of Yields for UK Gilts 2007-2008

Note that the Standard Deviation of Yields is greater at shorter maturities.

Now the Yield Beta is used to compute a new value for the number of contracts to be sold. Note that since the duration of the portfolio was less than the duration of the CTD Gilt, the number of futures to sell is actually greater than in the first case.

```
NumContracts = (BenchmarkDuration - PortfolioDuration)*PortfolioValue./...
    (CTDBondPrice*ContractSize*CTDDuration)*CTDConversionFactor*YieldBeta;

disp(['To achieve the target duration using a Yield Beta-modified hedge, '
    num2str(abs(round(NumContracts))) ...
    ' Long Gilt Futures must be sold.'])
```

```
To achieve the target duration using a Yield Beta-modified hedge, 193 Long
```

## Bibliography

This example is based on the following books and papers:

[1] Burghardt, G., T. Belton, M. Lane and J. Papa. The Treasury Bond Basis. New York, NY: McGraw-Hill, 2005.

[2] Krgin, D. Handbook of Global Fixed Income Calculations. New York, NY: John Wiley & Sons, 2002.

[3] CFA Program Curriculum, Level III, Volume 4, Reading 31. CFA Institute, 2009.

# Fitting the Diebold Li Model

This example shows how to construct a Diebold Li model of the US yield curve for each month from 1990 to 2010 and how to forecast future yield curves by fitting an autoregressive model to the time series of each parameter.

The paper can be found here:

http://www.ssc.upenn.edu/~fdiebold/papers/paper49/Diebold-Li.pdf

### Load the Data

The data used are monthly Treasury yields from 1990 through 2010 for tenors of 1 Mo, 3 Mo, 6 Mo, 1 Yr, 2 Yr, 3 Yr, 5 Yr, 7 Yr, 10 Yr, 20 Yr, 30 Yr.

Daily data can be found here:

http://www.treasury.gov/resource-center/data-chart-center/interest-rates/Pages/TextView.

Data is stored in a MATLAB data file as a MATLAB dataset object.

```
load Data_USYieldCurve

% Extract data for the last day of each month
MonthYearMat = repmat((1990:2010)',1,12)';
EOMDates = lbusdate(MonthYearMat(:),repmat((1:12)',21,1));
MonthlyIndex = find(ismember(get(Dataset,'ObsNames'),datestr(EOMDates)));
Estimationdataset = Dataset(MonthlyIndex,:);
EstimationData = double(Estimationdataset);
```

### Diebold Li Model

Diebold and Li start with the Nelson Siegel model

$$y = \beta_0 + (\beta_1 + \beta_2)\frac{\tau}{m}(1 - e^{\frac{-m}{\tau}}) - \beta_2 e^{\frac{-m}{\tau}}$$

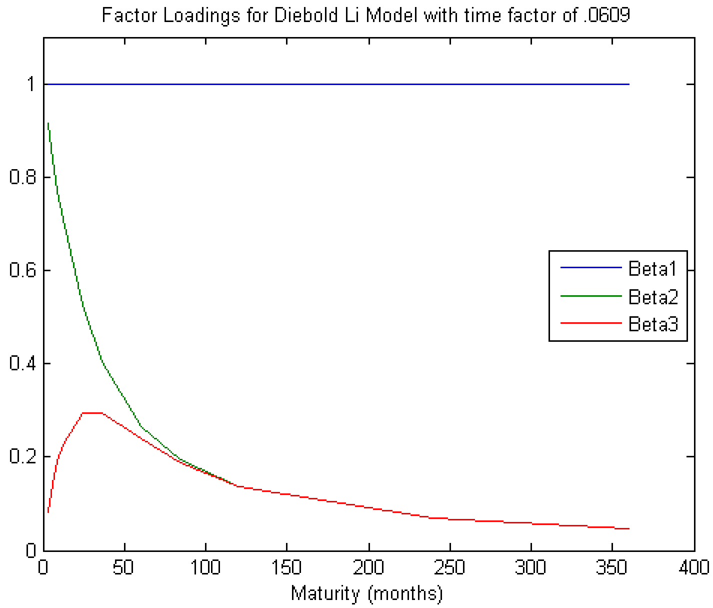
and rewrite it to be the following:

$$y_t(\tau) = \beta_{1t} + \beta_{2t}\left(\frac{1-e^{-\lambda_t\tau}}{\lambda_t\tau}\right) + \beta_{3t}\left(\frac{1-e^{-\lambda_t\tau}}{\lambda_t\tau} - e^{-\lambda_t\tau}\right)$$

The above model allows the factors to be interpreted in the following way: Beta1 corresponds to the long term/level of the yield curve, Beta2 corresponds to the short term/slope, and Beta3 corresponds to the medium term/curvature.

Diebold and Li advocate setting the time factor, lambda, to maximize the loading on the medium term factor, Beta3, at 30 months. This also transforms the problem from a nonlinear fitting one to a simple linear regression.

```
% Explicitly set the time factor lambda
lambda_t = .0609;

% Construct a matrix of the factor loadings
% Tenors associated with data
TimeToMat = [3 6 9 12 24 36 60 84 120 240 360]';
X = [ones(size(TimeToMat)) (1 - exp(-lambda_t*TimeToMat))./(lambda_t*TimeTo
    ((1 - exp(-lambda_t*TimeToMat))./(lambda_t*TimeToMat) - exp(-lambda_t*T

% Plot the factor loadings
plot(TimeToMat,X)
title('Factor Loadings for Diebold Li Model with time factor of .0609')
xlabel('Maturity (months)')
ylim([0 1.1])
legend({'Beta1','Beta2','Beta3'},'location','east')
```



### Fit the Model

A DieboldLi object has been developed to facilitate fitting the model from yield data. The DieboldLi object inherits from the IRCurve object, so the getZeroRates, getDiscountFactors, getParYields, getForwardRates and toRateSpec methods are all implemented. Additionally, the method fitYieldsFromBetas has been implemented to estimate the Beta parameters given a lambda parameter for observed market yields.

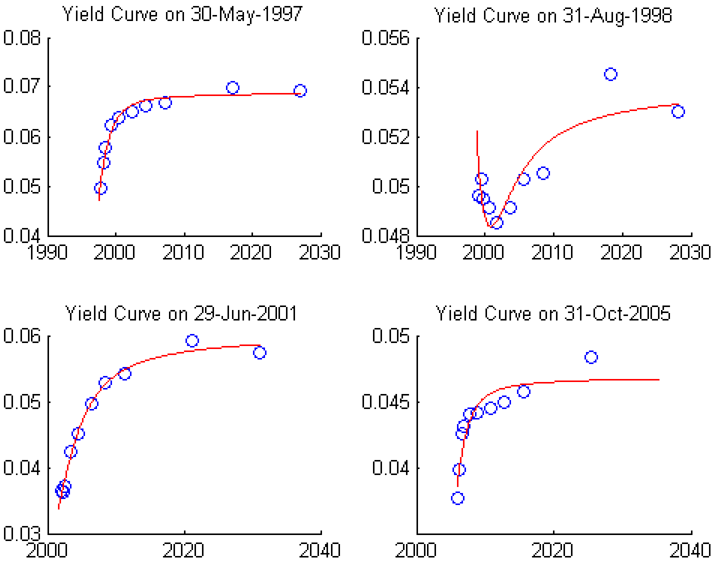
The DieboldLi object will be used to fit a Diebold Li model for each month from 1990 through 2010.

```
% Preallocate the Betas
Beta = zeros(size(EstimationData,1),3);

% Loop through and fit each end of month yield curve
for jdx=1:size(EstimationData,1)
    tmpCurveModel = DieboldLi.fitBetasFromYields(EOMDates(jdx),lambda_t*12,
    Beta(jdx,:) = [tmpCurveModel.Beta1 tmpCurveModel.Beta2 tmpCurveModel.Be
```

```
end
```

The Diebold Li fits on selected dates are included here

```
PlotSettles = datenum({'30-May-1997','31-Aug-1998','29-Jun-2001','31-Oct-20
figure
for jdx=1:length(PlotSettles)
    subplot(2,2,jdx)
    tmpIdx = find(strcmpi(get(Estimationdataset,'ObsNames'),datestr(PlotSet
    tmpCurveModel = DieboldLi.fitBetasFromYields(PlotSettles(jdx),lambda_t*
        daysadd(PlotSettles(jdx),30*TimeToMat),EstimationData(tmpIdx,:)');
    scatter(daysadd(PlotSettles(jdx),30*TimeToMat),EstimationData(tmpIdx,:)
    hold on
    PlottingDates = (PlotSettles(jdx)+30:30:PlotSettles(jdx)+30*360)';
    plot(PlottingDates,tmpCurveModel.getParYields(PlottingDates),'r-')
    title(['Yield Curve on ' datestr(PlotSettles(jdx))])
    datetick
end
```



### Forecasting

The Diebold Li model can be used to forecast future yield curves. Diebold and Li propose fitting an AR(1) model to the time series of each Beta parameter. This fitted model can then be used to forecast future values of each parameter, and by extension, future yield curves.

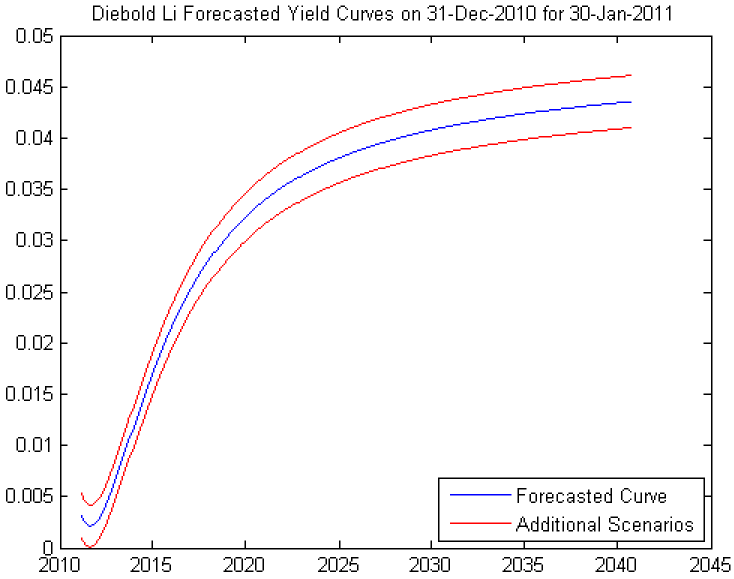
For this example the MATLAB function REGRESS is used to estimate the parameters for an AR(1) model for each Beta.

The confidence intervals for the regression fit are also used to generate two additional yield curve forecasts that serve as additional possible scenarios for the yield curve.

The MonthsLag variable can be adjusted to make different period ahead forecasts. For example, changing the value from 1 to 6 would change the forecast from a 1 month ahead to 6 month ahead forecast.

```
MonthsLag = 1;

[tmpBeta,bint] = regress(Beta(MonthsLag+1:end,1),[ones(size(Beta(MonthsLag+
ForecastBeta(1,1) = [1 Beta(end,1)]*tmpBeta;
ForecastBeta_Down(1,1) = [1 Beta(end,1)]*bint(:,1);
ForecastBeta_Up(1,1) = [1 Beta(end,1)]*bint(:,2);
[tmpBeta,bint]  = regress(Beta(MonthsLag+1:end,2),[ones(size(Beta(MonthsLag
ForecastBeta(1,2) = [1 Beta(end,2)]*tmpBeta;
ForecastBeta_Down(1,2) = [1 Beta(end,2)]*bint(:,1);
ForecastBeta_Up(1,2) = [1 Beta(end,2)]*bint(:,2);
[tmpBeta,bint]  = regress(Beta(MonthsLag+1:end,3),[ones(size(Beta(MonthsLag
ForecastBeta(1,3) = [1 Beta(end,3)]*tmpBeta;
ForecastBeta_Down(1,3) = [1 Beta(end,3)]*bint(:,1);
ForecastBeta_Up(1,3) = [1 Beta(end,3)]*bint(:,2);

% Forecasted yield curve
figure
Settle = daysadd(EOMDates(end),30*MonthsLag);
DieboldLi_Forecast = DieboldLi('ParYield',Settle,[ForecastBeta lambda_t*12]
DieboldLi_Forecast_Up = DieboldLi('ParYield',Settle,[ForecastBeta_Up lambda
DieboldLi_Forecast_Down = DieboldLi('ParYield',Settle,[ForecastBeta_Down la
PlottingDates = (Settle+30:30:Settle+30*360)';
```

```
plot(PlottingDates,DieboldLi_Forecast.getParYields(PlottingDates),'b-')
hold on
plot(PlottingDates,DieboldLi_Forecast_Up.getParYields(PlottingDates),'r-')
plot(PlottingDates,DieboldLi_Forecast_Down.getParYields(PlottingDates),'r-'
title(['Diebold Li Forecasted Yield Curves on ' datestr(EOMDates(end)) ' fo
legend({'Forecasted Curve','Additional Scenarios'},'location','southeast')
datetick
```



### Bibliography

This example is based on the following paper:

[1] Francis X. Diebold, Canlin Li, Forecasting the term structure of government bond yields, Journal of Econometrics, Volume 130, Issue 2, February 2006, Pages 337-364

# 8

# Credit Derivatives

# Credit Default Swap (CDS)

A credit default swap (CDS) is a contract that protects against losses resulting from credit defaults. The transaction involves two parties, the protection buyer and the protection seller, and also a reference entity, usually a bond. The protection buyer pays a stream of premiums to the protection seller, who in exchange offers to compensate the buyer for the loss in the bond's value if a credit event occurs. The stream of premiums is called the premium leg, and the compensation when a credit event occurs is called the protection leg. Credit events usually include situations in which the bond issuer goes bankrupt, misses coupon payments, or enters a restructuring process. Financial Instruments Toolbox software supports:

**CDS Functions**

| Function | Purpose |
|---|---|
| `cdsbootstrap` | Compute default probability parameters from CDS market quotes. |
| `cdsspread` | Compute breakeven spreads for the CDS contracts. |
| `cdsprice` | Compute the price for the CDS contracts. |

## Bootstrapping a Default Probability Curve

In a typical workflow, pricing a new CDS contract involves first estimating a default probability term structure using `cdsbootstrap`. This requires market quotes of existing CDS contracts, or quotes of CDS indices (e.g., iTraxx). The estimated default probability curve is then used as input to `cdsspread` or `cdsprice`. If the default probability information is already known, `cdsbootstrap` can be bypassed and `cdsspread` or `cdsprice` can be called directly.

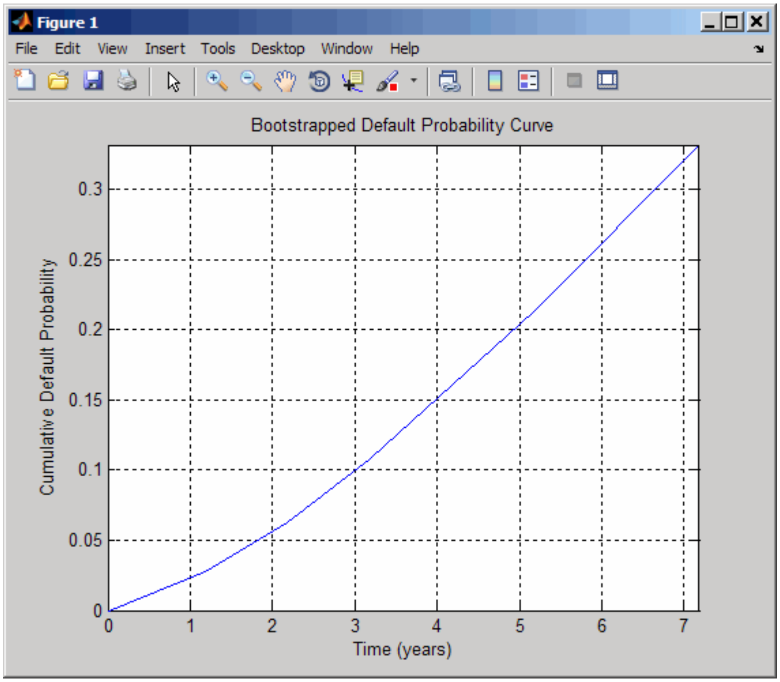
The market information in this example is provided in the form of running spreads of CDS contracts maturing on the CDS standard payment dates closest to 1, 2, 3, 5, and 7 years from the valuation date.

```
Settle = '17-Jul-2009';
MarketDates = datenum({'20-Sep-10','20-Sep-11','20-Sep-12','20-Sep-14',...
```

```
'20-Sep-16'});
MarketSpreads = [140 175 210 265 310]';
MarketData = [MarketDates MarketSpreads];
ZeroDates = datenum({'17-Jan-10','17-Jul-10','17-Jul-11','17-Jul-12',...
'17-Jul-13','17-Jul-14'});
ZeroRates = [1.35 1.43 1.9 2.47 2.936 3.311]'/100;
ZeroData = [ZeroDates ZeroRates];

[ProbData,HazData] = cdsbootstrap(ZeroData,MarketData,Settle);
```

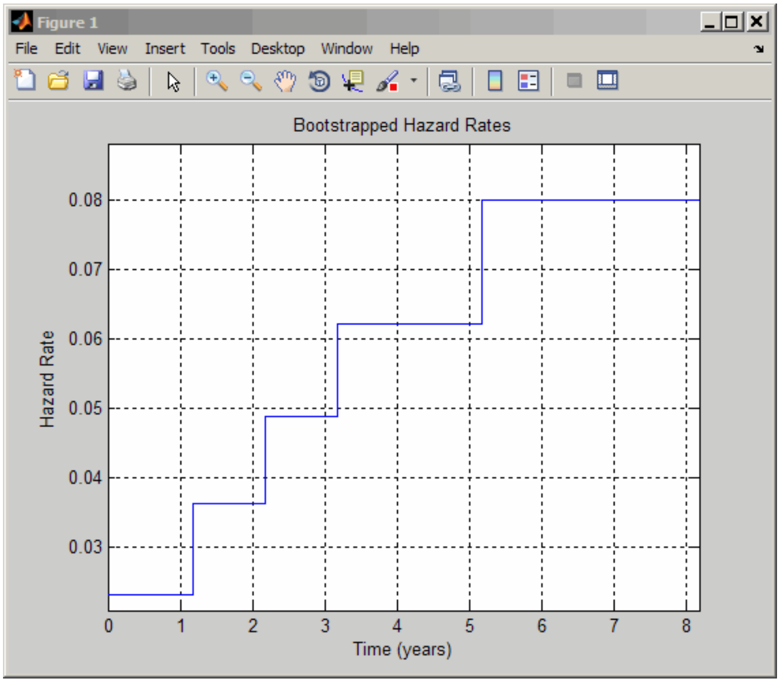
The bootstrapped default probability curve is plotted against time, in years, from the valuation date.

```
ProbTimes = yearfrac(Settle,ProbData(:,1));
figure
plot([0; ProbTimes],[0; ProbData(:,2)])
grid on
axis([0 ProbTimes(end,1) 0 ProbData(end,2)])
xlabel('Time (years)')
ylabel('Cumulative Default Probability')
title('Bootstrapped Default Probability Curve')
```



The associated hazard rates are returned as an optional output. The convention is that the first hazard rate applies from the settlement date to the first market date, the second hazard rate from the first to the second market date, etc., and the last hazard rate applies from the second-to-last market date onwards. The following plot displays the bootstrapped hazard rates, plotted against time, in years, from the valuation date:

```
HazTimes = yearfrac(Settle,HazData(:,1));
figure
stairs([0; HazTimes(1:end-1,1); HazTimes(end,1)+1],...
[HazData(:,2);HazData(end,2)])
grid on
axis([0 HazTimes(end,1)+1 0.9*HazData(1,2) 1.1*HazData(end,2)])
xlabel('Time (years)')
ylabel('Hazard Rate')
title('Bootstrapped Hazard Rates')
```



## Finding Breakeven Spread for New CDS Contract

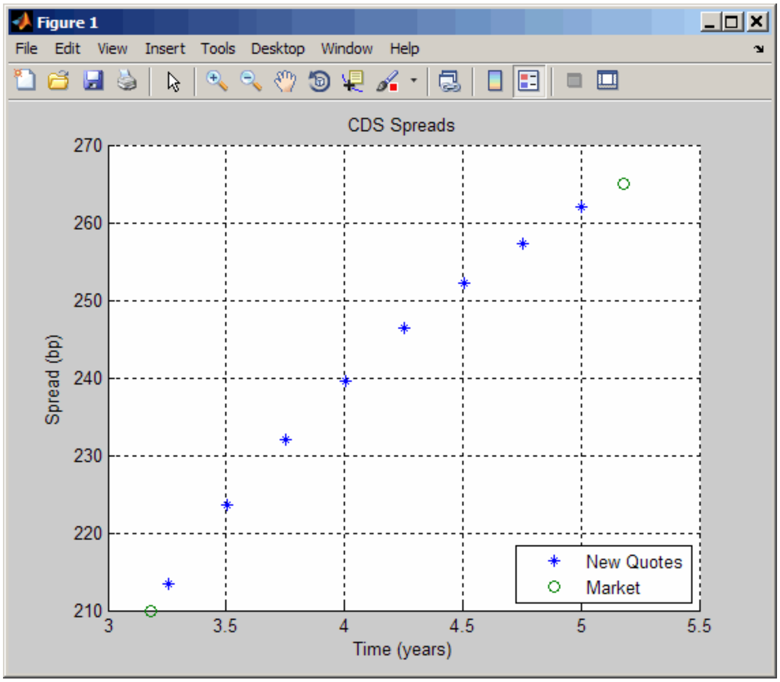
The breakeven, or running, spread is the premium a protection buyer must pay, with no upfront payments involved, to receive protection for credit events associated to a given reference entity. Spreads are expressed in basis points (bp). There is a notional amount associated to the CDS contract to determine the monetary amounts of the premium payments.

New quotes for CDS contracts can be obtained with `cdsspread`. After obtaining a default probability curve using `cdsbootstrap`, you get quotes that are consistent with current market conditions.

In this example, instead of standard CDS payment dates, define new maturity dates. Using the period from 3 and 5 years (CDS standard dates), maturities are defined within this range spaced at quarterly intervals (measuring time from the valuation date):

```
Settle = '17-Jul-2009';
MarketDates = datenum({'20-Sep-10','20-Sep-11','20-Sep-12','20-Sep-14',...
'20-Sep-16'});
MarketSpreads = [140 175 210 265 310]';
MarketData = [MarketDates MarketSpreads];
ZeroDates = datenum({'17-Jan-10','17-Jul-10','17-Jul-11','17-Jul-12',...
'17-Jul-13','17-Jul-14'});
ZeroRates = [1.35 1.43 1.9 2.47 2.936 3.311]'/100;
ZeroData = [ZeroDates ZeroRates];

[ProbData,HazData] = cdsbootstrap(ZeroData,MarketData,Settle);

Maturity1 = datestr(daysadd('17-Jul-09',360*(3.25:0.25:5),1));
Spread1 = cdsspread(ZeroData,ProbData,Settle,Maturity1);

figure
scatter(yearfrac(Settle,Maturity1),Spread1,'*')
hold on
scatter(yearfrac(Settle,MarketData(3:4,1)),MarketData(3:4,2))
hold off
grid on
xlabel('Time (years)')
ylabel('Spread (bp)')
title('CDS Spreads')
legend('New Quotes','Market','location','SouthEast')
```

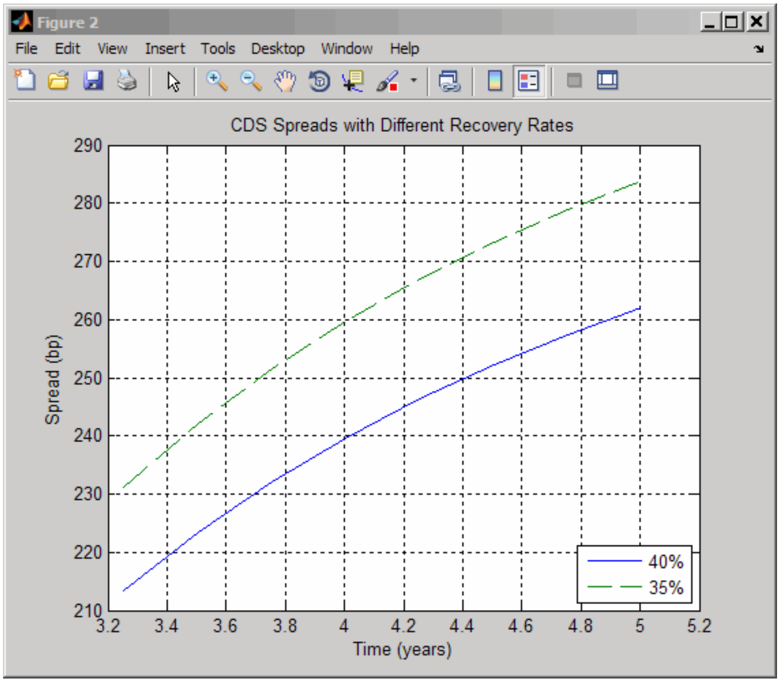
This plot displays the resulting spreads:



To evaluate the effect of the recovery rate parameter, instead of 40% (default value), use a recovery rate of 35%:

```
Spread1Rec35 = cdsspread(ZeroData,ProbData,Settle,Maturity1,...
'RecoveryRate',0.35);

figure
plot(yearfrac(Settle,Maturity1),Spread1,...
yearfrac(Settle,Maturity1),Spread1Rec35,'--')
grid on
xlabel('Time (years)')
ylabel('Spread (bp)')
title('CDS Spreads with Different Recovery Rates')
legend('40%','35%','location','SouthEast')
```

The resulting plot shows that smaller recovery rates produce higher premia, as expected, since in the event of default, the protection payments will be higher:



## Valuing an Existing CDS Contract

The current value, or mark-to-market, of an existing CDS contract is the amount of money the contract holder would receive (if positive) or pay (if negative) to unwind this contract. The upfront of the contract is the current value expressed as a fraction of the notional amount of the contract, and it is commonly used to quote market values.

The value of existing CDS contracts is obtained with `cdsprice`. By default, `cdsprice` treats contracts as long positions. Whether a contract position is long or short is determined from a protection standpoint, that is, long means protection was bought, and short means protection was sold. In the following example, an existing CDS contract pays a premium that is lower than current market conditions. The price is positive, as expected, since it would be more costly to buy the same type of protection today.

```
Settle = '17-Jul-2009';
MarketDates = datenum({'20-Sep-10','20-Sep-11','20-Sep-12','20-Sep-14',...
```

```
'20-Sep-16'});
MarketSpreads = [140 175 210 265 310]';
MarketData = [MarketDates MarketSpreads];

ZeroDates = datenum({'17-Jan-10','17-Jul-10','17-Jul-11','17-Jul-12',...
'17-Jul-13','17-Jul-14'});
ZeroRates = [1.35 1.43 1.9 2.47 2.936 3.311]'/100;
ZeroData = [ZeroDates ZeroRates];

[ProbData,HazData] = cdsbootstrap(ZeroData,MarketData,Settle);

Maturity2 = '20-Sep-2012';
Spread2 = 196;

[Price,AccPrem,PaymentDates,PaymentTimes,PaymentCF] = cdsprice(ZeroData,...
ProbData,Settle,Maturity2,Spread2);

fprintf('Dirty Price: %8.2f\n',Price);
fprintf('Accrued Premium: %8.2f\n',AccPrem);
fprintf('Clean Price: %8.2f\n',Price-AccPrem);
fprintf('\nPayment Schedule:\n\n');
fprintf('Date \t\t Time Frac \t Amount\n');
for k = 1:length(PaymentDates)
   fprintf('%s \t %5.4f \t %8.2f\n',datestr(PaymentDates(k)),...
      PaymentTimes(k),PaymentCF(k));
end
```

This resulting payment schedule is:

```
Dirty Price: 41630.75
Accrued Premium: 15244.44
Clean Price: 26386.30

Payment Schedule:

Date    Time Frac   Amount
20-Sep-2009   0.1806   35388.89
20-Dec-2009   0.2528   49544.44
20-Mar-2010   0.2500   49000.00
20-Jun-2010   0.2556   50088.89
```

```
20-Sep-2010   0.2556   50088.89
20-Dec-2010   0.2528   49544.44
20-Mar-2011   0.2500   49000.00
20-Jun-2011   0.2556   50088.89
20-Sep-2011   0.2556   50088.89
20-Dec-2011   0.2528   49544.44
20-Mar-2012   0.2528   49544.44
20-Jun-2012   0.2556   50088.89
20-Sep-2012   0.2556   50088.89
```

Additionally, you can use `cdsprice` to value a portfolio of CDS contracts. In the following example, a simple hedged position with two vanilla CDS contracts, one long, one short, with slightly different spreads is priced in a single call and the value of the portfolio is the sum of the returned prices:

```
[Price2,AccPrem2] = cdsprice(ZeroData,ProbData,Settle,...
repmat(datenum(Maturity2),2,1),[Spread2;Spread2+3],...
'Notional',[1e7; -1e7]);

fprintf('Contract \t Dirty Price \t Acc Premium \t  Clean Price\n');
fprintf('    Long \t $ %9.2f \t $ %9.2f \t $ %9.2f \t\n',...
   Price2(1), AccPrem2(1), Price2(1) - AccPrem2(1));
fprintf('   Short \t $ %8.2f \t $ %8.2f \t $ %8.2f \t\n',...
   Price2(2), AccPrem2(2), Price2(2) - AccPrem2(2));
fprintf('Mark-to-market of hedged position: $ %8.2f\n',sum(Price2));
```

This resulting value of the portfolio is:

```
Contract   Dirty Price   Acc Premium    Clean Price
    Long   $  41630.75   $  15244.44   $  26386.30
   Short   $ -32709.87   $ -15477.78   $ -17232.10
Mark-to-market of hedged position: $  8920.87
```

## Converting from Running to Upfront

A CDS market quote is given in terms of a standard spread (usually 100 bp or 500 bp) and an upfront payment, or in terms of an equivalent running or breakeven spread, with no upfront payment. The functions `cdsbootstrap`, `cdsspread`, and `cdsprice` perform upfront to running or running to upfront conversions.

For example, to convert the market quotes to upfront quotes for a standard spread of 100 bp:

```
Settle = '17-Jul-2009';
MarketDates = datenum({'20-Sep-10','20-Sep-11','20-Sep-12','20-Sep-14',...
'20-Sep-16'});
MarketSpreads = [140 175 210 265 310]';
MarketData = [MarketDates MarketSpreads];

ZeroDates = datenum({'17-Jan-10','17-Jul-10','17-Jul-11','17-Jul-12',...
'17-Jul-13','17-Jul-14'});
ZeroRates = [1.35 1.43 1.9 2.47 2.936 3.311]'/100;
ZeroData = [ZeroDates ZeroRates];

[ProbData,HazData] = cdsbootstrap(ZeroData,MarketData,Settle);

Maturity3 = MarketData(:,1);
Spread3Run = MarketData(:,2);
Spread3Std = 100*ones(size(Maturity3));
Price3 = cdsprice(ZeroData,ProbData,Settle,Maturity3,Spread3Std);
Upfront3 = Price3/10000000; % Standard notional of 10MM
display(Upfront3);
```

This resulting value is:

```
Upfront3 =

    0.0047
    0.0158
    0.0327
    0.0737
    0.1182
```

The conversion can be reversed to convert upfront quotes to market quotes:

```
ProbData3Upf = cdsbootstrap(ZeroData,[Maturity3 Upfront3 Spread3Std],Settle);
Spread3RunFromUpf = cdsspread(ZeroData,ProbData3Upf,Settle,Maturity3);
display([Spread3Run Spread3RunFromUpf]);
```

Comparing the results of this conversion to the original market spread demonstrates the reversal:

```
ans =

  140.0000  140.0000
  175.0000  175.0000
  210.0000  210.0000
  265.0000  265.0000
  310.0000  310.0000
```

Under the flat-hazard rate (FHR) quoting convention, a single market quote is used to calibrate a probability curve. This convention yields a single point in the probability curve, and a single hazard rate value. For example, assume a 4-year (standard dates) CDS contract with a current FHR-based running spread of 550 bp needs conversion to a CDS contract with a standard spread of 500 bp:

```
Maturity4 = datenum('20-Sep-13');
Spread4Run = 550;
ProbData4Run = cdsbootstrap(ZeroData,[Maturity4 Spread4Run],Settle);
Spread4Std = 500;
Price4 = cdsprice(ZeroData,ProbData4Run,Settle,Maturity4,Spread4Std);
Upfront4 = Price4/10000000;
fprintf('A running spread of %5.2f is equivalent to\n',Spread4Run);
fprintf('   a standard spread of %5.2f with an upfront of %8.7f\n',...
   Spread4Std,Upfront4);

A running spread of 550.00 is equivalent to
   a standard spread of 500.00 with an upfront of 0.0167583
```

To reverse the conversion:

```
ProbData4Upf = cdsbootstrap(ZeroData,[Maturity4 Upfront4 Spread4Std],Settle);
Spread4RunFromUpf = cdsspread(ZeroData,ProbData4Upf,Settle,Maturity4);
fprintf('A standard spread of %5.2f with an upfront of %8.7f\n',...
   Spread4Std,Upfront4);
fprintf('    is equivalent to a running spread of %5.2f\n',Spread4RunFromUpf);

A standard spread of 500.00 with an upfront of 0.0167583
    is equivalent to a running spread of 550.00
```

As discussed in Beumee et. al., 2009 (see “Credit Derivatives” on page C-14), the FHR approach is a quoting convention only, and leads to quotes inconsistent with market data. For example, calculating the upfront for the 3-year (standard dates) CDS contract with a standard spread of 100 bp using the FHR approach and comparing the results to the upfront amounts previously calculated, demonstrates that the FHR-based approach yields a different upfront amount:

```
Maturity5 = MarketData(3,1);
Spread5Run = MarketData(3,2);
ProbData5Run = cdsbootstrap(ZeroData,[Maturity5 Spread5Run],Settle);
Spread5Std = 100;
Price5 = cdsprice(ZeroData,ProbData5Run,Settle,Maturity5,Spread5Std);
Upfront5 = Price5/10000000;
fprintf('Relative error of FHR-based upfront amount: %3.1f%%\n',...
   ((Upfront5-Upfront3(3))/Upfront3(3))*100);

Relative error of FHR-based upfront amount: -0.8%
```

## Bootstrapping from Inverted Market Curves

The following two examples demonstrate the behavior of bootstrapping with inverted CDS market curves, that is, market quotes with higher spreads for short-term CDS contracts. The first example is handled normally by `cdsbootstrap`:

```
Settle = '17-Jul-2009';
MarketDates = datenum({'20-Sep-10','20-Sep-11','20-Sep-12','20-Sep-14',...
'20-Sep-16'});

ZeroDates = datenum({'17-Jan-10','17-Jul-10','17-Jul-11','17-Jul-12',...
'17-Jul-13','17-Jul-14'});
ZeroRates = [1.35 1.43 1.9 2.47 2.936 3.311]'/100;
ZeroData = [ZeroDates ZeroRates];

MarketSpreadsInv1 = [750 650 550 500 450]';
MarketDataInv1 = [MarketDates MarketSpreadsInv1];
[ProbDataInv1,HazDataInv1] = cdsbootstrap(ZeroData,MarketDataInv1,Settle);
```

In the second example, `cdsbootstrap` generates a warning:

```
MarketSpreadsInv2 = [800 550 400 250 100]';
MarketDataInv2 = [MarketDates MarketSpreadsInv2];
```

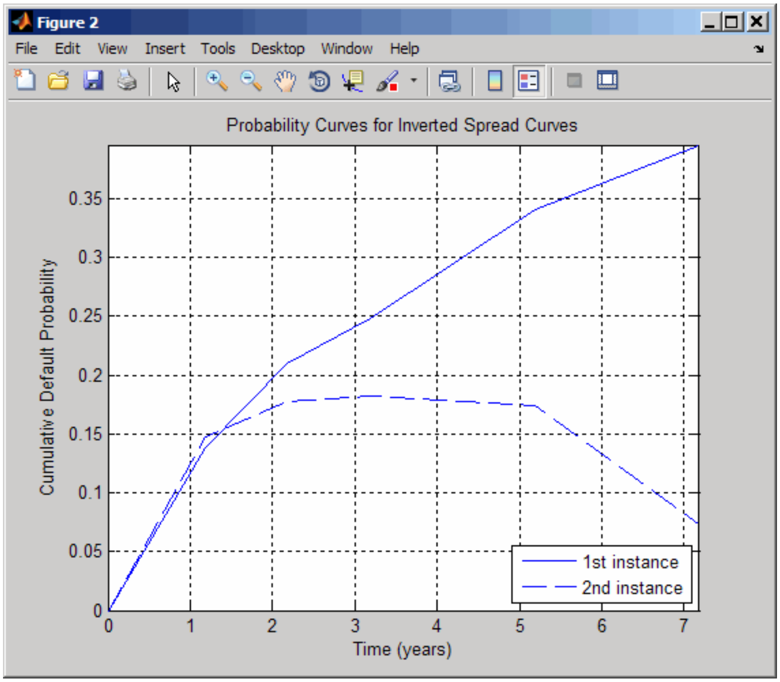
```
[ProbDataInv2,HazDataInv2] = cdsbootstrap(ZeroData,MarketDataInv2,Settle);

Warning: Found non-monotone default probabilities (negative hazard rates)
```

A non-monotone bootstrapped probability curve implies negative default probabilities and negative hazard rates for certain time intervals. Extreme market conditions can lead to these types of situations. In these cases, you must assess the reliability and usefulness of the bootstrapped results.

The following plot illustrates these bootstrapped probability curves. The curves are concave, meaning that the marginal default probability decreases with time. This result is consistent with the market information that indicates a higher default risk in the short term. The second bootstrapped curve is non-monotone, as indicated by the warning.

```
ProbTimes = yearfrac(Settle, MarketDates);
figure
plot([0; ProbTimes],[0; ProbDataInv1(:,2)])
hold on
plot([0; ProbTimes],[0; ProbDataInv2(:,2)],'--')
hold off
grid on
axis([0 ProbTimes(end,1) 0 ProbDataInv1(end,2)])
xlabel('Time (years)')
ylabel('Cumulative Default Probability')
title('Probability Curves for Inverted Spread Curves')
legend('1st instance','2nd instance','location','SouthEast')
```

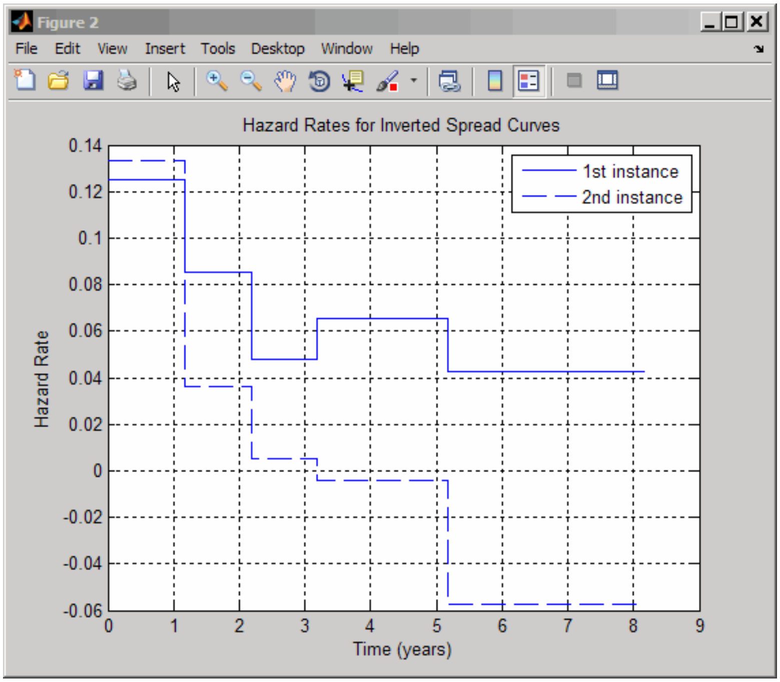
The resulting plot



Also in line with the previous plot, the hazard rates for these bootstrapped curves are decreasing because the short-term risk is higher. Some bootstrapped parameters in the second curve are negative, as indicated by the warning.

```
HazTimes = yearfrac(Settle, MarketDates);
figure
stairs([0; HazTimes(1:end-1,1); HazTimes(end,1)+1],...
   [HazDataInv1(:,2);HazDataInv1(end,2)])
hold on
stairs([0; HazTimes(1:end-1,1); HazTimes(end,1)+1],...
   [HazDataInv2(:,2);HazDataInv2(end,2)],'--')
hold off
grid on
xlabel('Time (years)')
ylabel('Hazard Rate')
title('Hazard Rates for Inverted Spread Curves')
legend('1st instance','2nd instance','location','NorthEast')
```

The resulting plot shows the hazard rates for both bootstrapped curves:



For further discussion on inverted curves, and their relationship to arbitrage, see O'Kane and Turnbull, 2003 ("Credit Derivatives" on page C-14).

# Counterparty Credit Risk and CVA

This example shows how to compute the unilateral Credit Value (Valuation) Adjustment (CVA) for a bank holding a portfolio of vanilla interest rate swaps with several counterparties. CVA is the expected loss on an over-the-counter instrument or portfolio of instruments due to counterparty default. The CVA for a particular counterparty is defined as the sum over all points in time of the discounted expected exposure at each moment multiplied by the probability that the counterparty defaults at that moment, all multiplied by 1 minus the recovery rate. The Credit Value (Valuation) Adjustment (CVA) formula is:

$$CVA = (1 - R)\int_0^T discEE(t)dPD(t)$$

Where `R` is the recovery, `discEE` the discounted expected exposure at time t, and `PD` the default probability distribution.

The expected exposure is computed by first simulating many future scenarios of risk factors for the given instrument or portfolio. Risk factors can be interest rates, FX rates, equity or commodity prices, or anything that will affect the market value of the instruments. Once a sufficient set of scenarios has been simulated, the contract or portfolio can be priced on a series of future dates for each scenario. The result is a matrix, or "cube", of instrument values.

These prices are converted into exposures after taking into account collateral agreements that the bank might have in place as well as netting agreements where the values of several instruments may offset each other, lowering their total exposure.

The instrument values for each scenario are discounted to compute the discounted exposures. The discounted expected exposures can then be computed by a simple average of the discounted exposures at each simulation date.

Finally, counterparty default probabilities are typically derived from credit default swap market quotes and the CVA for the counterparty can be computed according to the above formula.

For this example we will work with a portfolio of vanilla interest rate swaps with the goal of computing the CVA for a particular counterparty.

This example can run slowly on some machines. If you have the Parallel Computing Toolbox™ installed it can improve the performance.

#### Read Swap Portfolio

The portfolio of swaps is close to zero value at time t=0. Each swap is associated with a counterparty and may or may not be included in a netting agreement.

```
% Read swaps from spreadsheet
swapFile = 'cva-swap-portfolio.xls';
swapData = dataset('XLSFile',swapFile);

swaps = struct(...
    'Counterparty',[],...
    'NettingID',[],...
    'Principal',[],...
    'Maturity',[],...
    'LegRate',[],...
    'LegType',[],...
    'LatestFloatingRate',[],...
    'LastFloatingDate',[]);

swaps.Counterparty = swapData.CounterpartyID;
swaps.NettingID    = swapData.NettingID;
swaps.Principal    = swapData.Principal;
swaps.Maturity     = swapData.Maturity;
swaps.LegType      = [swapData.LegType ~swapData.LegType];
swaps.LegRate      = [swapData.LegRateReceiving swapData.LegRatePaying];
swaps.LatestFloatingRate = swapData.LatestFloatingRate;
swaps.Period       = swapData.Period;

numSwaps = numel(swaps.Counterparty);
numCounterparties = max(swaps.Counterparty);
```

#### Create RateSpec from the Interest Rate Curve

```
Settle   = datenum('14-Dec-2007');
Tenor  = [3 6 12 5*12 7*12 10*12 20*12 30*12]';
ZeroRates = [0.033 0.034 0.035 0.040 0.042 0.044 0.048 0.0475]';
```

```
ZeroDates = datemnth(Settle,Tenor);
Compounding = 2;
Basis = 0;
RateSpec = intenvset('StartDates', Settle,'EndDates', ZeroDates,...
    'Rates', ZeroRates,'Compounding',Compounding,'Basis',Basis);

% Create an IRCurve object.  We will use this for computing instantaneous
% forward rates during the calculation of the Hull-White short rate path.
RateCurveObj = IRDataCurve('Zero',Settle,ZeroDates,ZeroRates,...
    'Compounding', Compounding,'Basis', Basis);
```

### Setup Tunable Simulation Parameters

We can vary the number of simulated interest rate scenarios we generate by tuning the variable here. We set our simulation dates to be more frequent at first, then turning less frequent further in the future.

```
% Number of Monte Carlo simulations
numScenarios = 64;

% Compute monthly simulation dates, then quarterly dates later.
simulationDates = datemnth(Settle+1,1:12);
simulationDates = [simulationDates datemnth(simulationDates(end),3:3:64)];
```

### Compute Initial Prices for All Swaps

```
currentPrices = swapbyzero(RateSpec,...
    swaps.LegRate,...
    Settle,...
    swaps.Maturity,...
    'Principal',swaps.Principal,...
    'LegType',swaps.LegType,...
    'LatestFloatingRate',swaps.LatestFloatingRate);

% For each simulation date, compute last floating reset date per swap
floatDates = cfdates(Settle-360,swaps.Maturity,swaps.Period);
swaps.LastFloatingDate = zeros(numSwaps,numel(simulationDates));
for i = numel(simulationDates):-1:1
    thisDate = simulationDates(i);
    floatDates(floatDates > thisDate) = 0;
```

```
    swaps.LastFloatingDate(:,i) = max(floatDates,[],2);
end
```

### Setup Hull-White Single Factor Model

The risk factor we will simulate to value our instruments is the zero curve. For this example we will model the interest rate term structure using the one-factor Hull-White model. This is a model of the short rate and is defined as:

$$dr = [\theta(t) - ar]dt + \sigma dz$$

where

- $dr$ : Change in the short rate after a small change in time, $dt$
- $a$ : Mean reversion rate
- $\sigma$ : Volatility of the short rate
- $dz$ : A Weiner process (a standard normal process)
- $\theta(t)$ : Drift function defined as:

$$\theta(t) = F_t(0,t) + aF(0,t) + \frac{\sigma^2}{2a}(1 - e^{-2at})$$

$F(0,t)$ : Instantaneous forward rate at time $t$

$F_t(0,t)$ : Partial derivative of $F$ with respect to time

Once we have simulated a path of the short rate we generate a full yield curve at each simulation date using the formula:

$$R(t,T) = -\frac{1}{(T-t)}\ln A(t,T) + \frac{1}{(T-t)}B(t,T)r(t)$$

$$\ln A(t,T) = \ln\frac{P(0,T)}{P(0,t)} + B(t,T)F(0,t) - \frac{1}{4a^3}\sigma^2(e^{-aT} - e^{-at})^2(e^{2at} - 1)$$

$$B(t,T) = \frac{1 - e^{-a(T-t)}}{a}$$

$R(t,T)$: Zero rate at time $t$ for a period of $T-t$

$P(t,T)$: Price of a zero coupon bond at time $t$ that pays one dollar at time $T$

Each scenario contains the full term structure moving forward through time, modeled at each of our selected simulation dates.

Refer to "Calibrating the Hull-White Model Using Market Data" example in the Financial Instruments Toolbox Users' Guide for more details on Hull-White single factor model calibration.

```
Alpha = 0.2;
Sigma = 0.015;

r0 = RateCurveObj.getZeroRates(Settle+1,'Compounding',-1);
t0 = Settle;

% Construct SDE object
hullwhite1 = hwv(Alpha,@(t,x) hw1LevelFun(t0,t,RateCurveObj,Alpha,Sigma),..
    Sigma,'StartState',r0);

% Store all model calibration information
calibration.RateCurveObj = RateCurveObj;
calibration.Tenor = Tenor;
calibration.ShortRateModel = hullwhite1;
calibration.Alpha = Alpha;
calibration.Sigma = Sigma;
```

#### Simulate Scenarios and Compute Mark-To-Market Values

For each scenario the swap portfolio is priced at each future simulation date. Prices are computed using `swapbyzero` and the simulated zero curve at each date. The prices are then aggregated into a "cube" of vales which contains all future instrument values at each simulation date for each scenario. The resulting cube of instrument prices is a 3 dimensional matrix where each row represents an instrument, each column a simulation date, and each "page" a different simulated scenario.

```
% Allocate cube of simulated values: rows correspond to instruments,
% columns to valuation dates, "pages" to scenarios
```

```
simulatedValues = zeros(numSwaps,numel(simulationDates),numScenarios);

% Pre-allocate scenarios data structure
sampleScenario = hgenerateScenario(calibration,Settle,simulationDates);
scenarios = repmat(sampleScenario,numScenarios,1);

initialOneYearRate = RateCurveObj.getZeroRates(Settle + 365,'Compounding',-
```

For each scenario, we model a future interest rate curve at each valuation date. Using the complete future interest rate path, we compute the value of all instruments at each valuation date.

Since the simulation dates do not correspond to the swaps cash flow dates (where the floating rates are reset) we estimate the latest floating rate with the 1-year rate (all of these swaps have period 1 year) interpolated between the nearest simulated rate curves.

The scenario generation and pricing are done in parallel using the `parfor` loop if the Parallel Computing Toolbox is installed. The scenarios and their respective instrument values are computed in parallel across all MATLAB workers.

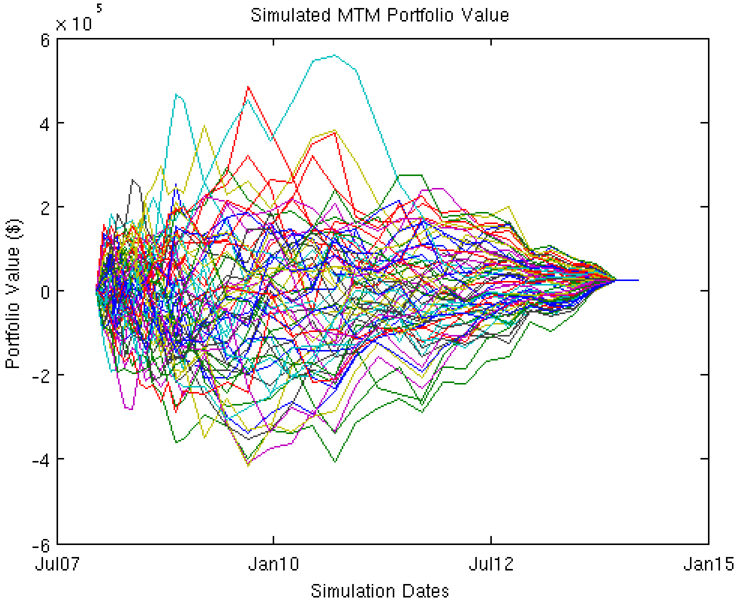
```
% Use reproducible random number generator (vary the seed to produce
% different random scenarios)
stream = RandStream.create('mrg32k3a','NumStreams',numScenarios,...
    'Seed',0);

% Simulate all scenarios and compute instrument values in parallel.  If you
% have the Parallel Computing Toolbox, you should open a matlabpool before
% running this section.
parfor scenarioIdx = 1:numScenarios

    % Save state of random number generator
    defaultStreamLocal = RandStream.getGlobalStream();
    savedStateLocal = defaultStreamLocal.State;

    % Setup new random number generator state for each scenario
    RandStream.setGlobalStream(stream);
    set(stream,'Substream',scenarioIdx);
```

```
    % Create a scenario
    scenarios(scenarioIdx) = hgenerateScenario(calibration,...
        Settle,simulationDates);

    % Compute all mark-to-market values for this scenario
    thisScenarioValues = hcomputeMTMValues(swaps,simulationDates,...
        scenarios(scenarioIdx),Settle,initialOneYearRate);

    % Aggregate data
    simulatedValues(:,:,scenarioIdx) = thisScenarioValues;

    % Restore random number generator state
    RandStream.setGlobalStream(defaultStreamLocal);
    defaultStreamLocal.State = savedStateLocal;

end
```

### Visualize Simulated Portfolio Values

We plot the total portfolio value for each scenario of our simulation. As each scenario moves forward in time the values of the instruments will move up or down depending on how the modeled interest rate term structure changes. As the swaps get closer to maturity, their values will begin to approach zero since the aggregate value of all remaining cash flows will decrease after each cash flow date.

```
% Append initial prices/date to our simulation data
values = cat(2,repmat(currentPrices,[1 1 numScenarios]),simulatedValues);
dates = [Settle simulationDates];

% View portfolio value over time
figure;
totalPortValues = squeeze(sum(values));
plot(dates,totalPortValues);
title('Simulated MTM Portfolio Value');
datetick('x','mmmyy')
ylabel('Portfolio Value ($)')
xlabel('Simulation Dates')
```



**Compute Exposure by Counterparty**

The exposure of a particular contract (i) at time t is the maximum of the contract value (Vi) and 0:

$$E_i(t) = \max\{V_i(t), 0\}$$

And the exposure for a particular counterparty is simply a sum of the individual contract exposures:

$$E(t) = \sum E_i(t) = \sum \max\{V_i(t), 0\}$$

In the presence of netting agreements, however, contracts are aggregated together and can offset each other. Therefore the total exposure of all instruments in a netting agreement is

$$E(t) = \max\{\sum V_i(t), 0\}$$

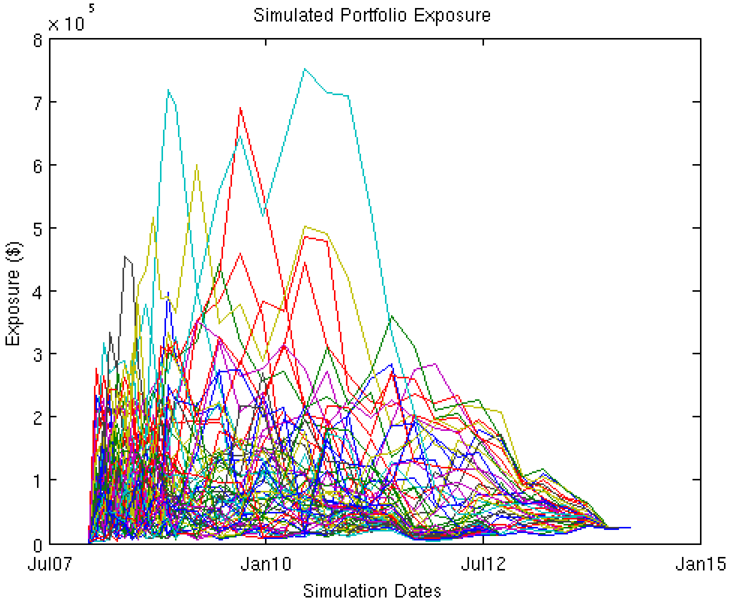
We compute these exposures for each counterparty at each simulation date.

```
% Additive exposure is computed at the netting set level.  Exposure of an
% unnetted instrument is equal to the market value of the instrument if the
% instrument has positive value, otherwise it is zero.
instrument_exposures = zeros(size(values));
unnettedIdx = swaps.NettingID == 0;
instrument_exposures(unnettedIdx,:,:) = max(values(unnettedIdx,:,:),0);

% Instruments included in a netting agreement have exposure equal to their
% value when the netting agreement has positive aggregate value, otherwise
% their exposure is zero.  We compute this per netting agreement, but in
% this case each counterparty has only a single netting agreement.
for j = 1:numCounterparties

    nettedIdx = swaps.NettingID == j;

    % Exposures for instruments under netting agreements
    nettedValues = values(nettedIdx,:,:);
    nettedExposure = max(sum(nettedValues,1),0);
    positiveIdx = nettedExposure > 0;
    instrument_exposures(nettedIdx,positiveIdx) = values(nettedIdx,positive

end

% Sum the instrument exposures for each counterparty
exposures = zeros(numCounterparties,numel(dates),numScenarios);
for j = 1:numCounterparties

    cpSwapIdx = swaps.Counterparty == j;
    exposures(j,:,:) = squeeze(sum(instrument_exposures(cpSwapIdx,:,:),1));

end
```

We plot the total portfolio exposure for each scenario in our simulation. Similar to the plot of instrument values, the exposures for each scenario will approach zero as the swaps mature.

```
% View portfolio exposure over time
figure
```

```
totalPortExposure = squeeze(sum(exposures,1));
plot(dates,totalPortExposure);
title('Simulated Portfolio Exposure');
datetick('x','mmmyy')
ylabel('Exposure ($)')
xlabel('Simulation Dates')
```



### Exposure Profiles

Several exposure profiles are useful when analyzing the potential future exposure of a bank to a counterparty. Here we compute several (non-discounted) exposure profiles per counterparty as well as for the entire portfolio.

- `PE` : Peak Exposure : A high percentile (95%) of the distribution of exposures at any particular future date
- `MPE` : Maximum Peak Exposure : The maximum peak exposure across all dates

- `EE` : Expected Exposure : The mean (average) of the distribution of exposures at each date
- `EPE` : Expected Positive Exposure : Weighted average over time of the expected exposure
- `EffEE` : Effective Expected Exposure : The maximum expected exposure at any time, t, or previous time
- `EffEPE` : Effective Expected Positive Exposure : The weighted average of the effective expected exposure

For further definitions, see for example Basel II document in references.

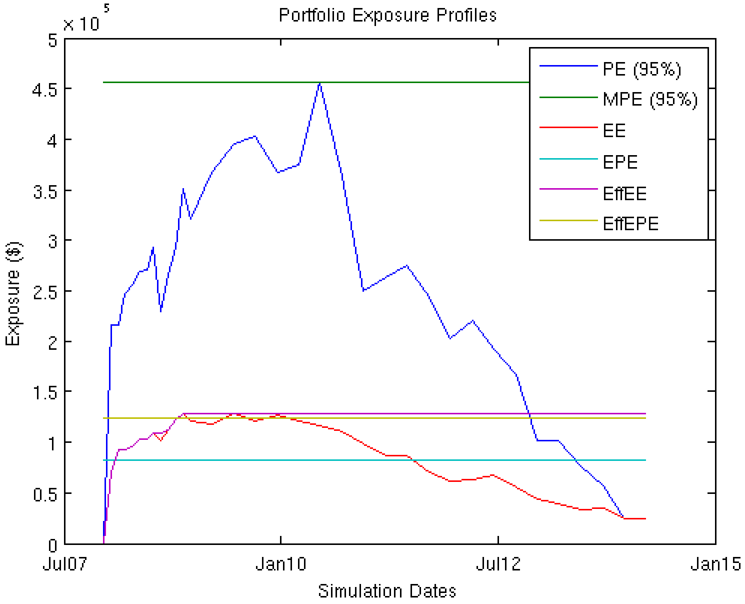
```
% Compute entire portfolio exposure
expPort = squeeze(sum(exposures))';

% Peak Exposure (same as Potential Future Exposure)
PEcp   = prctile(exposures,95,3);
PEport = prctile(expPort,95);

% Maximum Peak Exposure
MPEcp   = max(PEcp,[],2);
MPEport = max(PEport);

% Expected Exposure
EEcp   = mean(exposures,3);
EEport = mean(expPort);

% Expected Positive Exposure: Weighted average over time of EE
% * In continuous time, this is the average expected exposures over time,
%   an integral of EE(t) over the time interval, divided by the length of
%   the interval
% * Compute using a "trapezoidal" approach here
simTimeInterval = yearfrac(Settle, dates, 1);
simTotalTime = simTimeInterval(end)-simTimeInterval(1);
EPEcp   = 0.5*(EEcp(:,1:end-1)+EEcp(:,2:end))*diff(simTimeInterval)'/simTot
EPEport = 0.5*(EEport(1:end-1)+EEport(2:end))*diff(simTimeInterval)'/simTot

% Effective Expected Exposure: Max EE up to time simTimeInterval
EffEEcp = zeros(size(EEcp));
for i = 1:size(EEcp,1)
```

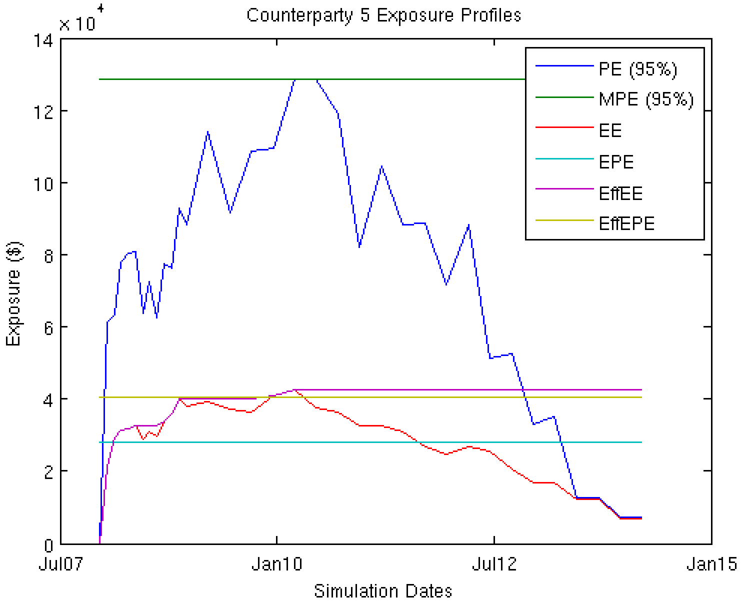
```
    % Compute cumulative maximum
    m = EEcp(i,1);
    for j = 1:numel(dates)
        if EEcp(i,j) > m
            m = EEcp(i,j);
        end
        EffEEcp(i,j) = m;
    end

end

% Compute cumulative maximum for portfolio
EffEEport = zeros(size(EEport));
m = EEport(1);
for j = 1:numel(dates)
    if EEport(j) > m
        m = EEport(j);
    end
    EffEEport(j) = m;
end

% Effective Expected Positive Exposure: Weighted average over time of EffEE
EffEPEcp   = 0.5*(EffEEcp(:,1:end-1)+EffEEcp(:,2:end))*diff(simTimeInterval
EffEPEport = 0.5*(EffEEport(1:end-1)+EffEEport(2:end))*diff(simTimeInterval
```

We visualize the exposure profiles, first for the entire portfolio, then for a particular counterparty.

```
% Visualize portfolio exposure profiles
figure
plot(dates,PEport,...
    dates,MPEport*ones(size(PEport)),...
    dates,EEport,...
    dates,EPEport*ones(size(PEport)),...
    dates,EffEEport,...
    dates,EffEPEport*ones(size(PEport)))
legend({'PE (95%)','MPE (95%)','EE','EPE','EffEE','EffEPE'})
datetick('x','mmmyy')
title('Portfolio Exposure Profiles');
```

```
ylabel('Exposure ($)')
xlabel('Simulation Dates')

% Visualize exposure profiles for a particular counterparty
cpIdx = 5;
figure
plot(dates,PEcp(cpIdx,:),...
    dates,MPEcp(cpIdx,:)*ones(size(PEcp(cpIdx,:))),...
    dates,EEcp(cpIdx,:),...
    dates,EPEcp(cpIdx,:)*ones(size(PEcp(cpIdx,:))),...
    dates,EffEEcp(cpIdx,:),...
    dates,EffEPEcp(cpIdx,:)*ones(size(PEcp(cpIdx,:))))
legend({'PE (95%)','MPE (95%)','EE','EPE','EffEE','EffEPE'})
datetick('x','mmmyy')
title(sprintf('Counterparty %d Exposure Profiles',cpIdx));
ylabel('Exposure ($)')
xlabel('Simulation Dates')
```





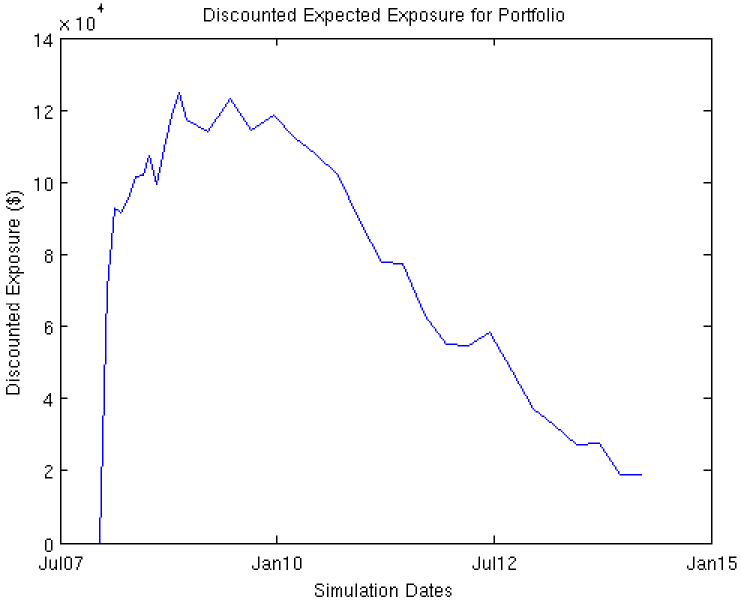
### Discounted Exposures

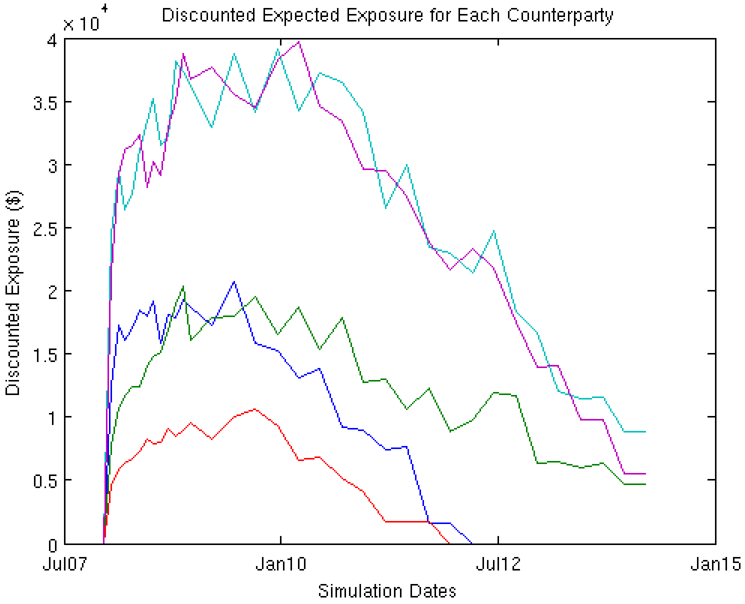
We compute the discounted expected exposures using the discount factors from each simulated interest rate scenario. The discount factor for a given valuation date in a given scenario is the product of the incremental discount factors from one simulation date to the next, along the interest rate path of that scenario.

```
% Get discounted exposures per counterparty, for each scenario
discExp = zeros(size(exposures));
for i=1:numScenarios
    discExp(:,:,i) = bsxfun(@times,scenarios(i).Disc,exposures(:,:,i));
end

% Discounted expected exposure
discEE = mean(discExp,3);
```

We plot the discounted expected exposures for the aggregate portfolio as well as for each counterparty.

```
% Portfolio discounted EE
figure;
plot(dates,sum(discEE))
datetick('x','mmmyy')
title('Discounted Expected Exposure for Portfolio');
ylabel('Discounted Exposure ($)')
xlabel('Simulation Dates')

% Counterparty discounted EE
figure;
plot(dates,discEE)
datetick('x','mmmyy')
title('Discounted Expected Exposure for Each Counterparty');
ylabel('Discounted Exposure ($)')
xlabel('Simulation Dates')
```





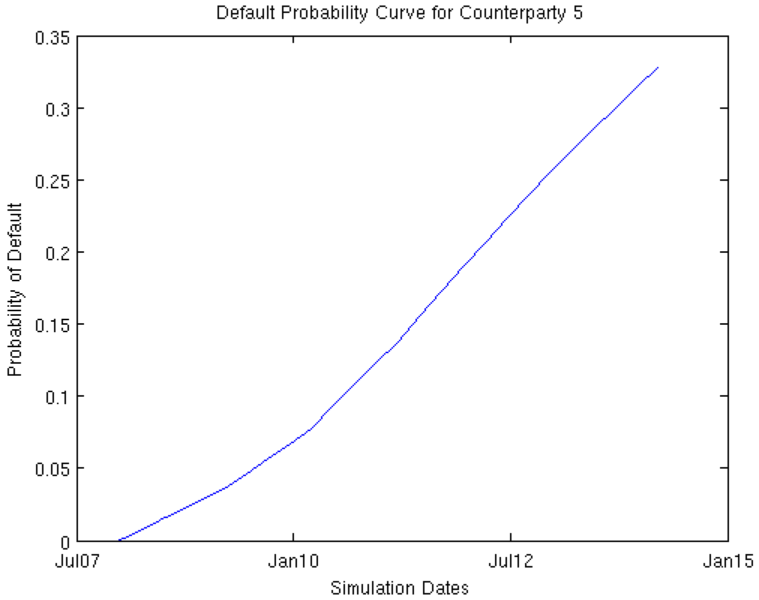
### Calibrating Probability of Default Curve for One Counterparty

The default probability for a given counterparty is implied by the current market spreads of the counterparty's credit default swaps. We use the function `cdsbootstrap` to generate the cumulative probability of default at each simulation date.

```
% CDS Market Information for the counterparty cpIdx
CDSDates = datenum({'20-Mar-08','20-Mar-09','20-Mar-10','20-Mar-11',...
    '20-Mar-12'});
CDSSpreads = [140 175 210 265 310]';
CDSData = [CDSDates CDSSpreads];

ZeroData = [RateSpec.EndDates RateSpec.Rates];

% Calibrate Default Probability to CDS Quotes
DefProbData = cdsbootstrap(ZeroData,CDSData,Settle,...
    'ProbDates',dates');
```

We plot of the cumulative probability of default for the counterparty in question.

```
figure
plot(DefProbData(:,1),DefProbData(:,2))
title(sprintf('Default Probability Curve for Counterparty %d',cpIdx));
xlabel('Date')
ylabel('Cumulative Probability')
datetick('x','mmmyy')
ylabel('Probability of Default')
xlabel('Simulation Dates')
```



### CVA Computation

The Credit Value (Valuation) Adjustment (CVA) formula is:

$$CVA = (1 - R) \int_0^T discEE(t) dPD(t)$$

Where `R` is the recovery, `discEE` the discounted expected exposure at time t, and `PD` the default probability distribution. This assumes the exposure is independent of default (no wrong-way risk), and it also assumes the exposure were obtained using risk-neutral probabilities.

Here we approximate the integral with a finite sum over the valuation dates as:

$$CVA(approx) = (1 - R) \sum_{i=2}^{n} discEE(t_i)(PD(t_i) - PD(t_{i-1}))$$

where t_1 is todays date, t_2,...,t_n the future valuation dates.

We assume CDS info corresponds to counterparty with index cpIdx. The computed CVA is the present market value of our credit exposure to counterparty cpIdx. For this example we set the recovery rate at 40%.

```
Recovery = 0.4;
CVA = (1-Recovery)*sum(discEE(cpIdx,2:end)'.*diff(DefProbData(:,2)));
fprintf('CVA for counterparty %d = $%.2f\n',cpIdx,CVA)
```

```
CVA for counterparty 5 = $4684.57
```

**References**

1. Pykhtin, Michael, and Steven Zhu, *A Guide to Modelling Counterparty Credit Risk*, GARP, July/August 2007, issue 37, pp. 16-22. Available at: http://papers.ssrn.com/sol3/papers.cfm?abstract_id=1032522.
2. Basel II: http://www.bis.org/publ/bcbs128.pdf page 256

# Credit Default Swap Option

A credit default swap (CDS) option, or credit default swaption, is a contract that provides the holder with the right, but not the obligation, to enter into a credit default swap in the future. CDS options can either be payer swaptions or receiver swaptions. If a payer swaption, the option holder has the right to enter into a CDS where they pay premiums; and, if a receiver swaption, the option holder receives premiums. Financial Instruments Toolbox software provides `cdsoptprice` for pricing payer and receiver credit default swaptions.

In addition, the optional `knockout` argument for `cdsoptprice` supports two variations of the mechanics of a CDS option. CDS options can be knockout or non-knockout options.

- A knockout option cancels with no payments if there is a credit event before the option expiry date.
- A non-knockout option does not cancel if there is a credit event before the option expiry date. In this case, the option holder of a non-knockout payer swaption can take delivery of the underlying long protection CDS on the option expiry date and exercise the protection, delivering a defaulted obligation in return for par.

# 9

# Interest-Rate Curve Objects

# Interest-Rate Curve Objects

| In this section... |
|---|
| “Class Structure” on page 9-2 |
| “Workflow Using Interest-Rate Curve Objects” on page 9-3 |

## Class Structure

Financial Instruments Toolbox class structure supports interest-rate curve objects. The class structure supports five classes.

| Class Name | Description |
|---|---|
| “@IRCurve” on page A-4 | Base abstract class for interest-rate curves. `IRCurve` is an abstract class; you cannot create instances of it directly. You can create `IRFunctionCurve` and `IRDataCurve` objects that are derived from this class. |
| “@IRDataCurve” on page A-7 | Creates a representation of an interest-rate curve with dates and data. `IRDataCurve` is constructed directly by specifying dates and corresponding interest rates or discount factors, or you can bootstrap an `IRDataCurve` object from market data. |
| “@IRFunctionCurve” on page A-12 | Creates a representation of an interest-rate curve with a function. `IRFunctionCurve` is constructed directly by specifying a function handle, or you can fit a function to market data using methods of the `IRFunctionCurve` object. |
| “@IRBootstrapOptions” on page A-2 | The `IRBootstrapOptions` object lets you specify options relating to the bootstrapping of an `IRDataCurve` object. |
| “@IRFitOptions” on page A-10 | The `IRFitOptions` object lets you specify options relating to the fitting process for an `IRFunctionCurve` object. |

## Workflow Using Interest-Rate Curve Objects

The supported workflow model for using interest-rate curve objects is:

1. Create an interest-rate curve based on an `IRDataCurve` object or an `IRFunctionCurve` object.
    - To create an `IRDataCurve` object:
        - Use vectors of dates and data with interpolation methods.
        - Use bootstrapping based on market instruments.
    - To create an `IRFunctionCurve` object:
        - Specify a function handle.
        - Fit a function using the Nelson-Siegel model, Svensson model, or smoothing spline model.
        - Fit a custom function.
2. Use methods of the `IRDataCurve` or `IRFunctionCurve` objects to extract forward, zero, discount factor, or par yield curves for the interest-rate curve object.
3. Convert an interest-rate curve from an `IRDataCurve` or `IRFunctionCurve` object to a `RateSpec` structure. This `RateSpec` structure is identical to the `RateSpec` produced by the Financial Instruments Toolbox function `intenvset`. Using the `RateSpec` for an interest-rate curve object, you can then use Financial Instruments Toolbox functions to model an interest-rate structure and price.

# Creating Interest-Rate Curve Objects

Depending on your data and purpose for analysis, you can create an interest-rate curve object by using an `IRDataCurve` or `IRFunctionCurve` object.

To create an `IRDataCurve` object, you can:

- Use the `IRDataCurve` constructor.
- Use the `IRDataCurve` method `bootstrap`.

Using an `IRDataCurve` object, you can use the following methods to determine:

- Forward rate curve — `getForwardRates`
- Zero rate curve — `getZeroRates`
- Discount rate curve — `getDiscountFactors`
- Par yield curve — `getParYields`

Alternatively, to create an `IRFunctionCurve` object, you can:

- Use the `IRFunctionCurve` constructor and directly specify a function handle.
- Use `IRFunctionCurve` methods:
  - `fitNelsonSiegel` fits a "Fitting IRFunctionCurve Object Using Nelson-Siegel Method" on page 9-18 to market data for bonds.
  - `fitSvensson` fits a "Fitting IRFunctionCurve Object Using Svensson Method" on page 9-20 to market data for bonds.
  - `fitSmoothingSpline` fits a "Fitting IRFunctionCurve Object Using Smoothing Spline Method" on page 9-22 function to market data for bonds.
  - `fitFunction` custom fits an interest-rate curve object to market data for bonds.

Using an `IRFunctionCurve` object, you can use the following method to determine:

- Forward rate curve — `getForwardRates`
- Zero rate curve — `getZeroRates`
- Discount rate curve — `getDiscountFactors`
- Par yield curve — `getParYields`

In addition, you can convert an `IRDataCurve` or `IRFunctionCurve` to a `RateSpec` structure. For more information, see "Converting an IRDataCurve or IRFunctionCurve Object" on page 9-37.

# Creating an IRDataCurve Object

## IRDataCurve Constructor with Dates and Data

Use the `IRDataCurve` constructor with vectors of dates and data to create an interest-rate curve object. When constructing the `IRDataCurve` object, you can also use optional inputs to define how the interest-rate curve is constructed from the dates and data.

### Example

In this example, you create the vectors for `Dates` and `Data` for an interest-rate curve:

```
Data = [2.09 2.47 2.71 3.12 3.43 3.85 4.57 4.58]/100;
Dates = daysadd(today,[360 2*360 3*360 5*360 7*360 10*360 20*360 30*360],1);
```

Use the `IRDataCurve` constructor to build interest-rate objects based on the `constant` and `pchip` interpolation methods:

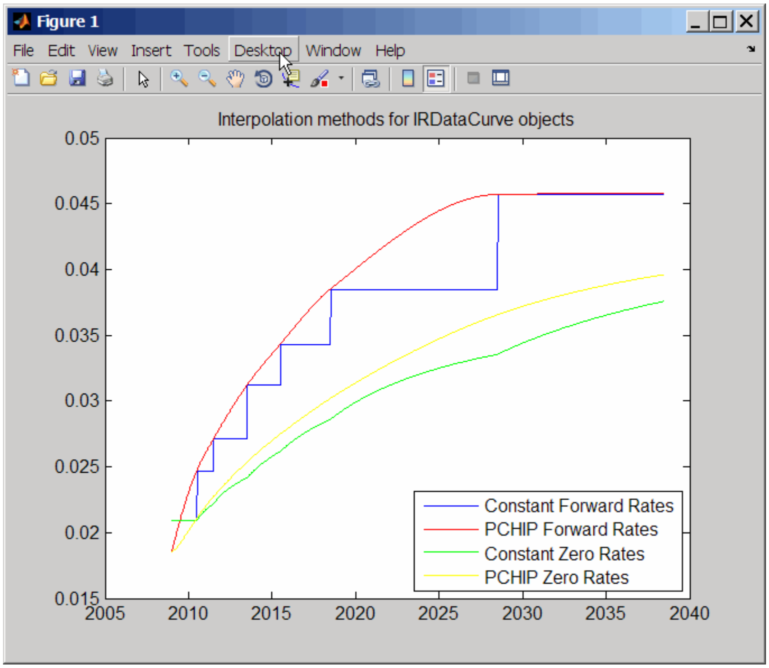
```
irdc_const = IRDataCurve('Forward',today,Dates,Data,'InterpMethod','constant');
irdc_pchip = IRDataCurve('Forward',today,Dates,Data,'InterpMethod','pchip');
```

Plot the forward and zero rate curves for the two `IRDataCurve` objects based on `constant` and `pchip` interpolation methods:

```
PlottingDates = daysadd(today,180:10:360*30,1);
plot(PlottingDates,irdc_const.getForwardRates(PlottingDates),'b')
hold on
plot(PlottingDates,irdc_pchip.getForwardRates(PlottingDates),'r')
plot(PlottingDates,irdc_const.getZeroRates(PlottingDates),'g')
plot(PlottingDates,irdc_pchip.getZeroRates(PlottingDates),'yellow')
legend({'Constant Forward Rates','PCHIP Forward Rates','Constant Zero Rates',...
'PCHIP Zero Rates'},'location','SouthEast')
title('Interpolation methods for IRDataCurve objects')
```

```
datetick
```

The plot demonstrates the relationship of the forward and zero rate curves.



## IRDataCurve Bootstrapping Based on Market Instruments

Use the bootstrapping method, based on market instruments, to create an interest-rate curve object. When bootstrapping, you also have the option to define a range of interpolation methods (`linear`, `spline`, `constant`, and `pchip`).

### Example 1

In this example, you bootstrap a swap curve from deposits, Eurodollar Futures and swaps. The input market data for this example is hard-coded and specified as two cell arrays of data; one cell array indicates the type of instrument and the other contains the `Settle`, `Maturity` values and a market quote for the instrument. For deposits and swaps, the quote is a rate; for the EuroDollar futures, the quote is a price. Although bonds are not used in this example, a bond would also be quoted with a price.

```
InstrumentTypes = {'Deposit';'Deposit';'Deposit';'Deposit';'Deposit'; ...
    'Futures';'Futures'; ...
    'Futures';'Futures';'Futures'; ...
    'Futures';'Futures';'Futures'; ...
    'Futures';'Futures';'Futures'; ...
    'Futures';'Futures';'Futures'; ...
    'Futures';'Futures';'Futures'; ...
    'Swap';'Swap';'Swap';'Swap';'Swap';'Swap';'Swap'};

Instruments = [datenum('08/10/2007'),datenum('08/17/2007'),.0532063; ...
    datenum('08/10/2007'),datenum('08/24/2007'),.0532000; ...
    datenum('08/10/2007'),datenum('09/17/2007'),.0532000; ...
    datenum('08/10/2007'),datenum('10/17/2007'),.0534000; ...
    datenum('08/10/2007'),datenum('11/17/2007'),.0535866; ...
    datenum('08/08/2007'),datenum('19-Dec-2007'),9485; ...
    datenum('08/08/2007'),datenum('19-Mar-2008'),9502; ...
    datenum('08/08/2007'),datenum('18-Jun-2008'),9509.5; ...
    datenum('08/08/2007'),datenum('17-Sep-2008'),9509; ...
    datenum('08/08/2007'),datenum('17-Dec-2008'),9505.5; ...
    datenum('08/08/2007'),datenum('18-Mar-2009'),9501; ...
    datenum('08/08/2007'),datenum('17-Jun-2009'),9494.5; ...
    datenum('08/08/2007'),datenum('16-Sep-2009'),9489; ...
    datenum('08/08/2007'),datenum('16-Dec-2009'),9481.5; ...
    datenum('08/08/2007'),datenum('17-Mar-2010'),9478; ...
    datenum('08/08/2007'),datenum('16-Jun-2010'),9474; ...
    datenum('08/08/2007'),datenum('15-Sep-2010'),9469.5; ...
    datenum('08/08/2007'),datenum('15-Dec-2010'),9464.5; ...
    datenum('08/08/2007'),datenum('16-Mar-2011'),9462.5; ...
    datenum('08/08/2007'),datenum('15-Jun-2011'),9456.5; ...
    datenum('08/08/2007'),datenum('21-Sep-2011'),9454; ...
    datenum('08/08/2007'),datenum('21-Dec-2011'),9449.5; ...
    datenum('08/08/2007'),datenum('08/08/2014'),.0530; ...
    datenum('08/08/2007'),datenum('08/08/2017'),.0545; ...
    datenum('08/08/2007'),datenum('08/08/2019'),.0551; ...
    datenum('08/08/2007'),datenum('08/08/2022'),.0559; ...
    datenum('08/08/2007'),datenum('08/08/2027'),.0565; ...
```

```
    datenum('08/08/2007'),datenum('08/08/2032'),.0566; ...
    datenum('08/08/2007'),datenum('08/08/2037'),.0566];
```

The `bootstrap` method is called as a static method of the "@IRDataCurve" on page A-7 class. Inputs to this method include the curve `Type` (`zero` or `forward`), `Settle` date, `InstrumentTypes`, and `Instrument` data. The `bootstrap` method also supports optional arguments, including an interpolation method, compounding, basis, and an options structure for bootstrapping. For example, you are passing in an "@IRBootstrapOptions" on page A-2 object which includes information for the `ConvexityAdjustment` to forward rates.

```
IRsigma = .01;
CurveSettle = datenum('08/10/2007');
bootModel = IRDataCurve.bootstrap('Forward', CurveSettle, ...
InstrumentTypes, Instruments,'InterpMethod','pchip',...
'Compounding',-1,'IRBootstrapOptions',...
IRBootstrapOptions('ConvexityAdjustment',@(t) .5*IRsigma^2.*t.^2))

bootModel =

IRDataCurve

    Type: Forward
     Settle: 733264 (10-Aug-2007)
   Compounding: -1
   Basis: 0 (actual/actual)
  InterpMethod: pchip
   Dates: [29x1 double]
    Data: [29x1 double]
```

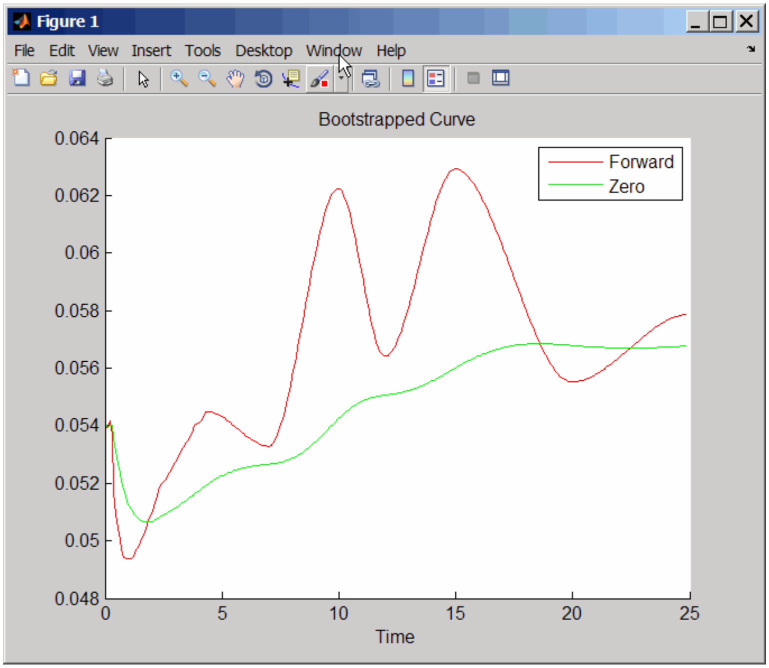
The `bootstrap` method uses an Optimization Toolbox function to solve for any bootstrapped rates.

Plot the forward and zero curves:

```
PlottingDates = (CurveSettle+20:30:CurveSettle+365*25)';
TimeToMaturity = yearfrac(CurveSettle,PlottingDates);
BootstrappedForwardRates = bootModel.getForwardRates(PlottingDates);
BootstrappedZeroRates = bootModel.getZeroRates(PlottingDates);
```

```
figure
hold on
plot(TimeToMaturity,BootstrappedForwardRates,'r')
plot(TimeToMaturity,BootstrappedZeroRates,'g')
title('Bootstrapped Curve')
xlabel('Time')
legend({'Forward','Zero'})
```

The plot demonstrates the forward and zero rate curves for the market data.



## Example 2

In this example, you bootstrap a swap curve from deposits, Eurodollar futures and swaps. The input market data for this example is hard-coded and specified as two cell arrays of data; one cell array indicates the type of instrument and the other cell array contains the `Settle`, `Maturity` values and a market quote for the instrument. This example of bootstrapping also demonstrates the use of an `InstrumentBasis` for each `Instrument` type:

```
InstrumentTypes = {'Deposit';'Deposit';...
'Futures';'Futures';'Futures';'Futures';'Futures';'Futures';...
'Swap';'Swap';'Swap';'Swap';};

Instruments = [datenum('08/10/2007'),datenum('09/17/2007'),.0532000; ...
datenum('08/10/2007'),datenum('11/17/2007'),.0535866; ...
datenum('08/08/2007'),datenum('19-Dec-2007'),9485; ...
datenum('08/08/2007'),datenum('19-Mar-2008'),9502; ...
datenum('08/08/2007'),datenum('18-Jun-2008'),9509.5; ...
datenum('08/08/2007'),datenum('17-Sep-2008'),9509; ...
datenum('08/08/2007'),datenum('17-Dec-2008'),9505.5; ...
datenum('08/08/2007'),datenum('18-Mar-2009'),9501; ...
datenum('08/08/2007'),datenum('08/08/2014'),.0530; ...
datenum('08/08/2007'),datenum('08/08/2019'),.0551; ...
datenum('08/08/2007'),datenum('08/08/2027'),.0565; ...
datenum('08/08/2007'),datenum('08/08/2037'),.0566];

CurveSettle = datenum('08/10/2007');
```

The `bootstrap` method is called as a static method of the “@IRDataCurve” on page A-7 class. Inputs to this method include the curve `Type` (`zero` or `forward`), `Settle` date, `InstrumentTypes`, and `Instrument` data. The `bootstrap` method also supports optional arguments, including an interpolation method, compounding, basis, and an options structure for bootstrapping. In this example, you are passing an additional `Basis` value for each instrument type:

```
bootModel=IRDataCurve.bootstrap('Forward',CurveSettle,InstrumentTypes, ...
Instruments,'InterpMethod','pchip','InstrumentBasis',[repmat(2,8,1);repmat(0,4,1)])

bootModel =

 IRDataCurve

    Type: Forward
     Settle: 733264 (10-Aug-2007)
   Compounding: 2
```

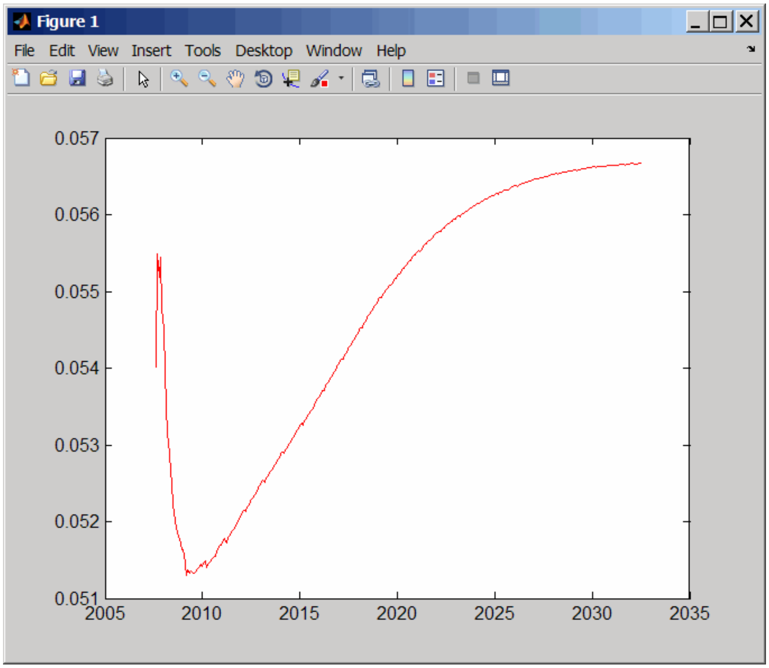
```
        Basis: 0 (actual/actual)
 InterpMethod: pchip
        Dates: [12x1 double]
         Data: [12x1 double]
```

The `bootstrap` method uses an Optimization Toolbox function to solve for any bootstrapped rates.

Plot the par yields curve using the `getParYields` method:

```
PlottingDates = (datenum('08/11/2007'):30:CurveSettle+365*25)';
plot(PlottingDates,bootModel.getParYields(PlottingDates),'r')
datetick
```

The plot demonstrates the par yields curve for the market data.



# Bootstrapping a Swap Curve

This example shows how to bootstrap an interest rate curve, often referred to as a swap curve, using the new IRDataCurve object. The static bootstrap method takes as inputs a cell array of market instruments (which can be deposits, interest rate futures, swaps, and bonds) and bootstraps an interest rate curve of either the forward or the zero curve. It is also possible to specify multiple interpolation methods, including piecewise constant, linear, and Piecewise Cubic Hermite Interpolating Polynomial (PCHIP)

### Obtain Data

A curve is bootstrapped from market data. In this example, we will bootstrap a swap curve from deposits, Eurodollar Futures and swaps.

For this example, we have hard-coded the input market data, which is simply specified as 2 cell arrays of data, one which indicates the type of instrument and a second cell array containing the Settle, Maturity, and Market Quote for the instrument. For deposits and swaps, the quote is a rate and for the EuroDollar Futurus, the quote is a price. Although bonds are not used in this example, a bond would be quoted with a price.

```
InstrumentTypes = {'Deposit';'Deposit';'Deposit';'Deposit';'Deposit'; ...
    'Futures';'Futures'; ...
    'Futures';'Futures';'Futures'; ...
    'Futures';'Futures';'Futures'; ...
    'Futures';'Futures';'Futures'; ...
    'Futures';'Futures';'Futures'; ...
    'Futures';'Futures';'Futures'; ...
    'Swap';'Swap';'Swap';'Swap';'Swap';'Swap';'Swap'};

Instruments = [datenum('08/10/2007'),datenum('08/17/2007'),.0532063; ...
    datenum('08/10/2007'),datenum('08/24/2007'),.0532000; ...
    datenum('08/10/2007'),datenum('09/17/2007'),.0532000; ...
    datenum('08/10/2007'),datenum('10/17/2007'),.0534000; ...
    datenum('08/10/2007'),datenum('11/17/2007'),.0535866; ...
    datenum('08/08/2007'),datenum('19-Dec-2007'),9485; ...
    datenum('08/08/2007'),datenum('19-Mar-2008'),9502; ...
    datenum('08/08/2007'),datenum('18-Jun-2008'),9509.5; ...
    datenum('08/08/2007'),datenum('17-Sep-2008'),9509; ...
```

```
    datenum('08/08/2007'),datenum('17-Dec-2008'),9505.5; ...
    datenum('08/08/2007'),datenum('18-Mar-2009'),9501; ...
    datenum('08/08/2007'),datenum('17-Jun-2009'),9494.5; ...
    datenum('08/08/2007'),datenum('16-Sep-2009'),9489; ...
    datenum('08/08/2007'),datenum('16-Dec-2009'),9481.5; ...
    datenum('08/08/2007'),datenum('17-Mar-2010'),9478; ...
    datenum('08/08/2007'),datenum('16-Jun-2010'),9474; ...
    datenum('08/08/2007'),datenum('15-Sep-2010'),9469.5; ...
    datenum('08/08/2007'),datenum('15-Dec-2010'),9464.5; ...
    datenum('08/08/2007'),datenum('16-Mar-2011'),9462.5; ...
    datenum('08/08/2007'),datenum('15-Jun-2011'),9456.5; ...
    datenum('08/08/2007'),datenum('21-Sep-2011'),9454; ...
    datenum('08/08/2007'),datenum('21-Dec-2011'),9449.5; ...
    datenum('08/08/2007'),datenum('08/08/2014'),.0530; ...
    datenum('08/08/2007'),datenum('08/08/2017'),.0545; ...
    datenum('08/08/2007'),datenum('08/08/2019'),.0551; ...
    datenum('08/08/2007'),datenum('08/08/2022'),.0559; ...
    datenum('08/08/2007'),datenum('08/08/2027'),.0565; ...
    datenum('08/08/2007'),datenum('08/08/2032'),.0566; ...
    datenum('08/08/2007'),datenum('08/08/2037'),.0566];
```

### Construct the Curve via Bootstrapping

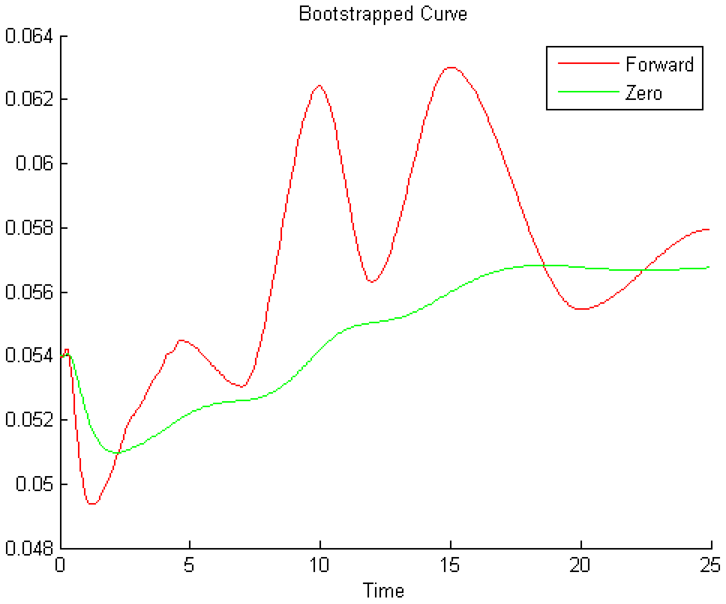
The bootstrap method is called as a static method of the IRDataCurve class. Inputs to this method include the curve type (Zero or Forward), settle date, instrument types, instrument data and optional arguments including an interpolation method, compounding and an options structure for bootstrapping. Note that in this example, we are passing in an IRBootstrapOptions object which includes information for the convexity adjustment to forward rates.

```
IRsigma = .01;
CurveSettle = datenum('08/10/2007');
bootModel = IRDataCurve.bootstrap('Forward', CurveSettle, ...
    InstrumentTypes, Instruments,'InterpMethod','pchip',...
    'Compounding',-1,'IRBootstrapOptions',...
    IRBootstrapOptions('ConvexityAdjustment',@(t) .5*IRsigma^2.*t.^2));
```

### Plot

We can now plot both the forward and zero curves

```
PlottingDates = (CurveSettle+20:30:CurveSettle+365*25)';
TimeToMaturity = yearfrac(CurveSettle,PlottingDates);
BootstrappedForwardRates = bootModel.getForwardRates(PlottingDates);
BootstrappedZeroRates = bootModel.getZeroRates(PlottingDates);

figure
hold on
plot(TimeToMaturity,BootstrappedForwardRates,'r')
plot(TimeToMaturity,BootstrappedZeroRates,'g')
title('Bootstrapped Curve')
xlabel('Time')
legend({'Forward','Zero'})
```



#### Bibliography

This example draws from the following papers and journal articles:

[1] Hagan, P., West, G. (2006), "Interpolation Methods for Curve Construction", Applied Mathematical Finance, Vol 13, No. 2

[2] Ron, Uri(2000), "A Practical Guide to Swap Curve Construction", Working Papers 00-17, Bank of Canada.

# Creating an IRFunctionCurve Object

**In this section...**

## Fitting IRFunctionCurve Object Using a Function Handle

You can use the constructor `IRFunctionCurve` with a MATLAB function handle to define an interest-rate curve. For more information on defining a function handle, see the MATLAB Programming Fundamentals documentation.

### Example

This example uses a `FunctionHandle` argument with a value `@(t) t.^2` to construct an interest-rate curve:

```
rr = IRFunctionCurve('Zero',today,@(t) t.^2);
rr =

  Properties:
    FunctionHandle: @(t)t.^2
              Type: 'Zero'
            Settle: 733600
       Compounding: 2
             Basis: 0
```

## Fitting IRFunctionCurve Object Using Nelson-Siegel Method

Use the method, `fitNelsonSiegel`, for the Nelson-Siegel model that fits the empirical form of the yield curve with a prespecified functional form of the spot rates which is a function of the time to maturity of the bonds.

The Nelson-Siegel model represents a dynamic three-factor model: level, slope, and curvature. However, the Nelson-Siegel factors are unobserved, or latent, which allows for measurement error, and the associated loadings have economic restrictions (forward rates are always positive, and the discount factor approaches zero as maturity increases). For more information, see "Zero-coupon yield curves: technical documentation," *BIS Papers*, Bank for International Settlements, Number 25, October, 2005.

### Example

This example uses `IRFunctionCurve` to model the default-free term structure of interest rates in the United Kingdom.

Load the data:

```
load ukdata20080430
```

Convert repo rates to be equivalent zero coupon bonds:

```
RepoCouponRate = repmat(0,size(RepoRates));
RepoPrice = bndprice(RepoRates, RepoCouponRate, RepoSettle, RepoMaturity);
```

Aggregate the data:

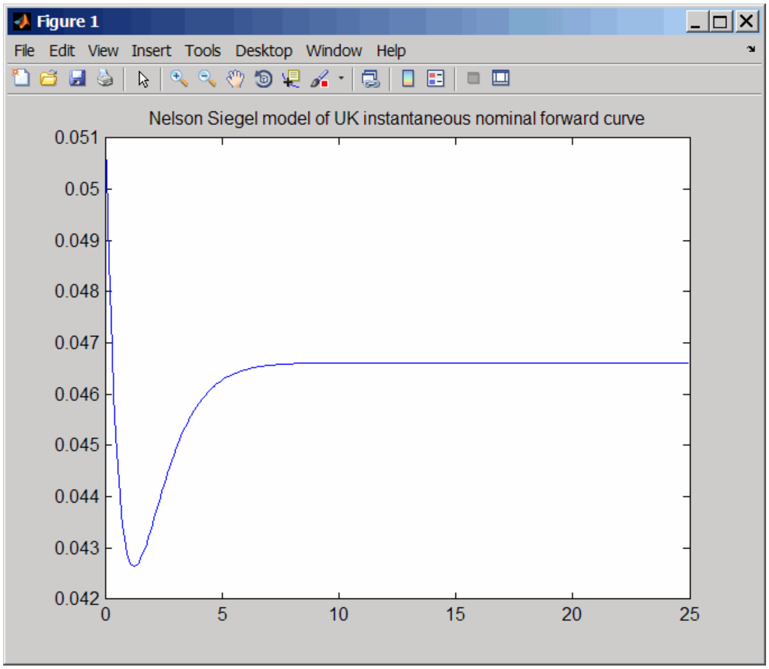
```
Settle = [RepoSettle;BondSettle];
Maturity = [RepoMaturity;BondMaturity];
CleanPrice = [RepoPrice;BondCleanPrice];
CouponRate = [RepoCouponRate;BondCouponRate];
Instruments = [Settle Maturity CleanPrice CouponRate];
InstrumentPeriod = [repmat(0,6,1);repmat(2,31,1)];
CurveSettle = datenum('30-Apr-2008');
```

The `IRFunctionCurve` object provides the capability to fit a Nelson-Siegel curve to observed market data with the `fitNelsonSiegel` method. The fitting is done by calling the Optimization Toolbox function `lsqnonlin`. This method has required inputs of `Type`, `Settle`, and a matrix of instrument data.

```
NSModel = IRFunctionCurve.fitNelsonSiegel('Zero',CurveSettle,...
Instruments,'Compounding',-1,'InstrumentPeriod',InstrumentPeriod);
```

Plot the Nelson-Siegel interest-rate curve for forward rates:

```
PlottingDates = CurveSettle+20:30:CurveSettle+365*25;
TimeToMaturity = yearfrac(CurveSettle,PlottingDates);
NSForwardRates = NSModel.getForwardRates(PlottingDates);
plot(TimeToMaturity,NSForwardRates)
title('Nelson Siegel model of UK instantaneous nominal forward curve')
```



## Fitting IRFunctionCurve Object Using Svensson Method

Use the method, `fitSvensson`, for the Svensson model to improve the flexibility of the curves and the fit for a Nelson-Siegel model. In 1994, Svensson extended Nelson and Siegel's function by adding a further term that allows for a second "hump." The extra precision is achieved at the cost of adding two more parameters, $\beta 3$ and $\tau 2$, which have to be estimated.

### Example

In this example of using the `fitSvensson` method, an `IRFitOptions` structure, previously defined using the `IRFitOptions` constructor, is used. Thus, you must specify `FitType`, `InitialGuess`, `UpperBound`, `LowerBound`, and the `OptOptions` optimization parameters for `lsqnonlin`.

Load the data:

```
load ukdata20080430
```

Convert repo rates to be equivalent zero coupon bonds:

```
RepoCouponRate = repmat(0,size(RepoRates));
RepoPrice = bndprice(RepoRates, RepoCouponRate, RepoSettle, RepoMaturity);
```

Aggregate the data:

```
Settle = [RepoSettle;BondSettle];
Maturity = [RepoMaturity;BondMaturity];
CleanPrice = [RepoPrice;BondCleanPrice];
CouponRate = [RepoCouponRate;BondCouponRate];
Instruments = [Settle Maturity CleanPrice CouponRate];
InstrumentPeriod = [repmat(0,6,1);repmat(2,31,1)];
CurveSettle = datenum('30-Apr-2008');
```

Define `OptOptions` for the `IRFitOptions` constructor:

```
OptOptions = optimset('lsqnonlin');
OptOptions = optimset(OptOptions,'MaxFunEvals',1000);
fIRFitOptions = IRFitOptions([5.82 -2.55 -.87 0.45 3.9 0.44],...
'FitType','durationweightedprice','OptOptions',OptOptions,...
'LowerBound',[0 -Inf -Inf -Inf 0 0],'UpperBound',[Inf Inf Inf Inf Inf Inf]);
```

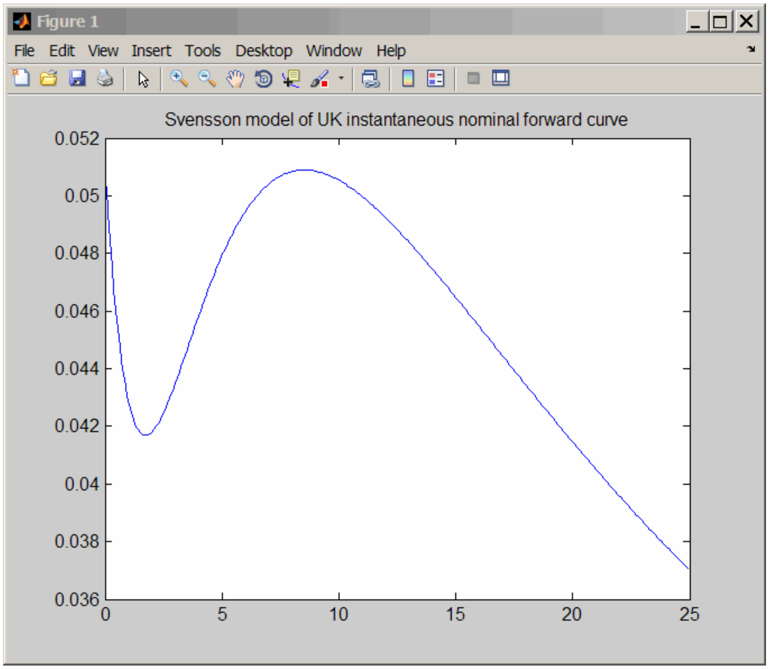
Fit the interest-rate curve using a Svensson model:

```
SvenssonModel = IRFunctionCurve.fitSvensson('Zero',CurveSettle,...
Instruments,'IRFitOptions',fIRFitOptions, 'Compounding',-1,...
'InstrumentPeriod',InstrumentPeriod);

Local minimum possible.

lsqnonlin stopped because the final change in the sum of squares relative to
its initial value is less than the selected value of the function tolerance.
```

The status message, output from `lsqnonlin`, indicates that the optimization to find parameters for the Svensson equation terminated successfully.

Plot the Svensson interest-rate curve for forward rates:

```
PlottingDates = CurveSettle+20:30:CurveSettle+365*25;
TimeToMaturity = yearfrac(CurveSettle,PlottingDates);
SvenssonForwardRates = SvenssonModel.getForwardRates(PlottingDates);
plot(TimeToMaturity,SvenssonForwardRates)
title('Svensson model of UK instantaneous nominal forward curve')
```



## Fitting IRFunctionCurve Object Using Smoothing Spline Method

Use the method, `fitSmoothingSpline`, to model the term structure with a spline, specifically, the term structure represents the forward curve with a cubic spline.

---

**Note** You must have a license for Curve Fitting Toolbox software to use the `fitSmoothingSpline` method.

---

### Example

The `IRFunctionCurve` object is used to fit a smoothing spline representation of the forward curve with a penalty function. Required inputs are `Type`, `Settle`, the matrix of `Instruments`, and `Lambdafun`, a function handle containing the penalty function

Load the data:

```
load ukdata20080430
```

Convert repo rates to be equivalent zero coupon bonds:

```
RepoCouponRate = repmat(0,size(RepoRates));
RepoPrice = bndprice(RepoRates, RepoCouponRate, RepoSettle, RepoMaturity);
```

Aggregate the data:

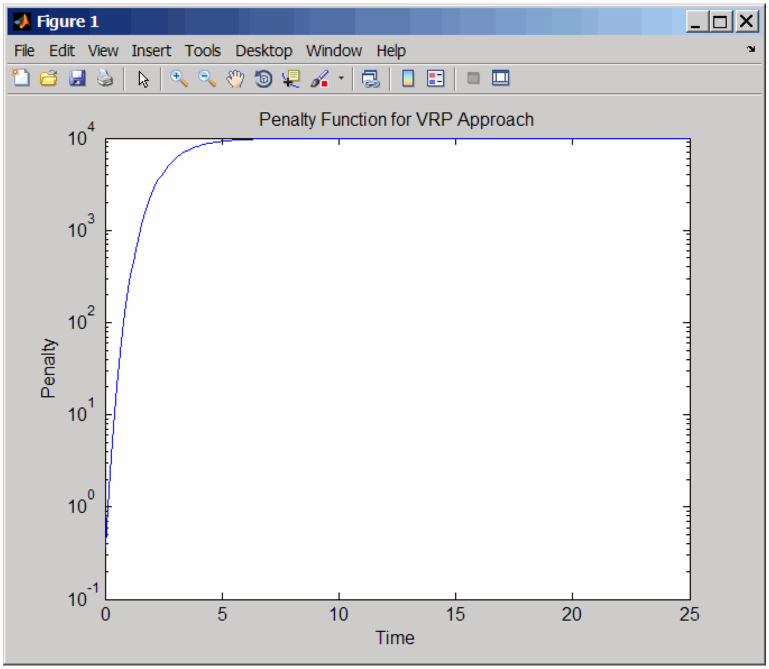
```
Settle = [RepoSettle;BondSettle];
Maturity = [RepoMaturity;BondMaturity];
CleanPrice = [RepoPrice;BondCleanPrice];
CouponRate = [RepoCouponRate;BondCouponRate];
Instruments = [Settle Maturity CleanPrice CouponRate];
InstrumentPeriod = [repmat(0,6,1);repmat(2,31,1)];
CurveSettle = datenum('30-Apr-2008');
```

Choose parameters for `Lambdafun`:

```
L = 9.2;
S = -1;
mu = 1;
```

Define the `Lambdafun` penalty function:

```
lambdafun = @(t) exp(L - (L-S)*exp(-t/mu));
t = 0:.1:25;
y = lambdafun(t);
figure
semilogy(t,y);
title('Penalty Function for VRP Approach')
ylabel('Penalty')
xlabel('Time')
```

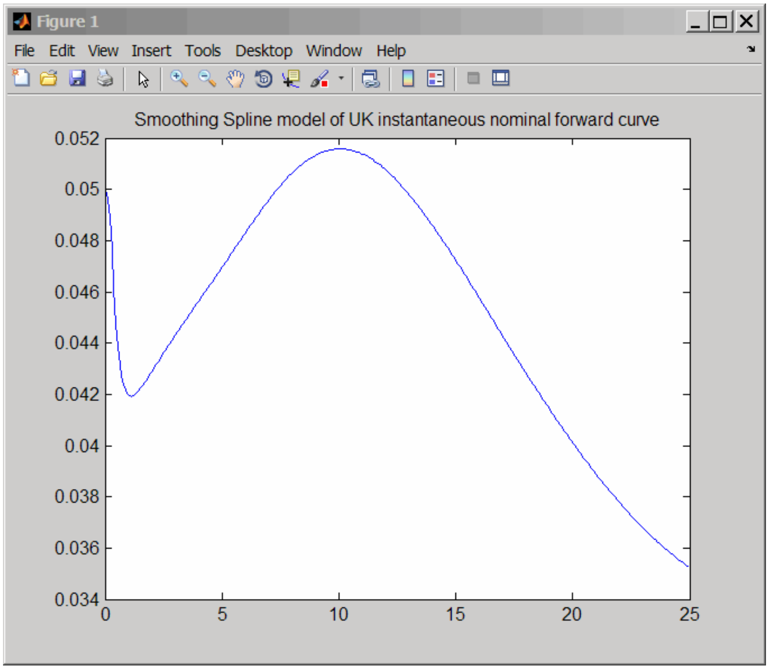


Use the `fitSmoothinSpline` method to fit the interest-rate curve and model the `Lambdafun` penalty function:

```
VRPModel = IRFunctionCurve.fitSmoothingSpline('Forward',CurveSettle,...
Instruments,lambdafun,'Compounding',-1, 'InstrumentPeriod',InstrumentPeriod);
```

Plot the smoothing spline interest-rate curve for forward rates:

```
PlottingDates = CurveSettle+20:30:CurveSettle+365*25;
TimeToMaturity = yearfrac(CurveSettle,PlottingDates);
VRPForwardRates = VRPModel.getForwardRates(PlottingDates);
plot(TimeToMaturity,VRPForwardRates)
title('Smoothing Spline model of UK instantaneous nominal forward curve')
```



## Using fitFunction to Create Custom Fitting Function

When using an `IRFunctionCurve` object, you can create a custom fitting function with the `fitFunction` method. To use `fitFunction`, you must define a `FunctionHandle`. In addition, you must also use the constructor `IRFitOptions` to define `IRFitOptionsObj` to support an `InitialGuess` for the parameters of the curve function.

### Example

The following example demonstrates the use of `fitFunction` with a `FunctionHandle` and an `IRFitOptionsObj`:

```
Settle = repmat(datenum('30-Apr-2008'),[6 1]);
Maturity = [datenum('07-Mar-2009');datenum('07-Mar-2011');...
datenum('07-Mar-2013');datenum('07-Sep-2016');...
datenum('07-Mar-2025');datenum('07-Mar-2036')];
```

```
CleanPrice = [100.1;100.1;100.8;96.6;103.3;96.3];
CouponRate = [0.0400;0.0425;0.0450;0.0400;0.0500;0.0425];
Instruments = [Settle Maturity CleanPrice CouponRate];
CurveSettle = datenum('30-Apr-2008');
```

Define the `FunctionHandle`:

```
functionHandle = @(t,theta) polyval(theta,t);
```

Define the `OptOptions` for `IRFitOptions`:

```
OptOptions = optimset('lsqnonlin');
OptOptions = optimset(OptOptions,'display','iter');
```

Define `fitFunction`:

```
CustomModel = IRFunctionCurve.fitFunction('Zero', CurveSettle, ...
functionHandle,Instruments, IRFitOptions([.05 .05 .05],'FitType','price',...
'OptOptions',OptOptions));

                                         Norm of      First-order
 Iteration  Func-count     f(x)          step          optimality   CG-iterations
     0           4         38036.7                      4.92e+004
     1           8         38036.7             10       4.92e+004            0
     2          12         38036.7            2.5       4.92e+004            0
     3          16         38036.7          0.625       4.92e+004            0
     4          20         38036.7        0.15625       4.92e+004            0
     5          24         30741.5      0.0390625       1.72e+005            0
     6          28         30741.5       0.078125       1.72e+005            0
     7          32         30741.5      0.0195312       1.72e+005            0
     8          36         28713.6     0.00488281       2.33e+005            0
     9          40         20323.3     0.00976562       9.47e+005            0
    10          44         20323.3      0.0195312       9.47e+005            0
    11          48         20323.3     0.00488281       9.47e+005            0
    12          52         20323.3      0.0012207       9.47e+005            0
    13          56         19698.8    0.000305176       1.08e+006            0
    14          60           17493    0.000610352          7e+006            0
    15          64           17493      0.0012207          7e+006            0
    16          68           17493    0.000305176          7e+006            0
    17          72         15455.1    7.62939e-005      2.25e+007            0
    18          76         15455.1    0.000177558       2.25e+007            0
```

```
    19          80        13317.1    3.8147e-005      3.18e+007            0
    20          84        12867.9    7.62939e-005     7.84e+007            0
    21          88        11779.8    7.62939e-005     7.58e+006            0
    22          92        11747.6    0.000152588      1.46e+005            0
    23          96        11720.9    0.000305176      2.48e+005            0
    24         100        11667.2    0.000610352      1.48e+005            0
    25         104        11558.5      0.0012207      4.47e+005            0
    26         108        11335.4     0.00244141      1.58e+005            0
    27         112          10864     0.00488281      1.61e+005            0
    28         116        9797.68     0.00976562      6.85e+005            0
    29         120        6884.03      0.0195312      5.79e+005            0
    30         124        6884.03       0.037498      5.79e+005            0
    31         128        3216.51     0.00937449      1.75e+006            0
    32         132        607.317       0.018749      2.94e+006            0
    33         136        12.7284      0.0253662         3e+006            0
    34         140      0.0760939     0.00153457      4.88e+004            0
    35         144      0.0731652    3.58678e-006          24.6            0
    36         148      0.0731652    6.04329e-008        0.0213            0

Local minimum possible.

lsqnonlin stopped because the final change in the sum of squares relative to
its initial value is less than the selected value of the function tolerance.
```

Plot the custom function that is defined using `fitFunction`:

```
Yields = bndyield(CleanPrice,CouponRate,Settle(1),Maturity);
scatter(Maturity,Yields);
PlottingPoints = min(Maturity):30:max(Maturity);
hold on;
plot(PlottingPoints,CustomModel.getParYields(PlottingPoints),'r');
datetick
legend('Market Yields','Fitted Yield Curve')
title('Custom Function fit to Market Data')
```

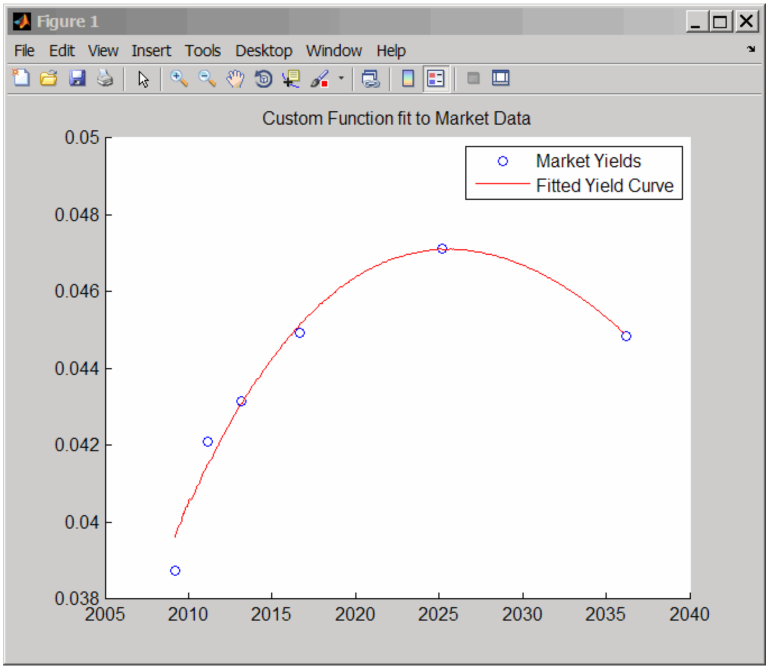


Figure 1
File Edit View Insert Tools Desktop Window Help
Custom Function fit to Market Data
Market Yields
Fitted Yield Curve
0.05
0.048
0.046
0.044
0.042
0.04
0.038
2005
2010
2015
2020
2025
2030
2035
2040

# Fitting Interest Rate Curve Functions

This example shows how to use objects to model the term structure of interest rates (also referred to as the yield curve). This can be contrasted with modeling the term structure with vectors of dates and data and interpolating between the points (which can currently be done with the function PRBYZERO).

The term structure can refer to at least three different curves: the discount curve, zero curve, or forward curve.

The new object IRFunctionCurve allows the user to model an interest rate curve as a function.

This example will explore using IRFunctionCurve to model the default-free term structure of interest rates in the United Kingdom. Three different forms for the term structure will be implemented, which will be discussed in more detail later:

- Nelson-Siegel
- Svensson
- Smoothing Cubic Spline with a so-called Variable Roughness Penalty (VRP)

### Choosing the Data

The first question in modeling the yield curve is what data should be used. To model a default-free yield curve, default-free, option-free market instruments must be used. The most significant component of the data set will be UK Government Bonds (known as Gilts). Historical data was retrieved from the following site:

http://www.dmo.gov.uk

Repo data will be used to construct the short end of the curve. Repo data was retrieved from the following site:

http://www.bba.org.uk

Note also that the data must be specified as a matrix where the columns are Settle, Maturity, CleanPrice and CouponRate -- and that instruments must be bonds or synthetically converted to bonds.

Market data for a close date of April 30, 2008, has been downloaded and saved to the following data file, which can be loaded into MATLAB with the following command

```
% Load the data
load ukdata20080430

% Convert repo rates to be equivalent zero coupon bonds
RepoCouponRate = repmat(0,size(RepoRates));
RepoPrice = bndprice(RepoRates, RepoCouponRate, RepoSettle, RepoMaturity);

% Aggregate the data
Settle = [RepoSettle;BondSettle];
Maturity = [RepoMaturity;BondMaturity];
CleanPrice = [RepoPrice;BondCleanPrice];
CouponRate = [RepoCouponRate;BondCouponRate];
Instruments = [Settle Maturity CleanPrice CouponRate];
InstrumentPeriod = [repmat(0,6,1);repmat(2,31,1)];

CurveSettle = datenum('30-Apr-2008');
```

#### Fit Nelson-Siegel Model to Market Data

The Nelson-Siegel model proposes that the instantaneous forward curve can be modeled with the following:

$$f = \beta_0 + \beta_1 e^{\frac{-m}{\tau}} + \beta_2 e^{\frac{-m}{\tau}} \frac{m}{\tau}$$

This can be integrated to derive an equation for the zero curve (see [6] for more information on the equations and the derivation):

$$s = \beta_0 + (\beta_1 + \beta_2)\frac{\tau}{m}(1 - e^{\frac{-m}{\tau}}) - \beta_2 e^{\frac{-m}{\tau}}$$

See [1] for more information.

The IRFunctionCurve object provides the capability to fit a Nelson Siegel curve to observed market data with the FITNELSONSIEGEL method. The fitting is done by calling the Optimization Toolbox™ function LSQNONLIN.

This method has required inputs: Curve Type, Curve Settle, and a matrix of instrument data.

Optional input arguments, specified in parameter value pairs, are:

- IRFitOptions structure: Provides capability to choose which quantity to be minimized (price, yield, or duration weighted price) and other optimization parameters (e.g.: upper and lower bounds for parameters)
- Curve Compounding and Basis (Day Count Convention)
- Additional instrument parameters, Period, Basis, FirstCouponDate, etc.

```
NSModel = IRFunctionCurve.fitNelsonSiegel('Zero',CurveSettle,...
    Instruments,'InstrumentPeriod',InstrumentPeriod);
```

### Fit Svensson Model

A very similar model to the Nelson-Siegel is the Svensson model, which adds two additional parameters to account for greater flexibility in the term structure. This model proposes that the forward rate can be modeled with the following form:

$$f = \beta_0 + \beta_1 e^{\frac{-m}{\tau_1}} + \beta_2 e^{\frac{-m}{\tau_1}} \frac{m}{\tau_1} + \beta_3 e^{\frac{-m}{\tau_2}} \frac{m}{\tau_2}$$

As above, this can be integrated to derive an equation for the zero curve:

$$s = \beta_0 + \beta_1 (1 - e^{\frac{-m}{\tau_1}})(-\frac{\tau_1}{m}) + \beta_2 ((1 - e^{\frac{-m}{\tau_1}})\frac{\tau_1}{m} - e^{\frac{m}{\tau_1}}) + \beta_3 ((1 - e^{\frac{-m}{\tau_2}})\frac{\tau_2}{m} - e^{\frac{m}{\tau_2}})$$

See [2] for more information.

Fitting the parameters to this model proceeds in a similar fashion to the Nelson-Siegel model.

```
SvenssonModel = IRFunctionCurve.fitSvensson('Zero',CurveSettle,...
    Instruments,'InstrumentPeriod',InstrumentPeriod);
```

#### Fit Smoothing Spline

The term structure can also be modeled with a spline -- specifically, one way to model the term structure is by representing the forward curve with a cubic spline. To ensure that the spline is sufficiently smooth, a penalty is imposed relating to the curvature (second derivative) of the spline:

$$\sum_{i=1}^{N}[\frac{P_i - \hat{P}_i(f)}{D_i}]^2 + \int_0^M \lambda_t(m)[f''(m)]^2 dm$$

where the first term is the difference between the observed price P and the predicted price, P_hat, (weighted by the bond's duration, D) summed over all bonds in our data set and the second term is the penalty term (where lambda is a penalty function and f is the spline)

See [3], [4], [5] below.

There have been different proposals for the specification of the penalty function lambda. One approach, advocated by [4], and currently used by the UK Debt Management Office, is a penalty function of the following form:

$$log(\lambda(m)) = L - (L - S)e^{\frac{-m}{\mu}}$$

The parameters L, S, and mu are typically estimated from historical data.

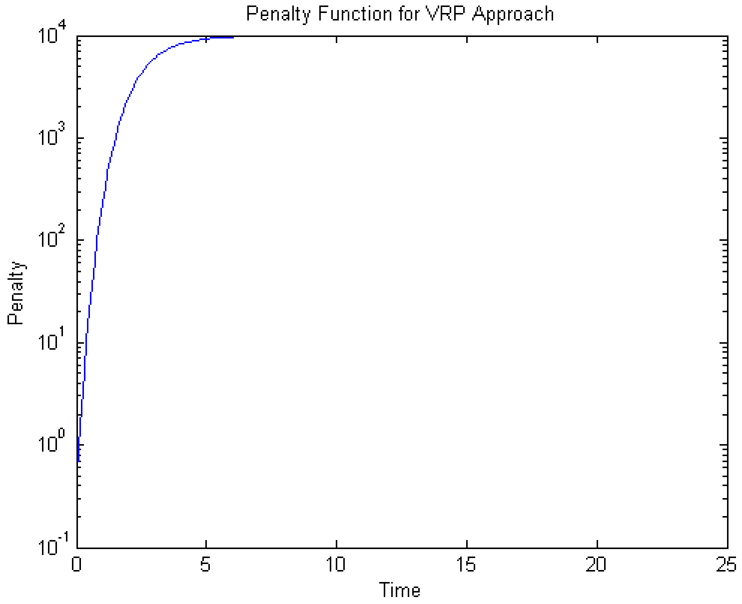
The IRFunctionCurve object can be used to fit a smoothing spline representation of the forward curve with a penalty function.

Required inputs, like for methods above, are a Curve Type, Curve Settle, Instrument matrix, and new for this method, a function handle containing the penalty function

Optional parameters are similar to FITNELSONSIEGEL and FITSVENSSON

```
% Parameters chosen to be roughly similar to [4] below.
L = 9.2;
```

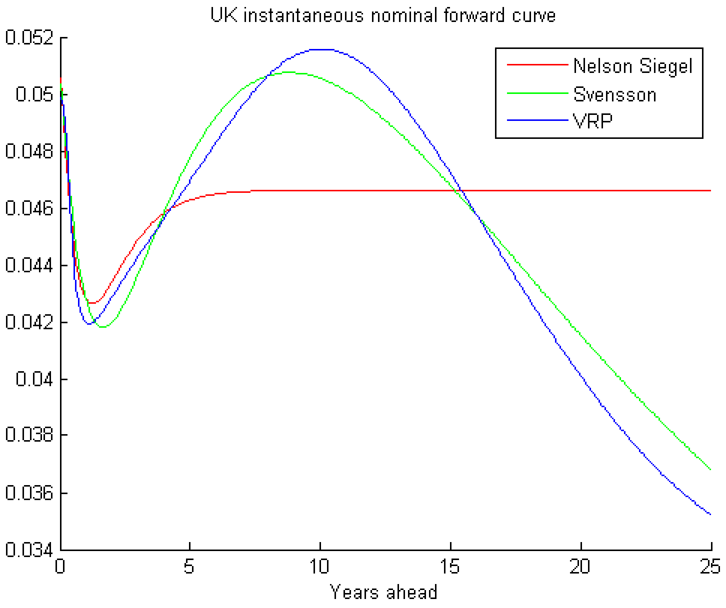
```
S = -1;
mu = 1;

lambdafun = @(t) exp(L - (L-S)*exp(-t/mu)); % Construct penalty function
t = 0:.1:25; % Construct data to plot penalty function
y = lambdafun(t);
figure
semilogy(t,y);
title('Penalty Function for VRP Approach')
ylabel('Penalty')
xlabel('Time')

VRPModel = IRFunctionCurve.fitSmoothingSpline('Forward',CurveSettle,...
    Instruments,lambdafun,'Compounding',-1,...
    'InstrumentPeriod',InstrumentPeriod);
```



**Use Fitted Curves and Plot Results**

Once a curve has been constructed, methods can be called to extract the Forward and Zero Rates and the Discount Factors. This curve can also be converted into a RateSpec structure, to be used with functions in the Financial Instruments Toolbox™

```
PlottingDates = CurveSettle+20:30:CurveSettle+365*25;
TimeToMaturity = yearfrac(CurveSettle,PlottingDates);

NSForwardRates = NSModel.getForwardRates(PlottingDates);
SvenssonForwardRates = SvenssonModel.getForwardRates(PlottingDates);
VRPForwardRates = VRPModel.getForwardRates(PlottingDates);

figure
hold on
plot(TimeToMaturity,NSForwardRates,'r')
plot(TimeToMaturity,SvenssonForwardRates,'g')
plot(TimeToMaturity,VRPForwardRates,'b')
title('UK instantaneous nominal forward curve')
xlabel('Years ahead')
legend({'Nelson Siegel','Svensson','VRP'})
```



**Compare with this Link**

This link provides a live look at the derived yield curve published by the UK

http://www.bankofengland.co.uk

**Bibliography**

This example is based on the following papers and journal articles:

[1] Nelson, C.R., Siegel, A.F., (1987), "Parsimonious modelling of yield curves", Journal of Business, 60, pp 473-89

[2] Svensson, L.E.O. (1994), "Estimating and interpreting forward interest rates: Sweden 1992-4", International Monetary Fund, IMF Working Paper, 1994/114

[3] Fisher, M., Nychka, D., Zervos, D. (1995), "Fitting the term structure of interest rates with smoothing splines", Board of Governors of the Federal Reserve System, Federal Reserve Board Working Paper 95-1

[4] Anderson, N., Sleath, J. (1999), "New estimates of the UK real and nominal yield curves", Bank of England Quarterly Bulletin, November, pp 384-92

[5] Waggoner, D. (1997), "Spline Methods for Extracting Interest Rate Curves from Coupon Bond Prices", Federal Reserve Board Working Paper 97-10

[6] "Zero-coupon yield curves: technical documentation", BIS Papers No. 25 October 2005

[7] Bolder, D.J., Gusba,S (2002), "Exponentials, Polynomials, and Fourier Series: More Yield Curve Modelling at the Bank of Canada," Working Papers 02-29, Bank of Canada

[8] Bolder, D.J., Streliski, D (1999), "Yield Curve Modelling at the Bank of Canada," Technical Reports 84, Bank of Canada

# Converting an IRDataCurve or IRFunctionCurve Object

**In this section...**

## Introduction

The `IRDataCurve` and `IRFunctionCurve` objects for interest-rate curves support conversion to:

- A `RateSpec` structure. The `RateSpec` generated from an `IRDataCurve` or `IRFunctionCurve` object, using the `toRateSpec` method, is identical to the `RateSpec` structure created with `intenvset` using Financial Instruments Toolbox software.
- A vector of dates and data from an `IRDataCurve` object acceptable to `prbyzero`, `bkcall`, `bkput`, `tfutbyprice`, and `tfutbyyield` or any function that requires a term structure of interest rates.

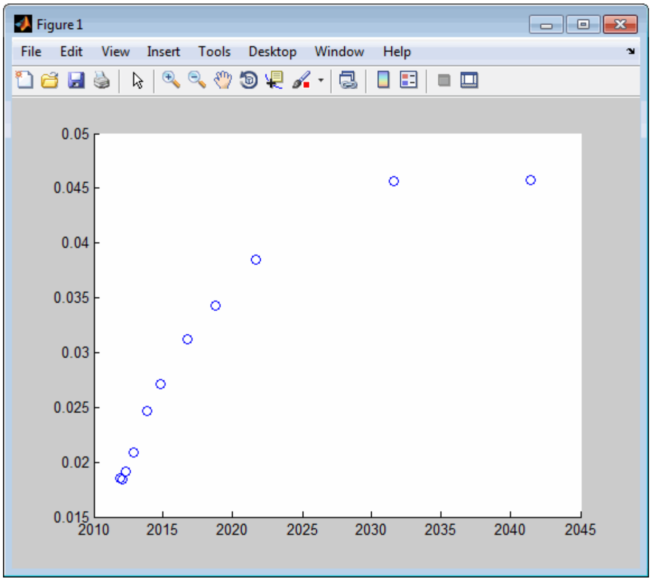
## Using the toRateSpec Method

To convert an `IRDataCurve` or `IRFunctionCurve` object to a `RateSpec` structure, you must first create an interest-rate curve object. Then, use the `toRateSpec` method for an `IRDataCurve` object or the`toRateSpec` method for an `IRFunctionCurve` object.

### Example

Create a data vector from the following data: `http://www.ustreas.gov/offices/domestic-finance/debt-management/interest-rate/yield.shtml`:

```
Data = [1.85 1.84 1.91 2.09 2.47 2.71 3.12 3.43 3.85 4.57 4.58]/100;
Dates = daysadd(today,[30 90 180 360 2*360 3*360 5*360 7*360 10*360 20*360 30*360],2);
scatter(Dates,Data)
datetick
```



Create an `IRDataCurve` interest-rate curve object:

```
rr = IRDataCurve('Zero',today,Dates,Data);
```

Convert to a `RateSpec`:

```
rr.toRateSpec(today+30:30:today+365)
ans =
           FinObj: 'RateSpec'
      Compounding: 2
             Disc: [12x1 double]
            Rates: [12x1 double]
         EndTimes: [12x1 double]
       StartTimes: [12x1 double]
         EndDates: [12x1 double]
       StartDates: 733569
    ValuationDate: 733569
            Basis: 0
     EndMonthRule: 1
```

## Using Vector of Dates and Data Methods

You can use the `getZeroRates` method for an `IRDataCurve` object with a `Dates` property to create a vector of dates and data acceptable for `prbyzero` in Financial Toolbox software and `bkcall`, `bkput`, `tfutbyprice`, and `tfutbyyield` in Financial Instruments Toolbox software.

### Example

This is an example of using the `IRDataCurve` method `getZeroRates` with `prbyzero`:

```
Data = [2.09 2.47 2.71 3.12 3.43 3.85 4.57 4.58]/100;
Dates = daysadd(today,[360 2*360 3*360 5*360 7*360 10*360 20*360 30*360],1);
irdc = IRDataCurve('Zero',today,Dates,Data,'InterpMethod','pchip');
Maturity = daysadd(today,8*360,1);
CouponRate = .055;
ZeroDates = daysadd(today,180:180:8*360,1);
ZeroRates = irdc.getZeroRates(ZeroDates);
BondPrice = prbyzero([Maturity CouponRate], today, ZeroRates, ZeroDates)
BondPrice =
  113.9221
```

# Class Reference

# @IRBootstrapOptions

Create specific options for bootstrapping an interest-rate curve object

**In this section...**

## Hierarchy

**Superclasses:** None

**Subclasses:** None

## Constructor

`IRBootstrapOptions`

## Public Read-Only Properties

| Name | Description |
|---|---|
| `ConvexityAdjustment` | Controls the convexity adjustment to interest rate futures. This can be specified as a function handle that takes time to maturity as an input and returns a value which is `ConvexityAdjustment`. Alternatively, you can define `ConvexityAdjustment` as an `N`-by-`1` vector of values, where `N` is the number of interest rate futures. In either case, the `ConvexityAdjustment` is subtracted from the futures rate.<br>For more information on defining a function handle, see the MATLAB Programming Fundamentals documentation. |

## Methods

There are no methods.

# @IRCurve

Base abstract class for interest-rate curve objects

**In this section...**

## Hierarchy

**Superclasses:** None

**Subclasses:** `@IRDataCurve`, `@IRFunctionCurve`

## Description

`IRCurve` is an abstract class; you cannot create instances of it directly. You can create `IRDataCurve` and `IRFunctionCurve` objects that are derived from this class.

## Constructor

`@IRCurve` is an abstract class. To construct an `IRCurve` object, use one of the subclass constructors, `IRDataCurve` or `IRFunctionCurve`.

## Public Read-Only Properties

| Name | Description |
|---|---|
| `Type` | Type of interest-rate curve: `zero`, `forward`, or `discount`. |
| `Settle` | Scalar or column vector of settlement dates. |

| Name | Description |
|---|---|
| `Compounding` | Scalar that sets the compounding frequency per year for the `IRCurve` object:<br>• -1 = Continuous compounding<br>• 1 = Annual compounding<br>• 2 = Semiannual compounding (default)<br>• 3 = Compounding three times per year<br>• 4 = Quarterly compounding<br>• 6 = Bimonthly compounding<br>• 12 = Monthly compounding |
| `Basis` | Day-count basis of the interest-rate curve. A vector of integers.<br>• 0 = actual/actual (default)<br>• 1 = 30/360 (SIA)<br>• 2 = actual/360<br>• 3 = actual/365<br>• 4 = 30/360 (BMA)<br>• 5 = 30/360 (ISDA)<br>• 6 = 30/360 (European)<br>• 7 = actual/365 (Japanese)<br>• 8 = actual/actual (ICMA)<br>• 9 = actual/360 (ICMA)<br>• 10 = actual/365 (ICMA)<br>• 11 = 30/360E (ICMA)<br>• 12 = actual/actual (ISDA)<br>• 13 = BUS/252 |

## Methods

Classes that inherit from the `IRCurve` abstract class must implement the following methods.

| Method | Description |
|---|---|
| `getForwardRates` | Returns forward rates for input dates. |
| `getZeroRates` | Returns zero rates for input dates. |
| `getDiscountFactors` | Returns discount factors for input dates. |
| `getParYields` | Returns par yields for input dates. |
| `toRateSpec` | Converts to be a `RateSpec` object. This is identical to the `RateSpec` structure produced by the Financial Instruments Toolbox function `intenvset`. |

# @IRDataCurve

Represent interest-rate curve object based on vector of dates and data

**In this section...**

## Hierarchy

**Superclasses:** `@IRCurve`

**Subclasses:** None

## Description

`IRDataCurve` is a representation of an interest-rate curve object with dates and data. You can construct this object directly by specifying dates and corresponding interest rates or discount factors; alternatively, you can bootstrap the object from market data. After an interest-rate curve object is constructed, you can:

- Calculate forward and zero rates and determine par yields.
- Extract the discount factors.
- Convert to a `RateSpec` structure that is identical to the `RateSpec` structure produced by the Financial Instruments Toolbox function `intenvset`.

## Constructor

`IRDataCurve`

## Public Read-Only Properties

| Name | Description |
|---|---|
| `Type` | Type of interest-rate curve: `zero`, `forward`, or `discount`. |
| `Settle` | Scalar or column vector of settlement dates. |
| `Compounding` | Scalar that sets the compounding frequency per year for the `IRCurve` object:<br>• -1 = Continuous compounding<br>• 1 = Annual compounding<br>• 2 = Semiannual compounding (default)<br>• 3 = Compounding three times per year<br>• 4 = Quarterly compounding<br>• 6 = Bimonthly compounding<br>• 12 = Monthly compounding |
| `Basis` | Day-count basis of the financial curve. A vector of integers.<br>• 0 = actual/actual (default)<br>• 1 = 30/360 (SIA)<br>• 2 = actual/360<br>• 3 = actual/365<br>• 4 = 30/360 (BMA)<br>• 5 = 30/360 (ISDA)<br>• 6 = 30/360 (European)<br>• 7 = actual/365 (Japanese)<br>• 8 = actual/actual (ICMA)<br>• 9 = actual/360 (ICMA)<br>• 10 = actual/365 (ICMA)<br>• 11 = 30/360E (ICMA) |

| Name | Description |
|---|---|
| | - 12 = actual/actual (ISDA)<br>- 13 = BUS/252 |
| `Dates` | Dates corresponding to rate data. |
| `Data` | Interest-rate data or discount factors for the curve object. |
| `InterpMethod` | Values are:<br>- `'linear'` — Linear interpolation (default).<br>- `'constant'` — Piecewise constant interpolation.<br>- `'pchip'` — Piecewise cubic Hermite interpolation.<br>- `'spline'` — Cubic spline interpolation. |

## Methods

The following table contains links to methods with supporting reference pages, including examples.

| Method | Description |
|---|---|
| `getForwardRates` | Returns forward rates for input dates. |
| `getZeroRates` | Returns zero rates for input dates. |
| `getDiscountFactors` | Returns discount factors for input dates. |
| `getParYields` | Returns par yields for input dates. |
| `toRateSpec` | Converts to be a `RateSpec` object. This structure is identical to the `RateSpec` |
| `bootstrap` | Bootstraps an interest rate curve from market data. |

# @IRFitOptions

Object to specify fitting options for an `IRFunctionCurve` interest-rate curve object

**In this section...**

## Hierarchy

**Superclasses:** None

**Subclasses:** None

## Description

The `IRFitOptions` object allows you to specify options relating to the fitting process for an `IRFunctionCurve` object. Input arguments are specified in parameter/value pairs. The `IRFitOptions` structure provides the capability to choose which quantity to be minimized and other optimization parameters.

## Constructor

`IRFitOptions`

## Public Read-Only Properties

| Name | Description |
|---|---|
| `FitType` | `Price`, `Yield`, or `DurationWeightedPrice` determines which is minimized in the curve fitting process. `DurationWeightedPrice` is the default. |
| `InitialGuess` | Initial guess for the parameters of the curve function. |
| `UpperBound` | Upper bound for the parameters of the curve function. |
| `LowerBound` | Lower bound for the parameters of the curve function. |
| `OptOptions` | Optimization structure based on the output from the Optimization Toolbox function `optimset`. This optimization structure is evaluated by `lsqnonlin`. |

## Methods

There are no methods.

# @IRFunctionCurve

Represent an interest-rate curve object using a function

**In this section...**

## Hierarchy

**Superclasses:** `@IRCurve`

**Subclasses:** None

## Description

`IRFunctionCurve` is a representation of an interest-rate curve object. You can construct this object directly by specifying a function handle or a function can be fit to market data using methods of the object. After an interest-rate curve object is constructed; you can:

- Calculate forward and zero rates and determine par yields.
- Extract the discount factors.
- Convert to a `RateSpec` structure; this is identical to the `RateSpec` structure produced by the Financial Instruments Toolbox function `intenvset`.

## Constructor

`IRFunctionCurve`

## Public Read-Only Properties

| Name | Description |
|---|---|
| `Type` | Type of interest-rate curve: `zero`, `forward`, or `discount`. |
| `Settle` | Scalar or column vector of settlement dates. |
| `Compounding` | Scalar that sets the compounding frequency per year for the `IRCurve` object:<br>• -1 = Continuous compounding<br>• 1 = Annual compounding<br>• 2 = Semiannual compounding (default)<br>• 3 = Compounding three times per year<br>• 4 = Quarterly compounding<br>• 6 = Bimonthly compounding<br>• 12 = Monthly compounding |
| `Basis` | Day-count basis of the interest-rate curve. A vector of integers.<br>• 0 = actual/actual (default)<br>• 1 = 30/360 (SIA)<br>• 2 = actual/360<br>• 3 = actual/365<br>• 4 = 30/360 (BMA)<br>• 5 = 30/360 (ISDA)<br>• 6 = 30/360 (European)<br>• 7 = actual/365 (Japanese)<br>• 8 = actual/actual (ICMA)<br>• 9 = actual/360 (ICMA)<br>• 10 = actual/365 (ICMA) |

| Name | Description |
|---|---|
| | - 11 = 30/360E (ICMA)<br>- 12 = actual/actual (ISDA)<br>- 13 = BUS/252 |
| `FunctionHandle` | Function handle that defines the interest-rate curve. For more information on defining a function handle, see the MATLAB Programming Fundamentals documentation. |

## Methods

The following table contains links to methods with supporting reference pages, including examples.

| Method | Description |
|---|---|
| `getForwardRates` | Returns forward rates for input dates. |
| `getZeroRates` | Returns zero rates for input dates. |
| `getDiscountFactors` | Returns discount factors for input dates. |
| `getParYields` | Returns par yields for input dates. |
| `toRateSpec` | Converts to be a `RateSpec` object. This is identical to the `RateSpec` structure |
| `fitSvensson` | Fits a Svensson function to market data. |
| `fitNelsonSiegel` | Fits a Nelson-Siegel function to market data. |
| `fitSmoothingSpline` | Fits a smoothing spline function to market data. |
| `fitFunction` | Fits a custom function to market data. |

# 10

# Function Reference

| | |
|---|---|
| Interest-Rate Instruments (p. 10-3) | Work with interest-rate instruments |
| Interest-Rate Term Structure (p. 10-4) | Work with interest-rate term structure |
| Interest-Rate Tree Models (p. 10-5) | Work with interest-rate models |
| Interest-Rate Closed-Form Solutions (p. 10-12) | Work with interest-rate closed-form solutions |
| Equity Instruments (p. 10-13) | Work with interest-rate instruments |
| Equity Tree Models (p. 10-14) | Work with equity tree models |
| Equity Derivative Closed-Form Solutions (p. 10-18) | Work with equity derivatives closed-form solutions |
| Monte Carlo Simulation for Equity Derivatives (p. 10-22) | Work with Monte Carlo simulation for equity derivatives |
| Controlling Defaults and Options (p. 10-23) | Work with derivatives pricing options |
| Portfolio Handling for Interest and Equity Instruments (p. 10-24) | Work with portfolio handling for interest and equity instruments |
| Financial Object Structures (p. 10-25) | Work with financial structures |
| Hedging Portfolios (p. 10-26) | Work with hedge portfolios |
| Bond Futures (p. 10-27) | Work with bond futures |
| Certificates of Deposit (p. 10-28) | Work with certificates of deposit |
| Convertible Bonds (p. 10-29) | Work with convertible bonds |
| Credit Default Swaps (p. 10-30) | Work with credit default swaps |

| | |
|---|---|
| Derivative Securities (p. 10-31) | Work with derivative securities |
| Interest-Rate Curve Objects (p. 10-32) | Work with interest-rate curve objects |
| Mortgage-Backed Securities (p. 10-34) | Work with mortgage-backed securities |
| Option-Adjusted Spread Computations (p. 10-36) | Work with option-adjusted spread computations |
| Stepped-Coupon Bonds (p. 10-37) | Work with stepped-coupon bonds |
| Treasury Bills (p. 10-38) | Work with Treasury bills |
| Zero-Coupon Instruments (p. 10-39) | Work with zero-coupon instruments |

# Interest-Rate Instruments

| | |
|---|---|
| `instbond` | Construct bond instrument |
| `instcap` | Construct cap instrument |
| `instcf` | Construct cash flow instrument |
| `instfixed` | Construct fixed-rate instrument |
| `instfloat` | Construct floating-rate instrument |
| `instfloor` | Construct floor instrument |
| `instoptbnd` | Construct bond option |
| `instoptembnd` | Construct bond with embedded option |
| `instrangefloat` | Construct range note instrument |
| `instswap` | Construct swap instrument |
| `instswaption` | Construct swaption instrument |

# Interest-Rate Term Structure

| | |
|---|---|
| `bondbyzero` | Price bond from set of zero curves |
| `cfbyzero` | Price cash flows from set of zero curves |
| `date2time` | Time and frequency from dates |
| `datedisp` | Display date entries |
| `disc2rate` | Interest rates from cash flow discounting factors |
| `fixedbyzero` | Price fixed-rate note from set of zero curves |
| `floatbyzero` | Price floating-rate note from set of zero curves |
| `intenvget` | Properties of interest-rate structure |
| `intenvprice` | Price instruments from set of zero curves |
| `intenvsens` | Instrument price and sensitivities from set of zero curves |
| `intenvset` | Set properties of interest-rate structure |
| `rate2disc` | Discount factors from interest rates |
| `ratetimes` | Change time intervals defining interest-rate environment |
| `swapbyzero` | Price swap instrument from set of zero curves |
| `time2date` | Dates from time and frequency |

# Interest-Rate Tree Models

## Heath-Jarrow-Morton Trees

| | |
|---|---|
| `hjmtimespec` | Specify time structure for Heath-Jarrow-Morton interest-rate tree |
| `hjmtree` | Construct Heath-Jarrow-Morton interest-rate tree |
| `hjmvolspec` | Specify Heath-Jarrow-Morton interest-rate volatility process |

## Heath-Jarrow-Morton Tree Utililites

| | |
|---|---|
| `bondbyhjm` | Price bond from Heath-Jarrow-Morton interest-rate tree |
| `capbyhjm` | Price cap instrument from Heath-Jarrow-Morton interest-rate tree |
| `cfbyhjm` | Price cash flows from Heath-Jarrow-Morton interest-rate tree |
| `fixedbyhjm` | Price fixed-rate note from Heath-Jarrow-Morton interest-rate tree |
| `floatbyhjm` | Price floating-rate note from Heath-Jarrow-Morton interest-rate tree |
| `floorbyhjm` | Price floor instrument from Heath-Jarrow-Morton interest-rate tree |
| `hjmprice` | Instrument prices from Heath-Jarrow-Morton interest-rate tree |
| `hjmsens` | Instrument prices and sensitivities from Heath-Jarrow-Morton interest-rate tree |
| `mmktbyhjm` | Create money-market tree from Heath-Jarrow-Morton interest-rate tree |
| `oasbyhjm` | Determine option adjusted spread using Heath-Jarrow-Morton model |
| `optbndbyhjm` | Price bond option from Heath-Jarrow-Morton interest-rate tree |

| | |
|---|---|
| `optembndbyhjm` | Price bonds with embedded options by Heath-Jarrow-Morton interest-rate tree |
| `rangefloatbyhjm` | Price range floating note using Heath-Jarrow-Morton tree |
| `swapbyhjm` | Price swap instrument from Heath-Jarrow-Morton interest-rate tree |
| `swaptionbyhjm` | Price swaption from Heath-Jarrow-Morton interest-rate tree |

## Black-Derman-Toy Trees

| | |
|---|---|
| `bdttimespec` | Specify time structure for Black-Derman-Toy interest-rate tree |
| `bdttree` | Construct Black-Derman-Toy interest-rate tree |
| `bdtvolspec` | Specify Black-Derman-Toy interest-rate volatility process |

## Black-Derman-Toy Tree Utilities

| | |
|---|---|
| `bdtprice` | Instrument prices from Black-Derman-Toy interest-rate tree |
| `bdtsens` | Instrument prices and sensitivities from Black-Derman-Toy interest-rate tree |
| `bondbybdt` | Price bond from Black-Derman-Toy interest-rate tree |
| `capbybdt` | Price cap instrument from Black-Derman-Toy interest-rate tree |

| | |
|---|---|
| `cfbybdt` | Price cash flows from Black-Derman-Toy interest-rate tree |
| `fixedbybdt` | Price fixed-rate note from Black-Derman-Toy interest-rate tree |
| `floatbybdt` | Price floating-rate note from Black-Derman-Toy interest-rate tree |
| `floorbybdt` | Price floor instrument from Black-Derman-Toy interest-rate tree |
| `mmktbybdt` | Create money-market tree from Black-Derman-Toy interest-rate tree |
| `oasbybdt` | Determine option adjusted spread using Black-Derman-Toy model |
| `optbndbybdt` | Price bond option from Black-Derman-Toy interest-rate tree |
| `optembndbybdt` | Price bonds with embedded options by Black-Derman-Toy interest-rate tree |
| `rangefloatbybdt` | Price range floating note using Black-Derman-Toy tree |
| `swapbybdt` | Price swap instrument from Black-Derman-Toy interest-rate tree |
| `swaptionbybdt` | Price swaption from Black-Derman-Toy interest-rate tree |

## Hull-White Trees

| | |
|---|---|
| `hwtimespec` | Specify time structure for Hull-White interest-rate tree |
| `hwtree` | Construct Hull-White interest-rate tree |
| `hwvolspec` | Specify Hull-White interest-rate volatility process |

## Hull-White Tree Utilities

| | |
|---|---|
| `bondbyhw` | Price bond from Hull-White interest-rate tree |
| `capbyhw` | Price cap instrument from Hull-White interest-rate tree |
| `cfbyhw` | Price cash flows from Hull-White interest-rate tree |
| `fixedbyhw` | Price fixed-rate note from Hull-White interest-rate tree |
| `floatbyhw` | Price floating-rate note from Hull-White interest-rate tree |
| `floorbyhw` | Price floor instrument from Hull-White interest-rate tree |
| `hwcalbycap` | Calibrate Hull-White tree using caps |
| `hwcalbyfloor` | Calibrate Hull-White tree using floors |
| `hwprice` | Instrument prices from Hull-White interest-rate tree |
| `hwsens` | Instrument prices and sensitivities from Hull-White interest-rate tree |
| `oasbyhw` | Determine option adjusted spread using Hull-White model |
| `optbndbyhw` | Price bond option from Hull-White interest-rate tree |
| `optembndbyhw` | Price bonds with embedded options by Hull-White interest-rate tree |
| `rangefloatbyhw` | Price range floating note using Hull-White tree |
| `swapbyhw` | Price swap instrument from Hull-White interest-rate tree |
| `swaptionbyhw` | Price swaption from Hull-White interest-rate tree |

## Black-Karasinski Trees

| | |
|---|---|
| `bktimespec` | Specify time structure for Black-Karasinski tree |
| `bktree` | Construct Black-Karasinski interest-rate tree |
| `bkvolspec` | Specify Black-Karasinski interest-rate volatility process |

## Black-Karasinski Tree Utilities

| | |
|---|---|
| `bkprice` | Instrument prices from Black-Karasinski interest-rate tree |
| `bksens` | Instrument prices and sensitivities from Black-Karasinski interest-rate tree |
| `bondbybk` | Price bond from Black-Karasinski interest-rate tree |
| `capbybk` | Price cap instrument from Black-Karasinski interest-rate tree |
| `cfbybk` | Price cash flows from Black-Karasinski interest-rate tree |
| `fixedbybk` | Price fixed-rate note from Black-Karasinski interest-rate tree |
| `floatbybk` | Price floating-rate note from Black-Karasinski interest-rate tree |
| `floorbybk` | Price floor instrument from Black-Karasinski interest-rate tree |
| `oasbybk` | Determine option adjusted spread using Black-Karasinski model |

| | |
|---|---|
| `optbndbybk` | Price bond option from Black-Karasinski interest-rate tree |
| `optembndbybk` | Price bonds with embedded options by Black-Karasinski interest-rate tree |
| `rangefloatbybk` | Price range floating note using Black-Karasinski tree |
| `swapbybk` | Price swap instrument from Black-Karasinski interest-rate tree |
| `swaptionbybk` | Price swaption from Black-Karasinski interest-rate tree |

## Tree Manipulation

| | |
|---|---|
| `bushpath` | Extract entries from node of bushy tree |
| `bushshape` | Retrieve shape of bushy tree |
| `cvtree` | Convert inverse-discount tree to interest-rate tree |
| `mkbush` | Create bushy tree |
| `mktree` | Create recombining binomial tree |
| `mktrintree` | Create recombining trinomial tree |
| `treepath` | Entries from node of recombining binomial tree |
| `treeshape` | Shape of recombining binomial tree |
| `treeviewer` | Tree information |
| `trintreepath` | Entries from node of recombining trinomial tree |
| `trintreeshape` | Shape of recombining trinomial tree |

## Interest-Rate Closed-Form Solutions

| | |
|---|---|
| `capbyblk` | Price caps using Black option pricing model |
| `floorbyblk` | Price floors using Black option pricing model |
| `swaptionbyblk` | Price European swaption instrument using Black model |

# Equity Instruments

| | |
|---|---|
| `instasian` | Construct Asian option |
| `instbarrier` | Construct barrier option |
| `instcompound` | Construct compound option |
| `instlookback` | Construct lookback option |
| `instoptstock` | Construct stock option |

# Equity Tree Models

## Cox-Ross-Rubinstein Trees

| | |
|---|---|
| `crrtimespec` | Specify time structure for Cox-Ross-Rubinstein tree |
| `crrtree` | Construct Cox-Ross-Rubinstein stock tree |

## Cox-Ross-Rubinstein Tree Utilities

| | |
|---|---|
| `asianbycrr` | Price Asian option from Cox-Ross-Rubinstein binomial tree |
| `barrierbycrr` | Price barrier option from Cox-Ross-Rubinstein binomial tree |

| | |
|---|---|
| `compoundbycrr` | Price compound option from Cox-Ross-Rubinstein binomial tree |
| `crrprice` | Instrument prices from Cox-Ross-Rubinstein tree |
| `crrsens` | Instrument prices and sensitivities from Cox-Ross-Rubinstein tree |
| `lookbackbycrr` | Price lookback option from Cox-Ross-Rubinstein tree |
| `optstockbycrr` | Price stock option from Cox-Ross-Rubinstein tree |

## Equal Probabilities Binomial Trees

| | |
|---|---|
| `eqptimespec` | Specify time structure for Equal Probabilities binomial tree |
| `eqptree` | Construct Equal Probabilities stock tree |

## Equal Probabilities Binomial Tree Utilities

| | |
|---|---|
| `asianbyeqp` | Price Asian option from Equal Probabilities binomial tree |
| `barrierbyeqp` | Price barrier option from Equal Probabilities binomial tree |
| `compoundbyeqp` | Price compound option from Equal Probabilities binomial tree |
| `eqpprice` | Instrument prices from Equal Probabilities binomial tree |
| `eqpsens` | Instrument prices and sensitivities from Equal Probabilities binomial tree |

| | |
|---|---|
| `lookbackbyeqp` | Price lookback option from Equal Probabilities binomial tree |
| `optstockbyeqp` | Price stock option from Equal Probabilities binomial tree |

## Leisen-Reimer Trees

| | |
|---|---|
| `lrtimespec` | Specify time structure for Leisen-Reimer binomial tree |
| `lrtree` | Build Leisen-Reimer stock tree |

## Leisen-Reimer Tree Utilities

| | |
|---|---|
| `optstockbylr` | Price options on stocks using Leisen-Reimer binomial tree model |
| `optstocksensbylr` | Determine option prices and sensitivities using Leisen-Reimer binomial tree model |

## Implied Trinomial Trees

| | |
|---|---|
| `itttimespec` | Specify time structure using implied trinomial tree (ITT) |
| `itttree` | Build implied trinomial stock tree |

## Implied Trinomial Tree Utilities

| | |
|---|---|
| `asianbyitt` | Price Asian options using implied trinomial tree (ITT) |
| `barrierbyitt` | Price barrier options using implied trinomial tree (ITT) |

| | |
|---|---|
| `compoundbyitt` | Price compound options using implied trinomial tree (ITT) |
| `ittprice` | Price instruments using implied trinomial tree (ITT) |
| `ittsens` | Instrument sensitivities and prices using implied trinomial tree (ITT) |
| `lookbackbyitt` | Price lookback option using implied trinomial tree (ITT) |
| `optstockbyitt` | Price options on stocks using implied trinomial tree (ITT) |
| `stockoptspec` | Specify European stock option structure |

## Tree Manipulation

| | |
|---|---|
| `bushpath` | Extract entries from node of bushy tree |
| `bushshape` | Retrieve shape of bushy tree |
| `cvtree` | Convert inverse-discount tree to interest-rate tree |
| `mkbush` | Create bushy tree |
| `mktree` | Create recombining binomial tree |
| `mktrintree` | Create recombining trinomial tree |
| `treepath` | Entries from node of recombining binomial tree |
| `treeshape` | Shape of recombining binomial tree |
| `treeviewer` | Tree information |
| `trintreepath` | Entries from node of recombining trinomial tree |
| `trintreeshape` | Shape of recombining trinomial tree |

# Equity Derivative Closed-Form Solutions

## Black-Scholes Option Pricing Model

| | |
|---|---|
| `assetbybls` | Determine price of asset-or-nothing digital options using Black-Scholes model |
| `assetsensbybls` | Determine price and sensitivities of asset-or-nothing digital options using Black-Scholes model |
| `cashbybls` | Determine price of cash-or-nothing digital options using Black-Scholes model |
| `cashsensbybls` | Determine price and sensitivities of cash-or-nothing digital options using Black-Scholes model |
| `chooserbybls` | Price European simple chooser options using Black-Scholes model |
| `gapbybls` | Determine price of gap digital options using Black-Scholes model |

| | |
|---|---|
| `gapsensbybls` | Determine price and sensitivities of gap digital options using Black-Scholes model |
| `impvbybls` | Determine implied volatility using Black-Scholes option pricing model |
| `optstockbybls` | Price options using Black-Scholes option pricing model |
| `optstocksensbybls` | Determine option prices and sensitivities using Black-Scholes option pricing model |
| `supersharebybls` | Calculate price of supershare digital options using Black-Scholes model |
| `supersharesensbybls` | Calculate price and sensitivities of supershare digital options using Black-Scholes model |

## Black Option Pricing Model

| | |
|---|---|
| `impvbyblk` | Determine implied volatility using Black option pricing model |
| `optstockbyblk` | Price options on futures using Black option pricing model |
| `optstocksensbyblk` | Determine option prices and sensitivities on futures using Black pricing model |

## Role-Geske-Whaley Option Pricing Model

| | |
|---|---|
| `impvbyrgw` | Determine implied volatility using Roll-Geske-Whaley option pricing model for American call option |
| `optstockbyrgw` | Determine American call option prices using Roll-Geske-Whaley option pricing model |
| `optstocksensbyrgw` | Determine American call option prices and sensitivities using Roll-Geske-Whaley option pricing model |

## Bjerksund-Stensland Option Pricing Model

| | |
|---|---|
| `impvbybjs` | Determine implied volatility using Bjerksund-Stensland 2002 option pricing model |
| `optstockbybjs` | Price American options using Bjerksund-Stensland 2002 option pricing model |
| `optstocksensbybjs` | Determine American option prices and sensitivities using Bjerksund-Stensland 2002 option pricing model |

## Nengjiu Ju Approximation Pricing Model

| | |
|---|---|
| `basketbyju` | Price European basket options using Nengjiu Ju approximation model |
| `basketsensbyju` | Determine European basket options price and sensitivities using Nengjiu Ju approximation model |

## Stulz Option Pricing

| | |
|---|---|
| `maxassetbystulz` | Determine European rainbow option price on maximum of two risky assets using Stulz option pricing model |
| `maxassetsensbystulz` | Determine European rainbow option prices and sensitivities on maximum of two risky assets using Stulz pricing model |
| `minassetbystulz` | Determine European rainbow option prices on minimum of two risky assets using Stulz option pricing model |
| `minassetsensbystulz` | Determine European rainbow option prices and sensitivities on minimum of two risky assets using Stulz pricing model |

# Monte Carlo Simulation for Equity Derivatives

## Longstaff-Schwartz Option Pricing Model

| | |
|---|---|
| `basketbyls` | Price basket options using Longstaff-Schwartz model |
| `basketsensbyls` | Determine price and sensitivities for basket options using Longstaff-Schwartz model |
| `basketstockspec` | Specify basket stock structure using Longstaff-Schwartz model |

# Controlling Defaults and Options

| | |
|---|---|
| `derivget` | Get derivatives pricing options |
| `derivset` | Set or modify derivatives pricing options |

## Portfolio Handling for Interest and Equity Instruments

| | |
|---|---|
| `instadd` | Add types to instrument collection |
| `instaddfield` | Add new instruments to instrument collection |
| `instdelete` | Complement of instrument set by matching conditions |
| `instdisp` | Display instruments |
| `instfields` | List field names |
| `instfind` | Search instruments for matching conditions |
| `instget` | Data from instrument variable |
| `instgetcell` | Data and context from instrument variable |
| `instlength` | Count instruments |
| `instselect` | Create instrument subset by matching conditions |
| `instsetfield` | Add or reset data for existing instruments |
| `insttypes` | List types |

# Financial Object Structures

| | |
|---|---|
| `classfin` | Create financial structure or return financial structure class name |
| `isafin` | True if input argument is financial structure type or financial object class |
| `stockspec` | Create stock structure |

## Hedging Portfolios

| | |
|---|---|
| `hedgeopt` | Allocate optimal hedge for target costs or sensitivities |
| `hedgeslf` | Self-financing hedge |

# Bond Futures

| | |
|---|---|
| `bndfutimprepo` | Implied repo rates for bond future given price |
| `bndfutprice` | Price bond future given repo rates |
| `convfactor` | Bond conversion factors |
| `tfutbyprice` | Future prices of Treasury bonds given spot price |
| `tfutbyyield` | Future prices of Treasury bonds given current yield |
| `tfutimprepo` | Implied repo rates for Treasury bond future given price |
| `tfutpricebyrepo` | Implied repo rates given Treasury bond future price |
| `tfutyieldbyrepo` | Implied repo rates given Treasury bond future yield |

## Certificates of Deposit

| | |
|---|---|
| `cdai` | Accrued interest on certificate of deposit |
| `cdprice` | Price of certificate of deposit |
| `cdyield` | Yield on certificate of deposit (CD) |

# Convertible Bonds

| | |
|---|---|
| `cbprice` | Price convertible bond |

# Credit Default Swaps

| | |
|---|---|
| `cdsbootstrap` | Bootstrap default probability curve from credit default swap market quotes |
| `cdsoptprice` | Price payer and receiver credit default swap options |
| `cdsprice` | Determine price for credit default swap |
| `cdsspread` | Determine spread of credit default swap |

# Derivative Securities

| | |
|---|---|
| `bkcall` | Price European call option on bonds using Black model |
| `bkcaplet` | Price interest-rate caplet using Black model |
| `bkfloorlet` | Price interest-rate floorlet using Black model |
| `bkput` | Price European put option on bonds using Black model |
| `liborduration` | Duration of LIBOR-based interest-rate swap |
| `liborfloat2fixed` | Compute par fixed-rate of swap given 3-month LIBOR data |
| `liborprice` | Price swap given swap rate |

## Interest-Rate Curve Objects

| | |
|---|---|
| `bootstrap` | Bootstrap interest-rate curve from market data |
| `fitFunction` | Custom fit interest-rate curve object to bond market data |
| `fitNelsonSiegel` | Fit Nelson-Siegel function to bond market data |
| `fitSmoothingSpline` | Fit smoothing spline to bond market data |
| `fitSvensson` | Fit Svensson function to bond market data |
| `getDiscountFactors` | Get discount factors for input dates for `IRDataCurve` |
| `getDiscountFactors` | Get discount factors for input dates for `IRFunctionCurve` |
| `getForwardRates` | Get forward rates for input dates for `IRDataCurve` |
| `getForwardRates` | Get forward rates for input dates for `IRFunctionCurve` |
| `getParYields` | Get par yields for input dates for `IRDataCurve` |
| `getParYields` | Get par yields for input dates for `IRFunctionCurve` |
| `getZeroRates` | Get zero rates for input dates for `IRDataCurve` |
| `getZeroRates` | Get zero rates for input dates for `IRFunctionCurve` |
| `IRBootstrapOptions` | Construct specific options for bootstrapping interest-rate curve object |
| `IRDataCurve` | Construct interest-rate curve object from dates and data |

| | |
|---|---|
| `IRFitOptions` | Construct specific options for fitting interest-rate curve object |
| `IRFunctionCurve` | Construct interest-rate curve object from function handle or function and fit to market data |
| `toRateSpec` | Convert `IRDataCurve` object to `RateSpec` |
| `toRateSpec` | Convert `IRFunctionCurve` object to `RateSpec` |

## Mortgage-Backed Securities

| | |
|---|---|
| `cmosched` | Generate principal balance schedule for planned amortization class (PAC) or targeted amortization class (TAC) bond |
| `cmoschedcf` | Generate cash flows for scheduled collateralized mortgage obligation (CMO) using PAC or TAC model |
| `cmoseqcf` | Generate cash flows for sequential collateralized mortgage obligation (CMO) |
| `mbscfamounts` | Cash flow and time mapping for mortgage pool |
| `mbsconvp` | Convexity of mortgage pool given price |
| `mbsconvy` | Convexity of mortgage pool given yield |
| `mbsdurp` | Duration of mortgage pool given price |
| `mbsdury` | Duration of mortgage pool given yield |
| `mbsnoprepay` | End-of-month mortgage cash flows and balances without prepayment |
| `mbspassthrough` | Mortgage pool cash flows and balances with prepayment |
| `mbsprice` | Mortgage-backed security price given yield |
| `mbsprice2speed` | Implied PSA prepayment speeds given price |
| `mbswal` | Weighted average life of mortgage pool |

| | |
|---|---|
| `mbsyield` | Mortgage-backed security yield given price |
| `mbsyield2speed` | Implied PSA prepayment speeds given yield |
| `psaspeed2default` | Benchmark default |
| `psaspeed2rate` | Single monthly mortality rate given PSA speed |

# Option-Adjusted Spread Computations

| | |
|---|---|
| `agencyoas` | Determine option-adjusted spread of callable bond using Agency OAS model |
| `agencyprice` | Price callable bond using Agency OAS model |
| `mbsoas2price` | Price given option-adjusted spread |
| `mbsoas2yield` | Yield given option-adjusted spread |
| `mbsprice2oas` | Option-adjusted spread given price |
| `mbsyield2oas` | Option-adjusted spread given yield |

## Stepped-Coupon Bonds

| | |
|---|---|
| `stepcpncfamounts` | Cash flow amounts and times for bonds and stepped coupons |
| `stepcpnprice` | Price bond with stepped coupons |
| `stepcpnyield` | Yield to maturity of bond with stepped coupons |

## Treasury Bills

| | |
|---|---|
| `tbilldisc2yield` | Convert Treasury bill discount to equivalent yield |
| `tbillprice` | Price Treasury bill |
| `tbillrepo` | Break-even discount of repurchase agreement |
| `tbillval01` | Value of one basis point |
| `tbillyield` | Yield on Treasury bill |
| `tbillyield2disc` | Convert Treasury bill yield to equivalent discount |

# Zero-Coupon Instruments

| | |
|---|---|
| `zeroprice` | Price zero-coupon instruments given yield |
| `zeroyield` | Yield of zero-coupon instruments given price |

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# 12

# Functions — Alphabetical List

# asianbycrr

## Purpose

Price Asian option from Cox-Ross-Rubinstein binomial tree

## Syntax

```
Price = asianbycrr(CRRTree, OptSpec, Strike, Settle,
ExerciseDates, AmericanOpt, AvgType, AvgPrice, AvgDate)
```

## Arguments

| | |
|---|---|
| `CRRTree` | Stock tree structure created by `crrtree`. |
| `OptSpec` | `NINST`-by-`1` list of string values `'Call'` or `'Put'`. |
| `Strike` | `NINST`-by-`1` vector of strike price values. Each row is the schedule for one option. |
| `Settle` | `NINST`-by-`1` vector of `Settle` dates. The settle date for every Asian option is set to the valuation date of the stock tree. The Asian argument `Settle` is ignored. |
| `ExerciseDates` | For a European option (`AmericanOpt = 0`):<br>`NINST`-by-`1` vector of exercise dates. Each row is the schedule for one option. For a European option, there is only one exercise date, the option expiry date.<br>For an American option (`AmericanOpt = 1`):<br>`NINST`-by-`2` vector of exercise date boundaries. For each instrument, the option can be exercised on any tree date between or including the pair of dates on that row. If only one non-`NaN` date is listed, or if `ExerciseDates` is `NINST`-by-`1`, the option can be exercised between the valuation date of the stock tree and the single listed exercise date. |

| | |
|---|---|
| `AmericanOpt` | (Optional) If `AmericanOpt = 0`, `NaN`, or is unspecified, the option is a European option. If `AmericanOpt = 1`, the option is an American option. |
| `AvgType` | (Optional) String = `'arithmetic'` for arithmetic average (default) or `'geometric'` for geometric average. |
| `AvgPrice` | (Optional) Scalar representing the average price of the underlying asset at `Settle`. This argument is used when `AvgDate < Settle`. Default is the current stock price. |
| `AvgDate` | (Optional) Scalar representing the date on which the averaging period begins. Default = `Settle`. |

## Description

`Price = asianbycrr(CRRTree, OptSpec, Strike, Settle, ExerciseDates, AmericanOpt, AvgType, AvgPrice, AvgDate)` calculates the value of fixed- and floating-strike Asian options. To compute the value of a floating-strike Asian option, specify `Strike` as `NaN`. Fixed-strike Asian options are also known as average price options. Floating-strike Asian options are also known as average strike options.

`Price` is a `NINST`-by-`1` vector of expected prices at time 0.

Asian options are priced using Hull-White (1993). Consequently, for these options only the root node contains a unique price.

## Examples

Price a floating-strike Asian option using a CRR binomial tree.

Load the file `deriv.mat`, which provides `CRRTree`. The `CRRTree` structure contains the stock specification and time information needed to price the option.

```
load deriv.mat;
```

Set the required values. Other arguments will use defaults.

```
OptSpec = 'put';
Strike = NaN;
Settle = '01-Jan-2003';
ExerciseDates = '01-Jan-2004';
```

Use `asianbycrr` to compute the price of the option.

```
Price = asianbycrr(CRRTree, OptSpec, Strike, Settle, ...
ExerciseDates)

Price =

  1.2177
```

## References

Hull, J., and A. White, “Efficient Procedures for Valuing European and American Path-Dependent Options,” *Journal of Derivatives*, Volume 1, pp. 21-31.

## See Also

`crrtree` | `instasian`

## Purpose

Price Asian option from Equal Probabilities binomial tree

## Syntax

```
Price = asianbyeqp(EQPTree, OptSpec, Strike, Settle,
ExerciseDates, AmericanOpt, AvgType, AvgPrice, AvgDate)
```

## Arguments

| | |
|---|---|
| `EQPTree` | Stock tree structure created by `eqptree`. |
| `OptSpec` | `NINST`-by-`1` list of string values `'Call'` or `'Put'`. |
| `Strike` | `NINST`-by-`1` vector of strike price values. Each row is the schedule for one option. |
| `Settle` | `NINST`-by-`1` vector of `Settle` dates. The settle date for every Asian option is set to the valuation date of the stock tree. The Asian argument `Settle` is ignored. |
| `ExerciseDates` | For a European option (`AmericanOpt = 0`):<br>`NINST`-by-`1` vector of exercise dates. Each row is the schedule for one option. For a European option, there is only one exercise date, the option expiry date.<br>For an American option (`AmericanOpt = 1`):<br>`NINST`-by-`2` vector of exercise date boundaries. For each instrument, the option can be exercised on any tree date between or including the pair of dates on that row. If only one non-`NaN` date is listed, or if `ExerciseDates` is `NINST`-by-`1`, the option can be exercised between the valuation date of the stock tree and the single listed exercise date. |

| | |
|---|---|
| `AmericanOpt` | (Optional) If `AmericanOpt = 0`, `NaN`, or is unspecified, the option is a European option. If `AmericanOpt = 1`, the option is an American option. |
| `AvgType` | (Optional) String = `'arithmetic'` for arithmetic average (default) or `'geometric'` for geometric average. |
| `AvgPrice` | (Optional) Scalar representing the average price of the underlying asset at `Settle`. This argument is used when `AvgDate < Settle`. Default is the current stock price. |
| `AvgDate` | (Optional) Scalar representing the date on which the averaging period begins. |

## Description

`Price = asianbyeqp(EQPTree, OptSpec, Strike, Settle, ExerciseDates, AmericanOpt, AvgType, AvgPrice, AvgDate)` calculates the value of fixed- and floating-strike Asian options. To compute the value of a floating-strike Asian option, specify `Strike` as `NaN`. Fixed-strike Asian options are also known as average price options. Floating-strike Asian options are also known as average strike options.

`Price` is a `NINST`-by-`1` vector of expected prices at time 0.

## Examples

Price a floating-strike Asian option using an EQP equity tree.

Load the file `deriv.mat`, which provides `EQPTree`. The `EQPTree` structure contains the stock specification and time information needed to price the option.

```
load deriv.mat;
```

Set the required values. Other arguments will use defaults.

```
OptSpec = 'put';
Strike = NaN;
```

```
Settle = '01-Jan-2003';
ExerciseDates = '01-Jan-2004';
```

Use `asianbyeqp` to compute the price of the option.

```
Price = asianbyeqp(EQPTree, OptSpec, Strike, Settle, ...
ExerciseDates)

Price =

 1.2724
```

## References

Hull, J., and A. White, "Efficient Procedures for Valuing European and American Path-Dependent Options," *Journal of Derivatives*, Volume 1, pp. 21-31.

## See Also

`eqptree` | `instasian`

## Purpose

Price Asian options using implied trinomial tree (ITT)

## Syntax

```
Price = asianbyitt(ITTTree, OptSpec, Strike, Settle,
ExerciseDates, AmericanOpt, AvgType, AvgPrice, AvgDate)
```

## Arguments

| | |
|---|---|
| `ITTTree` | Stock tree structure created by `itttree`. |
| `OptSpec` | `NINST`-by-`1` list of string values `'call'` or `'put'`. |
| `Strike` | `NINST`-by-`1` vector of strike price values. Each row represents the schedule for one option. |
| `Settle` | `NINST`-by-`1` vector of `Settle` dates. The settle date for every Asian option is set to the valuation date of the stock tree. The Asian argument `Settle` is ignored. |
| `ExerciseDates` | For a European option (`AmericanOpt = 0`):<br>`NINST`-by-`1` vector of exercise dates. Each row is the schedule for one option. For a European option, there is only one exercise date which is the option expiry date.<br>For an American option (`AmericanOpt = 1`):<br>`NINST`-by-`2` vector of exercise date boundaries. For each instrument, the option can be exercised on any tree date between or including the pair of dates on that row. If only one non-`NaN` date is listed, or if `ExerciseDates` is `NINST`-by-`1`, the option can be exercised between the valuation date of the stock tree and the single listed exercise date. |

| | |
|---|---|
| `AmericanOpt` | (Optional) If `AmericanOpt = 0`, `NaN`, or is unspecified, the option is a European option. If `AmericanOpt = 1`, the option is an American option. |
| `AvgType` | (Optional) String = `'arithmetic'` for arithmetic average (default) or `'geometric'` for geometric average. |
| `AvgPrice` | (Optional) Scalar representing the average price of the underlying asset at `Settle`. This argument is used when `AvgDate < Settle`. Default is the current stock price. |
| `AvgDate` | (Optional) Scalar representing the date on which the averaging period begins. |

## Description

`Price = asianbyitt(ITTTree, OptSpec, Strike, Settle, ExerciseDates, AmericanOpt, AvgType, AvgPrice, AvgDate)` calculates the value of fixed- and floating-strike Asian options. To compute the value of a floating-strike Asian option, specify `Strike` as `NaN`. Fixed-strike Asian options are also known as average price options. Floating-strike Asian options are also known as average strike options.

`Price` is a `NINST`-by-`1` vector of expected prices at time 0.

---

**Note** The `Settle` date for every Asian option is set to the `ValuationDate` of the stock tree. The Asian argument, `Settle`, is ignored.

---

## Examples

Price a floating-strike Asian option using an ITT equity tree.

Load the file `deriv.mat` which provides the `ITTTree`. The `ITTTree` structure contains the stock specification and time information needed to price the option.

```
load deriv.mat;
```

Set the required values. Other arguments will use defaults.

```
OptSpec = 'put';
Strike = NaN;
Settle = '01-Jan-2006';
ExerciseDates = '01-Jan-2007';
```

Use `asianbyitt` to compute the price of the option.

```
Price = asianbyitt(ITTTree, OptSpec, Strike, Settle, ExerciseDates)

Price =

 1.0778
```

### References

Hull, J., and A. White, "Efficient Procedures for Valuing European and American Path-Dependent Options," *Journal of Derivatives*, Volume 1, 1993, pp. 21-31.

### See Also

`instasian` | `itttree`

## Purpose

Determine price of asset-or-nothing digital options using Black-Scholes model

## Syntax

```
Price = assetbybls(RateSpec, StockSpec, Settle, Maturity,
OptSpec, Strike)
```

## Arguments

| | |
|---|---|
| `RateSpec` | The annualized continuously compounded rate term structure. For information on the interest rate specification, see `intenvset`. |
| `StockSpec` | Stock specification. See `stockspec`. |
| `Settle` | `NINST`-by-`1` vector of settlement or trade dates. |
| `Maturity` | `NINST`-by-`1` vector of maturity dates. |
| `OptSpec` | `NINST`-by-`1` cell array of strings `'call'` or `'put'`. |
| `Strike` | `NINST`-by-`1` vector of payoff strike price values. |

## Description

`Price = assetbybls(RateSpec, StockSpec, Settle, Maturity, OptSpec, Strike)` computes asset-or-nothing option prices using the Black-Scholes option pricing model.

`Price` is a `NINST`-by-`1` vector of expected option prices.

## Examples

Consider two asset-or-nothing put options on a nondividend paying stock with a strike of 95 and 93 and expiring on January 30, 2009. On November 3, 2008 the stock is trading at 97.50. Using this data, calculate the price of the asset-or-nothing put options if the risk-free rate is 4.5% and the volatility is 22%.

Create the `RateSpec`:

```
Settle = 'Nov-3-2008';
Maturity = 'Jan-30-2009';
Rates = 0.045;
```

```
Compounding = -1;
RateSpec = intenvset('ValuationDate', Settle, 'StartDates', Settle,...
'EndDates', Maturity, 'Rates', Rates, 'Compounding', Compounding);
```

Define the `StockSpec`:

```
AssetPrice = 97.50;
Sigma = .22;
StockSpec = stockspec(Sigma, AssetPrice);
```

Define the put options:

```
OptSpec = {'put'};
Strike = [95;93];
```

Calculate the price:

```
Paon = assetbybls(RateSpec, StockSpec, Settle, Maturity, OptSpec, Strike)

Paon =

   33.7666
   26.9662
```

## See Also

`assetsensbybls` | `cashbybls` | `gapbybls` | `supersharebybls`

## Purpose

Determine price and sensitivities of asset-or-nothing digital options using Black-Scholes model

## Syntax

```
PriceSens = assetsensbybls(RateSpec, StockSpec, Settle,
Maturity, OptSpec, Strike)
PriceSens = assetsensbybls(RateSpec, StockSpec, Settle,
Maturity, OptSpec, Strike, OutSpec)
```

## Arguments

| | |
|---|---|
| `RateSpec` | The annualized, continuously compounded rate term structure. For information on the interest rate specification, see `intenvset`. |
| `StockSpec` | Stock specification. See `stockspec`. |
| `Settle` | `NINST`-by-`1` vector of settlement or trade dates. |
| `Maturity` | `NINST`-by-`1` vector of maturity dates. |
| `OptSpec` | `NINST`-by-`1` cell array of strings `'call'` or `'put'`. |
| `Strike` | `NINST`-by-`1` vector of strike price values. |
| `OutSpec` | (Optional) All optional inputs are specified as matching parameter name/value pairs. The parameter name is specified as a character string, followed by the corresponding parameter value. You can specify parameter name/value pairs in any order. Names are case-insensitive and partial string matches are allowed provided no ambiguities exist. Valid parameter names are:<br>• `NOUT`-by-`1` or `1`-by-`NOUT` cell array of strings indicating the nature and order of the outputs for the function. Possible values are `'Price'`, `'Delta'`, `'Gamma'`, `'Vega'`, `'Lambda'`, `'Rho'`, `'Theta'`, or `'All'`.<br>For example, `OutSpec = {'Price'; 'Lambda'; 'Rho'}` specifies that the output |

should be `Price`, `Lambda`, and `Rho`, in that order.

To invoke from a function: `[Price, Lambda, Rho] = assetsensbybls(..., 'OutSpec', {'Price', 'Lambda', 'Rho'})`

`OutSpec = {'All'}` specifies that the output should be `Delta`, `Gamma`, `Vega`, `Lambda`, `Rho`, `Theta`, and `Price`, in that order. This is the same as specifying `OutSpec` as `OutSpec = {'Delta', 'Gamma', 'Vega', 'Lambda', 'Rho', 'Theta', 'Price'};`.

- Default is `OutSpec` = `{'Price'}`.

## Description

`PriceSens = assetsensbybls(RateSpec, StockSpec, Settle, Maturity, OptSpec, Strike)` computes asset-or-nothing option prices using the Black-Scholes option pricing model.

`PriceSens = assetsensbybls(RateSpec, StockSpec, Settle, Maturity, OptSpec, Strike, OutSpec)` includes the parameter/value pairs defined for `OutSpec`, and computes asset-or-nothing option prices and sensitivities using the Black-Scholes option pricing model.

`PriceSens` is a `NINST`-by-`1` vector of expected option prices and sensitivities.

## Examples

Consider two asset-or-nothing put options on a nondividend paying stock with a strike of 95 and 93 and expiring on January 30, 2009. On November 3, 2008 the stock is trading at 97.50. Using this data, calculate the price and sensitivity of the asset-or-nothing put options if the risk-free rate is 4.5% and the volatility is 22%.

Create the `RateSpec`:

```
Settle = 'Nov-3-2008';
Maturity = 'Jan-30-2009';
```

```
Rates = 0.045;
Compounding = -1;
RateSpec = intenvset('ValuationDate', Settle, 'StartDates', Settle,...
'EndDates', Maturity, 'Rates', Rates, 'Compounding', Compounding);
```

Define the `StockSpec`:

```
AssetPrice = 97.50;
Sigma = .22;
StockSpec = stockspec(Sigma, AssetPrice);
```

Define the put options:

```
OptSpec = {'put'};
Strike = [95;93];
```

Calculate the delta, price, and gamma:

```
OutSpec = { 'delta';'price';'gamma'};
[Delta, Price, Gamma] = assetsensbybls(RateSpec, StockSpec, Settle,...
Maturity, OptSpec, Strike, 'OutSpec', OutSpec)

Delta =

   -3.0833
   -2.8337


Price =

   33.7666
   26.9662


Gamma =

    0.0941
    0.1439
```

**See Also** `assetbybls`

## Purpose

Price barrier option from Cox-Ross-Rubinstein binomial tree

## Syntax

```
[Price, PriceTree] = barrierbycrr(CRRTree, OptSpec, Strike,
Settle, ExerciseDates, AmericanOpt, BarrierSpec, Barrier,
Rebate, Options)
```

## Arguments

| | |
|---|---|
| `CRRTree` | Stock tree structure created by `crrtree`. |
| `OptSpec` | `NINST`-by-`1` list of string values `'Call'` or `'Put'`. |
| `Strike` | `NINST`-by-`1` vector of strike price values. Each row is the schedule for one option. |
| `Settle` | `NINST`-by-`1` vector of `Settle` dates. The settle date for every barrier option is set to the valuation date of the stock tree. The barrier argument `Settle` is ignored. |
| `ExerciseDates` | For a European option (`AmericanOpt = 0`):<br>`NINST`-by-`1` vector of exercise dates. Each row is the schedule for one option. For a European option, there is only one exercise date, the option expiry date.<br>For an American option (`AmericanOpt = 1`):<br>`NINST`-by-`2` vector of exercise date boundaries. For each instrument, the option can be exercised on any tree date between or including the pair of dates on that row. If only one non-`NaN` date is listed, or if `ExerciseDates` is `NINST`-by-`1`, the option can be exercised between the valuation date of the stock tree and the single listed exercise date. |

| | |
|---|---|
| `AmericanOpt` | If `AmericanOpt = 0`, `NaN`, or is unspecified, the option is a European option. If `AmericanOpt = 1`, the option is an American option. |
| `BarrierSpec` | List of string values:<br>`'UI'`: Up Knock In<br>`'UO'`: Up Knock Out<br>`'DI'`: Down Knock In<br>`'DO'`: Down Knock Out |
| `Barrier` | Vector of barrier values. |
| `Rebate` | (Optional) `NINST`-by-`1` matrix of rebate values. Default = 0. For Knock-in options, the rebate is paid at expiry. For Knock-out options, the rebate is paid when the barrier is reached. |
| `Options` | (Optional) Derivatives pricing options structure created with `derivset`. |

See `instbarrier` for a description of barrier contract arguments.

## Description

`[Price, PriceTree] = barrierbycrr(CRRTree, OptSpec, Strike, Settle, ExerciseDates, AmericanOpt, BarrierSpec, Barrier, Rebate, Options)` computes the price of barrier options using a CRR binomial tree.

`Price` is a `NINST`-by-`1` vector of expected prices at time 0.

`PriceTree` is a tree structure with a vector of instrument prices at each node.

## Examples

Price a barrier option using a CRR binomial tree.

Load the file `deriv.mat,` which provides `CRRTree`. The `CRRTree` structure contains the stock specification and time information needed to price the option.

```
load deriv.mat;
```

Set the required values. Other arguments will use defaults.

```
OptSpec = 'Call';
Strike = 105;
Settle = '01-Jan-2003';
ExerciseDates = '01-Jan-2006';
AmericanOpt = 1;
BarrierSpec = 'UI';
Barrier = 102;

Price = barrierbycrr(CRRTree, OptSpec, Strike, Settle, ...
ExerciseDates, AmericanOpt, BarrierSpec, Barrier)

Price =

 12.1272
```

## References

Derman, E., I. Kani, D. Ergener and I. Bardhan, “Enhanced Numerical Methods for Options with Barriers,” *Financial Analysts Journal*, (Nov. - Dec. 1995), pp. 65-74.

## See Also

`crrtree` | `instbarrier`

# barrierbyeqp

**Purpose** Price barrier option from Equal Probabilities binomial tree

**Syntax**

```
[Price, PriceTree] = barrierbyeqp(EQPTree, OptSpec, Strike,
ExerciseDates, AmericanOpt, BarrierSpec, Barrier, Rebate,
Options)
```

## Arguments

| | |
|---|---|
| `EQPTree` | Stock tree structure created by `eqptree`. |
| `OptSpec` | `NINST`-by-`1` list of string values `'Call'` or `'Put'`. |
| `Strike` | `NINST`-by-`1` vector of strike price values. Each row is the schedule for one option. |
| `Settle` | `NINST`-by-`1` vector of `Settle` dates. The settle date for every barrier option is set to the valuation date of the stock tree. The barrier argument `Settle` is ignored. |
| `ExerciseDates` | For a European option (`AmericanOpt = 0`):<br>`NINST`-by-`1` vector of exercise dates. Each row is the schedule for one option. For a European option, there is only one exercise date, the option expiry date.<br>For an American option (`AmericanOpt = 1`):<br>`NINST`-by-`2` vector of exercise date boundaries. For each instrument, the option can be exercised on any tree date between or including the pair of dates on that row. If only one non-`NaN` date is listed, or if `ExerciseDates` is `NINST`-by-`1`, the option can be exercised between the valuation date of the stock tree and the single listed exercise date. |

| | |
|---|---|
| `AmericanOpt` | If `AmericanOpt = 0`, `NaN`, or is unspecified, the option is a European option. If `AmericanOpt = 1`, the option is an American option. |
| `BarrierSpec` | List of string values:<br>`'UI'`: Up Knock In<br>`'UO'`: Up Knock Out<br>`'DI'`: Down Knock In<br>`'DO'`: Down Knock Out |
| `Barrier` | Vector of barrier values. |
| `Rebate` | (Optional) `NINST`-by-`1` matrix of rebate values. Default = 0. For Knock-in options, the rebate is paid at expiry. For Knock-out options, the rebate is paid when the barrier is reached. |
| `Options` | (Optional) Derivatives pricing options structure created with `derivset`. |

See `instbarrier` for a description of barrier contract arguments.

## Description

`[Price, PriceTree] = barrierbyeqp(EQPTree, OptSpec, Strike, ExerciseDates, AmericanOpt, BarrierSpec, Barrier, Rebate, Options)` computes the price of barrier options using an equal probabilities binomial tree.

`Price` is a `NINST`-by-`1` vector of expected prices at time 0.

`PriceTree` is a tree structure with a vector of instrument prices at each node.

## Examples

Price a barrier option using an EQP equity tree.

Load the file `deriv.mat,` which provides `EQPTree`. The `EQPTree` structure contains the stock specification and time information needed to price the option.

```
load deriv.mat;
```

Set the required values. Other arguments will use defaults.

```
OptSpec = 'Call';
Strike = 105;
Settle = '01-Jan-2003';
ExerciseDates = '01-Jan-2006';
AmericanOpt = 1;
BarrierSpec = 'UI';
Barrier = 102;

Price = barrierbyeqp(EQPTree, OptSpec, Strike, Settle, ...
ExerciseDates, AmericanOpt, BarrierSpec, Barrier)

Price =

 12.2632
```

## References

Derman, E., I. Kani, D. Ergener and I. Bardhan, "Enhanced Numerical Methods for Options with Barriers," *Financial Analysts Journal*, (Nov. - Dec. 1995), pp. 65-74.

## See Also

`eqptree` | `instbarrier`

## Purpose

Price barrier options using implied trinomial tree (ITT)

## Syntax

```
[Price, PriceTree] = barrierbyitt(ITTTree, OptSpec, Strike,
ExerciseDates, AmericanOpt, BarrierSpec, Barrier, Rebate,
Options)
```

## Arguments

| | |
|---|---|
| `ITTTree` | Stock tree structure created by `itttree`. |
| `OptSpec` | `NINST`-by-`1` list of string values `'Call'` or `'Put'`. |
| `Strike` | European and American option, `NINST`-by-`1` vector of strike price values. Each row is the schedule for one option. |
| `Settle` | `NINST`-by-`1` vector of `Settle` dates. The settle date for every barrier option is set to the valuation date of the stock tree. The barrier argument `Settle` is ignored. |
| `ExerciseDates` | For a European option (`AmericanOpt = 0`):<br>`NINST`-by-`1` vector of exercise dates. Each row is the schedule for one option. For a European option, there is only one exercise date, the option expiry date.<br>For an American option (`AmericanOpt = 1`):<br>`NINST`-by-`2` vector of exercise date boundaries. For each instrument, the option can be exercised on any tree date between or including the pair of dates on that row. If only one non-`NaN` date is listed, or if `ExerciseDates` is `NINST`-by-`1`, the option can be exercised between the valuation date of the stock tree and the single listed exercise date. |

| | |
|---|---|
| `AmericanOpt` | If `AmericanOpt = 0`, `NaN`, or is unspecified, the option is a European option. If `AmericanOpt = 1`, the option is an American option. |
| `BarrierSpec` | List of string values:<br>`'UI'`: Up Knock In<br>`'UO'`: Up Knock Out<br>`'DI'`: Down Knock In<br>`'DO'`: Down Knock Out |
| `Barrier` | Vector of barrier values. |
| `Rebate` | (Optional) `NINST`-by-`1` matrix of rebate values. Default = 0. For Knock In options, the rebate is paid at expiry. For Knock Out options, the rebate is paid when the barrier is reached. |
| `Options` | (Optional) Derivatives pricing options structure created with `derivset`. |

See `instbarrier` for a description of barrier contract arguments.

## Description

`[Price, PriceTree] = barrierbyitt(ITTTree, OptSpec, Strike, ExerciseDates, AmericanOpt, BarrierSpec, Barrier, Rebate, Options)` computes the price of barrier options using an implied trinomial tree.

`Price` is a `NINST`-by-`1` vector of expected prices at time 0.

`PriceTree` is a tree structure with a vector of instrument prices at each node.

---

**Note** The `Settle` date for every barrier option is set to the `ValuationDate` of the stock tree. The barrier argument, `Settle`, is ignored.

---

## Examples

Price a barrier option using an ITT tree.

Load the file `deriv.mat` which provides the `ITTTree`. The `ITTTree` structure contains the stock specification and time information needed to price the option.

```
load deriv.mat;
```

Set the required values. Other arguments will use defaults.

```
OptSpec = 'Call';
Strike =  85;
Settle = '01-Jan-2006';
ExerciseDates = '31-Dec-2008';
AmericanOpt = 1;
BarrierSpec = 'UI';
Barrier =  115;

Price = barrierbyitt(ITTTree,OptSpec,Strike,Settle,ExerciseDates,AmericanOpt,...
BarrierSpec,Barrier)

Price =

2.407
```

## References

Derman, E., I. Kani, D. Ergener, and I. Bardhan, "Enhanced Numerical Methods for Options with Barriers," *Financial Analysts Journal*, Nov.-Dec.,1995.

## See Also

`instbarrier` | `itttree`

# basketbyju

## Purpose

Price European basket options using Nengjiu Ju approximation model

## Syntax

```
Price = basketbyju(RateSpec, BasketStockSpec, OptSpec, Strike,
Settle, Maturity)
```

## Description

`Price = basketbyju(RateSpec, BasketStockSpec, OptSpec, Strike, Settle, Maturity)` prices European basket options using the Nengjiu Ju approximation model.

## Input Arguments

### RateSpec

Annualized, continuously compounded rate term structure. For more information on the interest rate specification, see `intenvset`.

### BasketStockSpec

`BasketStock` specification. For information on the basket of stocks specification, see `basketstockspec`.

### OptSpec

String or `2-by-1` cell array of the strings `'call'` or `'put'`.

### Strike

Scalar for the option strike price.

### Settle

Scalar of the settlement or trade date specified as a string or serial date number.

### Maturity

Maturity date specified as a string or serial date number.

## Output Arguments

### Price

Price of the basket option.

## Examples

Find a European call basket option of two stocks. Assume that the stocks are currently trading at $10 and $11.50 with annual volatilities of 20% and 25%, respectively. The basket contains one unit of the first stock and one unit of the second stock. The correlation between the assets is 30%. On January 1, 2009, an investor wants to buy a 1-year call option with a strike price of $21.50. The current annualized, continuously compounded interest rate is 5%. Use this data to compute the price of the call basket option with the Ju approximation model.

```
Settle = 'Jan-1-2009';
Maturity  = 'Jan-1-2010';

% Define RateSpec
Rate = 0.05;
Compounding = -1;
RateSpec = intenvset('ValuationDate', Settle, 'StartDates', ...
Settle, 'EndDates', Maturity, 'Rates', Rate, 'Compounding', Compounding);

% Define the Correlation matrix. Correlation matrices are symmetric, and
% have ones along the main diagonal.
Corr = [1 0.30; 0.30 1];

% Define BasketStockSpec
AssetPrice =  [10;11.50];
Volatility = [0.2;0.25];
Quantity = [1;1];
BasketStockSpec = basketstockspec(Volatility, AssetPrice, Quantity, Corr);

%Compute the price of the call basket option
OptSpec = {'call'};
Strike = 21.5;
PriceCorr30 = basketbyju(RateSpec, BasketStockSpec, OptSpec, Strike, Settle, Maturity)
```

This returns:

```
PriceCorr30 =

    2.12214
```

Compute the price of the basket instrument for these two stocks with a correlation of 60%. Then compare this cost to the total cost of buying two individual call options:

```
Corr = [1 0.60; 0.60 1];

% Define the new BasketStockSpec
BasketStockSpec = basketstockspec(Volatility, AssetPrice, Quantity, Corr);

%Compute the price of the call basket option with Correlation = -0.60
PriceCorr60 = basketbyju(RateSpec, BasketStockSpec, OptSpec, Strike, Settle, Maturity)
```

This returns:

```
PriceCorr60 =

   2.27566
```

The following table summarizes the sensitivity of the option to correlation changes. In general, the premium of the basket option decreases with lower correlation and increases with higher correlation.

| Correlation | -0.60 | -0.30 | 0 | 0.30 | 0.60 |
|---|---|---|---|---|---|
| Premium | 1.52830 | 1.76006 | 1.9527 | 2.1221 | 2.2756 |

Compute the cost of two vanilla 1-year call options using the Black-Scholes (BLS) model on the individual assets:

```
StockSpec = stockspec(Volatility, AssetPrice);
StrikeVanilla= [10;11.5];

PriceVanillaOption = optstockbybls(RateSpec, StockSpec, Settle, Maturity,...
OptSpec, StrikeVanilla)
```

This returns:

```
PriceVanillaOption =

    1.0451
    1.4186
```

Find the total cost of buying two individual call options:

```
sum(PriceVanillaOption)
```

This returns:

```
ans=2.4637
```

The total cost of purchasing two individual call options is $2.4637, compared to the maximum cost of the basket option of $2.27 with a correlation of 60%.

## References

Nengjiu Ju, "Pricing Asian and Basket Options Via Taylor Expansion", *Journal of Computational Finance*, Vol. 5, 2002.

## See Also

`basketstockspec` | `basketsensbyju`

## How To

- "Basket Option" on page 3-24

# basketbyls

## Purpose

Price basket options using Longstaff-Schwartz model

## Syntax

```
Price = basketbyls(RateSpec, BasketStockSpec, OptSpec, Strike,
Settle, ExerciseDates)
Price = basketbyls(RateSpec, BasketStockSpec, OptSpec, Strike,
Settle, ExerciseDates, 'ParameterName', ParameterValue ...)
```

## Description

`Price = basketbyls(RateSpec, BasketStockSpec, OptSpec, Strike, Settle, ExerciseDates)` prices basket options using the Longstaff-Schwartz model.

`Price = basketbyls(RateSpec, BasketStockSpec, OptSpec, Strike, Settle, ExerciseDates, 'ParameterName', ParameterValue ...)` accepts optional inputs as one or more comma-separated parameter/value pairs. `'ParameterName'` is the name of the parameter inside single quotes. `'ParameterValue` is the value corresponding to `'ParameterName'`. Specify parameter-value pairs in any order. Names are case-insensitive and partial string matches are allowable, if no ambiguities exist.

## Input Arguments

### RateSpec

Annualized, continuously compounded rate term structure. For more information on the interest rate specification, see `intenvset`.

### BasketStockSpec

`BasketStock` specification. For information on the basket of stocks specification, see `basketstockspec`.

### OptSpec

String or `2-by-1` cell array of the strings `'call'` or `'put'`.

### Strike

The option strike price:

- For a European or Bermuda option, `Strike` is a scalar (European) or `1`-by-`NSTRIKES` (Bermuda) vector of strike price.
- For an American option, `Strike` is a scalar vector of the strike price.

### Settle

Scalar of the settlement or trade date specified as a string or serial date number.

### ExerciseDates

The exercise date for the option:

- For a European or Bermuda option, `ExerciseDates` is a `1`-by-`1` (European) or `1`-by-`NSTRIKES` (Bermuda) vector of exercise dates. For a European option, there is only one `ExerciseDate` on the option expiry date.
- For an American option, `ExerciseDates` is a `1`-by-`2` vector of exercise date boundaries. The option exercises on any date between, or including, the pair of dates on that row. If there is only one non-`NaN` date, or if `ExerciseDates` is `1`-by-`1`, the option exercises between the `Settle` date and the single listed `ExerciseDate`.

## Parameter–Value Pairs

### AmericanOpt

Parameter values are a scalar flag.

- 0 — European/Bermuda
- 1 — American

**Default:** 0

### NumPeriods

Parameter value is a scalar number of simulation periods per trial. `NumPeriods` is considered only when pricing European basket options.

For American and Bermuda basket options, `NumPeriod` equals the number of exercise days during the life of the option.

**Default:** 100

### NumTrials

Parameter value is a scalar number of independent sample paths (simulation trials).

**Default:** 1000

## Output Arguments

### Price

Price of the basket option.

## Examples

Find an American call basket option of three stocks. The stocks are currently trading at $35, $40 and $45 with annual volatilities of 12%, 15% and 18%, respectively. The basket contains 33.33% of each stock. Assume the correlation between all pair of assets is 50%. On May 1, 2009, an investor wants to buy a three-year call option with a strike price of $42. The current annualized continuously compounded interest rate is 5%. Use this data to compute the price of the call basket option using the Longstaff-Schwartz model.

```
Settle = 'May-1-2009';
Maturity  = 'May-1-2012';

% Define RateSpec
Rate = 0.05;
Compounding = -1;
RateSpec = intenvset('ValuationDate', Settle, 'StartDates',...
Settle, 'EndDates', Maturity, 'Rates', Rate, 'Compounding', Compounding);

% Define the Correlation matrix. Correlation matrices are symmetric,
% and have ones along the main diagonal.
Corr = [1 0.50 0.50; 0.50 1 0.50;0.50 0.50 1];
```

```
% Define BasketStockSpec
AssetPrice =  [35;40;45];
Volatility = [0.12;0.15;0.18];
Quantity = [0.333;0.333;0.333];
BasketStockSpec = basketstockspec(Volatility, AssetPrice, Quantity, Corr);

% Compute the price of the call basket option
OptSpec = {'call'};
Strike = 42;
AmericanOpt = 1; % American option
Price = basketbyls(RateSpec, BasketStockSpec, OptSpec, Strike, Settle, Maturity,...
'AmericanOpt',AmericanOpt)
```

This returns:

```
Price =

    5.60499
```

Increase the number of simulation trials to 2000 to give the following results:

```
NumTrial = 2000;
Price = basketbyls(RateSpec, BasketStockSpec, OptSpec, Strike, Settle, Maturity,...
'AmericanOpt',AmericanOpt,'NumTrials',NumTrial)
Price =

   5.6665
```

## References

Longstaff, F.A., and E.S. Schwartz, “Valuing American Options by Simulation: A Simple Least-Squares Approach”, *The Review of Financial Studies*, Vol. 14, No. 1, Spring 2001, pp. 113–147.

## See Also

`basketstockspec` | `basketsensbyls`

## How To

- “Basket Option” on page 3-24

# basketsensbyju

## Purpose

Determine European basket options price and sensitivities using Nengjiu Ju approximation model

## Syntax

```
PriceSens = basketsensbyju(RateSpec, BasketStockSpec, OptSpec, Strike,
Settle, Maturity)
PriceSens = basketsensbyju(RateSpec, BasketStockSpec, OptSpec, Strike,
Settle, Maturity, 'ParameterName', ParameterValue ...)
```

## Description

`PriceSens = basketsensbyju(RateSpec, BasketStockSpec, OptSpec, Strike, Settle, Maturity)` calculates prices and sensitivities for basket options using the Nengjiu Ju approximation model.

`PriceSens = basketsensbyju(RateSpec, BasketStockSpec, OptSpec, Strike, Settle, Maturity, 'ParameterName', ParameterValue ...)` accepts optional inputs as one or more comma-separated parameter/value pairs. `'ParameterName'` is the name of the parameter inside single quotes. `'ParameterValue` is the value corresponding to `'ParameterName'`. Specify parameter-value pairs in any order. Names are case-insensitive and partial string matches are allowable, if no ambiguities exist.

## Input Arguments

### RateSpec

Annualized, continuously compounded rate term structure. For more information on the interest rate specification, see `intenvset`.

### BasketStockSpec

`BasketStock` specification. For information on the basket of stocks specification, see `basketstockspec`.

### OptSpec

String or `2`-by-`1` cell array of the strings `'call'` or `'put'`.

### Strike

Scalar of the option strike price.

**Settle**

Scalar of the settlement or trade date specified as a string or serial date number.

**Maturity**

Maturity date, specified as a string or serial date number.

## Parameter–Value Pairs

**OutSpec**

Parameter value is an `NOUT`-by-`1` or `1`-by-`NOUT` cell array of strings indicating the nature and order of the outputs for the function. Possible values are `'Price'`, `'Delta'`, `'Gamma'`, `'Vega'`, `'Lambda'`, `'Rho'`, `'Theta'`, and `'All'`. For example, `OutSpec = {'Price', 'Lambda', 'Rho'}` specifies that the output is `Price`, `Lambda`, and `Rho`, in that order.

`OutSpec = {'All'}` specifies that the output should be `Delta`, `Gamma`, `Vega`, `Lambda`, `Rho`, `Theta`, and `Price`, in that order. This is the same as specifying `OutSpec` as `OutSpec = {'Delta', 'Gamma', 'Vega', 'Lambda', 'Rho', 'Theta', 'Price'};`.

> **Default:** `OutSpec = {'Price'}`

**UndIdx**

Scalar of the indice of the underlying instrument to compute the sensitivity.

> **Default:** `UndIdx = []`

## Output Arguments

**PriceSens**

Expected prices or sensitivities values for the basket option.

## Examples

Find a European call basket option of five stocks. Assume that the basket contains:

- 5% of the first stock trading at $110
- 15% of the second stock trading at $75
- 20% of the third stock trading at $40
- 25% of the fourth stock trading at $125
- 35% of the fifth stock trading at $92

These stocks have annual volatilities of 20% and the correlation between the assets is zero. On May 1, 2009, an investor wants to buy a 1-year call option with a strike price of $90. The current annualized, continuously compounded interest is 5%. Use this data to compute price and delta of the call basket option with the Ju approximation model.

```
Settle = 'May-1-2009';
Maturity  = 'May-1-2010';

% Define RateSpec
Rate = 0.05;
Compounding = -1;
RateSpec = intenvset('ValuationDate', Settle, 'StartDates', ...
Settle, 'EndDates', Maturity, 'Rates', Rate, 'Compounding', Compounding);

% Define the Correlation matrix. Correlation matrices are symmetric, and
% have ones along the main diagonal.
NumInst  = 5;
InstIdx = ones(NumInst,1);
Corr = diag(ones(5,1), 0);

% Define BasketStockSpec
AssetPrice =  [110; 75; 40; 125; 92];
Volatility = 0.2;
Quantity = [0.05; 0.15; 0.2; 0.25; 0.35];
BasketStockSpec = basketstockspec(Volatility, AssetPrice, Quantity, Corr);
```

```
% Compute the price of the call basket option. Calculate also the delta
% of the first stock.
OptSpec = {'call'};
Strike = 90;
OutSpec = {'Price','Delta'};
UndIdx = 1; % First element in the basket
[Price, Delta] = basketsensbyju(RateSpec, BasketStockSpec, OptSpec, Strike, Settle, ...
Maturity, 'OutSpec', OutSpec,'UndIdx', UndIdx)
```

This returns:

```
Price =

   5.16098

Delta =

   0.02972
```

Compute `Delta` with respect to the second asset:

```
UndIdx = 2; % Second element in the basket
OutSpec = {'Delta'};
Delta = basketsensbyju(RateSpec, BasketStockSpec, OptSpec, Strike, Settle, Maturity, ...
'OutSpec',OutSpec,'UndIdx',UndIdx)

Delta =

   0.09063
```

## References

Nengjiu Ju, “Pricing Asian and Basket Options Via Taylor Expansion”, *Journal of Computational Finance*, Vol. 5, 2002.

## See Also

`basketstockspec` | `basketbyju`

## How To

- “Basket Option” on page 3-24

# basketsensbyls

## Purpose

Determine price and sensitivities for basket options using Longstaff-Schwartz model

## Syntax

```
PriceSens = basketsensbyls(RateSpec, BasketStockSpec, OptSpec, Strike,
Settle, ExerciseDates)
PriceSens = basketsensbyls(RateSpec, BasketStockSpec, OptSpec, Strike,
Settle, ExerciseDates, 'ParameterName', ParameterValue ...)
```

## Description

`PriceSens = basketsensbyls(RateSpec, BasketStockSpec, OptSpec, Strike, Settle, ExerciseDates)` prices basket options using the Longstaff-Schwartz model.

`PriceSens = basketsensbyls(RateSpec, BasketStockSpec, OptSpec, Strike, Settle, ExerciseDates, 'ParameterName', ParameterValue ...)` accepts optional inputs as one or more comma-separated parameter/value pairs. `'ParameterName'` is the name of the parameter inside single quotes. `'ParameterValue` is the value corresponding to `'ParameterName'`. Specify parameter-value pairs in any order. Names are case-insensitive and partial string matches are allowable, if no ambiguities exist.

## Input Arguments

### RateSpec

Annualized, continuously compounded rate term structure. For more information on the interest rate specification, see `intenvset`.

### BasketStockSpec

`BasketStock` specification. For information on the basket of stocks specification, see `basketstockspec`.

### OptSpec

String or 2-by-1 cell array of the strings `'call'` or `'put'`.

### Strike

The option strike price:

- For a European or Bermuda option, `Strike` is a scalar (European) or `1`-by-`NSTRIKES` (Bermuda) vector of strike price.
- For an American option, `Strike` is a scalar vector of strike price.

#### Settle

Scalar of settlement or trade date.

#### ExerciseDates

The exercise date for the option:

- For a European or Bermuda option, `ExerciseDates` is a `1`-by-`1` (European) or `1`-by-`NSTRIKES` (Bermuda) vector of exercise dates. For a European option, there is only one `ExerciseDate` on the option expiry date.
- For an American option, `ExerciseDates` is a `1`-by-`2` vector of exercise date boundaries. The option exercises on any date between or including the pair of dates on that row. If there is only one non-`NaN` date, or if `ExerciseDates` is `1`-by-`1`, the option exercises between the `Settle` date and the single listed `ExerciseDate`.

### Parameter–Value Pairs

#### AmericanOpt

Parameter values are a scalar flag.

- 0 — European/Bermuda
- 1 — American

**Default:** 0

#### NumPeriods

Parameter value is a scalar number of simulation periods. `NumPeriods` is considered only when pricing European basket options. For American and Bermuda basket options, `NumPeriod` equals the number of exercise days during the life of the option.

**Default:** 100

### NumTrials

Parameter value is a scalar number of independent sample paths (simulation trials).

**Default:** 1000

### OutSpec

Parameter value is an `NOUT`-by-`1` or `1`-by-`NOUT` cell array of strings indicating the nature and order of the outputs for the function. Possible values are `'Price'`, `'Delta'`, `'Gamma'`, `'Vega'`, `'Lambda'`, `'Rho'`, `'Theta'`, and `'All'`. For example, `OutSpec = {'Price', 'Lambda', 'Rho'}` specifies that the output is `Price`, `Lambda`, and `Rho`, in that order.

`OutSpec = {'All'}` specifies that the output should be `Delta`, `Gamma`, `Vega`, `Lambda`, `Rho`, `Theta`, and `Price`, in that order. This is the same as specifying `OutSpec` as `OutSpec = {'Delta', 'Gamma', 'Vega', 'Lambda', 'Rho', 'Theta', 'Price'};`.

**Default:** `OutSpec = {'Price'}`

### UndIdx

Scalar of the indice of the underlying instrument to compute the sensitivity.

**Default:** `UndIdx = []`

## Output Arguments

**PriceSens**

Expected prices or sensitivities values.

## Examples

Find a European put basket option of two stocks. The basket contains 50% of each stock. The stocks are currently trading at $90 and $75, with annual volatilities of 15%. Assume that the correlation between the assets is zero. On May 1, 2009, an investor wants to buy a one-year put option with a strike price of $80. The current annualized, continuously compounded interest is 5%. Use this data to compute price and delta of the put basket option with the Longstaff-Schwartz approximation model.

```
Settle = 'May-1-2009';
Maturity  = 'May-1-2010';

% Define RateSpec
Rate = 0.05;
Compounding = -1;
RateSpec = intenvset('ValuationDate', Settle, 'StartDates',...
Settle, 'EndDates', Maturity, 'Rates', Rate, 'Compounding', Compounding);

% Define the Correlation matrix. Correlation matrices are symmetric,
% and have ones along the main diagonal.
NumInst  = 2;
InstIdx = ones(NumInst,1);
Corr = diag(ones(NumInst,1), 0);

% Define BasketStockSpec
AssetPrice =  [90; 75];
Volatility = 0.15;
Quantity = [0.50; 0.50];
BasketStockSpec = basketstockspec(Volatility, AssetPrice, Quantity, Corr);

% Compute the price of the put basket option. Calculate also the delta
% of the first stock.
OptSpec = {'put'};
```

```
Strike = 80;
OutSpec = {'Price','Delta'};
UndIdx = 1; % First element in the basket

[PriceSens, Delta] = basketsensbyls(RateSpec, BasketStockSpec, OptSpec,...
Strike, Settle, Maturity,'OutSpec', OutSpec,'UndIdx', UndIdx)
```

This returns:

```
PriceSens =

   1.08519

Delta =

  -0.10311
```

Compute the `Price` and `Delta` of the basket with a correlation of -20%:

```
NewCorr = [1 -0.20; -0.20 1];

% Define the new BasketStockSpec.
BasketStockSpec = basketstockspec(Volatility, AssetPrice, Quantity, NewCorr);

% Compute the price and delta of the put basket option.
[PriceSens, Delta] = basketsensbyls(RateSpec, BasketStockSpec, OptSpec,...
Strike, Settle, Maturity,'OutSpec', OutSpec,'UndIdx', UndIdx)

PriceSens =

   0.83903

Delta =

  -0.08847
```

## References

Longstaff, F.A., and E.S. Schwartz, “Valuing American Options by Simulation: A Simple Least-Squares Approach”, *The Review of Financial Studies*,Vol. 14, No. 1, Spring 2001, pp. 113–147.

## See Also

`basketstockspec` | `basketbyls`

## How To

- “Basket Option” on page 3-24

# basketstockspec

## Purpose

Specify basket stock structure using Longstaff-Schwartz model

## Syntax

```
BasketStockSpec = basketstockspec(Sigma, AssetPrice, Quantity,
Correlation)
BasketStockSpec = basketstockspec(Sigma, AssetPrice, Quantity,
Correlation, 'ParameterName',ParameterValue ...)
```

## Description

`BasketStockSpec = basketstockspec(Sigma, AssetPrice, Quantity, Correlation)` creates a basket stock structure.

`BasketStockSpec = basketstockspec(Sigma, AssetPrice, Quantity, Correlation, 'ParameterName',ParameterValue ...)` accepts optional inputs as one or more comma-separated parameter/value pairs. `'ParameterName'` is the name of the parameter inside single quotes. `'ParameterValue` is the value corresponding to `'ParameterName'`. Specify parameter-value pairs in any order. Names are case-insensitive and partial string matches are allowable, if no ambiguities exist.

## Input Arguments

### Sigma

`NINST-by-1` vector of decimal annual price volatility of the underlying security.

### AssetPrice

`NINST-by-1` vector of underlying asset price values at time 0.

### Quantity

`NINST-by-1` vector of quantities of the instruments contained in the basket.

### Correlation

`NINST-by-NINST` matrix of correlation values.

### Parameter–Value Pairs

**DividendAmounts**

`NINST`-by-`1` cell array specifying the dividend amounts for basket instruments. Each element of the cell array is a `1`-by-`NDIV` row vector of cash dividends or a scalar representing a continuous annualized dividend yield for the corresponding instrument.

**DividendType**

`NINST`-by-`1` cell array of strings specifying each stock's dividend type. Dividend type must be either `cash` for actual dollar dividends or `continuous` for continuous dividend yield. .

**ExDividendDates**

`NINST`-by-`1` cell array specifying the ex-dividend dates for the basket instruments. Each row is a `1`-by-`NDIV` matrix of ex-dividend dates for `cash` type. For rows that correspond to basket instruments with `continuous` dividend type, the cell is empty. If none of the basket instruments pay `continuous` dividends, do not specify `ExDividendDates`.

## Output Arguments

**BasketStockSpec**

Structure encapsulating the properties of a basket stock structure.

## Examples

Find a basket option of three stocks. The stocks are currently trading at $56, $92 and $125 with annual volatilities of 20%, 12% and 15%, respectively. The basket option contains 25% of the first stock, 40% of the second stock, and 35% of the third. The first stock provides a continuous dividend of 1%, while the other two provide no dividends. The correlation between the first and second asset is 30%, between the second and third asset 11%, and between the first and third asset 16%. Use this data to create the `BasketStockSpec` structure:

```
AssetPrice = [56;92;125];
Sigma = [0.20;0.12;0.15];
```

```
% Create the Correlation matrix. Correlation matrices are symmetric and
% have ones along the main diagonal.
NumInst  = 3;
Corr = zeros(NumInst,1);
Corr(1,2) = .30;
Corr(2,3) = .11;
Corr(1,3) = .16;
Corr = triu(Corr,1) + tril(Corr',-1) + diag(ones(NumInst,1), 0);

% Define dividends
DivType = cell(NumInst,1);
DivType{1}='continuous';
DivAmounts = cell(NumInst,1);
DivAmounts{1} = 0.01;

Quantity = [0.25; 0.40; 0.35];

BasketStockSpec = basketstockspec(Sigma, AssetPrice, Quantity, Corr, ...
'DividendType', DivType, 'DividendAmounts', DivAmounts)
```

This returns:

```
BasketStockSpec =

             FinObj: 'BasketStockSpec'
              Sigma: [3x1 double]
         AssetPrice: [3x1 double]
           Quantity: [3x1 double]
        Correlation: [3x3 double]
       DividendType: {3x1 cell}
    DividendAmounts: {3x1 cell}
    ExDividendDates: {3x1 cell}
```

Examine the `BasketStockSpec` structure:

```
>>BasketStockSpec.Correlation
```

```
ans =

    1.0000    0.3000    0.1600
    0.3000    1.0000    0.1100
    0.1600    0.1100    1.0000
```

---

Find a basket option of two stocks. The stocks are currently trading at \$60 and \$55 with volatilities of 30% per annum. The basket option contains 50% of each stock. The first stock provides a cash dividend of \$0.25 on May 1, 2009 and September 1, 2009. The second stock provides a continuous dividend of 3%. The correlation between the assets is 40%. Use this data to create the structure `BasketStockSpec`:

```
AssetPrice = [60;55];
Sigma = [0.30;0.30];

% Create the Correlation matrix. Correlation matrices are symmetric and
% have ones along the main diagonal.
Correlation = [1 0.40;0.40 1];

% Define dividends
NumInst  = 2;
DivType = cell(NumInst,1);
DivType{1}='cash';
DivType{2}='continuous';

DivAmounts = cell(NumInst,1);
DivAmounts{1} = [0.25 0.25];
DivAmounts{2} = 0.03;

ExDates = cell(NumInst,1);
ExDates{1} = {'May-1-2009' 'Sept-1-2009'};

Quantity = [0.5; 0.50];

BasketStockSpec = basketstockspec(Sigma, AssetPrice, Quantity, Correlation, ...
```

```
'DividendType', DivType, 'DividendAmounts', DivAmounts, 'ExDividendDates',ExDates)
```

This returns:

```
BasketStockSpec =

             FinObj: 'BasketStockSpec'
              Sigma: [2x1 double]
         AssetPrice: [2x1 double]
           Quantity: [2x1 double]
        Correlation: [2x2 double]
       DividendType: {2x1 cell}
    DividendAmounts: {2x1 cell}
    ExDividendDates: {2x1 cell}
```

Examine the `BasketStockSpec` structure:

```
>>BasketStockSpec.DividendType

ans =

    'cash'
    'continuous'
```

## See Also

`basketbyls` | `basketbyju` | `basketsensbyju` | `basketsensbyls` | `stockspec` | `intenvset`

## How To

- “Basket Option” on page 3-24

## Purpose

Instrument prices from Black-Derman-Toy interest-rate tree

## Syntax

```
[Price, PriceTree] = bdtprice(BDTTree, InstSet, Options)
```

## Arguments

| | |
|---|---|
| `BDTTree` | Interest-rate tree structure created by `bdttree`. |
| `InstSet` | Variable containing a collection of `NINST` instruments. Instruments are categorized by type. Each type can have different data fields. The stored data field is a row vector or string for each instrument. |
| `Options` | (Optional) Derivatives pricing options structure created with `derivset`. |

## Description

`[Price, PriceTree] = bdtprice(BDTTree, InstSet, Options)` computes arbitrage-free prices for instruments using an interest-rate tree created with `bdttree`. All instruments contained in a financial instrument variable, `InstSet`, are priced.

`Price` is a number of instruments `(NINST)`-by-`1` vector of prices for each instrument. The prices are computed by backward dynamic programming on the interest-rate tree. If an instrument cannot be priced, `NaN` is returned.

`PriceTree` is a MATLAB structure of trees containing vectors of instrument prices and accrued interest, and a vector of observation times for each node.

`PriceTree.PTree` contains the clean prices.

`PriceTree.AITree` contains the accrued interest.

`PriceTree.tObs` contains the observation times.

`bdtprice` handles instrument types: `'Bond'`, `'CashFlow'`, `'OptBond'`, `'OptEmBond'`, `'Fixed'`, `'Float'`, `'Cap'`, `'Floor'`, `'RangeFloat'`, `'Swap'`. See `instadd` to construct defined types.

Related single-type pricing functions are:

- `bondbybdt`: Price a bond from a BDT tree.
- `capbybdt`: Price a cap from a BDT tree.
- `cfbybdt`: Price an arbitrary set of cash flows from a BDT tree.
- `fixedbybdt`: Price a fixed-rate note from a BDT tree.
- `floatbybdt`: Price a floating-rate note from a BDT tree.
- `floorbybdt`: Price a floor from a BDT tree.
- `optbndbybdt`: Price a bond option from a BDT tree.
- `optembndbybdt`: Price a bond with embedded option by a BDT tree.
- `rangefloatbybdt`: Price range floating note using a BDT tree.
- `swapbybdt`: Price a swap from a BDT tree.
- `swaptionbybdt`: Price a swaption from a BDT tree.

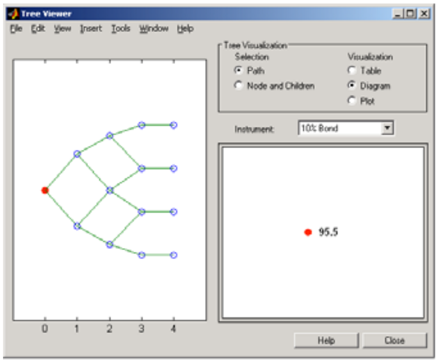
## Examples

Load the BDT tree and instruments from the data file `deriv.mat`. Price the cap and bond instruments contained in the instrument set.

```
load deriv.mat;
BDTSubSet = instselect(BDTInstSet,'Type', {'Bond', 'Cap'});

instdisp(BDTSubSet)

%Table of instrument portfolio partially displayed:
Index Type   CouponRate Settle      Maturity    Period ...   Name ...
1     Bond   0.1        01-Jan-2000 01-Jan-2003 1      ...  10% bond
2     Bond   0.1        01-Jan-2000 01-Jan-2004 2      ...  10% bond

Index Type Strike Settle      Maturity     CapReset ...  Name ...
3     Cap  0.15   01-Jan-2000 01-Jan-2004  1        ...  15% Cap

[Price, PriceTree] = bdtprice(BDTTree, BDTSubSet);
```

```
Warning: Not all cash flows are aligned with the tree. Result will
be approximated.

Price =

   95.5030
   93.9079
    1.4375
```

You can use `treeviewer` to see the prices of these three instruments along the price tree.



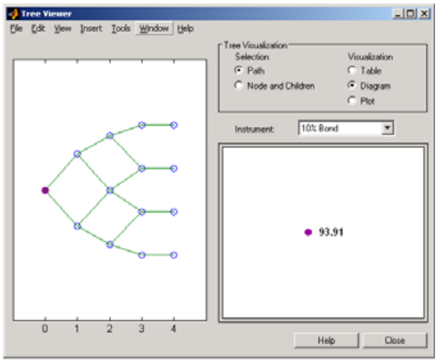
Selection: ● Path ○ Node and Children

Visualization: ○ Table ● Diagram ○ Plot

Instrument: 10% Bond

95.5

First 10% Bond (Maturity 2003)



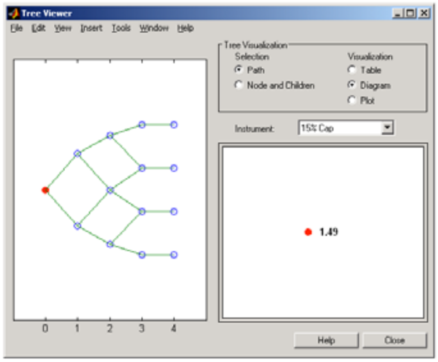
Selection: ● Path ○ Node and Children

Visualization: ○ Table ● Diagram ○ Plot

Instrument: 10% Bond

93.91

Second 10% Bond (Maturity 2004)



Selection: ● Path ○ Node and Children

Visualization: ○ Table ● Diagram ○ Plot

Instrument: 15% Cap

1.49

15% Cap

Price the following multi-stepped coupon bonds using the following data:

```
% The data for the interest rate term structure is as follows:
Rates = [0.035; 0.042147; 0.047345; 0.052707];
ValuationDate = 'Jan-1-2010';
StartDates = ValuationDate;
EndDates = {'Jan-1-2011'; 'Jan-1-2012'; 'Jan-1-2013'; 'Jan-1-2014'};
Compounding = 1;

% Create RateSpec
RS = intenvset('ValuationDate', ValuationDate, 'StartDates', StartDates,...
'EndDates', EndDates,'Rates', Rates, 'Compounding', Compounding);

% Create a portfolio of stepped coupon bonds with different maturities
Settle = '01-Jan-2010';
Maturity = {'01-Jan-2011';'01-Jan-2012';'01-Jan-2013';'01-Jan-2014'};
CouponRate = {{'01-Jan-2011' .042;'01-Jan-2012' .05; '01-Jan-2013' .06; '01-Jan-2014' .07}};

% Display the instrument portfolio
ISet = instbond(CouponRate, Settle, Maturity, 1);
instdisp(ISet)

%Table of instrument portfolio partially displayed:
Index Type CouponRate Settle  Maturity Period Basis EndMonthRule IssueDate FirstCouponDate ... Face
1 Bond [Cell]   01-Jan-2010  01-Jan-2011  1   0     1           NaN       NaN              ... 100
2 Bond [Cell]   01-Jan-2010  01-Jan-2012  1   0     1           NaN       NaN              ... 100
3 Bond [Cell]   01-Jan-2010  01-Jan-2013  1   0     1           NaN       NaN              ... 100
4 Bond [Cell]   01-Jan-2010  01-Jan-2014  1   0     1           NaN       NaN              ... 100


% Build the tree
% Assume the volatility to be 10%
Sigma = 0.1;
BDTTimeSpec = bdttimespec(ValuationDate, EndDates, Compounding);
BDTVolSpec = bdtvolspec(ValuationDate, EndDates, Sigma*ones(1, length(EndDates))');
BDTT = bdttree(BDTVolSpec, RS, BDTTimeSpec);

% Compute the price of the  stepped coupon bonds
```

```
PBDT = bdtprice(BDTT, ISet)

%Table of instrument portfolio partially displayed:
Index Type CouponRate Settle Maturity Period Basis EndMnthRule IssueDate FirstCouponDate ... Face
1  Bond [Cell]   01-Jan-2010  01-Jan-2011  1    0     1        NaN       NaN              ... 100
2  Bond [Cell]   01-Jan-2010  01-Jan-2012  1    0     1        NaN       NaN              ... 100
3  Bond [Cell]   01-Jan-2010  01-Jan-2013  1    0     1        NaN       NaN              ... 100
4  Bond [Cell]   01-Jan-2010  01-Jan-2014  1    0     1        NaN       NaN              ... 100

PBDT =

  100.6763
  100.7368
  100.9266
  101.0115
```

---

Price a portfolio of stepped callable bonds and stepped vanilla bonds using the following data:

```
% The data for the interest rate term structure is as follows:
Rates = [0.035; 0.042147; 0.047345; 0.052707];
ValuationDate = 'Jan-1-2010';
StartDates = ValuationDate;
EndDates = {'Jan-1-2011'; 'Jan-1-2012'; 'Jan-1-2013'; 'Jan-1-2014'};
Compounding = 1;

%Create RateSpec
RS = intenvset('ValuationDate', ValuationDate, 'StartDates', StartDates,...
'EndDates', EndDates,'Rates', Rates, 'Compounding', Compounding);

% Create an instrument portfolio of 3 stepped callable bonds and three
% stepped vanilla bonds
Settle = '01-Jan-2010';
Maturity = {'01-Jan-2012';'01-Jan-2013';'01-Jan-2014'};
CouponRate = {{'01-Jan-2011' .042;'01-Jan-2012' .05; '01-Jan-2013' .06; '01-Jan-2014' .07}};
```

```
OptSpec='call';
Strike=100;
ExerciseDates='01-Jan-2011'; %Callable in one year

% Bonds with embedded option
ISet = instoptembnd(CouponRate, Settle, Maturity, OptSpec, Strike,...
ExerciseDates, 'Period', 1);

% Vanilla bonds
ISet = instbond(ISet, CouponRate, Settle, Maturity, 1);

% Display the instrument portfolio
instdisp(ISet)

%Table of instrument portfolio partially displayed:
Index Type      CouponRate Settle         Maturity     OptSpec Strike ExerciseDates  ...  AmericanOpt
1   OptEmBond [Cell]     01-Jan-2010    01-Jan-2012    call    100    01-Jan-2011    ...     0
2   OptEmBond [Cell]     01-Jan-2010    01-Jan-2013    call    100    01-Jan-2011    ...     0
3   OptEmBond [Cell]     01-Jan-2010    01-Jan-2014    call    100    01-Jan-2011    ...     0

Index Type CouponRate Settle         Maturity      Period Basis EndMonthRule  ... Face
4     Bond [Cell]     01-Jan-2010    01-Jan-2012    1      0     1            ... 100
5     Bond [Cell]     01-Jan-2010    01-Jan-2013    1      0     1            ... 100
6     Bond [Cell]     01-Jan-2010    01-Jan-2014    1      0     1            ... 100

% Build the tree
% Assume the volatility to be 10%
Sigma = 0.1;
BDTTimeSpec = bdttimespec(ValuationDate, EndDates, Compounding);
BDTVolSpec = bdtvolspec(ValuationDate, EndDates, Sigma*ones(1, length(EndDates))');
BDTT = bdttree(BDTVolSpec, RS, BDTTimeSpec);

%The first three rows corresponds to the price of the stepped callable bonds and the
%last three rows corresponds to the price of the stepped vanilla bonds.
PBDT = bdtprice(BDTT, ISet)

PBDT =
```

```
100.4799
100.3228
100.0840
100.7368
100.9266
101.0115
```

---

Compute the price of a portfolio using the following data:

```
% The data for the interest rate term structure is as follows:
Rates = [0.035; 0.042147; 0.047345; 0.052707];
ValuationDate = 'Jan-1-2011';
StartDates = ValuationDate;
EndDates = {'Jan-1-2012'; 'Jan-1-2013'; 'Jan-1-2014'; 'Jan-1-2015'};
Compounding = 1;

%  Create RateSpec
RS = intenvset('ValuationDate', ValuationDate, 'StartDates', StartDates,...
'EndDates', EndDates,'Rates', Rates, 'Compounding', Compounding);

% Create an instrument portfolio with two range notes and a floating rate
% note with the following data:
Spread = 200;
Settle = 'Jan-1-2011';
Maturity = 'Jan-1-2014';

% First Range Note:
RateSched(1).Dates = {'Jan-1-2012'; 'Jan-1-2013'  ; 'Jan-1-2014'};
RateSched(1).Rates  = [0.045 0.055; 0.0525  0.0675; 0.06 0.08];

% Second Range Note:
RateSched(2).Dates = {'Jan-1-2012'; 'Jan-1-2013' ; 'Jan-1-2014'};
RateSched(2).Rates  = [0.048 0.059; 0.055  0.068 ; 0.07 0.09];

 Create InstSet
```

```
InstSet = instadd('RangeFloat', Spread, Settle, Maturity, RateSched);

% Add a floating-rate note
InstSet = instadd(InstSet, 'Float', Spread, Settle, Maturity);

% Display the portfolio instrument
instdisp(InstSet)

Index Type       Spread Settle       Maturity    RateSched FloatReset Basis Principal EndMonthRule
1     RangeFloat 200    01-Jan-2011  01-Jan-2014  [Struct]  1          0     100       1
2     RangeFloat 200    01-Jan-2011  01-Jan-2014  [Struct]  1          0     100       1

Index Type  Spread Settle         Maturity       FloatReset Basis Principal EndMonthRule
3     Float 200    01-Jan-2011    01-Jan-2014    1          0     100       1

% The data to build the tree is as follows:
% Assume the volatility to be 10%.
Sigma = 0.1;
BDTTS = bdttimespec(ValuationDate, EndDates, Compounding);
BDTVS = bdtvolspec(ValuationDate, EndDates, Sigma*ones(1, length(EndDates))');
BDTT = bdttree(BDTVS, RS, BDTTS);

% Price the portfolio
Price = bdtprice(BDTT, InstSet)

Price =

  100.2841
   98.0757
  105.5147
```

## See Also

`bdtsens` | `bdttree` | `instadd` | `intenvprice` | `intenvsens`

# bdtsens

## Purpose

Instrument prices and sensitivities from Black-Derman-Toy interest-rate tree

## Syntax

`[Delta, Gamma, Vega, Price] = bdtsens(BDTTree, InstSet, Options)`

## Arguments

| | |
|---|---|
| `BDTTree` | Interest-rate tree structure created by `bdttree`. |
| `InstSet` | Variable containing a collection of `NINST` instruments. Instruments are categorized by type. Each type can have different data fields. The stored data field is a row vector or string for each instrument. |
| `Options` | (Optional) Derivatives pricing options structure created with `derivset`. |

## Description

`[Delta, Gamma, Vega, Price] = bdtsens(BDTTree, InstSet, Options)` computes instrument sensitivities and prices for instruments using an interest-rate tree created with the `bdttree` function. `NINST` instruments from a financial instrument variable, `InstSet`, are priced. `bdtsens` handles instrument types: `'Bond'`, `'CashFlow'`, `'OptBond'`, `'OptEmBond'`, `'Fixed'`, `'Float'`, `'Cap'`, `'Floor'`, `'RangeFloat'`, `'Swap'`. See `instadd` for information on instrument types.

`Delta` is an `NINST`-by-`1` vector of deltas, representing the rate of change of instrument prices with respect to changes in the interest rate. `Delta` is computed by finite differences in calls to `bdttree`. See `bdttree` for information on the observed yield curve.

`Gamma` is an `NINST`-by-`1` vector of gammas, representing the rate of change of instrument deltas with respect to the changes in the interest rate. `Gamma` is computed by finite differences in calls to `bdttree`.

`Vega` is an `NINST`-by-`1` vector of vegas, representing the rate of change of instrument prices with respect to the changes in the volatility

$\sigma(t,T)$. `Vega` is computed by finite differences in calls to `bdttree`. See `bdtvolspec` for information on the volatility process.

---

**Note** All sensitivities are returned as dollar sensitivities. To find the per-dollar sensitivities, divide by the respective instrument price.

---

`Price` is an `NINST`-by-`1` vector of prices of each instrument. The prices are computed by backward dynamic programming on the interest-rate tree. If an instrument cannot be priced, `NaN` is returned.

`Delta` and `Gamma` are calculated based on yield shifts of 100 basis points. `Vega` is calculated based on a 1% shift in the volatility process.

## Examples

Load the tree and instruments from a data file. Compute `Delta` and `Gamma` for the cap and bond instruments contained in the instrument set.

```
load deriv.mat;
BDTSubSet = instselect(BDTInstSet,'Type', {'Bond', 'Cap'});

instdisp(BDTSubSet)

Index Type CouponRate Settle        Maturity     Period  Name  ...
1     Bond 0.1        01-Jan-2000   01-Jan-2003  1       10% Bond
2     Bond 0.1        01-Jan-2000   01-Jan-2004  2       10% Bond

Index Type Strike Settle      Maturity     CapReset...  Name ...
3     Cap  0.15   01-Jan-2000 01-Jan-2004  1            15% Cap

[Delta, Gamma] = bdtsens(BDTTree, BDTSubSet)

Warning: Not all cash flows are aligned with the tree. Result will
be approximated.

Delta =
```

```
 -232.6681
 -281.0517
   63.8102

Gamma =

  1.0e+03 *

    0.8037
    1.1819
    0.8535
```

**See Also** `bdtprice` | `bdttree` | `bdtvolspec` | `instadd`

## Purpose

Specify time structure for Black-Derman-Toy interest-rate tree

## Syntax

```
TimeSpec = bdttimespec(ValuationDate, Maturity, Compounding)
```

## Arguments

| | |
|---|---|
| `ValuationDate` | Scalar date marking the pricing date and first observation in the tree. Specify as serial date number or date string. |
| `Maturity` | Number of levels (depth) of the tree. A number of levels (`NLEVELS`)-by-`1` vector of dates marking the cash flow dates of the tree. Cash flows with these maturities fall on tree nodes. Maturity should be in increasing order. |
| `Compounding` | (Optional) Scalar value representing the rate at which the input zero rates were compounded when annualized. Default = `1`. This argument determines the formula for the discount factors:<br>`Compounding` = `1`, `2`, `3`, `4`, `6`, `12`<br>`Disc = (1 + Z/F)^(-T)`, where `F` is the compounding frequency, `Z` is the zero rate, and `T` is the time in periodic units; for example, `T = F` is 1 year.<br>`Compounding` = `365`<br>`Disc = (1 + Z/F)^(-T)`, where `F` is the number of days in the basis year and `T` is a number of days elapsed computed by basis.<br>`Compounding` = `-1`<br>`Disc = exp(-T*Z)`, where `T` is time in years. |

# bdttimespec

## Description

`TimeSpec = bdttimespec(ValuationDate, Maturity, Compounding)` sets the number of levels and node times for a BDT tree and determines the mapping between dates and time for rate quoting.

`TimeSpec` is a structure specifying the time layout for `bdttree`. The state observation dates are `[ValuationDate; Maturity(1:end-1)]`. Because a forward rate is stored at the last observation, the tree can value cash flows out to `Maturity`.

## Examples

Specify a five-period tree with annual nodes. Use annual compounding to report rates.

```
Compounding = 1;
ValuationDate = '01-01-2000';
Maturity = ['01-01-2001'; '01-01-2002'; '01-01-2003';
'01-01-2004'; '01-01-2005'];

TimeSpec = bdttimespec(ValuationDate, Maturity, Compounding)

TimeSpec =

           FinObj: 'BDTTimeSpec'
    ValuationDate: 730486
         Maturity: [5x1 double]
      Compounding: 1
            Basis: 0
     EndMonthRule: 1
```

## See Also

`bdttree` | `bdtvolspec`

## Purpose

Construct Black-Derman-Toy interest-rate tree

## Syntax

```
BDTTree = bdttree(VolSpec, RateSpec, TimeSpec)
```

## Arguments

| | |
|---|---|
| `VolSpec` | Volatility process specification. See `bdtvolspec` for information on the volatility process. |
| `RateSpec` | Interest-rate specification for the initial rate curve. See `intenvset` for information on declaring an interest-rate variable. |
| `TimeSpec` | Tree time layout specification. Defines the observation dates of the BDT tree and the `Compounding` rule for date to time mapping and price-yield formulas. See `bdttimespec` for information on the tree structure. |

## Description

`BDTTree = bdttree(VolSpec, RateSpec, TimeSpec)` creates a structure containing time and interest-rate information on a recombining tree.

## Examples

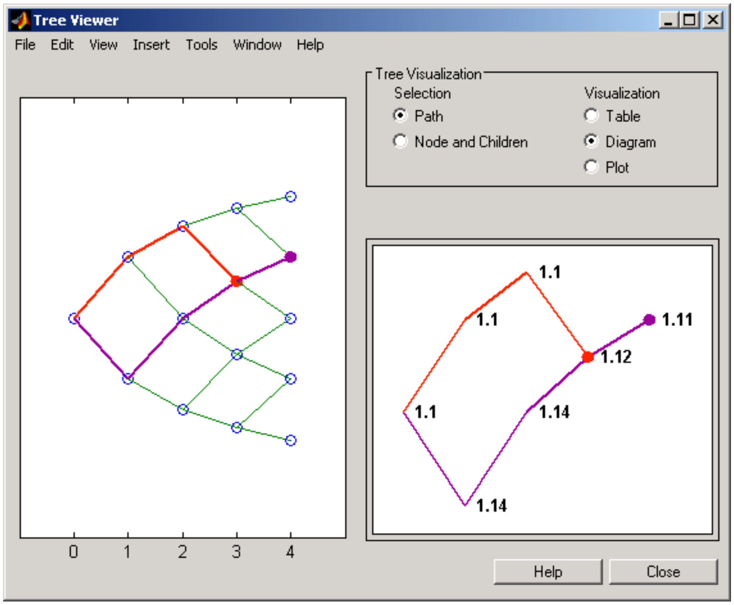
Using the data provided, create a BDT volatility specification (`VolSpec`), rate specification (`RateSpec`), and tree time layout specification (`TimeSpec`). Then use these specifications to create a BDT tree with `bdttree`.

```
Compounding = 1;
ValuationDate = '01-01-2000';
StartDate = ValuationDate;
EndDates = ['01-01-2001'; '01-01-2002'; '01-01-2003';
'01-01-2004'; '01-01-2005'];
Rates = [.1; .11; .12; .125; .13];
Volatility = [.2; .19; .18; .17; .16];

RateSpec = intenvset('Compounding', Compounding,...
```

```
      'ValuationDate', ValuationDate,...
      'StartDates', StartDate,...
      'EndDates', EndDates,...
      'Rates', Rates);

BDTTimeSpec = bdttimespec(ValuationDate, EndDates, Compounding);
BDTVolSpec = bdtvolspec(ValuationDate, EndDates, Volatility);
BDTTree = bdttree(BDTVolSpec, RateSpec, BDTTimeSpec);
```

Use `treeviewer` to observe the tree you have created.

```
treeviewer(BDTTree)
```



Tree Visualization

Selection
● Path
○ Node and Children

Visualization
○ Table
● Diagram
○ Plot

## See Also

`bdtprice` | `bdttimespec` | `bdtvolspec` | `intenvset`

## Purpose

Specify Black-Derman-Toy interest-rate volatility process

## Syntax

```
Volspec = bdtvolspec(ValuationDate, VolDates, VolCurve,
 InterpMethod)
```

## Arguments

| | |
|---|---|
| `ValuationDate` | Scalar value representing the observation date of the investment horizon. |
| `VolDates` | Number of points `(NPOINTS)`-by-`1` vector of yield volatility end dates. |
| `VolCurve` | `NPOINTS`-by-`1` vector of yield volatility values in decimal form. |
| `InterpMethod` | (Optional) Interpolation method. Default is `'linear'`. See `interp1` for more information. |

## Description

`Volspec = bdtvolspec(ValuationDate, VolDates, VolCurve, InterpMethod)` creates a structure specifying the volatility for `bdttree`.

## Examples

Using the data provided, create a BDT volatility specification (`VolSpec`).

```
ValuationDate = '01-01-2000';
EndDates = ['01-01-2001'; '01-01-2002'; '01-01-2003';
'01-01-2004'; '01-01-2005'];
Volatility = [.2; .19; .18; .17; .16];

BDTVolSpec = bdtvolspec(ValuationDate, EndDates, Volatility)

BDTVolSpec =
             FinObj: 'BDTVolSpec'
      ValuationDate: 730486
           VolDates: [5x1 double]
           VolCurve: [5x1 double]
    VolInterpMethod: 'linear'
```

**See Also** `bdttree` | `interp1`

## Purpose

Instrument prices from Black-Karasinski interest-rate tree

## Syntax

`[Price, PriceTree] = bkprice(BKTree, InstSet, Options)`

## Arguments

| | |
|---|---|
| `BKTree` | Interest-rate tree structure created by `bktree`. |
| `InstSet` | Variable containing a collection of `NINST` instruments. Instruments are categorized by type. Each type can have different data fields. The stored data field is a row vector or string for each instrument. |
| `Options` | (Optional) Derivatives pricing options structure created with `derivset`. |

## Description

`[Price, PriceTree] = bkprice(BKTree, InstSet, Options)` computes arbitrage-free prices for instruments using an interest-rate tree created with `bktree`. All instruments contained in a financial instrument variable, `InstSet`, are priced.

`Price` is a number of instruments `(NINST)`-by-`1` vector of prices for each instrument. The prices are computed by backward dynamic programming on the interest-rate tree. If an instrument cannot be priced, `NaN` is returned.

`PriceTree` is a MATLAB structure of trees containing vectors of instrument prices and accrued interest, and a vector of observation times for each node.

`PriceTree.PTree` contains the clean prices.

`PriceTree.AITree` contains the accrued interest.

`PriceTree.tObs` contains the observation times.

`bkprice` handles instrument types: `'Bond'`, `'CashFlow'`, `'OptBond'`, `'OptEmBond'`, `'Fixed'`, `'Float'`, `'Cap'`, `'Floor'`, `'RangeFloat'`, `'Swap'`. See `instadd` to construct defined types.

Related single-type pricing functions are:

- `bondbybk`: Price a bond from a Black-Karasinski tree.
- `capbybk`: Price a cap from a Black-Karasinski tree.
- `cfbybk`: Price an arbitrary set of cash flows from a Black-Karasinski tree.
- `fixedbybk`: Price a fixed-rate note from a Black-Karasinski tree.
- `floatbybk`: Price a floating-rate note from a Black-Karasinski tree.
- `floorbybk`: Price a floor from a Black-Karasinski tree.
- `optbndbybk`: Price a bond option from a Black-Karasinski tree.
- `optembndbybk`: Price a bond with embedded option by a Black-Karasinski tree.
- `rangefloatbybk`: Price range floating note from a Black-Karasinski tree.
- `swapbybk`: Price a swap from a Black-Karasinski tree.
- `swaptionbybk`: Price a swaption from a Black-Karasinski tree.

## Examples

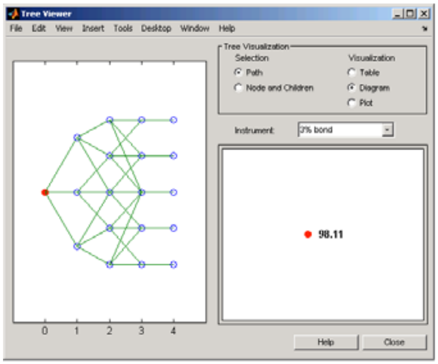
Load the BK tree and instruments from the data file `deriv.mat`. Price the cap and bond instruments contained in the instrument set.

```
load deriv.mat;
BKSubSet = instselect(BKInstSet,'Type', {'Bond', 'Cap'});

instdisp(BKSubSet)

%Table of instrument portfolio partially displayed:
Index Type   CouponRate Settle      Maturity    Period ...  Name ...
1     Bond   0.03       01-Jan-2004 01-Jan-2007 1      ... 3% bond
2     Bond   0.03       01-Jan-2004 01-Jan-2008 2      ...  3% bond

Index Type Strike Settle      Maturity     CapReset ...  Name ...
3     Cap  0.04   01-Jan-2004 01-Jan-2008  1        ...  4% Cap
```

```
[Price, PriceTree] = bkprice(BKTree, BKSubSet);

Price =

   98.1096
   95.6734
    2.2706
```

You can use `treeviewer` to see the prices of these three instruments along the price tree.

```
treeviewer(PriceTree, BKSubSet)
```



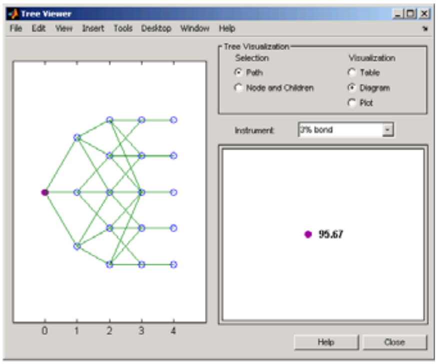
● Path ○ Node and Children

○ Table ● Diagram ○ Plot

Instrument: 3% bond

98.11

First 3% Bond (Maturity 2007)



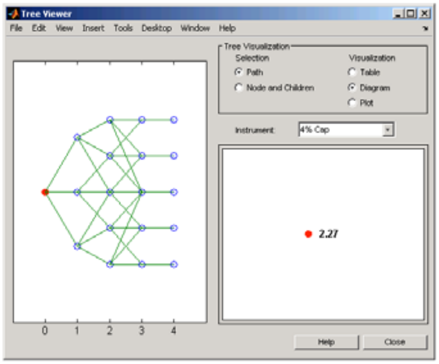
● Path ○ Node and Children

○ Table ● Diagram ○ Plot

Instrument: 3% bond

95.67

Second 3% Bond (Maturity 2008)



● Path ○ Node and Children

○ Table ● Diagram ○ Plot

Instrument: 4% Cap

2.27

4% Cap

---

Price the following multi-stepped coupon bonds using the following data:

```
% The data for the interest rate term structure is as follows:
Rates = [0.035; 0.042147; 0.047345; 0.052707];
ValuationDate = 'Jan-1-2010';
StartDates = ValuationDate;
EndDates = {'Jan-1-2011'; 'Jan-1-2012'; 'Jan-1-2013'; 'Jan-1-2014'};
Compounding = 1;

% Create RateSpec
RS = intenvset('ValuationDate', ValuationDate, 'StartDates', StartDates,...
'EndDates', EndDates,'Rates', Rates, 'Compounding', Compounding);

% Create a portfolio of stepped coupon bonds with different maturities
Settle = '01-Jan-2010';
Maturity = {'01-Jan-2011';'01-Jan-2012';'01-Jan-2013';'01-Jan-2014'};
CouponRate = {{'01-Jan-2011' .042;'01-Jan-2012' .05; '01-Jan-2013' .06; '01-Jan-2014' .07}};

ISet = instbond(CouponRate, Settle, Maturity, 1);
instdisp(ISet)

%Table of instrument portfolio partially displayed:
Index Type CouponRate Settle  Maturity  Period Basis EndMonthRule ... Face
1 Bond [Cell]  01-Jan-2010    01-Jan-2011   1    0     1           ... 100
2 Bond [Cell]  01-Jan-2010    01-Jan-2012   1    0     1           ... 100
3 Bond [Cell]  01-Jan-2010    01-Jan-2013   1    0     1           ... 100
4 Bond [Cell]  01-Jan-2010    01-Jan-2014   1    0     1           ... 100

% Build the tree with the following data
VolDates = ['1-Jan-2011'; '1-Jan-2012'; '1-Jan-2013'; '1-Jan-2014'];
VolCurve = 0.01;
AlphaDates = '01-01-2014';
AlphaCurve = 0.1;


BKVolSpec = bkvolspec(RS.ValuationDate, VolDates, VolCurve,...
AlphaDates, AlphaCurve);
BKTimeSpec = bktimespec(RS.ValuationDate, VolDates, Compounding);
```

```
BKT = bktree(BKVolSpec, RS, BKTimeSpec);

% Compute the price of the  stepped coupon bonds
PBK = bkprice(BKT, ISet)

PBK = bkprice(BKT, ISet)

%Table of instrument portfolio partially displayed:
Index Type CouponRate Settle Maturity   Period Basis EndMonthRule ... Face
1 Bond [Cell]  01-Jan-2010    01-Jan-2011    1   0     1          ... 100
2 Bond [Cell]  01-Jan-2010    01-Jan-2012    1   0     1          ... 100
3 Bond [Cell]  01-Jan-2010    01-Jan-2013    1   0     1          ... 100
4 Bond [Cell]  01-Jan-2010    01-Jan-2014    1   0     1          ... 100


PBK =

  100.6763
  100.7368
  100.9266
  101.0115
```

---

Price a portfolio of stepped callable bonds and stepped vanilla bonds using the following data:

```
% The data for the interest rate term structure is as follows:
Rates = [0.035; 0.042147; 0.047345; 0.052707];
ValuationDate = 'Jan-1-2010';
StartDates = ValuationDate;
EndDates = {'Jan-1-2011'; 'Jan-1-2012'; 'Jan-1-2013'; 'Jan-1-2014'};
Compounding = 1;

%Create RateSpec
RS = intenvset('ValuationDate', ValuationDate, 'StartDates', StartDates,...
'EndDates', EndDates,'Rates', Rates, 'Compounding', Compounding);
```

```
% Create an instrument portfolio of 3 stepped callable bonds and three
% stepped vanilla bonds
Settle = '01-Jan-2010';
Maturity = {'01-Jan-2012';'01-Jan-2013';'01-Jan-2014'};
CouponRate = {{'01-Jan-2011' .042;'01-Jan-2012' .05; '01-Jan-2013' .06; '01-Jan-2014' .07}};
OptSpec='call';
Strike=100;
ExerciseDates='01-Jan-2011'; %Callable in one year

% Bonds with embedded option
ISet = instoptembnd(CouponRate, Settle, Maturity, OptSpec, Strike,...
ExerciseDates, 'Period', 1);

% Vanilla bonds
ISet = instbond(ISet, CouponRate, Settle, Maturity, 1);

% Display the instrument portfolio
instdisp(ISet)

%Table of instrument portfolio partially displayed:
Index Type CouponRate       Settle     Maturity    OptSpec Strike ExerciseDates ... AmericanOpt
1     OptEmBond [Cell]   01-Jan-2010   01-Jan-2012   call    100  01-Jan-2011   ... 0
2     OptEmBond [Cell]   01-Jan-2010   01-Jan-2013   call    100  01-Jan-2011   ... 0
3     OptEmBond [Cell]   01-Jan-2010   01-Jan-2014   call    100  01-Jan-2011   ... 0

Index Type CouponRate Settle         Maturity       Period Basis EndMonthRule ... Face
4     Bond [Cell]     01-Jan-2010    01-Jan-2012    1      0     1            ... 100
5     Bond [Cell]     01-Jan-2010    01-Jan-2013    1      0     1            ... 100
6     Bond [Cell]     01-Jan-2010    01-Jan-2014    1      0     1            ... 100

% Build the tree with the following data
VolDates = ['1-Jan-2011'; '1-Jan-2012'; '1-Jan-2013'; '1-Jan-2014'];
VolCurve = 0.01;
AlphaDates = '01-01-2014';
AlphaCurve = 0.1;
```

```
BKVolSpec = bkvolspec(RS.ValuationDate, VolDates, VolCurve,...
AlphaDates, AlphaCurve);
BKTimeSpec = bktimespec(RS.ValuationDate, VolDates, Compounding);
BKT = bktree(BKVolSpec, RS, BKTimeSpec);

% The first three rows corresponds to the price of the stepped callable bonds
% and the last three rows corresponds to the price of the stepped vanilla bonds.

PBK = bkprice(BKT, ISet)

PBK =

  100.6735
  100.6763
  100.6763
  100.7368
  100.9266
  101.0115
```

---

Compute the price of a portfolio using the following data:

```
% The data for the interest rate term structure is as follows:
Rates = [0.035; 0.042147; 0.047345; 0.052707];
ValuationDate = 'Jan-1-2011';
StartDates = ValuationDate;
EndDates = {'Jan-1-2012'; 'Jan-1-2013'; 'Jan-1-2014'; 'Jan-1-2015'};
Compounding = 1;

%  Create RateSpec
RS = intenvset('ValuationDate', ValuationDate, 'StartDates',...
StartDates, 'EndDates', EndDates,'Rates', Rates, 'Compounding', Compounding);

% Create an instrument portfolio with two range notes and a floating rate
% note with the following data:
```

```
Spread = 200;
Settle = 'Jan-1-2011';
Maturity = 'Jan-1-2014';

% First Range Note:
RateSched(1).Dates = {'Jan-1-2012'; 'Jan-1-2013'  ; 'Jan-1-2014'};
RateSched(1).Rates  = [0.045 0.055; 0.0525  0.0675; 0.06 0.08];

% Second Range Note:
RateSched(2).Dates = {'Jan-1-2012'; 'Jan-1-2013' ; 'Jan-1-2014'};
RateSched(2).Rates  = [0.048 0.059; 0.055  0.068 ; 0.07 0.09];



% Create InstSet
InstSet = instadd('RangeFloat', Spread, Settle, Maturity, RateSched);

% Add a floating-rate note
InstSet = instadd(InstSet, 'Float', Spread, Settle, Maturity);

% Display the portfolio instrument
instdisp(InstSet)

Index Type       Spread Settle       Maturity    RateSched FloatReset Basis Principal EndMonthRule
1     RangeFloat 200    01-Jan-2011  01-Jan-2014 [Struct]  1          0     100       1
2     RangeFloat 200    01-Jan-2011  01-Jan-2014 [Struct]  1          0     100       1

Index Type  Spread Settle       Maturity       FloatReset Basis Principal EndMonthRule
3     Float 200    01-Jan-2011    01-Jan-2014    1          0     100       1

% The data to build the tree is as follows:
VolDates = ['1-Jan-2012'; '1-Jan-2013'; '1-Jan-2014';'1-Jan-2015'];
VolCurve = 0.01;
AlphaDates = '01-01-2015';
AlphaCurve = 0.1;

BKVS = bkvolspec(RS.ValuationDate, VolDates, VolCurve,...
AlphaDates, AlphaCurve);
```

```
BKTS = bktimespec(RS.ValuationDate, VolDates, Compounding);
BKT = bktree(BKVS, RS, BKTS);

% Price the portfolio
Price = bkprice(BKT, InstSet)

Price =

  105.5147
  101.4740
  105.5147
```

## See Also

`bksens` | `bktree` | `instadd` | `intenvprice` | `intenvsens`

## Purpose

Instrument prices and sensitivities from Black-Karasinski interest-rate tree

## Syntax

```
[Delta, Gamma, Vega, Price] = bksens(BKTree, InstSet,
Options)
```

## Arguments

| | |
|---|---|
| `BKTree` | Interest-rate tree structure created by `bktree`. |
| `InstSet` | Variable containing a collection of `NINST` instruments. Instruments are categorized by type. Each type can have different data fields. The stored data field is a row vector or string for each instrument. |
| `Options` | (Optional) Derivatives pricing options structure created with `derivset`. |

## Description

`[Delta, Gamma, Vega, Price] = bksens(BKTree, InstSet, Options)` computes instrument sensitivities and prices for instruments using an interest-rate tree created with the `bktree` function. `NINST` instruments from a financial instrument variable, `InstSet`, are priced. `bksens` handles instrument types: `'Bond'`, `'CashFlow'`, `'OptBond'`, `'OptEmBond'`, `'Fixed'`, `'Float'`, `'Cap'`, `'Floor'`, `'RangeFloat'`, `'Swap'`. See `instadd` for information on instrument types.

`Delta` is an `NINST`-by-`1` vector of deltas, representing the rate of change of instrument prices with respect to changes in the interest rate. `Delta` is computed by finite differences in calls to `bktree`. See `bktree` for information on the observed yield curve.

`Gamma` is an `NINST`-by-`1` vector of gammas, representing the rate of change of instrument deltas with respect to the changes in the interest rate. `Gamma` is computed by finite differences in calls to `bktree`.

`Vega` is an `NINST`-by-`1` vector of vegas, representing the rate of change of instrument prices with respect to the changes in the volatility $\sigma(t,T)$.

`Vega` is computed by finite differences in calls to `bktree`. See `bkvolspec` for information on the volatility process.

---

**Note** All sensitivities are returned as dollar sensitivities. To find the per-dollar sensitivities, divide by the respective instrument price.

---

`Price` is an `NINST`-by-`1` vector of prices of each instrument. The prices are computed by backward dynamic programming on the interest-rate tree. If an instrument cannot be priced, `NaN` is returned.

`Delta` and `Gamma` are calculated based on yield shifts of 100 basis points. `Vega` is calculated based on a 1% shift in the volatility process.

## Examples

Load the tree and instruments from a data file. Compute `Delta` and `Gamma` for the cap and bond instruments contained in the instrument set.

```
load deriv.mat;
BKSubSet = instselect(BKInstSet,'Type', {'Bond', 'Cap'});

instdisp(BKSubSet)

%Table of instrument portfolio partially displayed:
Index Type CouponRate Settle        Maturity     Period ...  Name...
1     Bond 0.03       01-Jan-2004   01-Jan-2007  1      ...  3% Bond
2     Bond 0.03       01-Jan-2004   01-Jan-2008  1      ...  3% Bond

Index Type Strike Settle      Maturity     CapReset ...  Name ...
3     Cap  0.04   01-Jan-2004 01-Jan-2008  1        ...  4% Cap

[Delta, Gamma] = bksens(BKTree, BKSubSet)

Delta =

 -285.7151
 -365.7048
```

```
  189.5319

Gamma =

  1.0e+003 *

    0.8456
    1.4345
    6.9999
```

## See Also

`bkprice` | `bktree` | `bkvolspec` | `instadd`

**Purpose** Specify time structure for Black-Karasinski tree

**Syntax** `TimeSpec = bktimespec(ValuationDate, Maturity, Compounding)`

**Arguments**

| | |
|---|---|
| `ValuationDate` | Scalar date marking the pricing date and first observation in the tree. Specify as a serial date number or date string. |
| `Maturity` | Number of levels (depth) of the tree. A number of levels (`NLEVELS`)-by-`1` vector of dates marking the cash flow dates of the tree. Cash flows with these maturities fall on tree nodes. Maturity should be in increasing order. |
| `Compounding` | (Optional) Scalar value representing the rate at which the input zero rates were compounded when annualized. Default = `-1` (continuous compounding). This argument determines the formula for the discount factors:<br>`Compounding` = `1`, `2`, `3`, `4`, `6`, `12`<br>`Disc = (1 + Z/F)^(-T)`, where `F` is the compounding frequency, `Z` is the zero rate, and `T` is the time in periodic units; for example, `T = F` is 1 year.<br>`Compounding` = `365`<br>`Disc = (1 + Z/F)^(-T)`, where `F` is the number of days in the basis year and `T` is a number of days elapsed computed by basis.<br>`Compounding` = `-1`<br>`Disc = exp(-T*Z)`, where `T` is time in years. |

## Description

`TimeSpec = bktimespec(ValuationDate, Maturity, Compounding)` sets the number of levels and node times for an BK tree and determines the mapping between dates and time for rate quoting.

`TimeSpec` is a structure specifying the time layout for `bktree`. The state observation dates are `[Settle; Maturity(1:end-1)]`. Because a forward rate is stored at the last observation, the tree can value cash flows out to `Maturity`.

## Examples

Specify a four-period tree with annual nodes. Use annual compounding to report rates.

```
ValuationDate = 'Jan-1-2004';
Maturity = ['12-31-2004'; '12-31-2005'; '12-31-2006';
'12-31-2007'];
Compounding = 1;
TimeSpec = bktimespec(ValuationDate, Maturity, Compounding)

TimeSpec =

          FinObj: 'BKTimeSpec'
   ValuationDate: 731947
        Maturity: [4x1 double]
     Compounding: 1
           Basis: 0
    EndMonthRule: 1
```

## See Also

`bktree` | `bkvolspec` | `hwtree`

## Purpose

Construct Black-Karasinski interest-rate tree

## Syntax

```
BKTree = bktree(VolSpec, RateSpec, TimeSpec)
BKTree = bktree(VolSpec, RateSpec, TimeSpec, Name,Value)
```

## Description

`BKTree = bktree(VolSpec, RateSpec, TimeSpec)` creates a structure containing time and interest-rate information on a recombining tree.

`BKTree = bktree(VolSpec, RateSpec, TimeSpec, Name,Value)` creates a structure containing time and interest-rate information on a recombining tree with additional options specified by one or more `Name,Value` pair arguments.

## Input Arguments

**VolSpec**

Volatility process specification. See `bkvolspec` for information on the volatility process.

**RateSpec**

Interest-rate specification for the initial rate curve. See `intenvset` for information on declaring an interest-rate variable.

**TimeSpec**

Tree time layout specification. Defines the observation dates of the BK tree and the compounding rule for date to time mapping and price-yield formulas. See `bktimespec` for information on the tree structure.

### Name-Value Pair Arguments

Specify optional comma-separated pairs of `Name,Value` arguments, where `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside single quotes (`' '`). You can specify several name and value pair arguments in any order as `Name1,Value1,...,NameN,ValueN`.

**Method**

String specifying the Hull-White method upon which the tree-node connectivity algorithm is based. Possible values are `HW1996` and `HW2000`.

---

**Note** `bktree` supports two tree-node connectivity algorithms. `HW1996` is based on the original paper published in the *Journal of Derivatives*, and `HW2000` is the general version of the algorithm, as specified in the paper published in August 2000.

---

**Default:** `HW1996`

## Output Arguments

### BKTree

Structure containing time and interest rate information of a trinomial recombining tree.

## Examples

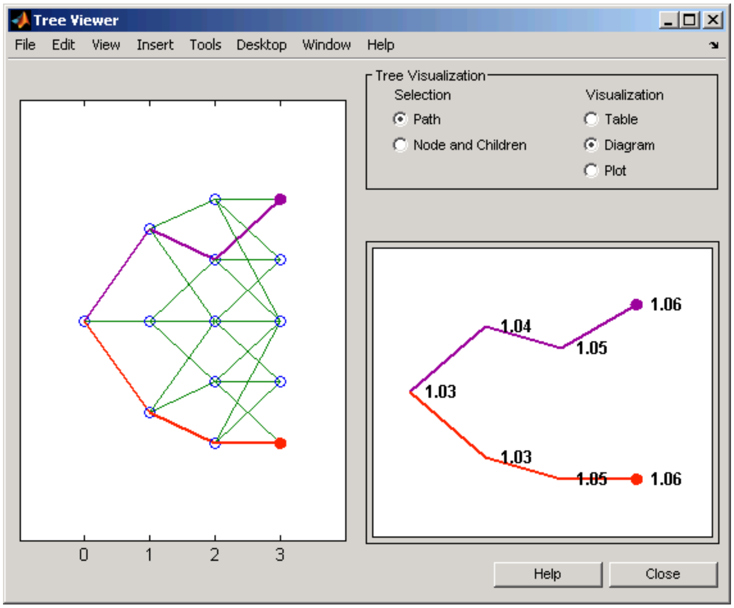
Using the data provided, create a BK volatility specification (`VolSpec`), rate specification (`RateSpec`), and tree time layout specification (`TimeSpec`). Then use these specifications to create a BK tree using `bktree`.

```
Compounding = -1;
ValuationDate = '01-01-2004';
StartDate = ValuationDate;
VolDates = ['12-31-2004'; '12-31-2005'; '12-31-2006';
'12-31-2007'];
VolCurve = 0.01;
AlphaDates = '01-01-2008';
AlphaCurve = 0.1;
Rates = [0.0275; 0.0312; 0.0363; 0.0415];

BKVolSpec = bkvolspec(ValuationDate, VolDates, VolCurve,...
AlphaDates, AlphaCurve);

RateSpec = intenvset('Compounding', Compounding,...
      'ValuationDate', ValuationDate,...
```

```
      'StartDates', ValuationDate,...
      'EndDates', VolDates,...
      'Rates', Rates);

BKTimeSpec = bktimespec(ValuationDate, VolDates, Compounding);

BKTree = bktree(BKVolSpec, RateSpec, BKTimeSpec)

BKTree =

      FinObj: 'BKFwdTree'
     VolSpec: [1x1 struct]
    TimeSpec: [1x1 struct]
    RateSpec: [1x1 struct]
        tObs: [0 0.9973 1.9973 2.9973]
        dObs: [731947 732312 732677 733042]
      CFlowT: {[4x1 double]  [3x1 double]  [2x1 double]  [3.9973]}
       Probs: {[3x1 double]  [3x3 double]  [3x5 double]}
     Connect: {[2]  [2 3 4]  [2 2 3 4 4]}
     FwdTree: {1x4 cell}
```

Use `treeviewer` to observe the tree you have created.

```
treeviewer(BKTree)
```



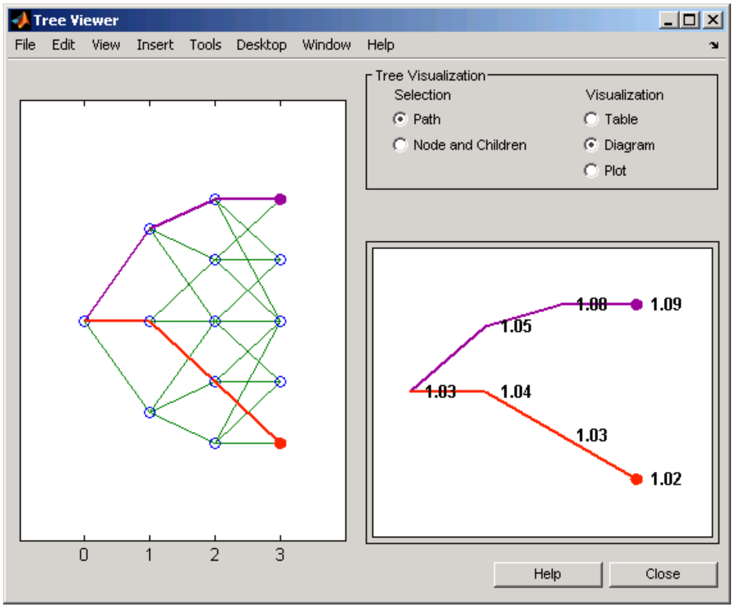
Using the data provided, create a Hull-White volatility specification (`VolSpec`), rate specification (`RateSpec`), and tree time layout specification (`TimeSpec`). Then use these specifications to create a Hull-White tree using `hwtree`.

```
Compounding = -1;
ValuationDate = '01-01-2004';
StartDate = ValuationDate;
VolDates = ['12-31-2004'; '12-31-2005'; '12-31-2006';
'12-31-2007'];
VolCurve = 0.01;
AlphaDates = '01-01-2008';
AlphaCurve = 0.1;
Rates = [0.0275; 0.0312; 0.0363; 0.0415];

HWVolSpec = hwvolspec(ValuationDate, VolDates, VolCurve,...
AlphaDates, AlphaCurve);
```

```
RateSpec = intenvset('Compounding', Compounding,...
      'ValuationDate', ValuationDate,...
      'StartDates', ValuationDate,...
      'EndDates', VolDates,...
      'Rates', Rates);

HWTimeSpec = hwtimespec(ValuationDate, VolDates, Compounding);
HWTree = hwtree(HWVolSpec, RateSpec, HWTimeSpec)

HWTree =

      FinObj: 'HWFwdTree'
     VolSpec: [1x1 struct]
    TimeSpec: [1x1 struct]
    RateSpec: [1x1 struct]
        tObs: [0 0.9973 1.9973 2.9973]
        dObs: [731947 732312 732677 733042]
      CFlowT: {[4x1 double]  [3x1 double]  [2x1 double]  [3.9973]}
       Probs: {[3x1 double]  [3x3 double]  [3x5 double]}
     Connect: {[2]  [2 3 4]  [2 2 3 4 4]}
     FwdTree: {1x4 cell}
```

Use `treeviewer` to observe the tree you have created.

```
treeviewer(HWTree)
```



## References

Hull, J., and A. White, "Using Hull-White Interest Rate Trees", *Journal of Derivatives*, 1996.

Hull, J., and A. White, "The General Hull-White Model and Super Calibration", August 2000.

## See Also

| `bkprice` | `bktimespec` | `bkvolspec` | `intenvset` |

## Tutorials

- "Calibrating Hull-White Model Using Market Data" on page 2-79

# bkvolspec

## Purpose

Specify Black-Karasinski interest-rate volatility process

## Syntax

```
Volspec = bkvolspec(ValuationDate, VolDates, VolCurve,
AlphaDates, AlphaCurve, InterpMethod)
```

## Arguments

| | |
|---|---|
| `ValuationDate` | Scalar value representing the observation date of the investment horizon. |
| `VolDates` | Number of points (`NPOINTS`)-by-`1` vector of yield volatility end dates. |
| `VolCurve` | `NPOINTS`-by-`1` vector of annualized yield volatility values in decimal form. Volatility is the standard deviation of proportional changes in the rate. |
| `AlphaDates` | `NPOINTS`-by-`1` vector of mean reversion end dates. |
| `AlphaCurve` | `NPOINTS`-by-`1` vector of positive mean reversion values in decimal form. |
| `InterpMethod` | (Optional) Interpolation method. Default is `'linear'`. See `interp1` for more information. |

## Description

`Volspec = bkvolspec(ValuationDate, VolDates, VolCurve, AlphaDates, AlphaCurve, InterpMethod)` creates a structure specifying the volatility for `bktree`.

## Examples

Using the data provided, create a Black-Karasinski volatility specification (`VolSpec`).

```
ValuationDate = '01-01-2004';
StartDate = ValuationDate;
VolDates = ['12-31-2004'; '12-31-2005'; '12-31-2006';
'12-31-2007'];
```

```
VolCurve = 0.01;
AlphaDates = '01-01-2008';
AlphaCurve = 0.1;
BKVolSpec = bkvolspec(ValuationDate, VolDates, VolCurve,...
AlphaDates, AlphaCurve)

BKVolSpec =

             FinObj: 'BKVolSpec'
      ValuationDate: 731947
           VolDates: [4x1 double]
           VolCurve: [4x1 double]
         AlphaCurve: 0.1000
         AlphaDates: 733408
    VolInterpMethod: 'linear'
```

## See Also

`bktree` | `interp1`

# bondbybdt

## Purpose

Price bond from Black-Derman-Toy interest-rate tree

## Syntax

```
[Price, PriceTree] = bondbybdt(BDTTree, CouponRate, Settle,
Maturity)
[Price, PriceTree] = bondbybdt(BDTTree, CouponRate, Settle,
Maturity, Period, Basis, EndMonthRule, IssueDate,
FirstCouponDate,  LastCouponDate, StartDate, Face,
Options)
[Price, PriceTree] = bondbybdt(BDTTree, CouponRate, Settle,
Maturity,Name,Value)
```

## Input Arguments

| | |
|---|---|
| `BDTTree` | Interest-rate tree structure created by `bdttree`. |
| `CouponRate` | Decimal annual rate. `CouponRate` is a `NINST`-by-`1` vector or `NINST`-by-`1` cell array of decimal annual rates, or decimal annual rate schedules. For the latter case of a variable coupon schedule, each element of the cell array is a `NumDates`-by-`2` cell array, where the first column is dates and the second column is its associated rate. The date indicates the last day that the coupon rate is valid. |
| `Settle` | Settlement date. A vector of serial date numbers or date strings. `Settle` must be earlier than `Maturity`. |
| `Maturity` | Maturity date. A vector of serial date numbers or date strings. |

The `Settle` date for every bond is set to the `ValuationDate` of the BDT tree. The bond argument `Settle` is ignored.

### Ordered Input or Name-Value Pair Arguments

Enter the following optional inputs using an ordered syntax or as name-value pair arguments. You cannot mix ordered syntax with name-value pair arguments.

#### Period

Coupons per year of the bond. A vector of integers. Values are `1`, `2`, `3`, `4`, `6`, and `12`.

**Default:** `2`

#### Basis

Day-count basis of the instrument. A vector of integers.

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (PSA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ISMA)
- 9 = actual/360 (ISMA)
- 10 = actual/365 (ISMA)
- 11 = 30/360E (ISMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see basis.

**Default:** `0` (actual/actual)

#### EndMonthRule

End-of-month rule. A `NINST`-by-`1` vector. This rule applies only when `Maturity` is an end-of-month date for a month having 30 or fewer days.

- `0` = Ignore rule, meaning that a bond coupon payment date is always the same numerical day of the month.
- `1` = Set rule on, meaning that a bond coupon payment date is always the last actual day of the month.

**Default:** `1`

#### IssueDate

Date when a bond was issued.

#### FirstCouponDateDate

Date when a bond makes its first coupon payment; used when bond has an irregular first coupon period. When `FirstCouponDate` and `LastCouponDate` are both specified, `FirstCouponDate` takes precedence in determining the coupon payment structure. If you do not specify a `FirstCouponDate`, the cash flow payment dates are determined from other inputs.

#### LastCouponDateDate

Last coupon date of a bond before the maturity date; used when bond has an irregular last coupon period. In the absence of a specified `FirstCouponDate`, a specified `LastCouponDate` determines the coupon structure of the bond. The coupon structure of a bond is truncated at the `LastCouponDate`, regardless of where it falls, and is followed only by the bond's maturity cash flow date. If you do not specify a `LastCouponDate`, the cash flow payment dates are determined from other inputs.

#### StartDate

Date when a bond actually starts (the date from which a bond cash flow is considered). To make an instrument forward-starting, specify this date as a future date. If you do not specify `StartDate`, the effective start date is the `Settle` date.

### Face

Face or par value. `Face` is a `NINST`-by-`1` vector or `NINST`-by-`1` cell array of face values, or face value schedules. For the latter case, each element of the cell array is a `NumDates`-by-`2` cell array, where the first column is dates and the second column is its associated face value. The date indicates the last day that the face value is valid.

**Default:** `100`

### Options

Derivatives pricing options structure created with `derivset`.

## Name-Value Pair Arguments

Specify optional comma-separated pairs of `Name,Value` arguments, where `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside single quotes (`' '`). You can specify several name and value pair arguments in any order as `Name1,Value1,...,NameN,ValueN`.

### AdjustCashFlowsBasis

Adjust the cash flows based on the actual period day count. `NINST`-by-`1` of logicals.

**Default:** `false`

### BusinessDayConvention

Require payment dates to be business dates. `NINST`-by-`1` cell array with possible choices of business day convention:

- `actual`

- `follow`
- `modifiedfollow`
- `previous`
- `modifiedprevious`

**Default:** `actual`

### Holidays

Holidays used for business day convention. `NHOLIDAYS`-by-`1` of MATLAB date numbers.

**Default:** If no dates are specified, `holidays.m` is used.

## Description

`[Price, PriceTree] = bondbybdt(BDTTree, CouponRate, Settle, Maturity)` computes the price of a bond from a BDT interest-rate tree.

`[Price, PriceTree] = bondbybdt(BDTTree, CouponRate, Settle, Maturity, Period, Basis, EndMonthRule, IssueDate, FirstCouponDate, LastCouponDate, StartDate, Face, Options)` computes the price of a bond from a BDT interest-rate tree using optional input arguments.

`[Price, PriceTree] = bondbybdt(BDTTree, CouponRate, Settle, Maturity,Name,Value)` computes the price of a bond from a BDT interest-rate tree with additional options specified by one or more `Name,Value` pair arguments.

`Price` is a number of instruments (`NINST`)-by-`1` matrix of expected prices at time 0.

`PriceTree` is a MATLAB structure of trees containing vectors of instrument prices and accrued interest, and a vector of observation times for each node. Within `PriceTree`:

- `PriceTree.PTree` contains the clean prices.
- `PriceTree.AITree` contains the accrued interest.

- `PriceTree.tObs` contains the observation times.

`bondbybdt` computes prices of vanilla bonds, stepped coupon bonds, and amortizing bonds with no market purchase option and no call provisions.

## Definitions

### Vanilla Bond

A vanilla coupon bond is a security representing an obligation to repay a borrowed amount at a designated time and to make periodic interest payments until that time. The issuer of a bond makes the periodic interest payments until the bond matures. At maturity, the issuer pays to the holder of the bond the principal amount owed (face value) and the last interest payment.

### Stepped Coupon Bond

A step-up and step-down bond is a debt security with a predetermined coupon structure over time. With these instruments, coupons increase (step up) or decrease (step down) at specific times during the life of the bond.

### Bond with an Amortization Schedule

An amortized bond is treated as an asset, with the discount amount being amortized to interest expense over the life of the bond.

## Examples

### Price a Bond Using a BDT Tree

Price a 10% bond using a BDT interest-rate tree.

Load `deriv.mat`, which provides `BDTTree`. The `BDTTree` structure contains the time and interest-rate information needed to price the bond.

```
load deriv.mat;
```

Define the bond using the required arguments. Other arguments use defaults.

```
CouponRate = 0.10;
Settle = '01-Jan-2000';
```

```
Maturity = '01-Jan-2003';
Period = 1;
```

Use `bondbybdt` to compute the price of the bond.

```
Price = bondbybdt(BDTTree, CouponRate, Settle, Maturity, Period)
```

```
Price =

  95.5030
```

### Price a Stepped Coupon Bond

Price single stepped coupon bonds using market data.

Define the interest-rate term structure.

```
Rates = [0.035; 0.042147; 0.047345; 0.052707];
ValuationDate = 'Jan-1-2010';
StartDates = ValuationDate;
EndDates = {'Jan-1-2011'; 'Jan-1-2012'; 'Jan-1-2013'; 'Jan-1-2014'};
Compounding = 1;
```

Create the `RateSpec`.

```
RS = intenvset('ValuationDate', ValuationDate, 'StartDates', StartDates, 'EndDates',...
EndDates, 'Rates', Rates, 'Compounding', Compounding)
```

```
RS =

         FinObj: 'RateSpec'
    Compounding: 1
           Disc: [4x1 double]
          Rates: [4x1 double]
       EndTimes: [4x1 double]
     StartTimes: [4x1 double]
       EndDates: [4x1 double]
     StartDates: 734139
```

```
    ValuationDate: 734139
            Basis: 0
     EndMonthRule: 1
```

Create the stepped bond instrument.

```
Settle = '01-Jan-2010';
Maturity = {'01-Jan-2011';'01-Jan-2012';'01-Jan-2013';'01-Jan-2014'};
CouponRate = {{'01-Jan-2012' .0425;'01-Jan-2014' .0750}};
Period = 1;
```

Build the BDT tree and assume the volatility to be 10% using the following market data:

```
Sigma = 0.1;
BDTTimeSpec = bdttimespec(ValuationDate, EndDates);
BDTVolSpec = bdtvolspec(ValuationDate, EndDates, Sigma*ones(1, length(EndDates))');
BDTT = bdttree(BDTVolSpec, RS, BDTTimeSpec)
```

Compute the price of the stepped coupon bonds.

```
PBDT= bondbybdt(BDTT, CouponRate, Settle,Maturity , Period)
```

```
PBDT =

  100.7246
  100.0945
  101.5900
  102.0820
```

### Price Two Bonds with Amortization Schedules

Price two bonds with amortization schedules using the `Face` input argument to define the schedule.

Define the interest-rate term structure.

```
Rates = 0.035;
ValuationDate = '1-Nov-2011';
```

```
StartDates = ValuationDate;
EndDates = '1-Nov-2017';
Compounding = 1;
```

Create the `RateSpec`.

```
RateSpec = intenvset('ValuationDate', ValuationDate,'StartDates', StartDates,...
'EndDates', EndDates,'Rates', Rates, 'Compounding', Compounding);
```

```
RateSpec =

           FinObj: 'RateSpec'
      Compounding: 1
             Disc: 0.8135
            Rates: 0.0350
         EndTimes: 6
       StartTimes: 0
         EndDates: 737000
       StartDates: 734808
    ValuationDate: 734808
            Basis: 0
     EndMonthRule: 1
```

Create the bond instrument. The bonds have a coupon rate of 4% and 3.85%, a period of one year, and mature on 1-Nov-2017.

```
CouponRate = [0.04; 0.0385];
Settle ='1-Nov-2011';
Maturity = '1-Nov-2017';
Period = 1;
```

Define the amortizing schedule.

```
Face = {{'1-Nov-2015' 100;'1-Nov-2016' 85;'1-Nov-2017' 70};
{'1-Nov-2015' 100;'1-Nov-2016' 90;'1-Nov-2017' 80}};
```

Build the BDT tree and assume the volatility to be 10%.

```
MatDates = {'1-Nov-2012'; '1-Nov-2013';'1-Nov-2014';'1-Nov-2015';'1-Nov-2016';'1-Nov-2017'};
```

```
BDTTimeSpec = bdttimespec(ValuationDate, MatDates);
Volatility = 0.1;
BDTVolSpec = bdtvolspec(ValuationDate, MatDates, Volatility*ones(1,length(MatDates))');
BDTT = bdttree(BDTVolSpec, RateSpec, BDTTimeSpec);
```

Compute the price of the amortizing bonds.

```
Price = bondbybdt(BDTT, CouponRate, Settle, Maturity, 'Period',Period,...
'Face', Face)
```

```
Price =

  102.4791
  101.7786
```

## See Also

`bdttree` | `bdtprice` | `cfamounts` | `instbond`

# bondbybk

## Purpose

Price bond from Black-Karasinski interest-rate tree

## Syntax

```
[Price, PriceTree] = bondbybk(BKTree, CouponRate,
Settle, Maturity)
[Price, PriceTree] = bondbybk(BKTree, CouponRate,
Settle, Maturity, Period, Basis, EndMonthRule, IssueDate,
FirstCouponDate, LastCouponDate, StartDate, Face,
Options)
[Price, PriceTree] = bondbybk(BKTree, CouponRate,
Settle, Maturity, Name,Value)
```

## Input Arguments

| | |
|---|---|
| `BKTree` | Forward rate tree structure created by `bktree`. |
| `CouponRate` | Decimal annual rate. `CouponRate` is a `NINST`-by-`1` vector or `NINST`-by-`1` cell array of decimal annual rates, or decimal annual rate schedules. For the latter case of a variable coupon schedule, each element of the cell array is a `NumDates`-by-`2` cell array, where the first column is dates and the second column is its associated rate. The date indicates the last day that the coupon rate is valid. |
| `Settle` | Settlement date. A vector of serial date numbers or date strings. `Settle` must be earlier than `Maturity`. |
| `Maturity` | Maturity date. A vector of serial date numbers or date strings. |

The `Settle` date for every bond is set to the `ValuationDate` of the BK tree. The bond argument `Settle` is ignored.

### Ordered Input or Name-Value Pair Arguments

Enter the following optional inputs using an ordered syntax or as name-value pair arguments. You cannot mix ordered syntax with name-value pair arguments.

#### Period

Coupons per year of the bond. A vector of integers. Values are `1`, `2`, `3`, `4`, `6`, and `12`.

**Default:** `2`

#### Basis

Day-count basis of the instrument. A vector of integers.

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (PSA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ISMA)
- 9 = actual/360 (ISMA)
- 10 = actual/365 (ISMA)
- 11 = 30/360E (ISMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see basis.

**Default:** `0` (actual/actual)

### EndMonthRule

End-of-month rule. A `NINST`-by-`1` vector. This rule applies only when `Maturity` is an end-of-month date for a month having 30 or fewer days.

- `0` = Ignore rule, meaning that a bond coupon payment date is always the same numerical day of the month.
- `1` = Set rule on, meaning that a bond coupon payment date is always the last actual day of the month.

**Default:** `1`

### IssueDate

Date when a bond was issued.

### FirstCouponDateDate

Date when a bond makes its first coupon payment; used when bond has an irregular first coupon period. When `FirstCouponDate` and `LastCouponDate` are both specified, `FirstCouponDate` takes precedence in determining the coupon payment structure. If you do not specify a `FirstCouponDate`, the cash flow payment dates are determined from other inputs.

### LastCouponDateDate

Last coupon date of a bond before the maturity date; used when bond has an irregular last coupon period. In the absence of a specified `FirstCouponDate`, a specified `LastCouponDate` determines the coupon structure of the bond. The coupon structure of a bond is truncated at the `LastCouponDate`, regardless of where it falls, and is followed only by the bond's maturity cash flow date. If you do not specify a `LastCouponDate`, the cash flow payment dates are determined from other inputs.

### StartDate

Date when a bond actually starts (the date from which a bond cash flow is considered). To make an instrument forward-starting, specify this date as a future date. If you do not specify `StartDate`, the effective start date is the `Settle` date.

### Face

Face or par value. `Face` is a `NINST`-by-`1` vector or `NINST`-by-`1` cell array of face values, or face value schedules. For the latter case, each element of the cell array is a `NumDates`-by-`2` cell array, where the first column is dates and the second column is its associated face value. The date indicates the last day that the face value is valid.

**Default:** `100`

### Options

Derivatives pricing options structure created with `derivset`.

## Name-Value Pair Arguments

Specify optional comma-separated pairs of `Name,Value` arguments, where `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside single quotes (`' '`). You can specify several name and value pair arguments in any order as `Name1,Value1,...,NameN,ValueN`.

### AdjustCashFlowsBasis

Adjust the cash flows based on the actual period day count. `NINST`-by-`1` of logicals.

**Default:** `false`

### BusinessDayConvention

Require payment dates to be business dates. `NINST`-by-`1` cell array with possible choices of business day convention:

- `actual`

- `follow`
- `modifiedfollow`
- `previous`
- `modifiedprevious`

**Default:** `actual`

### Holidays

Holidays used for business day convention. `NHOLIDAYS`-by-`1` of MATLAB date numbers.

**Default:** If no dates are specified, `holidays.m` is used.

## Description

`[Price, PriceTree] = bondbybk(BKTree, CouponRate, Settle, Maturity)` computes the price of a bond from a Black-Karasinski interest-rate tree.

`[Price, PriceTree] = bondbybk(BKTree, CouponRate, Settle, Maturity, Period, Basis, EndMonthRule, IssueDate, FirstCouponDate, LastCouponDate, StartDate, Face, Options)` computes the price of a bond from a Black-Karasinski interest-rate tree using optional input arguments.

`[Price, PriceTree] = bondbybk(BKTree, CouponRate, Settle, Maturity, Name,Value)` computes the price of a bond from a Black-Karasinski interest-rate tree with additional options specified by one or more `Name,Value` pair arguments.

`Price` is a number of instruments (`NINST`)-by-`1` matrix of expected prices at time 0.

`PriceTree` is a structure of trees containing vectors of instrument prices and accrued interest, and a vector of observation times for each node. Within `PriceTree`:

- `PriceTree.PTree` contains the clean prices.

- `PriceTree.AITree` contains the accrued interest.
- `PriceTree.tObs` contains the observation times.

`bondbybk` computes prices of vanilla bonds, stepped coupon bonds, and amortizing bonds with no market purchase option and no call provisions.

## Definitions

### Vanilla Bond

A vanilla coupon bond is a security representing an obligation to repay a borrowed amount at a designated time and to make periodic interest payments until that time. The issuer of a bond makes the periodic interest payments until the bond matures. At maturity, the issuer pays to the holder of the bond the principal amount owed (face value) and the last interest payment.

### Stepped Coupon Bond

A step-up and step-down bond is a debt security with a predetermined coupon structure over time. With these instruments, coupons increase (step up) or decrease (step down) at specific times during the life of the bond.

### Bond with an Amortization Schedule

An amortized bond is treated as an asset, with the discount amount being amortized to interest expense over the life of the bond.

## Examples

### Price a Bond Using a BK Tree

Price a 4% bond using a Black-Karasinski interest-rate tree.

Load `deriv.mat`, which provides `BKTree`. The `BKTree` structure contains the time and interest-rate information needed to price the bond.

```
load deriv.mat;
```

Define the bond using the required arguments. Other arguments use defaults.

```
CouponRate = 0.04;
```

```
Settle = '01-Jan-2004';
Maturity = '31-Dec-2008';
```

Use `bondbybk` to compute the price of the bond.

```
Price = bondbybk(BKTree, CouponRate, Settle, Maturity)
Warning: Not all cash flows are aligned with the tree. Result will
be approximated.

Price =

  98.0300
```

### Price a Stepped Coupon Bond

Price single stepped coupon bonds using market data.

Define the interest-rate term structure.

```
Rates = [0.035; 0.042147; 0.047345; 0.052707];
ValuationDate = 'Jan-1-2010';
StartDates = ValuationDate;
EndDates = {'Jan-1-2011'; 'Jan-1-2012';...
'Jan-1-2013'; 'Jan-1-2014'};
Compounding = 1;
```

Create the `RateSpec`.

```
RS = intenvset('ValuationDate', ValuationDate, 'StartDates', StartDates,...
'EndDates', EndDates,'Rates', Rates, 'Compounding', Compounding)

RS =

           FinObj: 'RateSpec'
      Compounding: 1
             Disc: [4x1 double]
            Rates: [4x1 double]
         EndTimes: [4x1 double]
```

```
        StartTimes: [4x1 double]
          EndDates: [4x1 double]
        StartDates: 734139
     ValuationDate: 734139
             Basis: 0
      EndMonthRule: 1
```

Create the stepped bond instrument.

```
Settle = '01-Jan-2010';
Maturity = {'01-Jan-2011';'01-Jan-2012';'01-Jan-2013';'01-Jan-2014'};
CouponRate = {{'01-Jan-2012' .0425;'01-Jan-2014' .0750}};
Period = 1;
```

Build the BK tree using the following market data:

```
VolDates = ['1-Jan-2011'; '1-Jan-2012'; '1-Jan-2013'; '1-Jan-2014'];
VolCurve = 0.01;
AlphaDates = '01-01-2014';
AlphaCurve = 0.1;

BKVolSpec = bkvolspec(RS.ValuationDate, VolDates, VolCurve,...
AlphaDates, AlphaCurve);
BKTimeSpec = bktimespec(RS.ValuationDate, VolDates, Compounding);
BKT = bktree(BKVolSpec, RS, BKTimeSpec);
```

Compute the price of the stepped coupon bonds.

```
PBK= bondbybk(BKT, CouponRate, Settle,Maturity , Period)

PBK =

  100.7246
  100.0945
  101.5900
  102.0820
```

### Price a Bond with an Amortization Schedule

Price a bond with an amortization schedule using the `Face` input argument to define the schedule.

Define the interest-rate term structure.

```
Rates = 0.065;
ValuationDate = '1-Jan-2011';
StartDates = ValuationDate;
EndDates=  '1-Jan-2017';
Compounding = 1;
```

Create the `RateSpec`.

```
RateSpec = intenvset('ValuationDate', ValuationDate,'StartDates', StartDates,...
'EndDates', EndDates,'Rates', Rates, 'Compounding', Compounding)
```

```
RateSpec =

           FinObj: 'RateSpec'
      Compounding: 1
             Disc: 0.6853
            Rates: 0.0650
         EndTimes: 6
       StartTimes: 0
         EndDates: 736696
       StartDates: 734504
    ValuationDate: 734504
            Basis: 0
     EndMonthRule: 1
```

Create the bond instrument. The bond has a coupon rate of 7%, a period of one year, and matures on 1-Jan-2017.

```
CouponRate = 0.07;
Settle ='1-Jan-2011';
Maturity = '1-Jan-2017';
Period = 1;
```

```
Face = {{'1-Jan-2015' 100;'1-Jan-2016' 90;'1-Jan-2017' 80}};
```

Build the BK tree with the following market data:

```
VolDates = ['1-Jan-2012'; '1-Jan-2013';...
'1-Jan-2014';'1-Jan-2015';'1-Jan-2016';'1-Jan-2017'];
VolCurve = 0.01;
AlphaDates = '01-01-2017';
AlphaCurve = 0.1;

BKVolSpec = bkvolspec(RateSpec.ValuationDate, VolDates, VolCurve,...
AlphaDates, AlphaCurve);
BKTimeSpec = bktimespec(RateSpec.ValuationDate, VolDates, Compounding);
BKT = bktree(BKVolSpec, RateSpec, BKTimeSpec);
```

Compute the price of the amortizing bond.

```
Price = bondbybk(BKT, CouponRate, Settle, Maturity, 'Period', Period,
'Face', Face)

Price =

  102.3155
```

Compare the results with price of a vanilla bond.

```
PriceVanilla = bondbybk(BKT, CouponRate, Settle, Maturity, Period)

PriceVanilla =

  102.4205
```

## See Also

bkprice | bktree | cfamounts | hwprice | hwtree | instbond

# bondbyhjm

## Purpose

Price bond from Heath-Jarrow-Morton interest-rate tree

## Syntax

```
[Price, PriceTree] = bondbyhjm(HJMTree, CouponRate, Settle,
Maturity)
[Price, PriceTree] = bondbyhjm(HJMTree, CouponRate, Settle,
Maturity, Period, Basis, EndMonthRule, IssueDate,
FirstCouponDate,  LastCouponDate, StartDate, Face,
Options)
[Price, PriceTree] = bondbyhjm(HJMTree, CouponRate,
Settle, Maturity, Name,Value)
```

## Input Arguments

| | |
|---|---|
| `HJMTree` | Forward rate tree structure created by `hjmtree`. |
| `CouponRate` | Decimal annual rate. `CouponRate` is a `NINST`-by-`1` vector or `NINST`-by-`1` cell array of decimal annual rates, or decimal annual rate schedules. For the latter case of a variable coupon schedule, each element of the cell array is a `NumDates`-by-`2` cell array, where the first column is dates and the second column is its associated rate. The date indicates the last day that the coupon rate is valid. |
| `Settle` | Settlement date. A vector of serial date numbers or date strings. `Settle` must be earlier than `Maturity`. |
| `Maturity` | Maturity date. A vector of serial date numbers or date strings. |

The `Settle` date for every bond is set to the `ValuationDate` of the HJM tree. The bond argument `Settle` is ignored.

### Ordered Input or Name-Value Pair Arguments

Enter the following optional inputs using an ordered syntax or as name-value pair arguments. You cannot mix ordered syntax with name-value pair arguments.

#### Period

Coupons per year of the bond. A vector of integers. Values are `1`, `2`, `3`, `4`, `6`, and `12`.

**Default:** `2`

#### Basis

Day-count basis of the instrument. A vector of integers.

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (PSA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ISMA)
- 9 = actual/360 (ISMA)
- 10 = actual/365 (ISMA)
- 11 = 30/360E (ISMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see basis.

**Default:** `0` (actual/actual)

### EndMonthRule

End-of-month rule. A `NINST`-by-`1` vector. This rule applies only when `Maturity` is an end-of-month date for a month having 30 or fewer days.

- `0` = Ignore rule, meaning that a bond coupon payment date is always the same numerical day of the month.
- `1` = Set rule on, meaning that a bond coupon payment date is always the last actual day of the month.

**Default:** `1`

### IssueDate

Date when a bond was issued.

### FirstCouponDateDate

Date when a bond makes its first coupon payment; used when bond has an irregular first coupon period. When `FirstCouponDate` and `LastCouponDate` are both specified, `FirstCouponDate` takes precedence in determining the coupon payment structure. If you do not specify a `FirstCouponDate`, the cash flow payment dates are determined from other inputs.

### LastCouponDateDate

Last coupon date of a bond before the maturity date; used when bond has an irregular last coupon period. In the absence of a specified `FirstCouponDate`, a specified `LastCouponDate` determines the coupon structure of the bond. The coupon structure of a bond is truncated at the `LastCouponDate`, regardless of where it falls, and is followed only by the bond's maturity cash flow date. If you do not specify a `LastCouponDate`, the cash flow payment dates are determined from other inputs.

### StartDate

Date when a bond actually starts (the date from which a bond cash flow is considered). To make an instrument forward-starting, specify this date as a future date. If you do not specify `StartDate`, the effective start date is the `Settle` date.

#### Face

Face or par value. `Face` is a `NINST`-by-`1` vector or `NINST`-by-`1` cell array of face values, or face value schedules. For the latter case, each element of the cell array is a `NumDates`-by-`2` cell array, where the first column is dates and the second column is its associated face value. The date indicates the last day that the face value is valid.

**Default:** `100`

#### Options

Derivatives pricing options structure created with `derivset`.

### Name-Value Pair Arguments

Specify optional comma-separated pairs of `Name,Value` arguments, where `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside single quotes (`' '`). You can specify several name and value pair arguments in any order as `Name1,Value1,...,NameN,ValueN`.

#### AdjustCashFlowsBasis

Adjust the cash flows based on the actual period day count. `NINST`-by-`1` of logicals.

**Default:** `false`

#### BusinessDayConvention

Require payment dates to be business dates. `NINST`-by-`1` cell array with possible choices of business day convention:

- `actual`

- `follow`
- `modifiedfollow`
- `previous`
- `modifiedprevious`

**Default:** `actual`

### Holidays

Holidays used for business day convention. `NHOLIDAYS-by-1` of MATLAB date numbers.

**Default:** If no dates are specified, `holidays.m` is used.

## Description

`[Price, PriceTree] = bondbyhjm(HJMTree, CouponRate, Settle, Maturity)` computes the price of a bond from an HJM forward-rate tree.

`[Price, PriceTree] = bondbyhjm(HJMTree, CouponRate, Settle, Maturity, Period, Basis, EndMonthRule, IssueDate, FirstCouponDate, LastCouponDate, StartDate, Face, Options)` computes the price of a bond from an HJM forward-rate tree with optional input arguments.

`[Price, PriceTree] = bondbyhjm(HJMTree, CouponRate, Settle, Maturity, Name,Value)` computes the price of a bond from an HJM forward-rate tree with additional options specified by one or more `Name,Value` pair arguments.

`Price` is a number of instruments (`NINST`)-by-`1` matrix of expected prices at time 0.

`PriceTree` is a structure of trees containing vectors of instrument prices and accrued interest, and a vector of observation times for each node. Within `PriceTree`:

- `PriceTree.PBush` contains the clean prices.
- `PriceTree.AIBush` contains the accrued interest.

- `PriceTree.tObs` contains the observation times.

`bondbyhjm` computes prices of vanilla bonds, stepped coupon bonds, and amortizing bonds with no market purchase option and no call provisions.

## Definitions

### Vanilla Bond

A vanilla coupon bond is a security representing an obligation to repay a borrowed amount at a designated time and to make periodic interest payments until that time. The issuer of a bond makes the periodic interest payments until the bond matures. At maturity, the issuer pays to the holder of the bond the principal amount owed (face value) and the last interest payment.

### Stepped Coupon Bond

A step-up and step-down bond is a debt security with a predetermined coupon structure over time. With these instruments, coupons increase (step up) or decrease (step down) at specific times during the life of the bond.

### Bond with an Amortization Schedule

An amortized bond is treated as an asset, with the discount amount being amortized to interest expense over the life of the bond.

## Examples

### Price a Bond Using an HJM Tree

Price a 4% bond using an HJM interest-rate tree.

Load `deriv.mat`, which provides `HJMTree`. The `HJMTree` structure contains the time and interest-rate information needed to price the bond.

```
load deriv.mat;
```

Define the bond using the required arguments. Other arguments use defaults.

```
CouponRate = 0.04;
Settle = '01-Jan-2000';
```

```
Maturity = '01-Jan-2004';
```

Use `bondbyhjm` to compute the price of the bond.

```
Price = bondbyhjm(HJMTree, CouponRate, Settle, Maturity)
Warning: Not all cash flows are aligned with the tree. Result will
be approximated.

Price =

  97.5280
```

### Price a Stepped Coupon Bond

Price single stepped coupon bonds using market data.

Define the interest-rate term structure.

```
Rates = [0.035; 0.042147; 0.047345; 0.052707];
ValuationDate = 'Jan-1-2010';
StartDates = ValuationDate;
EndDates = {'Jan-1-2011'; 'Jan-1-2012'; 'Jan-1-2013'; 'Jan-1-2014'};
Compounding = 1;
```

Create the `RateSpec`.

```
RS = intenvset('ValuationDate', ValuationDate, 'StartDates', StartDates,...
'EndDates', EndDates,'Rates', Rates, 'Compounding', Compounding);

RS =

          FinObj: 'RateSpec'
     Compounding: 1
            Disc: [4x1 double]
           Rates: [4x1 double]
        EndTimes: [4x1 double]
      StartTimes: [4x1 double]
        EndDates: [4x1 double]
      StartDates: 734139
```

```
    ValuationDate: 734139
            Basis: 0
     EndMonthRule: 1
```

Create the stepped bond instrument.

```
Settle = '01-Jan-2010';
Maturity = {'01-Jan-2011';'01-Jan-2012';'01-Jan-2013';'01-Jan-2014'};
CouponRate = {{'01-Jan-2012' .0425;'01-Jan-2014' .0750}};
Period = 1;
```

Build the HJM tree using the following market data:

```
Volatility = [.2; .19; .18; .17];
CurveTerm = [ 1;  2;   3;   4];
HJMTimeSpec = hjmtimespec(ValuationDate, EndDates);
HJMVolSpec = hjmvolspec('Proportional', Volatility, CurveTerm, 1e6);
HJMT = hjmtree(HJMVolSpec,RS,HJMTimeSpec);
```

Compute the price of the stepped coupon bonds.

```
PHJM= bondbyhjm(HJMT, CouponRate, Settle,Maturity , Period)
```

```
PHJM =

  100.7246
  100.0945
  101.5900
  102.0820
```

### Price a Bond with an Amortization Schedule

Price a bond with an amortization schedule using the `Face` input argument to define the schedule.

Define the interest-rate term structure.

```
Rates = 0.065;
ValuationDate = '1-Jan-2011';
StartDates = ValuationDate;
```

```
EndDates=  '1-Jan-2017';
Compounding = 1;
```

Create the `RateSpec`.

```
RateSpec = intenvset('ValuationDate', ValuationDate,'StartDates', StartDates,...
'EndDates', EndDates,'Rates', Rates, 'Compounding', Compounding)
```

```
RateSpec =

           FinObj: 'RateSpec'
      Compounding: 1
             Disc: 0.6853
            Rates: 0.0650
         EndTimes: 6
       StartTimes: 0
         EndDates: 736696
       StartDates: 734504
    ValuationDate: 734504
            Basis: 0
     EndMonthRule: 1
```

Create the bond instrument. The bond has a coupon rate of 7%, a period of one year, and matures on 1-Jan-2017.

```
CouponRate = 0.07;
Settle ='1-Jan-2011';
Maturity = '1-Jan-2017';
Period = 1;
Face = {{'1-Jan-2015' 100;'1-Jan-2016' 90;'1-Jan-2017' 80}};
```

Build the HJM tree using the following market data:

```
Volatility = [.2; .19; .18; .17];
CurveTerm = [ 1;  2;   3;   4];
MaTree = {'Jan-1-2012'; 'Jan-1-2013'; 'Jan-1-2014'; 'Jan-1-2015';...
'Jan-1-2016'; 'Jan-1-2017'};
HJMTimeSpec = hjmtimespec(ValuationDate, MaTree);
```

```
HJMVolSpec = hjmvolspec('Proportional', Volatility, CurveTerm, 1e6);
HJMT = hjmtree(HJMVolSpec,RateSpec,HJMTimeSpec);
```

Compute the price of the amortizing bond.

```
Price = bondbyhjm(HJMT, CouponRate, Settle, Maturity, 'Period',...
Period, 'Face' , Face)
```

```
Price =

  102.3155
```

Compare the results with price of a vanilla bond.

```
PriceVanilla = bondbyhjm(HJMT, CouponRate, Settle, Maturity, Period)
```

```
PriceVanilla =

  102.4205
```

## See Also

`hjmtree` | `cfamounts` | `hjmprice` | `instbond`

# bondbyhw

## Purpose

Price bond from Hull-White interest-rate tree

## Syntax

```
[Price, PriceTree] = bondbyhw(HWTree, CouponRate,
Settle, Maturity)
[Price, PriceTree] = bondbyhw(HWTree, CouponRate,
Settle, Maturity, Period, Basis, EndMonthRule, IssueDate,
FirstCouponDate, LastCouponDate, StartDate, Face,
Options)
[Price, PriceTree] = bondbyhw(HWTree, CouponRate,
Settle, Maturity, Name,Value)
```

## Input Arguments

| | |
|---|---|
| `HWTree` | Forward-rate tree structure created by `hwtree`. |
| `CouponRate` | Decimal annual rate. `CouponRate` is a `NINST`-by-`1` vector or `NINST`-by-`1` cell array of decimal annual rates, or decimal annual rate schedules. For the latter case of a variable coupon schedule, each element of the cell array is a `NumDates`-by-`2` cell array, where the first column is dates and the second column is its associated rate. The date indicates the last day that the coupon rate is valid. |
| `Settle` | Settlement date. A vector of serial date numbers or date strings. `Settle` must be earlier than `Maturity`. |
| `Maturity` | Maturity date. A vector of serial date numbers or date strings. |

The `Settle` date for every bond is set to the `ValuationDate` of the HW tree. The bond argument `Settle` is ignored.

### Ordered Input or Name-Value Pair Arguments

Enter the following optional inputs using an ordered syntax or as name-value pair arguments. You cannot mix ordered syntax with name-value pair arguments.

#### Period

Coupons per year of the bond. A vector of integers. Values are `1`, `2`, `3`, `4`, `6`, and `12`.

**Default:** `2`

#### Basis

Day-count basis of the instrument. A vector of integers.

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (PSA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ISMA)
- 9 = actual/360 (ISMA)
- 10 = actual/365 (ISMA)
- 11 = 30/360E (ISMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see basis.

**Default:** `0` (actual/actual)

**EndMonthRule**

End-of-month rule. `NINST`-by-`1` vector. This rule applies only when `Maturity` is an end-of-month date for a month having 30 or fewer days.

- `0` = Ignore rule, meaning that a bond coupon payment date is always the same numerical day of the month.
- `1` = Set rule on, meaning that a bond coupon payment date is always the last actual day of the month.

**Default:** `1`

**IssueDate**

Date when a bond was issued.

**FirstCouponDateDate**

Date when a bond makes its first coupon payment; used when bond has an irregular first coupon period. When `FirstCouponDate` and `LastCouponDate` are both specified, `FirstCouponDate` takes precedence in determining the coupon payment structure. If you do not specify a `FirstCouponDate`, the cash flow payment dates are determined from other inputs.

**LastCouponDateDate**

Last coupon date of a bond before the maturity date; used when bond has an irregular last coupon period. In the absence of a specified `FirstCouponDate`, a specified `LastCouponDate` determines the coupon structure of the bond. The coupon structure of a bond is truncated at the `LastCouponDate`, regardless of where it falls, and is followed only by the bond's maturity cash flow date. If you do not specify a `LastCouponDate`, the cash flow payment dates are determined from other inputs.

**StartDate**

Date when a bond actually starts (the date from which a bond cash flow is considered). To make an instrument forward-starting, specify this date as a future date. If you do not specify `StartDate`, the effective start date is the `Settle` date.

### Face

Face or par value. `Face` is a `NINST`-by-`1` vector or `NINST`-by-`1` cell array of face values, or face value schedules. For the latter case, each element of the cell array is a `NumDates`-by-`2` cell array, where the first column is dates and the second column is its associated face value. The date indicates the last day that the face value is valid.

**Default:** `100`

### Options

Derivatives pricing options structure created with `derivset`.

## Name-Value Pair Arguments

Specify optional comma-separated pairs of `Name,Value` arguments, where `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside single quotes (`' '`). You can specify several name and value pair arguments in any order as `Name1,Value1,...,NameN,ValueN`.

### AdjustCashFlowsBasis

Adjust the cash flows based on the actual period day count. `NINST`-by-`1` of logicals.

**Default:** `false`

### BusinessDayConvention

Require payment dates to be business dates. `NINST`-by-`1` cell array with possible choices of business day convention:

- `actual`

- `follow`
- `modifiedfollow`
- `previous`
- `modifiedprevious`

**Default:** `actual`

**Holidays**

Holidays used for business day convention. `NHOLIDAYS`-by-`1` of MATLAB date numbers.

**Default:** If no dates are specified, `holidays.m` is used.

## Description

`[Price, PriceTree] = bondbyhw(HWTree, CouponRate, Settle, Maturity)` computes the price of a bond from a Hull-White interest-rate tree.

`[Price, PriceTree] = bondbyhw(HWTree, CouponRate, Settle, Maturity, Period, Basis, EndMonthRule, IssueDate, FirstCouponDate, LastCouponDate, StartDate, Face, Options)` computes the price of a bond from a Hull-White interest-rate tree with optional input arguments.

`[Price, PriceTree] = bondbyhw(HWTree, CouponRate, Settle, Maturity, Name,Value)` computes the price of a bond from a Hull-White interest-rate tree with additional options specified by one or more `Name,Value` pair arguments.

`Price` is a number of instruments (`NINST`)-by-`1` matrix of expected prices at time 0.

`PriceTree` is a structure of trees containing vectors of instrument prices and accrued interest, and a vector of observation times for each node. Within `PriceTree`:

- `PriceTree.PTree` contains the clean prices.

- `PriceTree.AITree` contains the accrued interest.
- `PriceTree.tObs` contains the observation times.

`bondbyhw` computes prices of vanilla bonds, stepped coupon bonds, and amortizing bonds with no market purchase option and no call provisions.

## Definitions

### Vanilla Bond

A vanilla coupon bond is a security representing an obligation to repay a borrowed amount at a designated time and to make periodic interest payments until that time. The issuer of a bond makes the periodic interest payments until the bond matures. At maturity, the issuer pays to the holder of the bond the principal amount owed (face value) and the last interest payment.

### Stepped Coupon Bond

A step-up and step-down bond is a debt security with a predetermined coupon structure over time. With these instruments, coupons increase (step up) or decrease (step down) at specific times during the life of the bond.

### Bond with an Amortization Schedule

An amortized bond is treated as an asset, with the discount amount being amortized to interest expense over the life of the bond.

## Examples

### Price a Bond Using the HW Tree

Price a 4% bond using a Hull-White interest-rate tree.

Load `deriv.mat`, which provides `HWTree`. The `HWTree` structure contains the time and interest-rate information needed to price the bond.

```
load deriv.mat;
```

Define the bond using the required arguments. Other arguments use defaults.

```
CouponRate = 0.04;
```

```
Settle = '01-Jan-2004';
Maturity = '31-Dec-2008';
```

Use `bondbyhw` to compute the price of the bond.

```
Price = bondbyhw(HWTree, CouponRate, Settle, Maturity)
Warning: Not all cash flows are aligned with the tree. Result will
be approximated.

Price =

  98.0483
```

### Price a Stepped Coupon Bond

Price single stepped coupon bonds using market data.

Define the interest-rate term structure.

```
Rates = [0.035; 0.042147; 0.047345; 0.052707];
ValuationDate = 'Jan-1-2010';
StartDates = ValuationDate;
EndDates = {'Jan-1-2011'; 'Jan-1-2012'; 'Jan-1-2013'; 'Jan-1-2014'};
Compounding = 1;
```

Create the `RateSpec`.

```
RS = intenvset('ValuationDate', ValuationDate, 'StartDates', StartDates,...
'EndDates', EndDates,'Rates', Rates, 'Compounding', Compounding)

RS =

           FinObj: 'RateSpec'
      Compounding: 1
             Disc: [4x1 double]
            Rates: [4x1 double]
         EndTimes: [4x1 double]
       StartTimes: [4x1 double]
         EndDates: [4x1 double]
```

```
       StartDates: 734139
    ValuationDate: 734139
            Basis: 0
     EndMonthRule: 1
```

Create the stepped bond instrument.

```
Settle = '01-Jan-2010';
Maturity = {'01-Jan-2011';'01-Jan-2012';'01-Jan-2013';'01-Jan-2014'};
CouponRate = {{'01-Jan-2012' .0425;'01-Jan-2014' .0750}};
Period = 1;
```

Build the HW tree using the following market data:

```
VolDates = ['1-Jan-2011'; '1-Jan-2012'; '1-Jan-2013'; '1-Jan-2014'];
VolCurve = 0.01;
AlphaDates = '01-01-2014';
AlphaCurve = 0.1;

HWVolSpec = hwvolspec(RS.ValuationDate, VolDates, VolCurve,...
AlphaDates, AlphaCurve);
HWTimeSpec = hwtimespec(RS.ValuationDate, VolDates, Compounding);
HWT = hwtree(HWVolSpec, RS, HWTimeSpec);
```

Compute the price of the stepped coupon bonds.

```
PHW= bondbyhw(HWT, CouponRate, Settle,Maturity , Period)
```

```
PHW =

  100.7246
  100.0945
  101.5900
  102.0820
```

### Price Two Bonds with Amortization Schedules

Price two bonds with amortization schedules using the `Face` input argument to define the schedules.

Define the interest rate term structure.

```
Rates = 0.035;
ValuationDate = '1-Nov-2011';
StartDates = ValuationDate;
EndDates = '1-Nov-2017';
Compounding = 1;
```

Create the `RateSpec`.

```
RateSpec = intenvset('ValuationDate', ValuationDate,'StartDates', StartDates,...
'EndDates', EndDates,'Rates', Rates, 'Compounding', Compounding);
```

```
RateSpec =

           FinObj: 'RateSpec'
      Compounding: 1
             Disc: 0.8135
            Rates: 0.0350
         EndTimes: 6
       StartTimes: 0
         EndDates: 737000
       StartDates: 734808
    ValuationDate: 734808
            Basis: 0
     EndMonthRule: 1
```

Create the bond instrument. The bonds have a coupon rate of 4% and 3.85%, a period of one year, and mature on 1-Nov-2017.

```
CouponRate = [0.04; 0.0385];
Settle ='1-Nov-2011';
Maturity = '1-Nov-2017';
Period = 1;
```

Define the amortizing schedule.

```
Face = {{'1-Nov-2015' 100;'1-Nov-2016' 85;'1-Nov-2017' 70};
{'1-Nov-2015' 100;'1-Nov-2016' 90;'1-Nov-2017' 80}};
```

Build the HW tree and assume the volatility to be 10%.

```
VolDates = ['1-Nov-2012'; '1-Nov-2013';'1-Nov-2014';'1-Nov-2015';'1-Nov-2016';'1-Nov-2017'];
VolCurve = 0.1;
AlphaDates = '01-01-2017';
AlphaCurve = 0.1;

HWVolSpec = hwvolspec(RateSpec.ValuationDate, VolDates, VolCurve,...
AlphaDates, AlphaCurve);
HWTimeSpec = hwtimespec(RateSpec.ValuationDate, VolDates, Compounding);
HWT = hwtree(HWVolSpec, RateSpec, HWTimeSpec);
```

Compute the price of the amortizing bonds.

```
Price = bondbyhw(HWT, CouponRate, Settle, Maturity, 'Period',Period,...
'Face', Face)
```

```
Price =

  102.4791
  101.7786
```

## See Also

`bkprice` | `bktree` | `cfamounts` | `hwprice` | `hwtree` | `instbond`

# bondbyzero

## Purpose

Price bond from set of zero curves

## Syntax

```
[Price, PriceNoAI, CFlowAmounts, CFlowDates] =
bondbyzero(RateSpec, CouponRate, Settle, Maturity)
[Price, PriceNoAI, CFlowAmounts, CFlowDates] =
bondbyzero(RateSpec, CouponRate, Settle, Maturity,
Period, Basis, EndMonthRule, IssueDate,
FirstCouponDate, LastCouponDate, StartDate, Face)
[Price, PriceNoAI, CFlowAmounts, CFlowDates] =
bondbyzero(RateSpec, CouponRate, Settle,
Maturity, Name, Value)
```

## Description

`[Price, PriceNoAI, CFlowAmounts, CFlowDates] = bondbyzero(RateSpec, CouponRate, Settle, Maturity)` returns a `NINST`-by-`NUMCURVES` matrix of clean bond prices. Each column arises from one of the zero curves.

`[Price, PriceNoAI, CFlowAmounts, CFlowDates] = bondbyzero(RateSpec, CouponRate, Settle, Maturity, Period, Basis, EndMonthRule, IssueDate, FirstCouponDate, LastCouponDate, StartDate, Face)` returns a `NINST`-by-`NUMCURVES` matrix of clean bond prices. Each column arises from one of the zero curves.

`[Price, PriceNoAI, CFlowAmounts, CFlowDates] = bondbyzero(RateSpec, CouponRate, Settle, Maturity, Name, Value)` returns a `NINST`-by-`NUMCURVES` matrix of clean bond prices (each column arises from one of the zero curves) with additional options specified by one or more `Name, Value` pair arguments.

`bondbyzero` computes prices of vanilla bonds, stepped coupon bonds, and amortizing bonds with no market purchase option and no call provisions.

## Input Arguments

### RateSpec

Structure containing the properties of an interest-rate structure. See `intenvset` for information on creating `RateSpec`.

### CouponRate

Decimal annual rate. `CouponRate` is a `NINST`-by-`1` vector or `NINST`-by-`1` cell array of decimal annual rates, or decimal annual rate schedules. For the latter case of a variable coupon schedule, each element of the cell array is a `NumDates`-by-`2` cell array, where the first column is dates and the second column is its associated rate. The date indicates the last day that the coupon rate is valid.

### Settle

Settlement date. `Settle` must be either a scalar or `NINST`-by-`1` vector of serial date numbers or date strings of the same value which represent the settlement date for each bond. `Settle` must be earlier than `Maturity`.

### Maturity

Maturity date. `NINST`-by-`1` vector of dates representing the maturity date for each swap.

## Ordered Input or Name-Value Pair Arguments

Enter the following optional inputs using an ordered syntax or as name-value pair arguments. You cannot mix ordered syntax with name-value pair arguments.

### Period

Coupons per year of the bond. A vector of integers. Values are `1`, `2`, `3`, `4`, `6`, and `12`.

**Default:** `2`

### Basis

Day-count basis of the instrument. A vector of integers.

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (PSA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ISMA)
- 9 = actual/360 (ISMA)
- 10 = actual/365 (ISMA)
- 11 = 30/360E (ISMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see basis.

**Default:** `0` (actual/actual)

### EndMonthRule

End-of-month rule. A `NINST`-by-`1` vector. This rule applies only when `Maturity` is an end-of-month date for a month having 30 or fewer days.

- `0` = Ignore rule, meaning that a bond coupon payment date is always the same numerical day of the month.
- `1` = Set rule on, meaning that a bond coupon payment date is always the last actual day of the month.

**Default:** `1`

**IssueDate**

Date when a bond was issued.

**FirstCouponDateDate**

Date when a bond makes its first coupon payment; used when bond has an irregular first coupon period. When `FirstCouponDate` and `LastCouponDate` are both specified, `FirstCouponDate` takes precedence in determining the coupon payment structure. If you do not specify a `FirstCouponDate`, the cash flow payment dates are determined from other inputs.

**LastCouponDateDate**

Last coupon date of a bond before the maturity date; used when bond has an irregular last coupon period. In the absence of a specified `FirstCouponDate`, a specified `LastCouponDate` determines the coupon structure of the bond. The coupon structure of a bond is truncated at the `LastCouponDate`, regardless of where it falls, and is followed only by the bond's maturity cash flow date. If you do not specify a `LastCouponDate`, the cash flow payment dates are determined from other inputs.

**StartDate**

Date when a bond actually starts (the date from which a bond cash flow is considered). To make an instrument forward-starting, specify this date as a future date. If you do not specify `StartDate`, the effective start date is the `Settle` date.

**Face**

Face or par value. `Face` is a `NINST`-by-`1` vector or `NINST`-by-`1` cell array of face values, or face value schedules. For the latter case, each element of the cell array is a `NumDates`-by-`2` cell array, where the first column is dates and the second column is its associated face value. The date indicates the last day that the face value is valid.

**Default:** `100`

### Options

Derivatives pricing options structure created with `derivset`.

## Name-Value Pair Arguments

Specify optional comma-separated pairs of `Name,Value` arguments, where `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside single quotes (`' '`). You can specify several name and value pair arguments in any order as `Name1,Value1,...,NameN,ValueN`.

### AdjustCashFlowsBasis

Adjust the cash flows based on the actual period day count. `NINST`-by-`1` of logicals.

**Default:** `false`

### BusinessDayConvention

Require payment dates to be business dates. `NINST`-by-`1` cell array with possible choices of business day convention:

- `actual`
- `follow`
- `modifiedfollow`
- `previous`
- `modifiedprevious`

**Default:** `actual`

### Holidays

Holidays used for business day convention. `NHOLIDAYS`-by-`1` of MATLAB date numbers.

**Default:** If no dates are specified, `holidays.m` is used.

## Output Arguments

**Price**

`NINST`-by-`NUMCURVES` matrix of clean bond prices. Each column arises from one of the zero curves.

**PriceNoAI**

`NINST`-by-`NUMCURVES` matrix of dirty bond price (clean + accrued interest). Each column arises from one of the zero curves.

**CFlowAmounts**

`NINST`-by-`NUMCFS` matrix of cash flows for each bond

**CFlowDates**

`NUMCFS`-by-`1` matrix of payment dates for each bond

## Definitions

### Vanilla Bond

A vanilla coupon bond is a security representing an obligation to repay a borrowed amount at a designated time and to make periodic interest payments until that time. The issuer of a bond makes the periodic interest payments until the bond matures. At maturity, the issuer pays to the holder of the bond the principal amount owed (face value) and the last interest payment.

### Stepped Coupon Bond

A step-up and step-down bond is a debt security with a predetermined coupon structure over time. With these instruments, coupons increase (step up) or decrease (step down) at specific times during the life of the bond.

### Bond with an Amortization Schedule

An amortized bond is treated as an asset, with the discount amount being amortized to interest expense over the life of the bond.

## Examples

### Price a Vanilla Bond

Price a 4% bond using a zero curve.

Load `deriv.mat`, which provides `ZeroRateSpec`, the interest-rate term structure, needed to price the bond.

```
load deriv.mat;
CouponRate = 0.04;
Settle = '01-Jan-2000';
Maturity = '01-Jan-2004';
Price = bondbyzero(ZeroRateSpec, CouponRate, Settle, Maturity)
```

```
Price =

   97.5334
```

### Price a Stepped Coupon Bond

Price single stepped coupon bonds using market data.

Define data for the interest-rate term structure.

```
Rates = [0.035; 0.042147; 0.047345; 0.052707];
ValuationDate = 'Jan-1-2010';
StartDates = ValuationDate;
EndDates = {'Jan-1-2011'; 'Jan-1-2012'; 'Jan-1-2013'; 'Jan-1-2014'};
Compounding = 1;
```

Create the `RateSpec`.

```
RS = intenvset('ValuationDate', ValuationDate, 'StartDates', StartDates,...
'EndDates', EndDates,'Rates', Rates, 'Compounding', Compounding)
```

```
RS =

           FinObj: 'RateSpec'
      Compounding: 1
             Disc: [4x1 double]
```

```
           Rates: [4x1 double]
        EndTimes: [4x1 double]
      StartTimes: [4x1 double]
        EndDates: [4x1 double]
      StartDates: 734139
   ValuationDate: 734139
           Basis: 0
    EndMonthRule: 1
```

Create the stepped bond instrument.

```
Settle = '01-Jan-2010';
Maturity = {'01-Jan-2011';'01-Jan-2012';'01-Jan-2013';'01-Jan-2014'};
CouponRate = {{'01-Jan-2012' .0425;'01-Jan-2014' .0750}};
Period = 1;
```

Compute the price of the stepped coupon bonds.

```
PZero= bondbyzero(RS, CouponRate, Settle, Maturity ,Period)
```

```
PZero =

  100.7246
  100.0945
  101.5900
  102.0820
```

## Price a Bond with an Amortizing Schedule

Price a bond with an amortizing schedule using the `Face` input argument to define the schedule.

Define data for the interest-rate term structure.

```
Rates = 0.065;
ValuationDate = '1-Jan-2011';
StartDates = ValuationDate;
EndDates=  '1-Jan-2017';
Compounding = 1;
```

Create the `RateSpec`.

```
RateSpec = intenvset('ValuationDate', ValuationDate,'StartDates', StartDates,...
'EndDates', EndDates,'Rates', Rates, 'Compounding', Compounding)
```

```
RateSpec = 

           FinObj: 'RateSpec'
      Compounding: 1
             Disc: 0.6853
            Rates: 0.0650
         EndTimes: 6
       StartTimes: 0
         EndDates: 736696
       StartDates: 734504
    ValuationDate: 734504
            Basis: 0
     EndMonthRule: 1
```

Create and price the amortizing bond instrument. The bond has a coupon rate of 7%, a period of one year, and matures on 1-Jan-2017.

```
CouponRate = 0.07;
Settle ='1-Jan-2011';
Maturity = '1-Jan-2017';
Period = 1;
Face = {{'1-Jan-2015' 100;'1-Jan-2016' 90;'1-Jan-2017' 80}};
Price = bondbyzero(RateSpec, CouponRate, Settle, Maturity, 'Period',...
Period, 'Face', Face)
```

```
Price =

  102.3155
```

Compare the results with price of a vanilla bond.

```
PriceVanilla = bondbyzero(RateSpec, CouponRate, Settle, Maturity,Period)
```

```
PriceVanilla =

  102.4205
```

## See Also

| `swapbyzero` | `cfamounts` | `cfbyzero` | `fixedbyzero` | `floatbyzero`

# bushpath

## Purpose

Extract entries from node of bushy tree

## Syntax

```
Values = bushpath(Tree, BranchList)
```

## Arguments

| | |
|---|---|
| `Tree` | Bushy tree. |
| `BranchList` | Number of paths (`NUMPATHS`) by path length (`PATHLENGTH`) matrix containing the sequence of branchings. |

## Description

`Values = bushpath(Tree, BranchList)` extracts entries of a node of a bushy tree. The node path is described by the sequence of branchings taken, starting at the root. The top branch is number 1, the second-to-top is 2, and so on. Set the branch sequence to zero to obtain the entries at the root node.

`Values` is a number of values (`NUMVALS`)-by-`NUMPATHS` matrix containing the retrieved entries of a bushy tree.

## Examples

Create an HJM tree by loading the example file.

```
load deriv.mat;
```

Then

```
FwdRates = bushpath(HJMTree.FwdTree, [1 2 1])
```

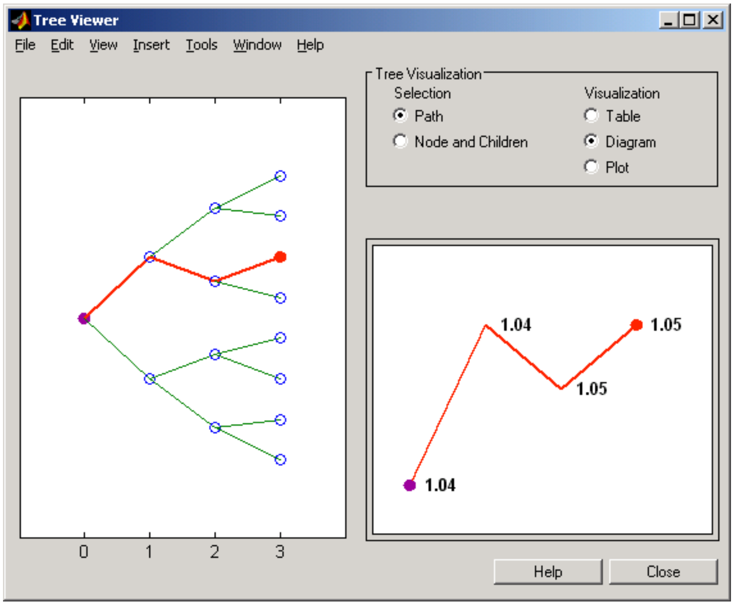
returns the rates at the tree nodes located by taking the up branch, then the down branch, and finally the up branch again.

```
FwdRates =

    1.0356
    1.0364
    1.0526
```

```
    1.0463
```

You can visualize this with the `treeviewer` function.

```
treeviewer(HJMTree)
```



Tree Visualization

Selection
● Path
○ Node and Children

Visualization
○ Table
● Diagram
○ Plot

## See Also

`bushshape` | `mkbush`

# bushshape

## Purpose

Retrieve shape of bushy tree

## Syntax

```
[NumLevels, NumChild, NumPos, NumStates,
Trim] = bushshape(Tree)
```

## Arguments

| | |
|---|---|
| `Tree` | Bushy tree. |

## Description

`[NumLevels, NumChild, NumPos, NumStates, Trim] = bushshape(Tree)` returns information on a bushy tree's shape.

`NumLevels` is the number of time levels of the tree.

`NumChild` is a `1`-by-number of levels (`NUMLEVELS`) vector with the number of branches (children) of the nodes in each level.

`NumPos` is a `1`-by-`NUMLEVELS` vector containing the length of the state vectors in each level.

`NumStates` is a `1`-by-`NUMLEVELS` vector containing the number of state vectors in each level.

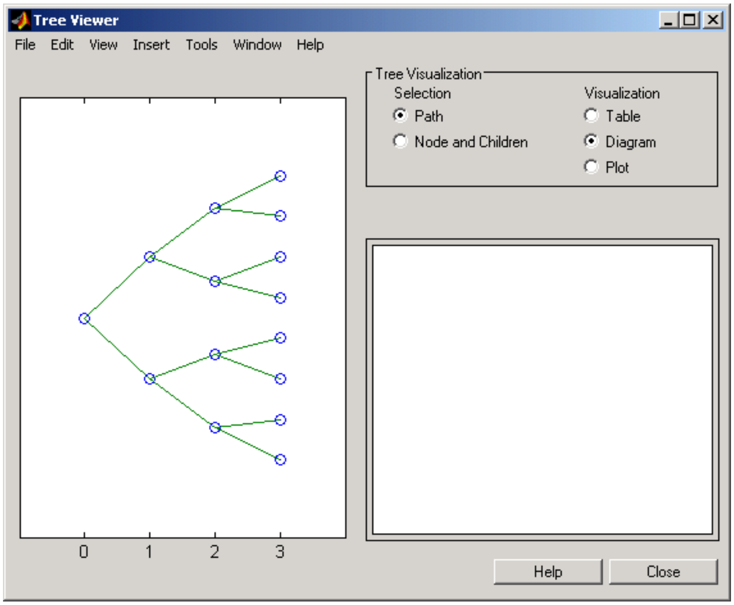
`Trim` is `1` if `NumPos` decreases by 1 when moving from one time level to the next. Otherwise, it is `0`.

## Examples

Create an HJM tree by loading the example file.

```
load deriv.mat;
```

With `treeviewer` you can see the general shape of the HJM interest-rate tree.



Tree Visualization

Selection
● Path
○ Node and Children

Visualization
○ Table
● Diagram
○ Plot

With this tree

```
[NumLevels, NumChild, NumPos, NumStates, Trim] =...
bushshape(HJMTree.FwdTree)
```

returns

```
NumLevels  =
     4

NumChild   =
     2     2     2     0

NumPos     =
     4     3     2     1

NumStates  =
     1     2     4     8
```

```
Trim =
     1
```

You can recreate this tree using the `mkbush` function.

```
Tree = mkbush(NumLevels, NumChild(1), NumPos(1), Trim);
Tree = mkbush(NumLevels, NumChild, NumPos);
```

**See Also** `bushpath` | `mkbush`

## Purpose

Price cap instrument from Black-Derman-Toy interest-rate tree

## Syntax

```
[Price, PriceTree] = capbybdt(BDTTree, Strike, Settle,
Maturity, Reset, Basis, Principal, Options)
```

## Arguments

| | |
|---|---|
| `BDTTree` | Interest-rate tree structure created by `bdttree`. |
| `Strike` | Number of instruments (`NINST`)-by-`1` vector of rates at which the cap is exercised. |
| `Settle` | Settlement dates. `NINST`-by-`1` vector of dates representing the settlement dates of the cap. |
| `Maturity` | `NINST`-by-`1` vector of dates representing the maturity dates of the cap. |
| `Reset` | (Optional) `NINST`-by-`1` vector representing the frequency of payments per year. Default = `1`. |
| `Basis` | (Optional) Day-count basis of the instrument. A vector of integers.<br>• 0 = actual/actual (default)<br>• 1 = 30/360 (SIA)<br>• 2 = actual/360<br>• 3 = actual/365<br>• 4 = 30/360 (BMA)<br>• 5 = 30/360 (ISDA)<br>• 6 = 30/360 (European)<br>• 7 = actual/365 (Japanese)<br>• 8 = actual/actual (ICMA)<br>• 9 = actual/360 (ICMA)<br>• 10 = actual/365 (ICMA) |

|  |  |
|---|---|
|  | • 11 = 30/360E (ICMA)<br>• 12 = actual/actual (ISDA)<br>• 13 = BUS/252<br>For more information, see basis. |
| `Principal` | (Optional) The notional principal amount. Default = `100`. |
| `Options` | (Optional) Derivatives pricing options structure created with `derivset`. |

## Description

`[Price, PriceTree] = capbybdt(BDTTree, Strike, Settle, Maturity, Reset, Basis, Principal, Options)` computes the price of a cap instrument from a BDT interest-rate tree.

`Price` is the expected price of the cap at time 0.

`PriceTree` is the tree structure with values of the cap at each node.

The `Settle` date for every cap is set to the `ValuationDate` of the BDT tree. The cap argument `Settle` is ignored.

## Examples

**Example 1**. Price a 3% cap instrument using a BDT interest-rate tree.

Load the file `deriv.mat`, which provides `BDTTree`. The `BDTTree` structure contains the time and interest-rate information needed to price the cap instrument.

```
load deriv.mat;
```

Set the required values. Other arguments will use defaults.

```
Strike = 0.03;
Settle = '01-Jan-2000';
Maturity = '01-Jan-2004';
```

Use `capbybdt` to compute the price of the cap instrument.

```
Price = capbybdt(BDTTree, Strike, Settle, Maturity)

Price =

  28.4001
```

**Example 2**. This example shows the pricing of a 10% cap instrument using a newly created BDT tree.

First set the required arguments for the three needed specifications.

```
Compounding = 1;
ValuationDate = '01-01-2000';
StartDate = ValuationDate;
EndDates = ['01-01-2001'; '01-01-2002'; '01-01-2003';
'01-01-2004'; '01-01-2005'];
Rates = [.1; .11; .12; .125; .13];
Volatility = [.2; .19; .18; .17; .16];
```

Next create the specifications.

```
RateSpec = intenvset('Compounding', Compounding,...
'ValuationDate', ValuationDate,...
'StartDates', StartDate,...
'EndDates', EndDates,...
'Rates', Rates);
BDTTimeSpec = bdttimespec(ValuationDate, EndDates, Compounding);
BDTVolSpec = bdtvolspec(ValuationDate, EndDates, Volatility);
```

Now create the BDT tree from the specifications.

```
BDTTree = bdttree(BDTVolSpec, RateSpec, BDTTimeSpec);
```

Set the cap arguments. Remaining arguments will use defaults.

```
CapStrike = 0.10;
Settlement = ValuationDate;
Maturity = '01-01-2002';
```

```
CapReset = 1;
```

Use `capbybdt` to find the price of the cap instrument.

```
Price= capbybdt(BDTTree, CapStrike, Settlement, Maturity,...
CapReset)

Price =

    1.7169
```

## See Also

`bdttree` | `cfbybdt` | `floorbybdt` | `swapbybdt`

## Purpose

Price cap instrument from Black-Karasinski interest-rate tree

## Syntax

```
[Price, PriceTree] = capbybk(BKTree, Strike, Settle, Maturity,
Reset, Basis, Principal, Options)
```

## Arguments

| | |
|---|---|
| `BKTree` | Interest-rate tree structure created by `bktree`. |
| `Strike` | Number of instruments (`NINST`)-by-`1` vector of rates at which the cap is exercised. |
| `Settle` | Settlement dates. `NINST`-by-`1` vector of dates representing the settlement dates of the cap. |
| `Maturity` | `NINST`-by-`1` vector of dates representing the maturity dates of the cap. |
| `Reset` | (Optional) `NINST`-by-`1` vector representing the frequency of payments per year. Default = `1`. |
| `Basis` | (Optional) Day-count basis of the instrument. A vector of integers.<br>• 0 = actual/actual (default)<br>• 1 = 30/360 (SIA)<br>• 2 = actual/360<br>• 3 = actual/365<br>• 4 = 30/360 (BMA)<br>• 5 = 30/360 (ISDA)<br>• 6 = 30/360 (European)<br>• 7 = actual/365 (Japanese)<br>• 8 = actual/actual (ICMA)<br>• 9 = actual/360 (ICMA)<br>• 10 = actual/365 (ICMA) |

| | |
|---|---|
| | - 11 = 30/360E (ICMA)<br>- 12 = actual/actual (ISDA)<br>- 13 = BUS/252<br>For more information, see basis. |
| `Principal` | (Optional) The notional principal amount. Default = `100`. |
| `Options` | (Optional) Derivatives pricing options structure created with `derivset`. |

## Description

`[Price, PriceTree] = capbybk(BKTree, Strike, Settle, Maturity, Reset, Basis, Principal, Options)` computes the price of a cap instrument from a Black-Karasinski interest-rate tree.

`Price` is the expected price of the cap at time 0.

`PriceTree` is the tree structure with values of the cap at each node.

The `Settle` date for every cap is set to the `ValuationDate` of the BK tree. The cap argument `Settle` is ignored.

## Examples

Price a 3% cap instrument using a Black-Karasinski interest-rate tree.

Load the file `deriv.mat`, which provides `BKTree`. The `BKTree` structure contains the time and interest-rate information needed to price the cap instrument.

```
load deriv.mat;
```

Set the required values. Other arguments will use defaults.

```
Strike = 0.03;
Settle = '01-Jan-2005';
Maturity = '01-Jan-2009';
```

Use `capbybk` to compute the price of the cap instrument.

```
Price = capbybk(BKTree, Strike, Settle, Maturity)

Price =

  6.8337
```

## See Also

`cfbybk` | `floorbybk` | `bktree` | `swapbybk`

**Purpose** Price caps using Black option pricing model

**Syntax**

```
[CapPrice, Caplets] = capbyblk(RateSpec, Strike, Settle,
Maturity, Volatility)
[CapPrice, Caplets] = capbyblk(RateSpec, Strike, Settle,
Maturity, Volatility, 'Name1', Value1...)
```

## Arguments

| | |
|---|---|
| `RateSpec` | The annualized, continuously compounded rate term structure. For more information, see `intenvset`. |
| `Strike` | `NINST`-by-`1` vector of rates at which the cap is exercised, as a decimal number. |
| `Settle` | Scalar representing the settle date of the cap. |
| `Maturity` | Scalar representing the maturity date of the cap. |
| `Volatility` | `NINST`-by-`1` vector of volatilities. |
| `Reset` | (Optional) `NINST`-by-`1` vector representing the frequency of payments per year. Default is `1`. |
| `Principal` | (Optional) `NINST`-by-`1` vector representing the notional principal amount. Default is `100`. |
| `Basis` | `NINST`-by-`1` vector representing the basis used when annualizing the input forward rate.<br>• 0 = actual/actual (default)<br>• 1 = 30/360 (SIA)<br>• 2 = actual/360<br>• 3 = actual/365<br>• 4 = 30/360 (BMA)<br>• 5 = 30/360 (ISDA)<br>• 6 = 30/360 (European) |

- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/actual (ISDA)
- 13 = BUS/252

| | |
|---|---|
| | For more information, see basis. |
| `ValuationDate` | (Optional) Scalar representing the observation date of the investment horizons. The default is the `Settle` date. |

---

**Note** All optional inputs are specified as matching parameter name/value pairs. The parameter name is specified as a character string, followed by the corresponding parameter value. You can specify parameter name/value pairs in any order. Names are case-insensitive and partial string matches are allowed provided no ambiguities exist.

---

## Description

```
[CapPrice, Caplets] = capbyblk(RateSpec, Strike, Settle,
Maturity, Volatility)
```

```
[CapPrice, Caplets] = capbyblk(RateSpec, Strike, Settle,
Maturity, Volatility, 'Name1', Value1...)
```

Use `capbyblk` to price caps using the Black option pricing model.

The outputs are:

- `CapPrice` — `NINST-by-1` expected prices of the cap.
- `Caplets` — `NINST-by-NCF` array of caplets, padded with `NaN`s.

## Examples

Consider an investor who gets into a contract that caps the interest rate on a $100,000 loan at 8% quarterly compounded for 3 months, starting on January 1, 2009. Assuming that on January 1, 2008 the zero rate is 6.9394% continuously compounded and the volatility is 20%, use this data to compute the cap price.

Calculate the `RateSpec`:

```
ValuationDate = 'Jan-01-2008';
EndDates ='April-01-2010';
Rates = 0.069394;
Compounding = -1;
Basis = 1;

RateSpec = intenvset('ValuationDate', ValuationDate, ...
'StartDates', ValuationDate,'EndDates', EndDates, ...
'Rates', Rates,'Compounding', Compounding,'Basis', Basis);
```

Compute the price of the cap:

```
Settle = 'Jan-01-2009'; % cap starts in a year
Maturity = 'April-01-2009';
Volatility = 0.20;
CapRate = 0.08;
CapReset = 4;
Principal=100000;

CapPrice = capbyblk(RateSpec,  CapRate, Settle, Maturity, Volatility,...
'Reset',CapReset,'ValuationDate',ValuationDate,'Principal', Principal,...
'Basis', Basis)

CapPrice =

   51.6125
```

## See Also

`floorbyblk`

## Purpose

Price cap instrument from Heath-Jarrow-Morton interest-rate tree

## Syntax

```
[Price, PriceTree] = capbyhjm(HJMTree, Strike,
Settle, Maturity, Reset, Basis, Principal, Options)
```

## Arguments

| | |
|---|---|
| `HJMTree` | Forward-rate tree structure created by `hjmtree`. |
| `Strike` | Number of instruments (`NINST`)-by-`1` vector of rates at which the cap is exercised. |
| `Settle` | Settlement dates. `NINST`-by-`1` vector of dates representing the settlement dates of the cap. |
| `Maturity` | `NINST`-by-`1` vector of dates representing the maturity dates of the cap. |
| `Reset` | (Optional) `NINST`-by-`1` vector representing the frequency of payments per year. Default = `1`. |
| `Basis` | (Optional) Day-count basis of the instrument. A vector of integers.<br>• 0 = actual/actual (default)<br>• 1 = 30/360 (SIA)<br>• 2 = actual/360<br>• 3 = actual/365<br>• 4 = 30/360 (BMA)<br>• 5 = 30/360 (ISDA)<br>• 6 = 30/360 (European)<br>• 7 = actual/365 (Japanese)<br>• 8 = actual/actual (ICMA)<br>• 9 = actual/360 (ICMA)<br>• 10 = actual/365 (ICMA) |

|  |  |
|---|---|
|  | - 11 = 30/360E (ICMA)<br>- 12 = actual/actual (ISDA)<br>- 13 = BUS/252<br>For more information, see basis. |
| `Principal` | (Optional) The notional principal amount. Default = `100`. |
| `Options` | (Optional) Derivatives pricing options structure created with `derivset`. |

## Description

`[Price, PriceTree] = capbyhjm(HJMTree, Strike, Settle, Maturity, Reset, Basis, Principal, Options)` computes the price of a cap instrument from an HJM tree.

`Price` is the expected price of the cap at time 0.

`PriceTree` is the tree structure with values of the cap at each node.

The `Settle` date for every cap is set to the `ValuationDate` of the HJM tree. The cap argument `Settle` is ignored.

## Examples

Price a 3% cap instrument using an HJM forward-rate tree.

Load the file `deriv.mat`, which provides `HJMTree`. The `HJMTree` structure contains the time and forward-rate information needed to price the cap instrument.

```
load deriv.mat;
```

Set the required values. Other arguments will use defaults.

```
Strike = 0.03;
Settle = '01-Jan-2000';
Maturity = '01-Jan-2004';
```

Use `capbyhjm` to compute the price of the cap instrument.

```
Price = capbyhjm(HJMTree, Strike, Settle, Maturity)

Price =

  6.2831
```

## See Also

`cfbyhjm` | `floorbyhjm` | `hjmtree` | `swapbyhjm`

# capbyhw

**Purpose** Price cap instrument from Hull-White interest-rate tree

**Syntax**

```
[Price, PriceTree] = capbyhw(HWTree, Strike, Settle, Maturity,
Reset, Basis, Principal, Options)
```

**Arguments**

| | |
|---|---|
| `HWTree` | Interest-rate tree structure created by `hwtree`. |
| `Strike` | Number of instruments (`NINST`)-by-`1` vector of rates at which the cap is exercised. |
| `Settle` | Settlement dates. `NINST`-by-`1` vector of dates representing the settlement dates of the cap. |
| `Maturity` | `NINST`-by-`1` vector of dates representing the maturity dates of the cap. |
| `Reset` | (Optional) `NINST`-by-`1` vector representing the frequency of payments per year. Default = `1`. |
| `Basis` | (Optional) Day-count basis of the instrument. A vector of integers.<br>• 0 = actual/actual (default)<br>• 1 = 30/360 (SIA)<br>• 2 = actual/360<br>• 3 = actual/365<br>• 4 = 30/360 (BMA)<br>• 5 = 30/360 (ISDA)<br>• 6 = 30/360 (European)<br>• 7 = actual/365 (Japanese)<br>• 8 = actual/actual (ICMA)<br>• 9 = actual/360 (ICMA)<br>• 10 = actual/365 (ICMA) |

| | |
|---|---|
| | - 11 = 30/360E (ICMA)<br>- 12 = actual/actual (ISDA)<br>- 13 = BUS/252<br>For more information, see basis. |
| `Principal` | (Optional) The notional principal amount. Default = `100`. |
| `Options` | (Optional) Derivatives pricing options structure created with `derivset`. |

## Description

`[Price, PriceTree] = capbyhw(HWTree, Strike, Settle, Maturity, Reset, Basis, Principal, Options)` computes the price of a cap instrument from a Hull-White interest-rate tree.

`Price` is the expected price of the cap at time 0.

`PriceTree` is the tree structure with values of the cap at each node.

The `Settle` date for every cap is set to the `ValuationDate` of the HW tree. The cap argument `Settle` is ignored.

## Examples

Price a 3% cap instrument using a Hull-White interest-rate tree.

Load the file `deriv.mat`, which provides `HWTree`. The `HWTree` structure contains the time and interest-rate information needed to price the cap instrument.

```
load deriv.mat;
```

Set the required values. Other arguments will use defaults.

```
Strike = 0.03;
Settle = '01-Jan-2005';
Maturity = '01-Jan-2009';
```

Use `capbyhw` to compute the price of the cap instrument.

```
Price = capbyhw(HWTree, Strike, Settle, Maturity)

Price =

  7.0707
```

## See Also

`cfbyhw` | `floorbyhw` | `hwtree` | `swapbyhw`

## Purpose

Determine price of cash-or-nothing digital options using Black-Scholes model

## Syntax

```
Price = cashbybls(RateSpec, StockSpec, Settle, Maturity,
OptSpec, Strike, Payoff)
```

## Arguments

| | |
|---|---|
| `RateSpec` | The annualized, continuously compounded rate term structure. For information on the interest rate specification, see `intenvset`. |
| `StockSpec` | Stock specification. See `stockspec`. |
| `Settle` | `NINST`-by-`1` vector of settlement or trade dates. |
| `Maturity` | `NINST`-by-`1` vector of maturity dates. |
| `OptSpec` | `NINST`-by-`1` cell array of strings `'call'` or `'put'`. |
| `Strike` | `NINST`-by-`1` vector of strike price values. |
| `Payoff` | `NINST`-by-`1` vector of payoff values or the amount to be paid at expiration. |

## Description

`Price = cashbybls(RateSpec, StockSpec, Settle, Maturity, OptSpec, Strike, Payoff)` computes cash-or-nothing option prices using the Black-Scholes option pricing model.

`Price` is a `NINST`-by-`1` vector of expected option prices.

## Examples

Consider a European call and put cash-or-nothing options on a futures contract with and exercise strike price of $90, a fixed payoff of $10 that expires on October 1, 2008. Assume that on January 1, 2008, the contract trades at $110, and has a volatility of 25% per annum and the risk-free rate is 4.5% per annum. Using this data, calculate the price of the call and put cash-or-nothing options on the futures contract.

Create the `RateSpec`:

```
Settle = 'Jan-1-2008';
Maturity = 'Oct-1-2008';
Rates = 0.045;
Compounding = -1;
Basis = 1;
RateSpec = intenvset('ValuationDate', Settle, 'StartDates', Settle,...
'EndDates', Maturity, 'Rates', Rates, 'Compounding', Compounding, 'Basis', Basis);
```

Define the `StockSpec`:

```
AssetPrice = 110;
Sigma = .25;
DivType = 'Continuous';
DivAmount = Rates;
StockSpec = stockspec(Sigma, AssetPrice, DivType, DivAmount);
```

Define the call and put options:

```
OptSpec = {'call'; 'put'};
Strike = 90;
Payoff = 10;
```

Calculate the price:

```
Pcon = cashbybls(RateSpec, StockSpec, Settle,...
Maturity, OptSpec, Strike, Payoff)

Pcon =

    7.6716
    1.9965
```

**See Also** `assetbybls` | `cashsensbybls` | `gapbybls` | `supersharebybls`

## Purpose

Determine price and sensitivities of cash-or-nothing digital options using Black-Scholes model

## Syntax

```
PriceSens = cashsensbybls(RateSpec, StockSpec, Settle,
Maturity, OptSpec, Strike, Payoff)
PriceSens = cashsensbybls(RateSpec, StockSpec, Settle,
Maturity, OptSpec, Strike, Payoff, OutSpec)
```

## Arguments

| | |
|---|---|
| `RateSpec` | The annualized, continuously compounded rate term structure. For information on the interest rate specification, see `intenvset`. |
| `StockSpec` | Stock specification. See `stockspec`. |
| `Settle` | `NINST`-by-`1` vector of settlement or trade dates. |
| `Maturity` | `NINST`-by-`1` vector of maturity dates. |
| `OptSpec` | `NINST`-by-`1` cell array of strings `'call'` or `'put'`. |
| `Strike` | `NINST`-by-`1` vector of strike price values. |
| `Payoff` | `NINST`-by-`1` vector of payoff values or the amount to be paid at expiration. |
| `OutSpec` | (Optional) All optional inputs are specified as matching parameter name/value pairs. The parameter name is specified as a character string, followed by the corresponding parameter value. You can specify parameter name/value pairs in any order. Names are case-insensitive and partial string matches are allowed provided no ambiguities exist. Valid parameter names are:<br>• `NOUT`-by-`1` or `1`-by-`NOUT` cell array of strings indicating the nature and order of the outputs for the function. Possible values are `'Price'`, `'Delta'`, `'Gamma'`, `'Vega'`, `'Lambda'`, `'Rho'`, `'Theta'`, or `'All'`. |

For example, `OutSpec = {'Price'; 'Lambda'; 'Rho'}` specifies that the output should be `Price`, `Lambda`, and `Rho`, in that order.

To invoke from a function: `[Price, Lambda, Rho] = cashsensbybls(..., 'OutSpec', {'Price', 'Lambda', 'Rho'})`

`OutSpec = {'All'}` specifies that the output should be `Delta`, `Gamma`, `Vega`, `Lambda`, `Rho`, `Theta`, and `Price`, in that order. This is the same as specifying `OutSpec` as `OutSpec = {'Delta', 'Gamma', 'Vega', 'Lambda', 'Rho', 'Theta', 'Price'};`.

- Default is `OutSpec` = `{'Price'}`.

## Description

`PriceSens = cashsensbybls(RateSpec, StockSpec, Settle, Maturity, OptSpec, Strike, Payoff)` computes cash-or-nothing option prices using the Black-Scholes option pricing model.

`PriceSens = cashsensbybls(RateSpec, StockSpec, Settle, Maturity, OptSpec, Strike, Payoff, OutSpec)` includes an `OutSpec` argument defined as parameter/value pairs, and computes cash-or-nothing option prices and sensitivities using the Black-Scholes option pricing model.

`PriceSens` is a `NINST-by-1` vector of expected option prices and sensitivities.

## Examples

Consider a European call and put cash-or-nothing options on a futures contract with an exercise price of $90, and a fixed payoff of $10 that expires on January 1, 2009. Assume that on October 1, 2008 the contract trades at $110, and has a volatility of 25% per annum and the risk-free rate is 4.5% per annum. Using this data, calculate the price and sensitivity of the call and put cash-or-nothing options on the futures contract.

Create the `RateSpec`:

```
Settle = 'Jan-1-2008';
Maturity = 'Oct-1-2008';
Rates = 0.045;
Compounding = -1;
Basis = 1;
RateSpec = intenvset('ValuationDate', Settle, 'StartDates', Settle,...
'EndDates', Maturity, 'Rates', Rates, 'Compounding', Compounding, 'Basis', Basis);
```

Define the `StockSpec`:

```
AssetPrice = 110;
Sigma = .25;
DivType = 'Continuous';
DivAmount = Rates;
StockSpec = stockspec(Sigma, AssetPrice, DivType, DivAmount);
```

Define the call and put options:

```
OptSpec = {'call'; 'put'};
Strike = 90;
Payoff = 10;
```

Compute the gamma, theta, and price:

```
OutSpec = { 'gamma';'theta';'price'};
[Gamma, Theta, Price] = cashsensbybls(RateSpec, StockSpec,...
Settle, Maturity, OptSpec, Strike, Payoff, 'OutSpec', OutSpec)

Gamma =

   -0.0050
    0.0050
```

```
Theta =

   -2.2489
    1.8139


Price =

    7.6716
    1.9965
```

**See Also** `cashbybls`

## Purpose

Price cash flows from Black-Derman-Toy interest-rate tree

## Syntax

```
[Price, PriceTree] = cfbybdt(BDTTree,
CFlowAmounts, CFlowDates, Settle, Basis, Options)
```

## Arguments

| | |
|---|---|
| `BDTTree` | Forward-rate tree structure created by `bdttree`. |
| `CFlowAmounts` | Number of instruments (`NINST`) by maximum number of cash flows (`MOSTCFS`) matrix of cash flow amounts. Each row is a list of cash flow values for one instrument. If an instrument has fewer than `MOSTCFS` cash flows, the end of the row is padded with `NaN`s. |
| `CFlowDates` | `NINST`-by-`MOSTCFS` matrix of cash flow dates. Each entry contains the serial date number of the corresponding cash flow in `CFlowAmounts`. |
| `Settle` | Settlement date. A vector of serial date numbers or date strings. The `Settle` date for every cash flow is set to the `ValuationDate` of the BDT tree. The cash flow argument, `Settle`, is ignored. |
| `Basis` | (Optional) Day-count basis of the instrument. A vector of integers.<br>• 0 = actual/actual (default)<br>• 1 = 30/360 (SIA)<br>• 2 = actual/360<br>• 3 = actual/365<br>• 4 = 30/360 (BMA)<br>• 5 = 30/360 (ISDA)<br>• 6 = 30/360 (European)<br>• 7 = actual/365 (Japanese) |

- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/actual (ISDA)
- 13 = BUS/252

For more information, see basis.

| | |
|---|---|
| `Options` | (Optional) Derivatives pricing options structure created with `derivset`. |

## Description

`[Price, PriceTree] = cfbybdt(BDTTree, CFlowAmounts, CFlowDates, Settle, Basis, Options)` prices cash flows from a BDT interest-rate tree.

`Price` is an `NINST`-by-`1` vector of expected prices at time 0.

`PriceTree` is a tree structure with a vector of instrument prices at each node.

## Examples

Price a portfolio containing two cash flow instruments paying interest annually over the four-year period from January 1, 2000 to January 1, 2004.

Load the file `deriv.mat`, which provides `BDTTree`. The `BDTTree` structure contains the time and interest-rate information needed to price the instruments.

```
load deriv.mat;
```

The valuation date (settle date) specified in `BDTTree` is January 1, 2000 (date number `730486`).

```
BDTTree.RateSpec.ValuationDate
```

```
ans =

      730486
```

Provide values for the other required arguments.

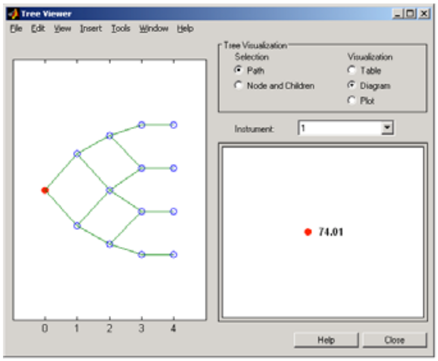
```
CFlowAmounts =[5 NaN 5.5 105; 5 0 6 105];
CFlowDates = [730852, NaN, 731582, 731947;
              730852, 731217, 731582, 731947];
```

Use this information to compute the prices of the two cash flow instruments.

```
[Price, PriceTree] = cfbybdt(BDTTree, CFlowAmounts, ...
CFlowDates, BDTTree.RateSpec.ValuationDate)

Price =

   74.0112
   74.3671

PriceTree =

    FinObj: 'BDTPriceTree'
      tObs: [0 1.00 2.00 3.00 4.00]
     PTree: {1x5 cell}
```

You can visualize the prices of the two cash flow instruments with the `treeviewer` function.

```
treeviewer(PriceTree)
```

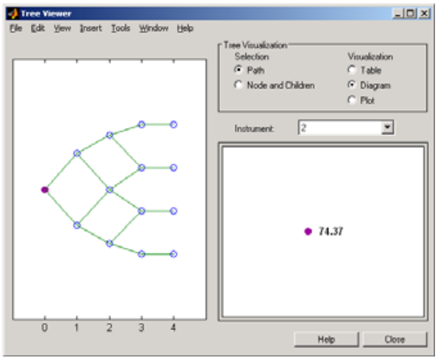


Selection: ● Path ○ Node and Children

Visualization: ○ Table ● Diagram ○ Plot

Instrument: 1

74.01



Selection: ● Path ○ Node and Children

Visualization: ○ Table ● Diagram ○ Plot

Instrument: 2

74.37

## See Also

`bdttree` | `bdtprice` | `cfamounts` | `instcf`

## Purpose

Price cash flows from Black-Karasinski interest-rate tree

## Syntax

```
[Price, PriceTree] = cfbybk(BKTree, CFlowAmounts, CFlowDates,
Settle, Basis, Options)
```

## Arguments

| | |
|---|---|
| `BKTree` | Forward-rate tree structure created by `bktree`. |
| `CFlowAmounts` | Number of instruments (`NINST`) by maximum number of cash flows (`MOSTCFS`) matrix of cash flow amounts. Each row is a list of cash flow values for one instrument. If an instrument has fewer than `MOSTCFS` cash flows, the end of the row is padded with `NaN`s. |
| `CFlowDates` | `NINST`-by-`MOSTCFS` matrix of cash flow dates. Each entry contains the serial date number of the corresponding cash flow in `CFlowAmounts`. |
| `Settle` | Settlement date. A vector of serial date numbers or date strings. The `Settle` date for every cash flow is set to the `ValuationDate` of the BK tree. The cash flow argument, `Settle`, is ignored. |
| `Basis` | (Optional) Day-count basis of the instrument. A vector of integers.<br>• 0 = actual/actual (default)<br>• 1 = 30/360 (SIA)<br>• 2 = actual/360<br>• 3 = actual/365<br>• 4 = 30/360 (BMA)<br>• 5 = 30/360 (ISDA)<br>• 6 = 30/360 (European)<br>• 7 = actual/365 (Japanese) |

|  |  |
|---|---|
|  | - 8 = actual/actual (ICMA)<br>- 9 = actual/360 (ICMA)<br>- 10 = actual/365 (ICMA)<br>- 11 = 30/360E (ICMA)<br>- 12 = actual/actual (ISDA)<br>- 13 = BUS/252<br>For more information, see basis. |
| `Options` | (Optional) Derivatives pricing options structure created with `derivset`. |

## Description

`[Price, PriceTree] = cfbybk(BKTree, CFlowAmounts, CFlowDates,`
`Settle, Basis, Options)` prices cash flows from a Black-Karasinski interest-rate tree.

`Price` is an `NINST`-by-`1` vector of expected prices at time 0.

`PriceTree` is a tree structure with a vector of instrument prices at each node.

## Examples

Price a portfolio containing two cash flow instruments paying interest annually over the four-year period from January 1, 2005 to January 1, 2009.

Load the file `deriv.mat`, which provides `BKTree`. The `BKTree` structure contains the time and interest-rate information needed to price the instruments.

```
load deriv.mat;
```

The valuation date (settle date) specified in `BKTree` is January 1, 2004 (date number `731947`).

```
BKTree.RateSpec.ValuationDate

ans =

      731947
```

Provide values for the other required arguments.

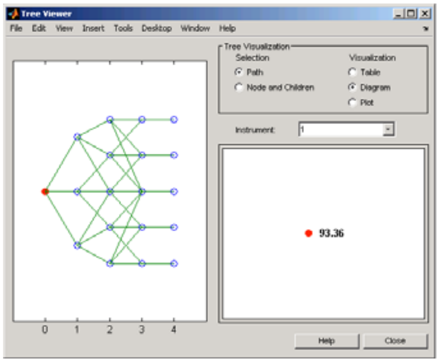
```
CFlowAmounts =[5 NaN 5.5 105; 5 0 6 105];
CFlowDates = [732678, NaN, 733408,733774;
              732678, 733034, 733408, 734774];
```

Use this information to compute the prices of the two cash flow instruments.

```
[Price, PriceTree] = cfbybk(BKTree, CFlowAmounts, CFlowDates,...
BKTree.RateSpec.ValuationDate)

Price =

   93.3600
   81.6218

PriceTree =

    FinObj: 'BKPriceTree'
       tObs: [0 1 2 3 4]
      PTree: {[2x1 double]  [2x3 double]  [2x5 double]  [2x5
double]                [2x5 double]}
    Connect: {[2]  [2 3 4]  [2 2 3 4 4]}
      Probs: {[3x1 double]  [3x3 double]  [3x5 double]}
```

You can visualize the prices of the two cash flow instruments with the `treeviewer` function.

```
treeviewer(PriceTree)
```

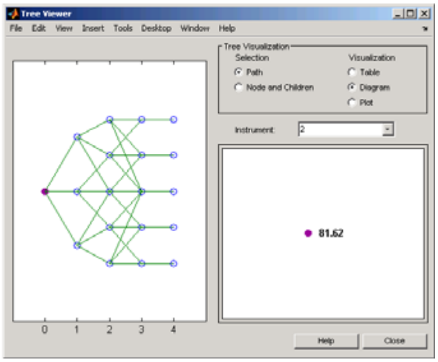


Selection: ● Path ○ Node and Children

Visualization: ○ Table ● Diagram ○ Plot

Instrument: 1

93.36



Selection: ● Path ○ Node and Children

Visualization: ○ Table ● Diagram ○ Plot

Instrument: 2

81.62

## See Also

`bktree` | `bkprice` | `cfamounts` | `instcf`

## Purpose

Price cash flows from Heath-Jarrow-Morton interest-rate tree

## Syntax

```
[Price, PriceTree] = cfbyhjm(HJMTree, CFlowAmounts,
CFlowDates, Settle, Basis, Options)
```

## Arguments

| | |
|---|---|
| `HJMTree` | Forward-rate tree structure created by `hjmtree`. |
| `CFlowAmounts` | Number of instruments (`NINST`) by maximum number of cash flows (`MOSTCFS`) matrix of cash flow amounts. Each row is a list of cash flow values for one instrument. If an instrument has fewer than `MOSTCFS` cash flows, the end of the row is padded with `NaN`s. |
| `CFlowDates` | `NINST`-by-`MOSTCFS` matrix of cash flow dates. Each entry contains the serial date number of the corresponding cash flow in `CFlowAmounts`. |
| `Settle` | Settlement date. A vector of serial date numbers or date strings. The `Settle` date for every cash flow is set to the `ValuationDate` of the HJM tree. The cash flow argument, `Settle`, is ignored. |
| `Basis` | (Optional) Day-count basis of the instrument. A vector of integers.<br>• 0 = actual/actual (default)<br>• 1 = 30/360 (SIA)<br>• 2 = actual/360<br>• 3 = actual/365<br>• 4 = 30/360 (BMA)<br>• 5 = 30/360 (ISDA)<br>• 6 = 30/360 (European)<br>• 7 = actual/365 (Japanese) |

| | |
|---|---|
| | - 8 = actual/actual (ICMA)<br>- 9 = actual/360 (ICMA)<br>- 10 = actual/365 (ICMA)<br>- 11 = 30/360E (ICMA)<br>- 12 = actual/actual (ISDA)<br>- 13 = BUS/252<br>For more information, see basis. |
| `Options` | (Optional) Derivatives pricing options structure created with `derivset`. |

## Description

`[Price, PriceTree] = cfbyhjm(HJMTree, CFlowAmounts, CFlowDates, Settle, Basis, Options)` prices cash flows from an HJM interest-rate tree.

`Price` is an `NINST`-by-`1` vector of expected prices at time 0.

`PriceTree` is a tree structure with a vector of instrument prices at each node.

## Examples

Price a portfolio containing two cash flow instruments paying interest annually over the four-year period from January 1, 2000 to January 1, 2004.

Load the file `deriv.mat`, which provides `HJMTree`. The `HJMTree` structure contains the time and forward-rate information needed to price the instruments.

```
load deriv.mat;
```

The valuation date (settle date) specified in `HJMTree` is January 1, 2000 (date number `730486`).

```
HWTree.RateSpec.ValuationDate
```

```
ans =

      730486
```

Provide values for the other required arguments.

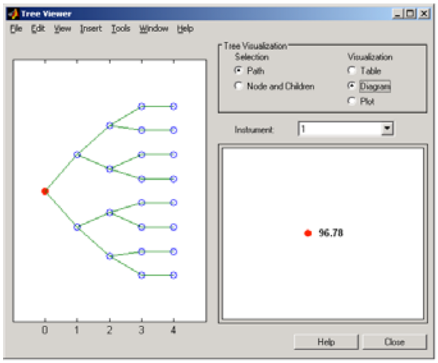
```
CFlowAmounts =[5 NaN 5.5 105; 5 0 6 105];
CFlowDates = [730852, NaN, 731582, 731947;
              730852, 731217, 731582, 731947];
```

Use this information to compute the prices of the two cash flow instruments.

```
[Price, PriceTree] = cfbyhjm(HJMTree, CFlowAmounts,...
CFlowDates, HJMTree.RateSpec.ValuationDate)

Price =

   96.7805
   97.2188

PriceTree =

    FinObj: 'HJMPriceTree'
      tObs: [0 1.00 2.00 3.00 4.00]
     PBush: {1x5 cell}
```

You can visualize the prices of the two cash flow instruments with the `treeviewer` function.

```
treeviewer(PriceTree)
```

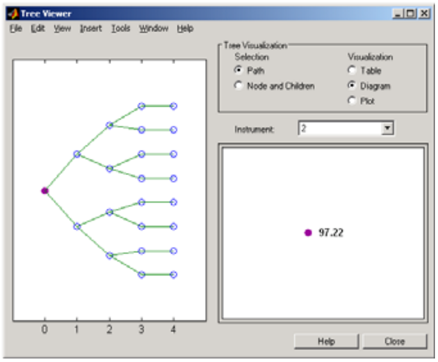


Selection: ● Path ○ Node and Children

Visualization: ○ Table ● Diagram ○ Plot

Instrument: 1

96.78



Selection: ● Path ○ Node and Children

Visualization: ○ Table ● Diagram ○ Plot

Instrument: 2

97.22

## See Also

`cfamounts` | `hjmprice` | `hjmtree` | `instcf`

## Purpose

Price cash flows from Hull-White interest-rate tree

## Syntax

```
[Price, PriceTree] = cfbyhw(HWTree, CFlowAmounts, CFlowDates,
Settle, Basis, Options)
```

## Arguments

| | |
|---|---|
| `HWTree` | Forward-rate tree structure created by `hwtree`. |
| `CFlowAmounts` | Number of instruments (`NINST`) by maximum number of cash flows (`MOSTCFS`) matrix of cash flow amounts. Each row is a list of cash flow values for one instrument. If an instrument has fewer than `MOSTCFS` cash flows, the end of the row is padded with `NaN`s. |
| `CFlowDates` | `NINST`-by-`MOSTCFS` matrix of cash flow dates. Each entry contains the serial date number of the corresponding cash flow in `CFlowAmounts`. |
| `Settle` | Settlement date. A vector of serial date numbers or date strings. The `Settle` date for every cash flow is set to the `ValuationDate` of the HW tree. The cash flow argument, `Settle`, is ignored. |
| `Basis` | (Optional) Day-count basis of the instrument. A vector of integers.<br>• 0 = actual/actual (default)<br>• 1 = 30/360 (SIA)<br>• 2 = actual/360<br>• 3 = actual/365<br>• 4 = 30/360 (BMA)<br>• 5 = 30/360 (ISDA)<br>• 6 = 30/360 (European)<br>• 7 = actual/365 (Japanese) |

- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/actual (ISDA)
- 13 = BUS/252

For more information, see basis.

| | |
|---|---|
| `Options` | (Optional) Derivatives pricing options structure created with `derivset`. |

## Description

`[Price, PriceTree] = cfbyhw(HWTree, CFlowAmounts, CFlowDates, Settle, Basis, Options)` prices cash flows from a Hull-White interest-rate tree.

`Price` is an `NINST`-by-`1` vector of expected prices at time 0.

`PriceTree` is a tree structure with a vector of instrument prices at each node.

## Examples

Price a portfolio containing two cash flow instruments paying interest annually over the four-year period from January 1, 2005 to January 1, 2009.

Load the file `deriv.mat`, which provides `HWTree`. The `HWTree` structure contains the time and interest-rate information needed to price the instruments.

```
load deriv.mat;
```

The valuation date (settle date) specified in `HWTree` is January 1, 2004 (date number `731947`).

```
HWTree.RateSpec.ValuationDate

ans =

      731947
```

Provide values for the other required arguments.

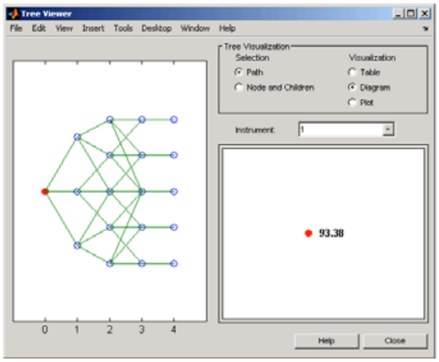
```
CFlowAmounts =[5 NaN 5.5 105; 5 0 6 105];
CFlowDates = [732678, NaN, 733408, 733774;
              732678, 733034, 733408, 734774];
```

Use this information to compute the prices of the two cash flow instruments.

```
[Price, PriceTree] = cfbyhw(HWTree, CFlowAmounts, CFlowDates,...
HWTree.RateSpec.ValuationDate)

Price =

   93.3789
   81.7651

PriceTree =

    FinObj: 'HWPriceTree'
       tObs: [0 1 2 3 4]
      PTree: {[2x1 double]  [2x3 double]  [2x5 double]  [2x5
double]                [2x5 double]}
    Connect: {[2]  [2 3 4]  [2 2 3 4 4]}
      Probs: {[3x1 double]  [3x3 double]  [3x5 double]}
```

You can visualize the prices of the two cash flow instruments with the `treeviewer` function.

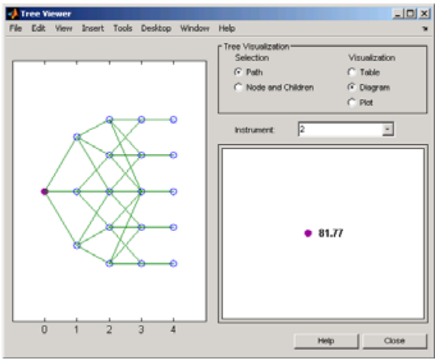
```
treeviewer(PriceTree)
```



Selection: ● Path ○ Node and Children

Visualization: ○ Table ● Diagram ○ Plot

Instrument: 1



Selection: ● Path ○ Node and Children

Visualization: ○ Table ● Diagram ○ Plot

Instrument: 2

## See Also

`cfamounts` | `hwtree` | `hwprice` | `instcf`

## Purpose

Price cash flows from set of zero curves

## Syntax

```
Price = cfbyzero(RateSpec, CFlowAmounts, CFlowDates, Settle,
Basis)
```

## Arguments

| | |
|---|---|
| `RateSpec` | Structure containing the properties of an interest-rate structure. See `intenvset` for information on creating `RateSpec`. |
| `CFlowAmounts` | Number of instruments (`NINST`) by maximum number of cash flows (`MOSTCFS`) matrix with entries listing cash flow amounts corresponding to each date in `CFlowDates`. Each row is a list of cash flow values for one instrument. If an instrument has fewer than `MOSTCFS` cash flows, the end of the row is padded with `NaN`s. |
| `CFlowDates` | `NINST`-by-`MOSTCFS` matrix of cash flow dates. Each entry contains the serial date of the corresponding cash flow in `CFlowAmounts`. |
| `Settle` | Settlement date on which the cash flows are priced. `Settle` must be either a scalar or `NINST`-by-`1` vector of serial date numbers or date strings of the same value which represent the settlement date for each cash flow. `Settle` must be earlier than `Maturity`. |
| `Basis` | (Optional) Day-count basis of the instrument. A vector of integers.<br>• 0 = actual/actual (default)<br>• 1 = 30/360 (SIA)<br>• 2 = actual/360<br>• 3 = actual/365 |

- 4 = 30/360 (BMA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/actual (ISDA)
- 13 = BUS/252

For more information, see basis.

## Description

`Price = cfbyzero(RateSpec, CFlowAmounts, CFlowDates, Settle, Basis)` computes `Price`, an `NINST`-by-`NUMCURVES` matrix of cash flows prices. Each column arises from one of the zero curves.

## Examples

Price a portfolio containing two cash flow instruments paying interest annually over the four-year period from January 1, 2000 to January 1, 2004.

Load the file `deriv.mat`, which provides `ZeroRateSpec`. The `ZeroRateSpec` structure contains the interest-rate information needed to price the instruments.

```
load deriv.mat
CFlowAmounts =[5 NaN 5.5 105;5 0 6 105];
CFlowDates = [730852, NaN, 731582,731947;
              730852, 731217, 731582, 731947];
Settle = 730486;
Price = cfbyzero(ZeroRateSpec, CFlowAmounts, CFlowDates, Settle)
```

```
Price =

   96.7804
   97.2187
```

## See Also

`bondbyzero` | `fixedbyzero` | `floatbyzero` | `swapbyzero`

# chooserbybls

## Purpose

Price European simple chooser options using Black-Scholes model

## Syntax

```
Price = chooserbybls(RateSpec, StockSpec, Settle,
Maturity, Strike)
```

## Arguments

| | |
|---|---|
| `RateSpec` | The annualized continuously compounded rate term structure. For information on the interest rate specification, see `intenvset`. |
| `StockSpec` | Stock specification. See `stockspec`. |
| `Settle` | `NINST`-by-`1` vector of settlement or trade dates. |
| `Maturity` | `NINST`-by-`1` vector of maturity dates. |
| `Strike` | `NINST`-by-`1` vector of strike price values. |
| `ChooseDate` | `NINST`-by-`1` vector of chooser dates. |

## Description

`Price = chooserbybls(RateSpec, StockSpec, Settle, Maturity, Strike)` computes the price for European simple chooser options using the Black-Scholes model.

`Price` is a `NINST`-by-`1` vector of expected prices.

**Note** Only dividends of type `continuous` can be considered for choosers.

## Examples

Consider a European chooser option with an exercise price of $60 on June 1, 2007. The option expires on December 2, 2007. Assume the underlying stock provides a continuous dividend yield of 5% per annum, is trading at $50, and has a volatility of 20% per annum. The annualized continuously compounded risk-free rate is 10% per annum. Assume that the choice must be made on August 31, 2007. Using this data:

```
AssetPrice = 50;
Strike = 60;
Settlement = 'Jun-1-2007';
Maturity = 'Dec-2-2007';
ChooseDate = 'Aug-31-2007';
RiskFreeRate = 0.1;
Sigma = 0.20;
Yield = 0.05
```

Define the `RateSpec` and `StockSpec`:

```
RateSpec = intenvset('Compounding', -1, 'Rates', RiskFreeRate, 'StartDates',...
Settlement, 'EndDates', Maturity);
StockSpec = stockspec(Sigma, AssetPrice,'continuous',Yield);
```

Price the chooser option:

```
Price  = chooserbybls(RateSpec, StockSpec, Settlement, Maturity,...
Strike, ChooseDate)

Price =

8.9308
```

## References

Rubinstein, Mark, “Options for the Undecided,” *Risk 4*, 1991.

## See Also

`blsprice` | `intenvset`

## Purpose

Create financial structure or return financial structure class name

## Syntax

```
Obj = classfin(ClassName)
Obj = classfin(Struct, ClassName)
ClassName = classfin(Obj)
```

## Arguments

| | |
|---|---|
| `ClassName` | String containing the name of a financial structure class. |
| `Struct` | MATLAB structure to be converted into a financial structure. |
| `Obj` | Name of a financial structure. |

## Description

`Obj = classfin(ClassName)` and `Obj = classfin(Struct, ClassName)` create a financial structure of class `ClassName`.

`ClassName = classfin(Obj)` returns a string containing a financial structure's class name.

## Examples

**Example 1**. Create an `HJMTimeSpec` financial structure and complete its fields. (Typically, the function `hjmtimespec` is used to create `HJMTimeSpec` structures).

```
TimeSpec = classfin('HJMTimeSpec');
TimeSpec.ValuationDate = datenum('Dec-10-1999');
TimeSpec.Maturity = datenum('Dec-10-2002');
TimeSpec.Compounding = 2;
TimeSpec.Basis = 0;
TimeSpec.EndMonthRule = 1;
TimeSpec =

           FinObj: 'HJMTimeSpec'
    ValuationDate: 730464
         Maturity: 731560
```

```
      Compounding: 2
            Basis: 0
     EndMonthRule: 1
```

**Example 2**. Convert an existing MATLAB structure into a financial structure.

```
TSpec.ValuationDate = datenum('Dec-10-1999');
TSpec.Maturity = datenum('Dec-10-2002');
TSpec.Compounding = 2;
TSpec.Basis = 0;
TSpec.EndMonthRule = 0;
TimeSpec = classfin(TSpec, 'HJMTimeSpec')

TimeSpec =

    ValuationDate: 730464
         Maturity: 731560
      Compounding: 2
            Basis: 0
     EndMonthRule: 0
           FinObj: 'HJMTimeSpec'
```

**Example 3**. Obtain a financial structure's class name.

```
load deriv.mat
ClassName = classfin(HJMTree)
ClassName =

HJMFwdTree
```

## See Also

`isafin`

# compoundbycrr

## Purpose

Price compound option from Cox-Ross-Rubinstein binomial tree

## Syntax

```
[Price, PriceTree] = compoundbycrr(CRRTree, UOptSpec, UStrike,
USettle, UExerciseDates, UAmericanOpt, COptSpec,
CStrike, CSettle,  CExerciseDates, CAmericanOpt)
```

## Arguments

| | |
|---|---|
| `CRRTree` | Stock tree structure created by `crrtree`. |
| `UOptSpec` | String = `'Call'` or `'Put'`. |
| `UStrike` | `1`-by-`1` vector of strike price values. |
| `USettle` | `1`-by-`1` vector of `Settle` dates. |
| `UExerciseDates` | For a European option (`UAmericanOpt = 0`):<br>`1`-by-`1` vector of exercise dates. For a European option, there is only one exercise date, the option expiry date.<br>For an American option (`UAmericanOpt = 1`):<br>`1`-by-`2` vector of exercise date boundaries. The option can be exercised on any tree date. If only one non-`NaN` date is listed, or if `ExerciseDates` is `1`-by-`1`, the option can be exercised between the valuation date of the stock tree and the single listed exercise date. |
| `UAmericanOpt` | If `UAmericanOpt = 0`, `NaN`, or is unspecified, the option is a European option. If `UAmericanOpt = 1`, the option is an American option. |
| `COptSpec` | `NINST`-by-`1` list of string values `'Call'` or `'Put'` of the compound option. |
| `CStrike` | `NINST`-by-`1` vector of strike price values. Each row is the schedule for one option. |

| | |
|---|---|
| `CSettle` | `1`-by-`1` vector containing the settlement or trade date. |
| `CExerciseDates` | For a European option (`CAmericanOpt = 0`):<br>`NINST`-by-`1` vector of exercise dates. Each row is the schedule for one option. For a European option, there is only one exercise date, the option expiry date.<br>For an American option (`CAmericanOpt = 1`):<br>`NINST`-by-`2` vector of exercise date boundaries. For each instrument, the option can be exercised on any tree date between or including the pair of dates on that row. If only one non-`NaN` date is listed, or if `ExerciseDates` is `NINST`-by-`1`, the option can be exercised between the valuation date of the stock tree and the single listed exercise date. |
| `CAmericanOpt` | (Optional) If `CAmericanOpt = 0`, `NaN`, or is unspecified, the option is a European option. If `CAmericanOpt = 1`, the option is an American option. |

## Description

`[Price, PriceTree] = compoundbycrr(CRRTree, UOptSpec, UStrike, USettle, UExerciseDates, UAmericanOpt, COptSpec, CStrike, CSettle, CExerciseDates, CAmericanOpt)` calculates the value of a compound option.

`Price` is a `NINST`-by-`1` vector of expected prices at time 0.

`PriceTree` is a tree structure with a vector of instrument prices at each node.

## Examples

Price a compound option using a CRR binomial tree.

Load the file `deriv.mat`, which provides `CRRTree`. The `CRRTree` structure contains the stock specification and time information needed to price the option.

```
load deriv.mat
```

Set the required values. Other arguments will use defaults.

```
UOptSpec = 'Call';
UStrike = 130;
USettle = '01-Jan-2003';
UExerciseDates = '01-Jan-2006';
UAmericanOpt = 1;
COptSpec = 'Put';
CStrike = 5;
CSettle = '01-Jan-2003';
CExerciseDates = '01-Jan-2005';

Price = compoundbycrr(CRRTree, UOptSpec, UStrike, USettle, ...
UExerciseDates, UAmericanOpt, COptSpec, CStrike, CSettle, ...
CExerciseDates)

Price =

 2.8482
```

## References

Rubinstein, Mark, “Double Trouble,” *Risk 5*, 1991, p. 73.

## See Also

`crrtree | instcompound`

## Purpose

Price compound option from Equal Probabilities binomial tree

## Syntax

```
[Price, PriceTree] = compoundbyeqp(EQPTree, UOptSpec, UStrike,
USettle, UExerciseDates, UAmericanOpt, COptSpec,
CStrike, CSettle,  CExerciseDates, CAmericanOpt)
```

## Arguments

| | |
|---|---|
| `EQPTree` | Stock tree structure created by `eqptree`. |
| `UOptSpec` | String = `'Call'` or `'Put'`. |
| `UStrike` | `1`-by-`1` vector of strike price values. |
| `USettle` | `1`-by-`1` vector of `Settle` dates. |
| `UExerciseDates` | For a European option (`UAmericanOpt = 0`):<br>`1`-by-`1` vector of exercise dates. For a European option, there is only one exercise date, the option expiry date.<br>For an American option (`UAmericanOpt = 1`):<br>`1`-by-`2` vector of exercise date boundaries. The option can be exercised on any tree date. If only one non-`NaN` date is listed, or if `ExerciseDates` is `1`-by-`1`, the option can be exercised between the valuation date of the stock tree and the single listed exercise date. |
| `UAmericanOpt` | If `UAmericanOpt = 0`, `NaN`, or is unspecified, the option is a European option. If `UAmericanOpt = 1`, the option is an American option. |
| `COptSpec` | `NINST`-by-`1` list of string values `'Call'` or `'Put'` of the compound option. |
| `CStrike` | `NINST`-by-`1` vector of strike price values. Each row is the schedule for one option. |

| | |
|---|---|
| `CSettle` | `1`-by-`1` vector containing the settlement or trade date. |
| `CExerciseDates` | For a European option (`CAmericanOpt = 0`):<br>`NINST`-by-`1` vector of exercise dates. Each row is the schedule for one option. For a European option, there is only one exercise date, the option expiry date.<br>For an American option (`CAmericanOpt = 1`):<br>`NINST`-by-`2` vector of exercise date boundaries. For each instrument, the option can be exercised on any tree date between or including the pair of dates on that row. If only one non-`NaN` date is listed, or if `ExerciseDates` is `NINST`-by-`1`, the option can be exercised between the valuation date of the stock tree and the single listed exercise date. |
| `CAmericanOpt` | If `CAmericanOpt = 0`, `NaN`, or is unspecified, the option is a European option. If `CAmericanOpt = 1`, the option is an American option. |

## Description

`[Price, PriceTree] = compoundbyeqp(EQPTree, UOptSpec, UStrike,`
`USettle, UExerciseDates, UAmericanOpt, COptSpec, CStrike, CSettle, CExerciseDates, CAmericanOpt)` calculates the value of a compound option.

`Price` is a `NINST`-by-`1` vector of expected prices at time 0.

`PriceTree` is a tree structure with a vector of instrument prices at each node.

## Examples

Price a compound option using an EQP equity tree.

Load the file `deriv.mat,` which provides `EQPTree`. The `EQPTree` structure contains the stock specification and time information needed to price the option.

```
load deriv.mat
```

Set the required values. Other arguments will use defaults.

```
UOptSpec = 'Call';
UStrike = 130;
USettle = '01-Jan-2003';
UExerciseDates = '01-Jan-2006';
UAmericanOpt = 1;
COptSpec = 'Put';
CStrike = 5;
CSettle = '01-Jan-2003';
CExerciseDates = '01-Jan-2005';

Price = compoundbyeqp(EQPTree, UOptSpec, UStrike, USettle, ...
UExerciseDates, UAmericanOpt, COptSpec, CStrike, CSettle, ...
CExerciseDates)

Price =

 3.3931
```

## References

Rubinstein, Mark, “Double Trouble,” *Risk 5*, 1991, p. 73

## See Also

`eqptree` | `instcompound`

# compoundbyitt

**Purpose** Price compound options using implied trinomial tree (ITT)

**Syntax**

```
[Price, PriceTree] = compoundbyitt(ITTTree, UOptSpec, UStrike,
USettle, UExerciseDates, UAmericanOpt, COptSpec,
CStrike, CSettle,  CExerciseDates, CAmericanOpt)
```

**Arguments**

| | |
|---|---|
| `ITTTree` | Stock tree structure created by `itttree`. |
| `UOptSpec` | String = `'call'` or `'put'`. |
| `UStrike` | `1`-by-`1` vector of strike price values. |
| `USettle` | `1`-by-`1` vector of `Settle` dates. |
| `UExerciseDates` | For a European option (`UAmericanOpt = 0`):<br>`1`-by-`1` vector of exercise dates. For a European option, there is only one exercise date, the option expiry date.<br>For an American option (`UAmericanOpt = 1`):<br>`1`-by-`2` vector of exercise date boundaries. The option can be exercised on any tree date. If only one non-`NaN` date is listed, or if `ExerciseDates` is `1`-by-`1`, the option can be exercised between the valuation date of the stock tree and the single listed exercise date. |
| `UAmericanOpt` | If `UAmericanOpt = 0`, `NaN`, or is unspecified, the option is a European option. If `UAmericanOpt = 1`, the option is an American option. |
| `COptSpec` | `NINST`-by-`1` list of string values `'Call'` or `'Put'` of the compound option. |
| `CStrike` | `NINST`-by-`1` vector of strike price values. Each row is the schedule for one option. |

| | |
|---|---|
| `CSettle` | `1`-by-`1` vector containing the settlement or trade date. |
| `CExerciseDates` | For a European option (`CAmericanOpt = 0`):<br>`NINST`-by-`1` vector of exercise dates. Each row is the schedule for one option. For a European option, there is only one exercise date, the option expiry date.<br>For an American option (`CAmericanOpt = 1`):<br>`NINST`-by-`2` vector of exercise date boundaries. For each instrument, the option can be exercised on any tree date between or including the pair of dates on that row. If only one non-`NaN` date is listed, or if `ExerciseDates` is `NINST`-by-`1`, the option can be exercised between the valuation date of the stock tree and the single listed exercise date. |
| `CAmericanOpt` | (Optional) If `CAmericanOpt = 0`, `NaN`, or is unspecified, the option is a European option. If `CAmericanOpt = 1`, the option is an American option. |

## Description

`[Price, PriceTree] = compoundbyitt(ITTTree, UOptSpec, UStrike, USettle, UExerciseDates, UAmericanOpt, COptSpec, CStrike, CSettle, CExerciseDates, CAmericanOpt)` calculates the value of a compound option by an ITT trinomial tree.

`Price` is a `NINST`-by-`1` vector of expected prices at time 0.

`PriceTree` is a tree structure with a vector of instrument prices at each node.

---

**Note** The `Settle` date is set to the `ValuationDate` of the stock tree.

---

## Examples

Price a compound option using an ITT tree.

Load the file `deriv.mat` which provides the `ITTTree`. The `ITTTree` structure contains the stock specification and time information needed to price the option.

```
load deriv.mat
```

Set the required values. Other arguments will use defaults.

```
UOptSpec = 'Call';
UStrike = 99;
USettle = '01-Jan-2006';
UExerciseDates = '01-Jan-2010';
UAmericanOpt = 1;
COptSpec = 'Put';
CStrike = 5;
CSettle = '01-Jan-2006';
CExerciseDates = '01-Jan-2010';

Price = compoundbyitt(ITTTree, UOptSpec, UStrike, USettle, ...
UExerciseDates, UAmericanOpt, COptSpec, CStrike, CSettle, ...
CExerciseDates)

Price =

 2.727
```

## References

Rubinstein, Mark, “Double Trouble,” *Risk 5*, 1991.

## See Also

`instcompound` | `itttree`

## Purpose

Instrument prices from Cox-Ross-Rubinstein tree

## Syntax

`[Price, PriceTree] = crrprice(CRRTree, InstSet, Options)`

## Arguments

| | |
|---|---|
| `CRRTree` | Stock price tree structure created by `crrtree`. |
| `InstSet` | Variable containing a collection of `NINST` instruments. Instruments are categorized by type; each type can have different data fields. The stored data field is a row vector or string for each instrument. For more information about how to create the `InstSet` structure, see `instadd`. |
| `Options` | (Optional) Derivatives pricing options structure created with `derivset`. |

## Description

`[Price, PriceTree] = crrprice(CRRTree, InstSet, Options)` computes stock option prices using a CRR binomial tree created with `crrtree`.

`Price` is a number of instruments `(NINST)`-by-`1` vector of prices for each instrument. The prices are computed by backward dynamic programming on the stock tree. If an instrument cannot be priced, `NaN` is returned.

`PriceTree` is a MATLAB structure of trees containing vectors of instrument prices and a vector of observation times for each node.

`PriceTree.PTree` contains the prices.

`PriceTree.tObs` contains the observation times.

`PriceTree.dObs` contains the observation dates.

`crrprice` handles instrument types: `'Asian'`, `'Barrier'`, `'Compound'`, `'Lookback'`, `'OptStock'`. See `instadd` to construct defined types.

Related single-type pricing functions are:

- `asianbycrr`: Price an Asian option from a CRR tree.
- `barrierbycrr`: Price a barrier option from a CRR tree.
- `compoundbycrr`: Price a compound option from a CRR tree.
- `lookbackbycrr`: Price a lookback option from a CRR tree.
- `optstockbycrr`: Price an American, Bermuda, or European option from a CRR tree.

## Examples

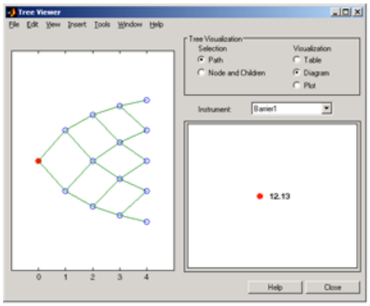
Load the CRR tree and instruments from the data file `deriv.mat`. Price the barrier and lookback options contained in the instrument set.

```
load deriv.mat;
CRRSubSet = instselect(CRRInstSet,'Type', ...
{'Barrier', 'Lookback'});

instdisp(CRRSubSet)

%Table of instrument portfolio partially displayed:
Index Type OptSpec Strike Settle   ExerciseDates AmericanOpt BarrierSpec ...
1     Barrier call 105   01-Jan-2003 01-Jan-2006 1           ui ...

Index Type     OptSpec Strike Settle     ExerciseDates AmericanOpt Name   Quantity
2     Lookback call    115   01-Jan-2003 01-Jan-2006   0           Lookback1 7
3     Lookback call    115   01-Jan-2003 01-Jan-2007   0           Lookback2 9

[Price, PriceTree] = crrprice(CRRTree, CRRSubSet)

Price =

   12.1272
    7.6015
   11.7772

PriceTree =
```

```
FinObj: 'BinPriceTree'
 PTree: {1x5 cell}
  tObs: [0 1 2 3 4]
  dObs: [731582 731947 732313 732678 733043]
```

You can use `treeviewer` to see the prices of these three instruments along the price tree.

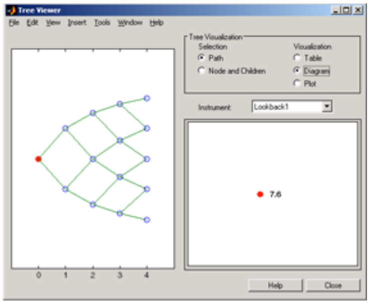
```
treeviewer(PriceTree, CRRSubSet)
```



Tree Visualization
Selection: ● Path ○ Node and Children
Visualization: ○ Table ● Diagram ○ Plot
Instrument: Barrier1

12.13

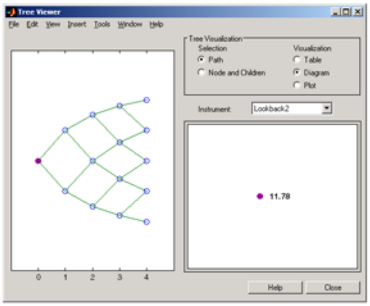
Barrier1



Tree Visualization
Selection: ● Path ○ Node and Children
Visualization: ○ Table ● Diagram ○ Plot
Instrument: Lookback1

7.6

Lookback1



Tree Visualization
Selection: ● Path ○ Node and Children
Visualization: ○ Table ● Diagram ○ Plot
Instrument: Lookback2

11.78

Lookback2

## See Also

`crrsens` | `crrtree` | `instadd`

## Purpose

Instrument prices and sensitivities from Cox-Ross-Rubinstein tree

## Syntax

`[Delta, Gamma, Vega, Price] = crrsens(CRRTree, InstSet, Options)`

## Arguments

| | |
|---|---|
| `CRRTree` | Interest-rate tree structure created by `crrtree`. |
| `InstSet` | Variable containing a collection of `NINST` instruments. Instruments are categorized by type; each type can have different data fields. The stored data field is a row vector or string for each instrument. |
| `Options` | (Optional) Derivatives pricing options structure created with `derivset`. |

## Description

`[Delta, Gamma, Vega, Price] = crrsens(CRRTree, InstSet, Options)` computes dollar sensitivities and prices for instruments using a binomial tree created with `crrtree`. `NINST` instruments from a financial instrument variable, `InstSet`, are priced. `crrsens` handles instrument types: `'Asian'`, `'Barrier'`, `'Compound'`, `'Lookback'`, `'OptStock'`. See `instadd` for information on instrument types.

`Delta` is an `NINST`-by-`1` vector of deltas, representing the rate of change of instrument prices with respect to changes in the stock price. `Delta` is computed by finite differences in calls to `crrtree`. See `crrtree` for information on the stock tree.

`Gamma` is an `NINST`-by-`1` vector of gammas, representing the rate of change of instrument deltas with respect to the changes in the stock price. `Gamma` is computed by finite differences in calls to `crrtree`.

`Vega` is an `NINST`-by-`1` vector of vegas, representing the rate of change of instrument prices with respect to the changes in the volatility of the stock. `Vega` is computed by finite differences in calls to `crrtree`.

---

**Note** All sensitivities are returned as dollar sensitivities. To find the per-dollar sensitivities, divide by the respective instrument price.

---

## Examples

Load the CRR tree and instruments from the data file `deriv.mat`. Compute the `Delta` and `Gamma` sensitivities of the barrier and lookback options contained in the instrument set.

```
load deriv.mat;
CRRSubSet = instselect(CRRInstSet,'Type', ...
{'Barrier', 'Lookback'});

instdisp(CRRSubSet)

%Table of instrument portfolio partially displayed:
Index Type OptSpec Strike Settle  ExerciseDates AmericanOpt BarrierSpec ...
1     Barrier call 105   01-Jan-2003 01-Jan-2006 1           ui ...

Index Type   OptSpec Strike Settle     ExerciseDates AmericanOpt Name Quantity
2     Lookback call    115   01-Jan-2003 01-Jan-2006   0           Lookback1 7
3     Lookback call    115   01-Jan-2003 01-Jan-2007   0           Lookback2 9

[Delta, Gamma] = crrsens(CRRTree, CRRSubSet)

Delta =

 0.6885
 0.6049
 0.8187

Gamma =

 0.0310
-0.0000
 0.0000
```

**See Also** `crrprice` | `crrtree`

## Purpose

Specify time structure for Cox-Ross-Rubinstein tree

## Syntax

```
TimeSpec = crrtimespec(ValuationDate, Maturity, NumPeriods)
```

## Arguments

| | |
|---|---|
| `ValuationDate` | Scalar date indicating the pricing date and first observation in the tree. A serial date number or date string. |
| `Maturity` | Scalar date indicating depth of the tree. |
| `NumPeriods` | Scalar determining number of time steps in the tree. |

## Description

`TimeSpec = crrtimespec(ValuationDate, Maturity, NumPeriods)` sets the number of levels and node times for a CRR binomial tree.

`TimeSpec` is a structure specifying the time layout for a CRR binomial tree.

## Examples

Specify a four-period CRR tree with time steps of 1 year.

```
ValuationDate = '1-July-2002';
Maturity = '1-July-2006';
TimeSpec = crrtimespec(ValuationDate, Maturity, 4)

TimeSpec =

           FinObj: 'BinTimeSpec'
    ValuationDate: 731398
         Maturity: 732859
       NumPeriods: 4
            Basis: 0
     EndMonthRule: 1
tObs: [0 1 2 3 4]
dObs: [1x5 double]
```

## See Also

`crrtree` | `stockspec`

## Purpose

Construct Cox-Ross-Rubinstein stock tree

## Syntax

`CRRTree = crrtree(StockSpec, RateSpec, TimeSpec)`

## Arguments

| | |
|---|---|
| `StockSpec` | Stock specification. See `stockspec` for information on creating a stock specification. |
| `RateSpec` | Interest-rate specification for the initial risk free rate curve. See `intenvset` for information on declaring an interest-rate variable. |
| `TimeSpec` | Tree time layout specification. Defines the observation dates of the CRR binomial tree. See `crrtimespec` for information on the tree structure. |

---

**Note** The standard CRR tree assumes a constant interest rate, but `RateSpec` allows you to specify an interest-rate curve with varying rates. If you specify variable interest rates, the resulting tree will not be a standard CRR tree.

---

## Description

`CRRTree = crrtree(StockSpec, RateSpec, TimeSpec)` creates a structure specifying the time layout for a CRR binomial tree.

## Examples

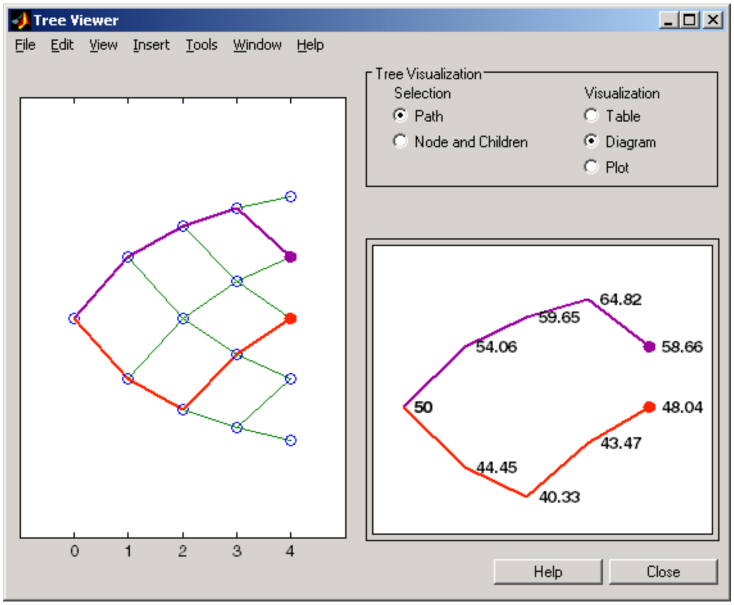
Using the data provided, create a stock specification (`StockSpec`), rate specification (`RateSpec`), and tree time layout specification (`TimeSpec`). Then use these specifications to create a CRR tree with `crrtree`.

```
Sigma = 0.20;
AssetPrice = 50;
DividendType = 'cash';
DividendAmounts = [0.50; 0.50; 0.50; 0.50];
ExDividendDates = {'03-Jan-2003'; '01-Apr-2003'; '05-July-2003';
```

```
'01-Oct-2003'}

StockSpec = stockspec(Sigma, AssetPrice, DividendType, ...
DividendAmounts, ExDividendDates)

StockSpec =

              FinObj: 'StockSpec'
               Sigma: 0.2000
          AssetPrice: 50
        DividendType: 'cash'
     DividendAmounts: [4x1 double]
     ExDividendDates: [4x1 double]

RateSpec = intenvset('Rates', 0.05, 'StartDates',...
'01-Jan-2003', 'EndDates', '31-Dec-2003')

RateSpec =

          FinObj: 'RateSpec'
     Compounding: 2
            Disc: 0.9519
           Rates: 0.0500
        EndTimes: 1.9945
      StartTimes: 0
        EndDates: 731946
      StartDates: 731582
   ValuationDate: 731582
           Basis: 0
    EndMonthRule: 1

ValuationDate = '1-Jan-2003';
Maturity = '31-Dec-2003';
TimeSpec = crrtimespec(ValuationDate, Maturity, 4)

TimeSpec =
```

```
          FinObj: 'BinTimeSpec'
   ValuationDate: 731582
        Maturity: 731946
      NumPeriods: 4
           Basis: 0
    EndMonthRule: 1
            tObs: [0 0.2493 0.4986 0.7479 0.9972]
            dObs: [731582 731673 731764 731855 731946]

CRRTree = crrtree(StockSpec, RateSpec, TimeSpec)

CRRTree =

       FinObj: 'BinStockTree'
       Method: 'CRR'
    StockSpec: [1x1 struct]
     TimeSpec: [1x1 struct]
     RateSpec: [1x1 struct]
         tObs: [0 0.2493 0.4986 0.7479 0.9972]
         dObs: [731582 731672 731763 731856 731946]
        STree: {1x5 cell}
      UpProbs: [0.5370 0.5370 0.5370 0.5370]
```

Use `treeviewer` to observe the tree you have created.



Tree Visualization

Selection
● Path
○ Node and Children

Visualization
○ Table
● Diagram
○ Plot

## See Also

`crrtimespec` | `intenvset` | `stockspec`

# cvtree

## Purpose

Convert inverse-discount tree to interest-rate tree

## Syntax

`RateTree = cvtree(Tree)`

## Arguments

| | |
|---|---|
| `Tree` | Heath-Jarrow-Morton, Black-Derman-Toy, Hull-White, or Black-Karasinski tree structure using inverse-discount notation for forward rates. |

## Description

`RateTree = cvtree(Tree)` converts a tree structure using inverse-discount notation to a tree structure using rate notation for forward rates.

## Examples

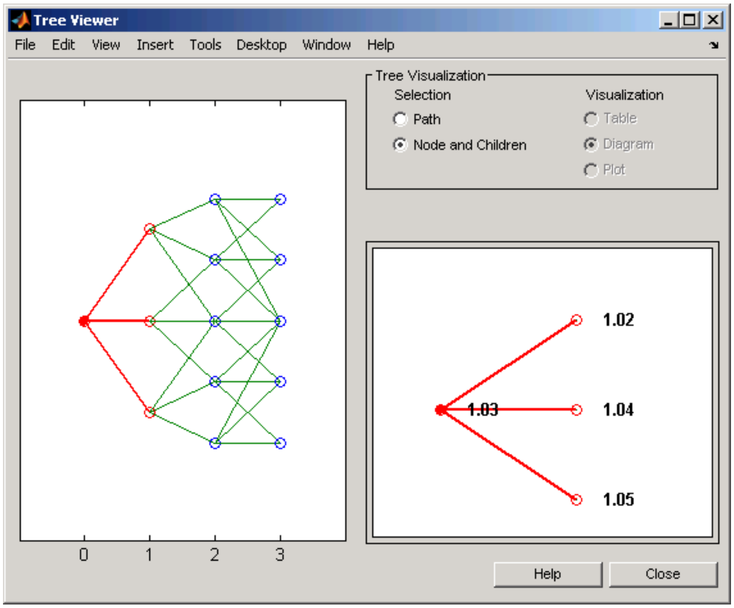
Convert a Hull-White tree using inverse-discount notation to a Hull-White tree displaying interest-rate notation.

```
load deriv.mat;

HWTree

HWTree =

      FinObj: 'HWFwdTree'
     VolSpec: [1x1 struct]
    TimeSpec: [1x1 struct]
    RateSpec: [1x1 struct]
        tObs: [0 1 2 3]
        dObs: [731947 732313 732678 733043]
      CFlowT: {[4x1 double]  [3x1 double]  [2x1 double]  [4]}
       Probs: {[3x1 double]  [3x3 double]  [3x5 double]}
     Connect: {[2]  [2 3 4]  [2 2 3 4 4]}
     FwdTree: {1x4 cell}

HWTree.FwdTree{1}
```

```
ans =
    1.0279

HWTree.FwdTree{2}

ans =
    1.0528    1.0356    1.0186
```

Use `treeviewer` to display the path of interest rates expressed in inverse-discount notation.

```
treeviewer(HWTree)
```



Tree Visualization

Selection
○ Path
● Node and Children

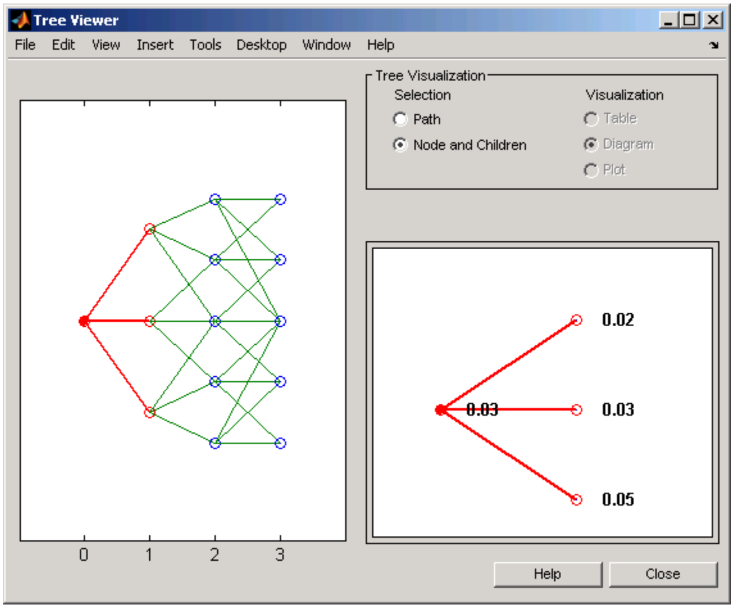
Visualization
○ Table
● Diagram
○ Plot

Use `cvtree` to convert the inverse-discount notation to interest-rate notation.

```
RTree = cvtree(HWTree)
```

```
RTree =


      FinObj: 'HWRateTree'
     VolSpec: [1x1 struct]
    TimeSpec: [1x1 struct]
    RateSpec: [1x1 struct]
        tObs: [0 1 2 3]
        dObs: [731947 732313 732678 733043]
      CFlowT: {[4x1 double]  [3x1 double]  [2x1 double]  [4]}
       Probs: {[3x1 double]  [3x3 double]  [3x5 double]}
     Connect: {[2]  [2 3 4]  [2 2 3 4 4]}
    RateTree: {1x4 cell}

RTree.RateTree{1}

ans =
    0.0275

RTree.RateTree{2}

ans =
    0.0514    0.0349    0.0185
```

Now use `treeviewer` to display the converted tree, showing the path of interest rates expressed as forward rates.



Tree Visualization

Selection
○ Path
● Node and Children

Visualization
○ Table
● Diagram
○ Plot

## See Also

`disc2rate` | `rate2disc`

# date2time

## Purpose

Time and frequency from dates

## Syntax

```
[Times, F] = date2time(Settle, Dates, Compounding, Basis,
EndMonthRule)
```

## Arguments

| | |
|---|---|
| `Settle` | Settlement date. A vector of serial date numbers or date strings. |
| `Dates` | Vector of dates corresponding to the compounding value. |
| `Compounding` | (Optional) Scalar value representing the rate at which the input zero rates were compounded when annualized. This argument determines the formula for the discount factors:<br>`Compounding = 1, 2, 3, 4, 6, 12` (Default = `2`.)<br>`Disc = (1 + Z/F)^(-T)`, where `F` is the compounding frequency, `Z` is the zero rate, and `T` is the time in periodic units; for example, `T = F` is 1 year.<br>`Compounding = 365`<br>`Disc = (1 + Z/F)^(-T)`, where `F` is the number of days in the basis year and `T` is a number of days elapsed computed by basis.<br>`Compounding = -1`<br>`Disc = exp(-T*Z)`, where `T` is time in years. |

| | |
|---|---|
| `Basis` | (Optional) Day-count basis of the instrument. A vector of integers.<br>• 0 = actual/actual (default)<br>• 1 = 30/360 (SIA)<br>• 2 = actual/360<br>• 3 = actual/365<br>• 4 = 30/360 (BMA)<br>• 5 = 30/360 (ISDA)<br>• 6 = 30/360 (European)<br>• 7 = actual/365 (Japanese)<br>• 8 = actual/actual (ICMA)<br>• 9 = actual/360 (ICMA)<br>• 10 = actual/365 (ICMA)<br>• 11 = 30/360E (ICMA)<br>• 12 = actual/actual (ISDA)<br>• 13 = BUS/252<br>For more information, see basis. |
| `EndMonthRule` | (Optional) End-of-month rule. A vector. This rule applies only when `Maturity` is an end-of-month date for a month having 30 or fewer days. `0` = ignore rule, meaning that a bond's coupon payment date is always the same numerical day of the month. `1` = set rule on (default), meaning that a bond's coupon payment date is always the last actual day of the month. |

## Description

`[Times, F] = date2time(Settle, Dates, Compounding, Basis, EndMonthRule)` computes time factors appropriate to compounded rate quotes beyond the settlement date.

`Times` is a vector of time factors.

`F` is a scalar of related compounding frequencies.

---

**Note** To obtain accurate results from this function, the `Basis` and `Dates` arguments must be consistent. If the `Dates` argument contains months that have 31 days, `Basis` must be one of the values that allow months to contain more than 30 days; for example, `Basis` = `0`, `3`, or `7`.

---

`date2time` is the inverse of `time2date`.

## See Also

`cftimes` | `disc2rate` | `rate2disc` | `time2date`

## Purpose

Display date entries

## Syntax

```
datedisp(NumMat, DateForm)
CharMat = datedisp(NumMat, DateForm)
```

## Arguments

| | |
|---|---|
| `NumMat` | Numeric matrix to display. |
| `DateForm` | (Optional) Date format. See `datestr` for available and default format flags. |

## Description

`datedisp(NumMat, DateForm)` displays the matrix with the serial dates formatted as date strings, using a matrix with mixed numeric entries and serial date number entries. Integers between `datenum('01-Jan-1900')` and `datenum('01-Jan-2200')` are assumed to be serial date numbers, while all other values are treated as numeric entries.

`CharMat` is a character array representing `NumMat`. If no output variable is assigned, the function prints the array to the display (`CharMat = datedisp(NumMat, DateForm)`).

## Examples

```
NumMat = [ 730730, 0.03, 1200, 730100;
           730731, 0.05, 1000, NaN]

NumMat =

   1.0e+05 *

    7.3073    0.0000    0.0120    7.3010
    7.3073    0.0000    0.0100       NaN

datedisp(NumMat)
    01-Sep-2000   0.03   1200   11-Dec-1998
    02-Sep-2000   0.05   1000       NaN
```

## Tips

This function is identical to the `datedisp` function in Financial Toolbox software.

## See Also

`datenum` | `datestr`

## Purpose

Get derivatives pricing options

## Syntax

```
Value = derivget(Options, 'Parameter')
```

## Arguments

| | |
|---|---|
| `Options` | Existing options specification structure, probably created from previous call to `derivset`. |
| *`Parameter`* | Must be `'Diagnostics'`, `'Warnings'`, `'ConstRate'`, or `'BarrierMethod'`. It is sufficient to type only the leading characters that uniquely identify the parameter. Case is ignored for parameter names. |

## Description

`Value = derivget(Options, 'Parameter')` extracts the value of the named parameter from the derivative options structure `Options`. Parameter values can be `'off'` or `'on'`, except for `'BarrierMethod'`, which can be `'unenhanced'` or `'interp'`. Specifying `'unenhanced'` uses no correction calculation. Specifying `'interp'` uses an enhanced valuation interpolating between nodes on barrier boundaries.

## Examples

**Example 1**. Create an Options structure with the value of `Diagnostics` set to `'on'`.

```
Options = derivset('Diagnostics','on')
```

Use `derivget` to extract the value of `Diagnostics` from the `Options` structure.

```
Value = derivget(Options, 'Diagnostics')

Value =

on
```

**Example 2**. Use `derivget` to extract the value of `ConstRate`.

```
Value   = derivget(Options, 'ConstRate')

Value =

on
```

Because the value of `'ConstRate'` was not previously set with `derivset`, the answer represents the default setting for `'ConstRate'`.

**Example 3**. Find the value of `'BarrierMethod'` in this structure.

```
derivget(Options ,'BarrierMethod')

ans =

unenhanced
```

## See Also

`barrierbycrr` | `barrierbyeqp` | `derivset`

## Purpose

Set or modify derivatives pricing options

## Syntax

```
Options = derivset(Options, 'Parameter1', Value1,
... 'Parameter4', Value4)
Options = derivset(OldOptions, NewOptions)
Options = derivset
derivset
```

## Arguments

| | |
|---|---|
| `Options` | (Optional) Existing options specification structure, probably created from a previous call to `derivset`. |
| `Parameter`*n* | The parameter must be `'Diagnostics'`, `'Warnings'`, `'ConstRate'`, or `'BarrierMethod'`. Parameters can be entered in any order. |
| `Value`*n* | (BDT, BK, HJM, or HW pricing only) The parameter values for the following three options can be `'on'` or `'off'`:<br>• `'Diagnostics' 'on'` generates diagnostic information. The default is `'Diagnostics' 'off'`.<br>• `'Warnings' 'on'` (default) displays a warning message when executing a pricing function.<br>• `'ConstRate' 'on'` (default) assumes a constant rate between tree nodes.<br>For pricing barrier options, the `'BarrierMethod'` pricing option can be `'unenhanced'` (default) or `'interp'`. Specifying `'unenhanced'` uses no correction calculation. Specifying `'interp'` uses an enhanced valuation interpolating between nodes on barrier boundaries. |

| | |
|---|---|
| `OldOptions` | Existing options specification structure. |
| `NewOptions` | New options specification structure. |

## Description

`Options = derivset(Options, 'Parameter1', Value1, ... 'Parameter4', Value4)` creates a derivatives pricing options structure `Options` in which the named parameters have the specified values. Any unspecified value is set to the default value for that parameter when `Options` is passed to the pricing function. It is sufficient to type only the leading characters that uniquely identify the parameter name. Case is also ignored for parameter names.

If the optional input argument `Options` is specified, `derivset` modifies an existing pricing options structure by changing the named parameters to the specified values.

**Note** For parameter *values*, correct case and the complete string are required; if an invalid string is provided, the default is used.

`Options = derivset(OldOptions, NewOptions)` combines an existing options structure `OldOptions` with a new options structure `NewOptions`. Any parameters in `NewOptions` with nonempty values overwrite the corresponding old parameters in `OldOptions`.

`Options = derivset` creates an options structure `Options` whose fields are set to the default values.

`derivset` with no input or output arguments displays all parameter names and information about their possible values.

## Examples

```
Options = derivset('Diagnostics','on')
```

enables the display of additional diagnostic information that appears when executing pricing functions.

```
Options = derivset(Options, 'ConstRate', 'off')
```

changes the `ConstRate` parameter in the existing `Options` structure so that the assumption of constant rates between tree nodes no longer applies.

With no input or output arguments `derivset` displays all parameter names and information about their possible values.

```
derivset
            Diagnostics: [ on   | {off} ]
               Warnings: [ {on} | off   ]
              ConstRate: [ {on} | off   ]
          BarrierMethod: [ {unenhanced} | interp   ]
```

## See Also

`barrierbycrr` | `barrierbyeqp` | `derivget`

**Purpose** Interest rates from cash flow discounting factors

**Syntax** **Usage 1**: Interval points are input as times in periodic units.

```
Rates = disc2rate(Compounding, Disc, EndTimes)

Rates = disc2rate(Compounding, Disc, EndTimes, StartTimes)
```

**Usage 2**: `ValuationDate` is passed and interval points are input as dates.

```
[Rates, EndTimes, StartTimes] = disc2rate(Compounding, Disc,
EndDates, StartDates, ValuationDate)
[Rates, EndTimes, StartTimes] = disc2rate(Compounding, Disc,
EndDates, StartDates, ValuationDate, Basis, EndMonthRule)
```

## Arguments

| | |
|---|---|
| `Compounding` | Scalar value representing the rate at which the input zero rates were compounded when annualized. This argument determines the formula for the discount factors:<br>`Compounding` = `1, 2, 3, 4, 6, 12`<br>`Disc = (1 + Z/F)^(-T)`, where `F` is the compounding frequency, `Z` is the zero rate, and `T` is the time in periodic units; for example, `T = F` is 1 year.<br>`Compounding` = `365`<br>`Disc = (1 + Z/F)^(-T)`, where `F` is the number of days in the basis year and `T` is a number of days elapsed computed by basis.<br>`Compounding` = `-1`<br>`Disc = exp(-T*Z)`, where `T` is time in years. |
| `Disc` | Number of points `(NPOINTS)` by number of curves `(NCURVES)` matrix of discounts. `Disc` are |

| | |
|---|---|
| | unit bond prices over investment intervals from `StartTimes`, when the cash flow is valued, to `EndTimes`, when the cash flow is received. |
| `EndTimes` | `NPOINTS`-by-`1` vector or scalar of times in periodic units ending the interval to discount over. |
| | **Note** When `ValuationDate` is not passed, the `EndTimes` and `StartTimes` arguments are interpreted as times. |
| `StartTimes` | (Optional) `NPOINTS`-by-`1` vector or scalar of times in periodic units starting the interval to discount over. Default = `0`. |
| `EndDates` | `NPOINTS`-by-`1` vector or scalar of serial maturity dates ending the interval to discount over. |
| | **Note** When `ValuationDate` is passed, `EndDates` and `StartDates` arguments are interpreted as dates. The date `ValuationDate` is used as the zero point for computing the times. |
| `StartDates` | (Optional) `NPOINTS`-by-`1` vector or scalar of serial dates starting the interval to discount over. Default = `ValuationDate`. `StartDates` must be earlier than `EndDates`. |
| `ValuationDate` | Scalar value in serial date number form representing the observation date of the investment horizons entered in `StartDates` and `EndDates`. Required in **Usage 2**. Omitted or passed as an empty matrix to invoke **Usage 1**. |

| | |
|---|---|
| `Basis` | (Optional) Day-count basis of the instrument when using **Usage 2**. A vector of integers.<br>• 0 = actual/actual (default)<br>• 1 = 30/360 (SIA)<br>• 2 = actual/360<br>• 3 = actual/365<br>• 4 = 30/360 (BMA)<br>• 5 = 30/360 (ISDA)<br>• 6 = 30/360 (European)<br>• 7 = actual/365 (Japanese)<br>• 8 = actual/actual (ICMA)<br>• 9 = actual/360 (ICMA)<br>• 10 = actual/365 (ICMA)<br>• 11 = 30/360E (ICMA)<br>• 12 = actual/actual (ISDA)<br>• 13 = BUS/252<br>For more information, see basis. |
| `EndMonthRule` | (Optional) End-of-month rule when using **Usage 2**. A vector. This rule applies only when `Maturity` is an end-of-month date for a month having 30 or fewer days. `0` = ignore rule, meaning that a bond's coupon payment date is always the same numerical day of the month. `1` = set rule on (default), meaning that a bond's coupon payment date is always the last actual day of the month. |

## Description

**Usage 1**: `Rates = disc2rate(Compounding, Disc, EndTimes)` or `Rates = disc2rate(Compounding, Disc, EndTimes, StartTimes)` where interval points are input as times in periodic units.

**Usage 2**: `[Rates, EndTimes, StartTimes] = disc2rate(Compounding, Disc, EndDates, StartDates, ValuationDate)` or `[Rates, EndTimes, StartTimes] = disc2rate(Compounding, Disc, EndDates, StartDates, ValuationDate, Basis, EndMonthRule)` where `ValuationDate` is passed and interval points are input as dates.

`disc2rate` computes the yields over a series of `NPOINTS` time intervals given the cash flow discounts over those intervals. `NCURVES` different rate curves can be translated at once if they have the same time structure. The time intervals can represent a zero or a forward curve.

`Rates` is an `NPOINTS`-by-`NCURVES` column vector of yields in decimal form over the `NPOINTS` time intervals.

Specify the investment intervals with either input times (**Usage 1**) or input dates (**Usage 2**). Entering `ValuationDate` invokes the date interpretation; omitting `ValuationDate` invokes the default time interpretations.

For **Usage 1**:

- `StartTimes` is an `NPOINTS`-by-`1` column vector of times starting the interval to discount over, measured in periodic units.

- `EndTimes` is an `NPOINTS`-by-`1` column vector of times ending the interval to discount over, measured in periodic units.

For **Usage 2**:

- `StartDates` is an `NPOINTS`-by-`1` column vector of serial dates starting the interval to discount over, measured in days.
- `EndDates` is an `NPOINTS`-by-`1` column vector of serial date ending the interval to discount over, measured in days.

If `Compounding = 365` (daily), `StartTimes` and `EndTimes` are measured in days for **Usage 2**. Otherwise, for **Usage 1**, the arguments contain values, `T`, computed from SIA semiannual time factors, `Tsemi`, by the formula `T = Tsemi/2 * F`, where `F` is the compounding frequency.

## See Also

`rate2disc` | `ratetimes`

## Purpose

Instrument prices from Equal Probabilities binomial tree

## Syntax

`[Price, PriceTree] = eqpprice(EQPTree, InstSet, Options)`

## Arguments

| | |
|---|---|
| `EQPTree` | Stock price tree structure created by `eqptree`. |
| `InstSet` | Variable containing a collection of `NINST` instruments. Instruments are categorized by type; each type can have different data fields. The stored data field is a row vector or string for each instrument. |
| `Options` | (Optional) Derivatives pricing options structure created with `derivset`. |

## Description

`[Price, PriceTree] = eqpprice(EQPTree, InstSet, Options)` computes stock option prices using an EQP binomial tree created with `eqptree`.

`Price` is a number of instruments `(NINST)`-by-`1` vector of prices for each instrument. The prices are computed by backward dynamic programming on the stock tree. If an instrument cannot be priced, `NaN` is returned.

`PriceTree` is a MATLAB structure of trees containing vectors of instrument prices and a vector of observation times for each node.

`PriceTree.PTree` contains the prices.

`PriceTree.tObs` contains the observation times.

`PriceTree.dObs` contains the observation dates.

`eqpprice` handles instrument types: `'Asian'`, `'Barrier'`, `'Compound'`, `'Lookback'`, `'OptStock'`. See `instadd` to construct defined types.

Related single-type pricing functions are:

- `asianbyeqp`: Price an Asian option from an EQP tree.

- `barrierbyeqp`: Price a barrier option from an EQP tree.
- `compoundbyeqp`: Price a compound option from an EQP tree.
- `lookbackbyeqp`: Price a lookback option from an EQP tree.
- `optstockbyeqp`: Price an American, Bermuda, or European option from an EQP tree.

## Examples

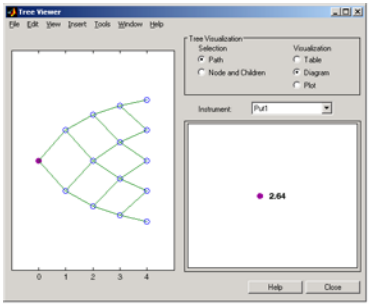
Load the EQP tree and instruments from the data file `deriv.mat`. Price the put options contained in the instrument set.

```
load deriv.mat;
EQPSubSet = instselect(EQPInstSet, 'FieldName', 'OptSpec', ...
'Data', 'put')

instdisp(EQPSubSet)

%Table of instrument portfolio partially displayed:
Index Type     OptSpec Strike Settle        ExerciseDates AmericanOpt Name...
1     OptStock put     105    01-Jan-2003 01-Jan-2006      0           Put 105...

Index Type  OptSpec Strike Settle      ExerciseDates AmericanOpt AvgType...
2     Asian put     110    01-Jan-2003 01-Jan-2006   0           arithmetic...
3     Asian put     110    01-Jan-2003 01-Jan-2007   0           arithmetic...

[Price, PriceTree] = eqpprice(EQPTree, EQPSubSet)

Price =

    2.6375
    4.7444
    3.9178

PriceTree =

    FinObj: 'BinPriceTree'
```

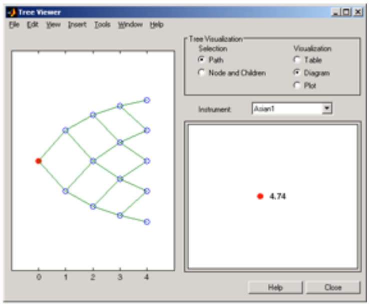
```
      PTree: {1x5 cell}
       tObs: [0 1 2 3 4]
       dObs: [731582 731947 732313 732678 733043]
```

You can use `treeviewer` to see the prices of these three instruments along the price tree.

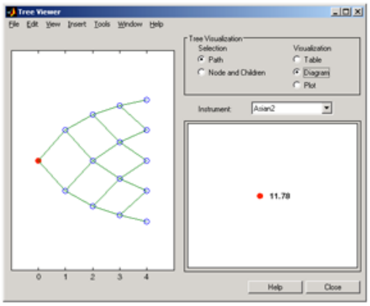
```
treeviewer(PriceTree, EQPSubSet)
```



Put1



Asian11



Asian2

## See Also

`eqpsens` | `eqptimespec` | `eqptree`

## Purpose

Instrument prices and sensitivities from Equal Probabilities binomial tree

## Syntax

```
[Delta, Gamma, Vega, Price] = eqpsens(EQPTree, InstSet,
Options)
```

## Arguments

| | |
|---|---|
| `EQPTree` | Interest-rate tree structure created by `eqptree`. |
| `InstSet` | Variable containing a collection of `NINST` instruments. Instruments are categorized by type; each type can have different data fields. The stored data field is a row vector or string for each instrument. |
| `Options` | (Optional) Derivatives pricing options structure created with `derivset`. |

## Description

`[Delta, Gamma, Vega, Price] = eqpsens(EQPTree, InstSet, Options)` computes dollar sensitivities and prices for instruments using a binomial tree created with `eqptree`. `NINST` instruments from a financial instrument variable, `InstSet`, are priced. `eqpsens` handles instrument types: `'Asian'`, `'Barrier'`, `'Compound'`, `'Lookback'`, and `'OptStock'`. See `instadd` for information on instrument types.

`Delta` is an `NINST`-by-`1` vector of deltas, representing the rate of change of instrument prices with respect to changes in the stock price. `Delta` is computed by finite differences in calls to `eqptree`. See `eqptree` for information on the stock tree.

`Gamma` is an `NINST`-by-`1` vector of gammas, representing the rate of change of instrument deltas with respect to the changes in the stock price. `Gamma` is computed by finite differences in calls to `eqptree`.

`Vega` is an `NINST`-by-`1` vector of vegas, representing the rate of change of instrument prices with respect to the changes in the volatility of the stock. `Vega` is computed by finite differences in calls to `eqptree`.

---

**Note** All sensitivities are returned as dollar sensitivities. To find the per-dollar sensitivities, divide by the respective instrument price.

---

## Examples

Load the EQP tree and instruments from the data file `deriv.mat`. Compute the `Delta` and `Gamma` sensitivities of the put options contained in the instrument set.

```
load deriv.mat;

EQPSubSet = instselect(EQPInstSet, 'FieldName', 'OptSpec', ...
'Data', 'put')

instdisp(EQPSubSet)

%Table of instrument portfolio partially displayed:
Index Type     OptSpec Strike Settle        ExerciseDates AmericanOpt Name...
1     OptStock put    105    01-Jan-2003 01-Jan-2006      0           Put 105...

Index Type  OptSpec Strike Settle     ExerciseDates AmericanOpt AvgType...
2     Asian put     110   01-Jan-2003 01-Jan-2006   0           arithmetic...
3     Asian put     110   01-Jan-2003 01-Jan-2007   0           arithmetic...

[Delta, Gamma] = eqpsens(EQPTree, EQPSubSet)

Delta =

 -0.2336
 -0.5443
 -0.4516

Gamma =

 0.0218
 0.0000
```

```
0.0000
```

## See Also

`eqpprice` | `eqptree`

# eqptimespec

## Purpose

Specify time structure for Equal Probabilities binomial tree

## Syntax

`TimeSpec = eqptimespec(ValuationDate, Maturity, NumPeriods)`

## Arguments

| | |
|---|---|
| `ValuationDate` | Scalar date indicating the pricing date and first observation in the tree. A serial date number or date string. |
| `Maturity` | Scalar date indicating depth of the tree. |
| `NumPeriods` | Scalar determining number of time steps in the tree. |

## Description

`TimeSpec = eqptimespec(ValuationDate, Maturity, NumPeriods)` sets the number of levels and node times for an equal probabilities tree.

`TimeSpec` is a structure specifying the time layout for an equal probabilities tree.

## Examples

Specify a four-period tree with time steps of 1 year.

```
ValuationDate = '1-July-2002';
Maturity = '1-July-2006';
TimeSpec = eqptimespec(ValuationDate, Maturity, 4)

TimeSpec =

           FinObj: 'BinTimeSpec'
    ValuationDate: 731398
         Maturity: 732859
       NumPeriods: 4
            Basis: 0
     EndMonthRule: 1
tObs: [0 1 2 3 4]
dObs: [1x5 double]
```

## See Also

`eqptree` | `stockspec`

# eqptree

## Purpose

Construct Equal Probabilities stock tree

## Syntax

```
EQPTree = eqptree(StockSpec, RateSpec, TimeSpec)
```

## Arguments

| | |
|---|---|
| `StockSpec` | Stock specification. See `stockspec` for information on creating a stock specification. |
| `RateSpec` | Interest-rate specification for the initial risk free rate curve. See `intenvset` for information on declaring an interest-rate variable. |
| `TimeSpec` | Tree time layout specification. Defines the observation dates of the equal probabilities binomial tree. See `eqptimespec` for information on the tree structure. |

**Note** The standard equal probabilities tree assumes a constant interest rate, but `RateSpec` allows you to specify an interest-rate curve with varying rates. If you specify variable interest rates, the resulting tree will not be a standard equal probabilities tree.

## Description

`EQPTree = eqptree(StockSpec, RateSpec, TimeSpec)` constructs an equal probabilities stock tree.

`EQPTree` is a MATLAB structure specifying the time layout for an equal probabilities stock tree.

## Examples

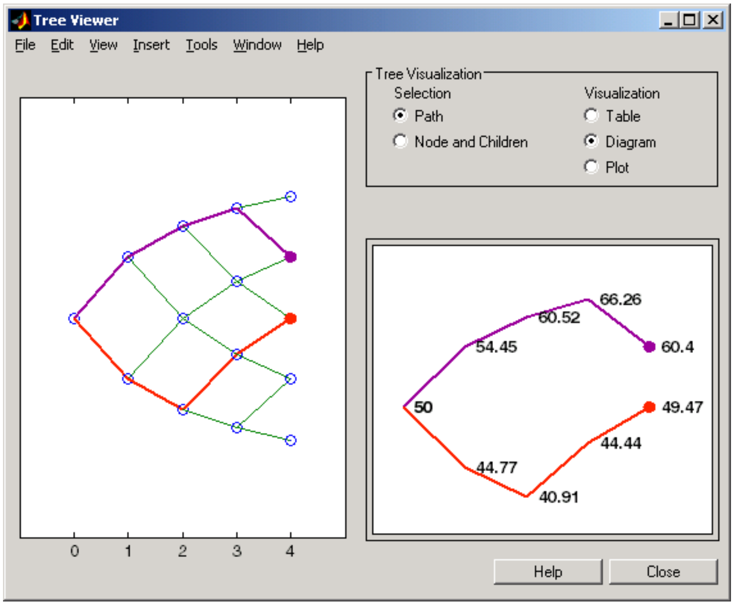
Using the data provided, create a stock specification (`StockSpec`), rate specification (`RateSpec`), and tree time layout specification (`TimeSpec`). Then use these specifications to create an EQP tree with `eqptree`.

```
Sigma = 0.20;
AssetPrice = 50;
```

```
DividendType = 'cash';
DividendAmounts = [0.50; 0.50; 0.50; 0.50];
ExDividendDates = {'03-Jan-2003'; '01-Apr-2003'; '05-July-2003';
'01-Oct-2003'}

StockSpec = stockspec(Sigma, AssetPrice, DividendType, ...
DividendAmounts, ExDividendDates)
StockSpec =

              FinObj: 'StockSpec'
               Sigma: 0.2000
          AssetPrice: 50
        DividendType: 'cash'
     DividendAmounts: [4x1 double]
     ExDividendDates: [4x1 double]

RateSpec = intenvset('Rates', 0.05, 'StartDates',...
'01-Jan-2003', 'EndDates', '31-Dec-2003')

RateSpec =

          FinObj: 'RateSpec'
     Compounding: 2
            Disc: 0.9519
           Rates: 0.0500
        EndTimes: 1.9945
      StartTimes: 0
        EndDates: 731946
      StartDates: 731582
   ValuationDate: 731582
           Basis: 0
    EndMonthRule: 1

ValuationDate = '1-Jan-2003';
Maturity = '31-Dec-2003';
TimeSpec = eqptimespec(ValuationDate, Maturity, 4)
```

```
TimeSpec =

           FinObj: 'BinTimeSpec'
    ValuationDate: 731582
         Maturity: 731946
       NumPeriods: 4
            Basis: 0
     EndMonthRule: 1
             tObs: [0 0.2493 0.4986 0.7479 0.9972]
             dObs: [731582 731673 731764 731855 731946]

EQPTree = eqptree(StockSpec, RateSpec, TimeSpec)

EQPTree =

       FinObj: 'BinStockTree'
       Method: 'EQP'
    StockSpec: [1x1 struct]
     TimeSpec: [1x1 struct]
     RateSpec: [1x1 struct]
         tObs: [0 0.2493 0.4986 0.7479 0.9972]
         dObs: [731582 731672 731763 731856 731946]
        STree: {1x5 cell}
      UpProbs: [0.5000 0.5000 0.5000 0.5000]
```

Use `treeviewer` to observe the tree you have created.



## See Also

`eqptimespec` | `intenvset` | `stockspec`

# fixedbybdt

## Purpose

Price fixed-rate note from Black-Derman-Toy interest-rate tree

## Syntax

```
[Price, PriceTree] = fixedbybdt(BDTTree, CouponRate, Settle,
Maturity)
[Price, PriceTree] = fixedbybdt(BDTTree, CouponRate, Settle,
Maturity, Reset, Basis, Principal, Options, EndMonthRule)
[Price, PriceTree] = fixedbybdt(BDTTree, CouponRate, Settle,
Maturity, Name,Value)
```

## Input Arguments

| | |
|---|---|
| `BDTTree` | Interest-rate tree structure created by `bdttree`. |
| `CouponRate` | Decimal annual rate. |
| `Settle` | Settlement dates. Number of instruments (`NINST`)-by-`1` vector of dates representing the settlement dates of the fixed-rate note. |
| `Maturity` | `NINST`-by-`1` vector of dates representing the maturity dates of the fixed-rate note. |

### Ordered Input or Name-Value Pair Arguments

Enter the following optional inputs using an ordered syntax or as name-value pair arguments. You cannot mix ordered syntax with name-value pair arguments.

**Reset**

`NINST`-by-`1` vector representing the frequency of payments per year.

**Default:** `1`

**Basis**

Day-count basis of the instrument. A vector of integers.

- 0 = actual/actual

- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (PSA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ISMA)
- 9 = actual/360 (ISMA)
- 10 = actual/365 (ISMA)
- 11 = 30/360E (ISMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see basis.

**Default:** `0` (actual/actual)

### Principal

The notional principal amount.

**Default:** `100`

### Options

Derivatives pricing options structure created with `derivset`.

### EndMonthRule

End-of-month rule. A `NINST`-by-`1` vector. This rule applies only when `Maturity` is an end-of-month date for a month having 30 or fewer days.

- `0` = Ignore rule, meaning that a bond coupon payment date is always the same numerical day of the month.
- `1` = Set rule on, meaning that a bond coupon payment date is always the last actual day of the month.

**Default:** `1`

## Name-Value Pair Arguments

Specify optional comma-separated pairs of `Name,Value` arguments, where `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside single quotes (`' '`). You can specify several name and value pair arguments in any order as `Name1,Value1,...,NameN,ValueN`.

### AdjustCashFlowsBasis

Adjust the cash flows based on the actual period day count. `NINST`-by-`1` of logicals.

**Default:** `False`

### BusinessDayConvention

Require payment dates to be business dates. `NINST`-by-`1` cell array with possible choices of business day convention:

- `actual`
- `follow`
- `modifiedfollow`
- `previous`
- `modifiedprevious`

**Default:** `actual`

### Holidays

Holidays used for business day convention. `NHOLIDAYS`-by-`1` of MATLAB date numbers.

**Default:** If no dates are specified, `holidays.m` is used.

## Description

`[Price, PriceTree] = fixedbybdt(BDTTree, CouponRate, Settle, Maturity)` computes the price of a fixed-rate note from a BDT interest-rate tree.

`[Price, PriceTree] = fixedbybdt(BDTTree, CouponRate, Settle, Maturity, Reset, Basis, Principal, Options, EndMonthRule)` computes the price of a fixed-rate note from a BDT interest-rate tree using additional input arguments.

`[Price, PriceTree] = fixedbybdt(BDTTree, CouponRate, Settle, Maturity, Name,Value)` computes the price of a price of a fixed-rate note from a BDT interest-rate tree with additional options specified by one or more `Name,Value` pair arguments.

`Price` is an `NINST`-by-`1` vector of expected prices of the fixed-rate note at time 0.

`PriceTree` is a structure of trees containing vectors of instrument prices and accrued interest, and a vector of observation times for each node.

`PriceTree.PTree` contains the clean prices.

`PriceTree.AITree` contains the accrued interest.

`PriceTree.tObs` contains the observation times.

The `Settle` date for every fixed-rate note is set to the `ValuationDate` of the BDT tree. The fixed-rate note argument `Settle` is ignored.

## Examples

Price a 10% fixed-rate note using a BDT interest-rate tree.

Load the file `deriv.mat`, which provides `BDTTree`. The `BDTTree` structure contains the time and interest-rate information needed to price the note.

```
load deriv.mat
```

Set the required values. Other arguments will use defaults.

```
CouponRate = 0.10;
Settle = '01-Jan-2000';
Maturity = '01-Jan-2004';
Reset = 1;
```

Use `fixedbybdt` to compute the price of the note.

```
Price = fixedbybdt(BDTTree, CouponRate, Settle, Maturity, Reset)

Price =

  92.9974
```

## See Also

`bdttree` | `bondbybdt` | `capbybdt` | `cfbybdt` | `floatbybdt` | `floorbybdt` | `swapbybdt`

## Purpose

Price fixed-rate note from Black-Karasinski interest-rate tree

## Syntax

```
[Price, PriceTree] = fixedbybk(BKTree, CouponRate, Settle,
Maturity)
[Price, PriceTree] = fixedbybk(BKTree, CouponRate, Settle,
Maturity, Reset, Basis, Principal, Options, EndMonthRule)
[Price, PriceTree] = fixedbybk(BKTree, CouponRate, Settle,
Maturity, Name,Value)
```

## Input Arguments

| | |
|---|---|
| `BKTree` | Interest-rate tree structure created by `bktree`. |
| `CouponRate` | Decimal annual rate. |
| `Settle` | Settlement dates. `NINST`-by-`1` vector of dates representing the settlement dates of the fixed-rate note. |
| `Maturity` | `NINST`-by-`1` vector of dates representing the maturity dates of the fixed-rate note. |

### Ordered Input or Name-Value Pair Arguments

Enter the following optional inputs using an ordered syntax or as name-value pair arguments. You cannot mix ordered syntax with name-value pair arguments.

**Reset**

`NINST`-by-`1` vector representing the frequency of payments per year.

**Default:** `1`

**Basis**

Day-count basis of the instrument. A vector of integers.

- 0 = actual/actual

- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (PSA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ISMA)
- 9 = actual/360 (ISMA)
- 10 = actual/365 (ISMA)
- 11 = 30/360E (ISMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see basis.

**Default:** `0` (actual/actual)

### Principal

The notional principal amount.

**Default:** `100`

### Options

Derivatives pricing options structure created with `derivset`.

### EndMonthRule

End-of-month rule. A `NINST`-by-`1` vector. This rule applies only when `Maturity` is an end-of-month date for a month having 30 or fewer days.

- `0` = Ignore rule, meaning that a bond coupon payment date is always the same numerical day of the month.
- `1` = Set rule on, meaning that a bond coupon payment date is always the last actual day of the month.

**Default:** `1`

### Name-Value Pair Arguments

Specify optional comma-separated pairs of `Name,Value` arguments, where `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside single quotes (`' '`). You can specify several name and value pair arguments in any order as `Name1,Value1,...,NameN,ValueN`.

#### AdjustCashFlowsBasis

Adjust the cash flows based on the actual period day count. `NINST`-by-`1` of logicals.

**Default:** `False`

#### BusinessDayConvention

Require payment dates to be business dates. `NINST`-by-`1` cell array with possible choices of business day convention:

- `actual`
- `follow`
- `modifiedfollow`
- `previous`
- `modifiedprevious`

**Default:** `actual`

#### Holidays

Holidays used for business day convention. `NHOLIDAYS`-by-`1` of MATLAB date numbers.

**Default:** If no dates are specified, `holidays.m` is used.

## Description

`[Price, PriceTree] = fixedbybk(BKTree, CouponRate, Settle, Maturity)` computes the price of a fixed-rate note from a Black-Karasinski tree.

`[Price, PriceTree] = fixedbybk(BKTree, CouponRate, Settle, Maturity, Reset, Basis, Principal, Options, EndMonthRule)` computes the price of a fixed-rate note from a Black-Karasinski tree using optional input arguments.

`[Price, PriceTree] = fixedbybk(BKTree, CouponRate, Settle, Maturity, Name,Value)` computes the price of a price of a fixed-rate note from a Black-Karasinski interest-rate tree with additional options specified by one or more `Name,Value` pair arguments.

`Price` is an `NINST`-by-`1` vector of expected prices of the fixed-rate note at time 0.

`PriceTree` is a structure of trees containing vectors of instrument prices and accrued interest, and a vector of observation times for each node.

`PriceTree.PTree` contains the clean prices.

`PriceTree.AITree` contains the accrued interest.

`PriceTree.tObs` contains the observation times.

The `Settle` date for every fixed-rate note is set to the `ValuationDate` of the BK tree. The fixed-rate note argument `Settle` is ignored.

## Examples

Price a 5% fixed-rate note using a Black-Karasinski interest-rate tree.

Load the file `deriv.mat`, which provides `BKTree`. The `BKTree` structure contains the time and interest-rate information needed to price the note.

```
load deriv.mat;
```

Set the required values. Other arguments will use defaults.

```
CouponRate = 0.05;
Settle = '01-Jan-2005';
Maturity = '01-Jan-2006';
```

Use `fixedbybk` to compute the price of the note.

```
Price = fixedbybk(BKTree, CouponRate, Settle, Maturity)

Price =

  103.5126
```

## See Also

`bktree` | `bondbybk` | `capbybk` | `cfbybk` | `floatbybk` | `floorbybk` | `swapbybk`

# fixedbyhjm

## Purpose

Price fixed-rate note from Heath-Jarrow-Morton interest-rate tree

## Syntax

```
[Price, PriceTree] = fixedbyhjm(HJMTree, CouponRate, Settle,
Maturity)
[Price, PriceTree] = fixedbyhjm(HJMTree, CouponRate, Settle,
Maturity, Reset, Basis, Principal, Options, EndMonthRule)
[Price, PriceTree] = fixedbyhjm(HJMTree, CouponRate, Settle,
Maturity, Name,Value)
```

## Input Arguments

| | |
|---|---|
| `HJMTree` | Forward-rate tree structure created by `hjmtree`. |
| `CouponRate` | Decimal annual rate. |
| `Settle` | Settlement dates. Number of instruments (`NINST`)-by-`1` vector of dates representing the settlement dates of the fixed-rate note. |
| `Maturity` | `NINST`-by-`1` vector of dates representing the maturity dates of the fixed-rate note. |

### Ordered Input or Name-Value Pair Arguments

Enter the following optional inputs using an ordered syntax or as name-value pair arguments. You cannot mix ordered syntax with name-value pair arguments.

**Reset**

`NINST`-by-`1` vector representing the frequency of payments per year.

**Default:** `1`

**Basis**

Day-count basis of the instrument. A vector of integers.

- 0 = actual/actual

- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (PSA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ISMA)
- 9 = actual/360 (ISMA)
- 10 = actual/365 (ISMA)
- 11 = 30/360E (ISMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see basis.

**Default:** `0` (actual/actual)

### Principal

The notional principal amount.

**Default:** `100`

### Options

Derivatives pricing options structure created with `derivset`.

### EndMonthRule

End-of-month rule. A `NINST`-by-`1` vector. This rule applies only when `Maturity` is an end-of-month date for a month having 30 or fewer days.

- `0` = Ignore rule, meaning that a bond coupon payment date is always the same numerical day of the month.
- `1` = Set rule on, meaning that a bond coupon payment date is always the last actual day of the month.

**Default:** `1`

### Name-Value Pair Arguments

Specify optional comma-separated pairs of `Name,Value` arguments, where `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside single quotes (`' '`). You can specify several name and value pair arguments in any order as `Name1,Value1,...,NameN,ValueN`.

#### AdjustCashFlowsBasis

Adjust the cash flows based on the actual period day count. `NINST`-by-`1` of logicals.

**Default:** `False`

#### BusinessDayConvention

Require payment dates to be business dates. `NINST`-by-`1` cell array with possible choices of business day convention:

- `actual`
- `follow`
- `modifiedfollow`
- `previous`
- `modifiedprevious`

**Default:** `actual`

#### Holidays

Holidays used for business day convention. `NHOLIDAYS`-by-`1` of MATLAB date numbers.

**Default:** If no dates are specified, `holidays.m` is used.

## Description

`[Price, PriceTree] = fixedbyhjm(HJMTree, CouponRate, Settle,`
`Maturity)` computes the price of a fixed-rate note from a HJM forward-rate tree.

`[Price, PriceTree] = fixedbyhjm(HJMTree, CouponRate, Settle,`
`Maturity, Reset, Basis, Principal, Options, EndMonthRule)` computes the price of a fixed-rate note from a HJM forward-rate tree using optional input asrguments.

`[Price, PriceTree] = fixedbyhjm(HJMTree, CouponRate, Settle,`
`Maturity, Name,Value)` computes the price of a price of a fixed-rate note from a HJM forward-rate tree with additional options specified by one or more `Name,Value` pair arguments.

`Price` is an `NINST`-by-`1` vector of expected prices of the fixed-rate note at time 0.

`PriceTree` is a structure of trees containing vectors of instrument prices and accrued interest, and a vector of observation times for each node.

`PriceTree.PBush` contains the clean prices.

`PriceTree.AIBush` contains the accrued interest.

`PriceTree.tObs` contains the observation times.

The `Settle` date for every fixed-rate note is set to the `ValuationDate` of the HJM tree. The fixed-rate note argument `Settle` is ignored.

## Examples

Price a 4% fixed-rate note using an HJM forward-rate tree.

Load the file `deriv.mat`, which provides `HJMTree`. The `HJMTree` structure contains the time and forward-rate information needed to price the note.

```
load deriv.mat
```

Set the required values. Other arguments will use defaults.

```
CouponRate = 0.04;
Settle = '01-Jan-2000';
Maturity = '01-Jan-2003';
```

Use `fixedbyhjm` to compute the price of the note.

```
Price = fixedbyhjm(HJMTree, CouponRate, Settle, Maturity)

Price =

  98.7159
```

## See Also

`bondbyhjm` | `capbyhjm` | `cfbyhjm` | `floatbyhjm` | `floorbyhjm` | `hjmtree` | `swapbyhjm`

## Purpose

Price fixed-rate note from Hull-White interest-rate tree

## Syntax

```
[Price, PriceTree] = fixedbyhw(HWTree, CouponRate, Settle,
Maturity)
[Price, PriceTree] = fixedbyhw(HWTree, CouponRate, Settle,
Maturity, Reset, Basis, Principal, Options, EndMonthRule)
[Price, PriceTree] = fixedbyhw(HWTree, CouponRate, Settle,
Maturity, Name,Value)
```

## Arguments

| | |
|---|---|
| `HWTree` | Interest-rate tree structure created by `hwtree`. |
| `CouponRate` | Decimal annual rate. |
| `Settle` | Settlement dates. `NINST`-by-`1` vector of dates representing the settlement dates of the fixed-rate note. |
| `Maturity` | `NINST`-by-`1` vector of dates representing the maturity dates of the fixed-rate note. |

### Ordered Input or Name-Value Pair Arguments

Enter the following optional inputs using an ordered syntax or as name-value pair arguments. You cannot mix ordered syntax with name-value pair arguments.

**Reset**

`NINST`-by-`1` vector representing the frequency of payments per year.

**Default:** `1`

**Basis**

Day-count basis of the instrument. A vector of integers.

- 0 = actual/actual

- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (PSA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ISMA)
- 9 = actual/360 (ISMA)
- 10 = actual/365 (ISMA)
- 11 = 30/360E (ISMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see basis.

**Default:** `0` (actual/actual)

### Principal

The notional principal amount.

**Default:** `100`

### Options

Derivatives pricing options structure created with `derivset`.

### EndMonthRule

End-of-month rule. A `NINST`-by-`1` vector. This rule applies only when `Maturity` is an end-of-month date for a month having 30 or fewer days.

- `0` = Ignore rule, meaning that a bond coupon payment date is always the same numerical day of the month.
- `1` = Set rule on, meaning that a bond coupon payment date is always the last actual day of the month.

**Default:** `1`

## Name-Value Pair Arguments

Specify optional comma-separated pairs of `Name,Value` arguments, where `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside single quotes (`' '`). You can specify several name and value pair arguments in any order as `Name1,Value1,...,NameN,ValueN`.

### AdjustCashFlowsBasis

Adjust the cash flows based on the actual period day count. `NINST`-by-`1` of logicals.

**Default:** `False`

### BusinessDayConvention

Require payment dates to be business dates. `NINST`-by-`1` cell array with possible choices of business day convention:

- `actual`
- `follow`
- `modifiedfollow`
- `previous`
- `modifiedprevious`

**Default:** `actual`

### Holidays

Holidays used for business day convention. `NHOLIDAYS`-by-`1` of MATLAB date numbers.

**Default:** If no dates are specified, `holidays.m` is used.

## Description

`[Price, PriceTree] = fixedbyhw(HWTree, CouponRate, Settle, Maturity)` computes the price of a fixed-rate note from a Hull-White tree.

`[Price, PriceTree] = fixedbyhw(HWTree, CouponRate, Settle, Maturity, Reset, Basis, Principal, Options, EndMonthRule)` computes the price of a fixed-rate note from a Hull-White tree using optional input arguments.

`[Price, PriceTree] = fixedbyhw(HWTree, CouponRate, Settle, Maturity, Name,Value)` computes the price of a price of a fixed-rate note from a Hull-White tree with additional options specified by one or more `Name,Value` pair arguments.

`Price` is an `NINST`-by-`1` vector of expected prices of the fixed-rate note at time 0.

`PriceTree` is a structure of trees containing vectors of instrument prices and accrued interest, and a vector of observation times for each node.

`PriceTree.PTree` contains the clean prices.

`PriceTree.AITree` contains the accrued interest.

`PriceTree.tObs` contains the observation times.

The `Settle` date for every fixed-rate note is set to the `ValuationDate` of the HW tree. The fixed-rate note argument `Settle` is ignored.

## Examples

Price a 5% fixed-rate note using a Hull-White interest-rate tree.

Load the file `deriv.mat`, which provides `HWTree`. The `HWTree` structure contains the time and interest-rate information needed to price the note.

```
load deriv.mat;
```

Set the required values. Other arguments will use defaults.

```
CouponRate = 0.05;
Settle = '01-Jan-2005';
Maturity = '01-Jan-2006';
```

Use `fixedbyhw` to compute the price of the note.

```
Price = fixedbyhw(HWTree, CouponRate, Settle, Maturity)

Price =

  103.5126
```

## See Also

`bondbyhw` | `capbyhw` | `cfbyhw` | `floatbyhw` | `floorbyhw` | `hwtree` | `swapbyhw`

## Purpose

Price fixed-rate note from set of zero curves

## Syntax

```
[Price, PriceNoAI, CFlowAmounts, CFlowDates] =
fixedbyzero(RateSpec, CouponRate, Settle, Maturity)
[Price, PriceNoAI, CFlowAmounts, CFlowDates] =
fixedbyzero(RateSpec, CouponRate, Settle, Maturity,
Reset, Basis,  Principal, EndMonthRule)
[Price PriceNoAI, CFlowAmounts, CFlowDates] =
fixedbyzero(RateSpec, CouponRate, Settle, Maturity,
Name, Value)
```

## Description

`[Price, PriceNoAI, CFlowAmounts, CFlowDates] = fixedbyzero(RateSpec, CouponRate, Settle, Maturity)` computes the price of a fixed-rate note from a set of zero curves. All inputs are either scalars or `NINST`-by-1 vectors unless otherwise specified. Any date may be a serial date number or date string. An optional argument may be passed as an empty matrix `[]`.

`[Price, PriceNoAI, CFlowAmounts, CFlowDates] = fixedbyzero(RateSpec, CouponRate, Settle, Maturity, Reset, Basis, Principal, EndMonthRule)` computes the price of a fixed-rate note from a set of zero curves using optional input arguments. All inputs are either scalars or `NINST`-by-1vectors unless otherwise specified. Any date may be a serial date number or date string. An optional argument may be passed as an empty matrix `[]`.

`[Price PriceNoAI, CFlowAmounts, CFlowDates] = fixedbyzero(RateSpec, CouponRate, Settle, Maturity, Name, Value)` computes the price of a fixed-rate note from a set of zero curves with additional options specified by one or more `Name, Value` pair arguments.

## Input Arguments

### RateSpec

Structure containing the properties of an interest-rate structure. See `intenvset` for information on creating `RateSpec`.

### CouponRate

Decimal annual rate.

#### Settle

Settlement date. `Settle` must be either a scalar or `NINST`-by-`1` vector of serial date numbers or date strings of the same value which represent the settlement date for each bond. `Settle` must be earlier than `Maturity`.

#### Maturity

Maturity date.

### Ordered Input or Name-Value Pair Arguments

Enter the following optional inputs using an ordered syntax or as name-value pair arguments. You cannot mix ordered syntax with name-value pair arguments.

#### Reset

`NINST`-by-`1` vector representing the frequency of payments per year.

**Default:** `1`

#### Basis

Day-count basis of the instrument. A vector of integers.

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (PSA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)

- 8 = actual/actual (ISMA)
- 9 = actual/360 (ISMA)
- 10 = actual/365 (ISMA)
- 11 = 30/360E (ISMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see basis.

**Default:** `0` (actual/actual)

### Principal

The notional principal amount.

**Default:** `100`

### Options

Derivatives pricing options structure created with `derivset`.

### EndMonthRule

End-of-month rule. A `NINST`-by-`1` vector. This rule applies only when `Maturity` is an end-of-month date for a month having 30 or fewer days.

- `0` = Ignore rule, meaning that a bond coupon payment date is always the same numerical day of the month.
- `1` = Set rule on, meaning that a bond coupon payment date is always the last actual day of the month.

**Default:** `1`

## Name-Value Pair Arguments

Specify optional comma-separated pairs of `Name,Value` arguments, where `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside single quotes (`' '`). You can specify several name and value pair arguments in any order as `Name1,Value1,...,NameN,ValueN`.

### AdjustCashFlowsBasis

Adjust the cash flows based on the actual period day count. `NINST`-by-`1` of logicals.

**Default:** `False`

### BusinessDayConvention

Require payment dates to be business dates. `NINST`-by-`1` cell array with possible choices of business day convention:

- `actual`
- `follow`
- `modifiedfollow`
- `previous`
- `modifiedprevious`

**Default:** `actual`

### Holidays

Holidays used for business day convention. `NHOLIDAYS`-by-`1` of MATLAB date numbers.

**Default:** If no dates are specified, `holidays.m` is used.

## Output Arguments

### Price

A number of instruments (`NINST`) by number of curves (`NUMCURVES`) matrix of fixed-rate note prices. Each column arises from one of the zero curves.

**PriceNoAI**

A `NINST`-by-`NUMCURVES` matrix of dirty bond price (clean + accrued interest). Each column arises from one of the zero curves.

**CFlowAmounts**

A `NINST`-by-`NUMCFS` matrix of cash flows for each bond.

**CFlowDates**

A `NINST`-by-`NUMCFS` matrix of payment dates for each bond.

## Examples

Price a 4% fixed-rate note using a set of zero curves. Load the file `deriv.mat`, which provides `ZeroRateSpec`, the interest-rate term structure needed to price the note.

```
load deriv.mat
```

Set the required values. Other arguments will use defaults.

```
CouponRate = 0.04;
Settle = '01-Jan-2000';
Maturity = '01-Jan-2003';
```

Use `fixedbyzero` to compute the price of the note:

```
Price = fixedbyzero(ZeroRateSpec, CouponRate, Settle, Maturity)

Price =

  98.7159
```

## See Also

| `bondbyzero` | `cfbyzero` | `floatbyzero` | `swapbyzero`

## Purpose

Price floating-rate note from Black-Derman-Toy interest-rate tree

## Syntax

```
[Price, PriceTree] = floatbybdt(BDTTree, Spread,
Settle, Maturity)
[Price, PriceTree] = floatbybdt(BDTTree, Spread,
Settle, Maturity, Reset, Basis, Principal, Options,
EndMonthRule)
[Price, PriceTree] = floatbybdt(BDTTree, Spread, Settle,
Maturity,Name,Value)
```

## Input Arguments

| | |
|---|---|
| `BDTTree` | Interest-rate tree structure created by `bdttree`. |
| `Spread` | Number of instruments (`NINST`)-by-`1` vector of number of basis points over the reference rate. |
| `Settle` | Settlement dates. `NINST`-by-`1` vector of dates representing the settlement dates of the floating-rate note.<br>**Note** The `Settle` date for every floating-rate note is set to the `ValuationDate` of the BDT Tree. The floating-rate note argument `Settle` is ignored. |
| `Maturity` | `NINST`-by-`1` vector of dates representing the maturity dates of the floating-rate note. |

### Ordered Input or Name-Value Pair Arguments

Enter the following optional inputs using an ordered syntax or as name-value pair arguments. You cannot mix ordered syntax with name-value pair arguments.

**Reset**

`NINST`-by-`1` vector representing the frequency of payments per year.

---

**Note** Payments on floating-rate notes (FRNs) are determined by the effective interest-rate between reset dates. If the reset period for a FRN spans more than one tree level, calculating the payment becomes impossible due to the recombining nature of the tree. That is, the tree path connecting the two consecutive reset dates can not be uniquely determined because there will be more than one possible path for connecting the two payment dates.

---

**Default:** `1`

### Basis

Day-count basis of the instrument. A vector of integers.

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (PSA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ISMA)
- 9 = actual/360 (ISMA)
- 10 = actual/365 (ISMA)
- 11 = 30/360E (ISMA)
- 12 = actual/365 (ISDA)

- 13 = BUS/252

For more information, see basis.

**Default:** `0` (actual/actual)

### Principal

`NINST`-by-`1` vector of notional principal amounts or `NINST`-by-`1` cell array. For the latter case, each element of the cell array is a `NumDates`-by-`2` matrix where the first column is dates and the second column is associated principal amount. The date indicates the last day that the principal value is valid.

**Default:** `100`

### Options

Derivatives pricing options structure created with `derivset`.

### EndMonthRule

End-of-month rule. `NINST`-by-`1` vector. This rule applies only when `Maturity` is an end-of-month date for a month having 30 or fewer days.

- `0` = Ignore rule, meaning that a bond coupon payment date is always the same numerical day of the month.
- `1` = Set rule on, meaning that a bond coupon payment date is always the last actual day of the month.

**Default:** `1`

## Name-Value Pair Arguments

Specify optional comma-separated pairs of `Name,Value` arguments, where `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside single quotes (`' '`). You can specify several name and value pair arguments in any order as `Name1,Value1,...,NameN,ValueN`.

### AdjustCashFlowsBasis

Adjust the cash flows based on the actual period day count. `NINST`-by-`1` of logicals.

**Default:** `false`

### BusinessDayConvention

Require payment dates to be business dates. `NINST`-by-`1` cell array with possible choices of business day convention:

- `actual`
- `follow`
- `modifiedfollow`
- `previous`
- `modifiedprevious`

**Default:** `actual`

### Holidays

Holidays used for business day convention. `NHOLIDAYS`-by-`1` of MATLAB date numbers.

**Default:** If no dates are specified, `holidays.m` is used.

### CapRate

`NINST`-by-`1` decimal annual rate or `NINST`-by-`1` cell array, where each element is a `NumDates`-by-`2` cell array, and the cell array first column is dates, and the second column is associated cap rates. The date indicates the last day that the cap rate is valid.

### FloorRate

`NINST`-by-`1` decimal annual rate or `NINST`-by-`1` cell array, where each element is a `NumDates`-by-`2` cell array, and the cell array first column is dates, and the second column is associated floor rates. The date indicates the last day that the floor rate is valid.

## Description

`[Price, PriceTree] = floatbybdt(BDTTree, Spread, Settle, Maturity)` computes the price of a floating-rate note from a BDT tree.

`[Price, PriceTree] = floatbybdt(BDTTree, Spread, Settle, Maturity, Reset, Basis, Principal, Options, EndMonthRule)` computes the price of a floating-rate note with optional inputs from a BDT tree.

`[Price, PriceTree] = floatbybdt(BDTTree, Spread, Settle, Maturity,Name,Value)` computes the price of a floating-rate note from a BDT tree with additional options specified by one or more `Name,Value` pair arguments..

`Price` is an `NINST`-by-`1` vector of expected prices of the floating-rate note at time 0.

`PriceTree` is a structure of trees containing vectors of instrument prices and accrued interest, and a vector of observation times for each node.

`PriceTree.PTree` contains the clean prices.

`PriceTree.AITree` contains the accrued interest.

`PriceTree.tObs` contains the observation times.

The `Settle` date for every floating-rate note is set to the `ValuationDate` of the BDT tree. The floating-rate note argument `Settle` is ignored.

## Examples

### Price a Floating-Rate Note Using a BDT Tree

Price a 20-basis point floating-rate note using a BDT interest-rate tree.

Load the file `deriv.mat`, which provides `BDTTree`. The `BDTTree` structure contains the time and interest-rate information needed to price the note.

```
load deriv.mat;
```

Define the floating-rate note using the required arguments. Other arguments use defaults.

```
Spread = 20;
Settle = '01-Jan-2000';
Maturity = '01-Jan-2003';
```

Use `floatbybdt` to compute the price of the note.

```
Price = floatbybk(BKTree, Spread, Settle, Maturity)
```

```
Price =

  100.4865
```

### Price an Amortizing Floating-Rate Note

Price an amortizing floating-rate note using the `Principal` input argument to define the amortization schedule.

Create the `RateSpec`.

```
Rates = [0.03583; 0.042147; 0.047345; 0.052707; 0.054302];
ValuationDate = '15-Nov-2011';
StartDates = ValuationDate;
EndDates = {'15-Nov-2012';'15-Nov-2013';'15-Nov-2014' ;'15-Nov-2015';'15-Nov-2016'};
Compounding = 1;
RateSpec = intenvset('ValuationDate', ValuationDate,'StartDates', StartDates,...
'EndDates', EndDates,'Rates', Rates, 'Compounding', Compounding)
```

```
RateSpec =

          FinObj: 'RateSpec'
     Compounding: 1
            Disc: [5x1 double]
           Rates: [5x1 double]
        EndTimes: [5x1 double]
```

```
         StartTimes: [5x1 double]
           EndDates: [5x1 double]
         StartDates: 734822
      ValuationDate: 734822
              Basis: 0
       EndMonthRule: 1
```

Create the floating-rate instrument using the following data:

```
Settle ='15-Nov-2011';
Maturity = '15-Nov-2015';
Spread = 15;
```

Define the floating-rate note amortizing schedule.

```
Principal ={{'15-Nov-2012' 100;'15-Nov-2013' 70;'15-Nov-2014' 40;'15-Nov-2015' 10}};
```

Build the BDT tree and assume volatility is 10%.

```
MatDates = {'15-Nov-2012'; '15-Nov-2013';'15-Nov-2014';'15-Nov-2015';'15-Nov-2016';'15-Nov-2017'};
BDTTimeSpec = bdttimespec(ValuationDate, MatDates);
Volatility = 0.10;
BDTVolSpec = bdtvolspec(ValuationDate, MatDates, Volatility*ones(1,length(MatDates))');
BDTT = bdttree(BDTVolSpec, RateSpec, BDTTimeSpec);
```

Compute the price of the amortizing floating-rate note.

```
Price = floatbybdt(BDTT, Spread, Settle, Maturity, 'Principal', Principal)
```

```
Price =

  100.3059
```

### Price a Collar with a Floating-Rate Note

Price a collar with a floating-rate note using the `CapRate` and `FloorRate` input argument to define the collar pricing.

Create the `RateSpec`.

```
Rates = [0.0287; 0.03024; 0.03345; 0.03861; 0.04033];
ValuationDate = '1-April-2012';
StartDates = ValuationDate;
EndDates = {'1-April-2013';'1-April-2014';'1-April-2015' ;...
'1-April-2016';'1-April-2017'};
Compounding = 1;
```

Create the `RateSpec`.

```
RateSpec = intenvset('ValuationDate', ValuationDate,'StartDates', StartDates,...
'EndDates', EndDates,'Rates', Rates, 'Compounding', Compounding);
```

Build the BDT tree and assume volatility is 5%.

```
MatDates = {'1-April-2013'; '1-April-2014';'1-April-2015';'1-April-2016';'1-April-2017';'1-April-2018'};
BDTTimeSpec = bdttimespec(ValuationDate, MatDates);
Volatility = 0.05;
BDTVolSpec = bdtvolspec(ValuationDate, MatDates, Volatility*ones(1,length(MatDates))');
BDTT = bdttree(BDTVolSpec, RateSpec, BDTTimeSpec);
```

Create the floating rate note instrument.

```
Settle ='1-April-2012';
Maturity = '1-April-2016';
Spread = 10;
Principal = 100;
```

Compute the price of a collared floating-rate note.

```
CapStrike = {{'1-April-2013' 0.03; '1-April-2015' 0.055}};
FloorStrike = {{'1-April-2013' 0.025; '1-April-2015' 0.04}};

Price = floatbybdt(BDTT, Spread, Settle, Maturity, 'CapRate',...
CapStrike, 'FloorRate', FloorStrike)
```

```
Price =

  101.2414
```

## See Also

`bdttree` | `bondbybdt` | `capbybdt` | `cfbybdt` | `fixedbybdt` | `floorbybdt` | `swapbybdt`

**Purpose** Price floating-rate note from Black-Karasinski interest-rate tree

**Syntax**

```
[Price, PriceTree] = floatbybk(BKTree, Spread,
Settle, Maturity)
[Price, PriceTree] = floatbybk(BKTree, Spread,
Settle, Maturity, Reset, Basis, Principal, Options,
EndMonthRule)
[Price, PriceTree] = floatbybk(BKTree, Spread, Settle,
Maturity,Name,Value)
```

## Input Arguments

| | |
|---|---|
| `BKTree` | Interest-rate tree structure created by `bktree`. |
| `Spread` | Number of instruments (`NINST`)-by-1 vector of number of basis points over the reference rate. |
| `Settle` | Settlement dates. `NINST`-by-`1` vector of dates representing the settlement dates of the floating-rate note.<br>**Note** The `Settle` date for every floating-rate note is set to the `ValuationDate` of the BK tree. The floating-rate note argument `Settle` is ignored. |
| `Maturity` | `NINST`-by-`1` vector of dates representing the maturity dates of the floating-rate note. |

### Ordered Input or Name-Value Pair Arguments

Enter the following optional inputs using an ordered syntax or as name-value pair arguments. You cannot mix ordered syntax with name-value pair arguments.

**Reset**

`NINST`-by-`1` vector representing the frequency of payments per year.

---

**Note** Payments on floating-rate notes (FRNs) are determined by the effective interest-rate between reset dates. If the reset period for a FRN spans more than one tree level, calculating the payment becomes impossible due to the recombining nature of the tree. That is, the tree path connecting the two consecutive reset dates can not be uniquely determined because there will be more than one possible path for connecting the two payment dates.

---

**Default:** `1`

### Basis

Day-count basis of the instrument. A vector of integers.

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (PSA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ISMA)
- 9 = actual/360 (ISMA)
- 10 = actual/365 (ISMA)
- 11 = 30/360E (ISMA)
- 12 = actual/365 (ISDA)

- 13 = BUS/252

For more information, see basis.

**Default:** `0` (actual/actual)

### Principal

`NINST`-by-`1` vector of notional principal amounts or `NINST`-by-`1` cell array. For the latter case, each element of the cell array is a `NumDates`-by-`2` matrix where the first column is dates and the second column is associated principal amount. The date indicates the last day that the principal value is valid.

**Default:** `100`

### Options

Derivatives pricing options structure created with `derivset`.

### EndMonthRule

End-of-month rule. `NINST`-by-`1` vector. This rule applies only when `Maturity` is an end-of-month date for a month having 30 or fewer days.

- `0` = Ignore rule, meaning that a bond coupon payment date is always the same numerical day of the month.
- `1` = Set rule on, meaning that a bond coupon payment date is always the last actual day of the month.

**Default:** `1`

## Name-Value Pair Arguments

Specify optional comma-separated pairs of `Name,Value` arguments, where `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside single quotes (`' '`). You can specify several name and value pair arguments in any order as `Name1,Value1,...,NameN,ValueN`.

**AdjustCashFlowsBasis**

Adjust the cash flows based on the actual period day count. `NINST`-by-`1` of logicals.

**Default:** `false`

**BusinessDayConvention**

Require payment dates to be business dates. `NINST`-by-`1` cell array with possible choices of business day convention:

- `actual`
- `follow`
- `modifiedfollow`
- `previous`
- `modifiedprevious`

**Default:** `actual`

**Holidays**

Holidays used for business day convention. `NHOLIDAYS`-by-`1` of MATLAB date numbers.

**Default:** If no dates are specified, `holidays.m` is used.

**CapRate**

`NINST`-by-`1` decimal annual rate or `NINST`-by-`1` cell array, where each element is a `NumDates`-by-`2` cell array, and the cell array first column is dates, and the second column is associated cap rates. The date indicates the last day that the cap rate is valid.

**FloorRate**

`NINST`-by-`1` decimal annual rate or `NINST`-by-`1` cell array, where each element is a `NumDates`-by-`2` cell array, and the cell array first column is dates, and the second column is associated floor rates. The date indicates the last day that the floor rate is valid.

## Description

`[Price, PriceTree] = floatbybk(BKTree, Spread, Settle, Maturity)` computes the price of a floating-rate note from a Black-Karasinski tree.

`[Price, PriceTree] = floatbybk(BKTree, Spread, Settle, Maturity, Reset, Basis, Principal, Options, EndMonthRule)` computes the price of a floating-rate note with optional inputs from a Black-Karasinski tree.

`[Price, PriceTree] = floatbybk(BKTree, Spread, Settle, Maturity,Name,Value)` computes the price of a floating-rate note from a Black-Karasinski tree with additional options specified by one or more `Name,Value` pair arguments.

`Price` is an `NINST`-by-`1` vector of expected prices of the floating-rate note at time 0.

`PriceTree` is a structure of trees containing vectors of instrument prices and accrued interest, and a vector of observation times for each node.

`PriceTree.PTree` contains the clean prices.

`PriceTree.AITree` contains the accrued interest.

`PriceTree.tObs` contains the observation times.

The `Settle` date for every floating-rate note is set to the `ValuationDate` of the BK tree. The floating-rate note argument `Settle` is ignored.

## Examples

### Price a Floating-Rate Note Using a Black-Karasinski Tree

Price a 20–basis point floating-rate note using a Black-Karasinski interest-rate tree.

Load the file `deriv.mat`, which provides `BKTree`. The `BKTree` structure contains the time and interest-rate information needed to price the note.

```
load deriv.mat;
```

Define the floating-rate note using the required arguments. Other arguments use defaults.

```
Spread = 20;
Settle = '01-Jan-2005';
Maturity = '01-Jan-2006';
```

Use `floatbybk` to compute the price of the note.

```
Price = floatbybk(BKTree, Spread, Settle, Maturity)
```

```
Price =

  100.3825
```

### Price an Amortizing Floating-Rate Note

Price an amortizing floating-rate note using the `Principal` input argument to define the amortization schedule.

Create the `RateSpec`.

```
Rates = [0.03583; 0.042147; 0.047345; 0.052707; 0.054302];
ValuationDate = '15-Nov-2011';
StartDates = ValuationDate;
EndDates = {'15-Nov-2012';'15-Nov-2013';'15-Nov-2014' ;'15-Nov-2015';'15-Nov-2016'};
Compounding = 1;
RateSpec = intenvset('ValuationDate', ValuationDate,'StartDates', StartDates,...
'EndDates', EndDates,'Rates', Rates, 'Compounding', Compounding)
```

```
RateSpec =

           FinObj: 'RateSpec'
      Compounding: 1
             Disc: [5x1 double]
            Rates: [5x1 double]
```

```
          EndTimes: [5x1 double]
        StartTimes: [5x1 double]
          EndDates: [5x1 double]
        StartDates: 734822
     ValuationDate: 734822
             Basis: 0
      EndMonthRule: 1
```

Create the floating-rate instrument using the following data:

```
Settle ='15-Nov-2011';
Maturity = '15-Nov-2015';
Spread = 15;
```

Define the floating-rate note amortizing schedule.

```
Principal ={{'15-Nov-2012' 100;'15-Nov-2013' 70;'15-Nov-2014' 40;'15-Nov-2015' 10}};
```

Build the BK tree and assume the volatility is 10%.

```
VolDates = ['15-Nov-2012'; '15-Nov-2013';'15-Nov-2014';'15-Nov-2015';'15-Nov-2016';'15-Nov-2017'];
VolCurve = 0.1;
AlphaDates = '15-Nov-2017';
AlphaCurve = 0.1;

BKVolSpec = bkvolspec(RateSpec.ValuationDate, VolDates, VolCurve,...
AlphaDates, AlphaCurve);
BKTimeSpec = bktimespec(RateSpec.ValuationDate, VolDates, Compounding);
BKT = bktree(BKVolSpec, RateSpec, BKTimeSpec);
```

Compute the price of the amortizing floating-rate note.

```
Price = floatbybk(BKT, Spread, Settle, Maturity, 'Principal', Principal)
```

```
Price =

  100.3059
```

### Price a Collar with a Floating-Rate Note

Price a collar with a floating-rate note using the `CapRate` and `FloorRate` input argument to define the collar pricing.

Price a portfolio of collared floating-rate notes using the following data:

```
Rates = [0.0287; 0.03024; 0.03345; 0.03861; 0.04033];
ValuationDate = '1-April-2012';
StartDates = ValuationDate;
EndDates = {'1-April-2013';'1-April-2014';'1-April-2015' ;...
'1-April-2016';'1-April-2017'};
Compounding = 1;
```

Create the `RateSpec`.

```
RateSpec = intenvset('ValuationDate', ValuationDate,'StartDates', StartDates,...
'EndDates', EndDates,'Rates', Rates, 'Compounding', Compounding);
```

Build the BK tree and assume the volatility to be 5%.

```
VolDates = ['1-April-2013';'1-April-2014';'1-April-2015';'1-April-2016';...
'1-April-2017';'1-April-2018'];
VolCurve = 0.05;
AlphaDates = '15-Nov-2018';
AlphaCurve = 0.1;

BKVolSpec = bkvolspec(RateSpec.ValuationDate, VolDates, VolCurve,...
AlphaDates, AlphaCurve);
BKTimeSpec = bktimespec(RateSpec.ValuationDate, VolDates, Compounding);
BKT = bktree(BKVolSpec, RateSpec, BKTimeSpec);
```

Create the floating-rate note instrument.

```
Settle ='1-April-2012';
Maturity = '1-April-2016';
Spread = [15;10];
Principal = 100;
```

Compute the price of the two vanilla floaters.

```
Price = floatbybk(BKT, Spread, Settle, Maturity)
```

```
Price =

 100.5519
 100.3680
```

Compute the price of the collared floating-rate notes.

```
CapStrike = {{'1-April-2013' 0.045; '1-April-2014' 0.05;...
'1-April-2015' 0.06}; 0.06};

FloorStrike = {{'1-April-2013' 0.035; '1-April-2014' 0.04;...
'1-April-2015' 0.05}; 0.03};
PriceCollared = floatbybk(BKT, Spread, Settle, Maturity,...
'CapRate', CapStrike,'FloorRate', FloorStrike)
```

```
PriceCollared =

 102.8537
 100.4910
```

## See Also

`bktree` | `bondbybk` | `capbybk` | `cfbybk` | `fixedbybk` | `floorbybk` | `swapbybk`

## Purpose

Price floating-rate note from Heath-Jarrow-Morton interest-rate tree

## Syntax

```
[Price, PriceTree] = floatbybk(HJMTree, Spread,
Settle, Maturity)
[Price, PriceTree] = floatbybk(HJMTree, Spread,
Settle, Maturity, Reset, Basis, Principal, Options,
EndMonthRule)
[Price, PriceTree] = floatbybk(HJMTree, Spread, Settle,
Maturity,Name,Value)
```

## Input Arguments

| | |
|---|---|
| `HJMTree` | Forward-rate tree structure created by `hjmtree`. |
| `Spread` | Number of instruments (`NINST`)-by-1 vector of number of basis points over the reference rate. |
| `Settle` | Settlement dates. `NINST`-by-`1` vector of dates representing the settlement dates of the floating-rate note.<br>**Note** The `Settle` date for every floating-rate note is set to the `ValuationDate` of the HJM tree. The floating-rate note argument `Settle` is ignored. |
| `Maturity` | `NINST`-by-`1` vector of dates representing the maturity dates of the floating-rate note. |

### Ordered Input or Name-Value Pair Arguments

Enter the following optional inputs using an ordered syntax or as name-value pair arguments. You cannot mix ordered syntax with name-value pair arguments.

**Reset**

`NINST`-by-`1` vector representing the frequency of payments per year.

**Default:** `1`

### Basis

Day-count basis of the instrument. A vector of integers.

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (PSA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ISMA)
- 9 = actual/360 (ISMA)
- 10 = actual/365 (ISMA)
- 11 = 30/360E (ISMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see basis.

**Default:** `0` (actual/actual)

### Principal

`NINST`-by-`1` vector of notional principal amounts or `NINST`-by-`1` cell array. For the latter case, each element of the cell array is a `NumDates`-by-`2` matrix where the first column is dates and the second column is associated principal amount. The date indicates the last day that the principal value is valid.

**Default:** `100`

### Options

Derivatives pricing options structure created with `derivset`.

### EndMonthRule

End-of-month rule. `NINST`-by-`1` vector. This rule applies only when `Maturity` is an end-of-month date for a month having 30 or fewer days.

- `0` = Ignore rule, meaning that a bond coupon payment date is always the same numerical day of the month.
- `1` = Set rule on, meaning that a bond coupon payment date is always the last actual day of the month.

**Default:** `1`

## Name-Value Pair Arguments

Specify optional comma-separated pairs of `Name,Value` arguments, where `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside single quotes (`' '`). You can specify several name and value pair arguments in any order as `Name1,Value1,...,NameN,ValueN`.

### AdjustCashFlowsBasis

Adjust the cash flows based on the actual period day count. `NINST`-by-`1` of logicals.

**Default:** `false`

### BusinessDayConvention

Require payment dates to be business dates. `NINST`-by-`1` cell array with possible choices of business day convention:

- `actual`
- `follow`
- `modifiedfollow`
- `previous`
- `modifiedprevious`

**Default:** `actual`

### Holidays

Holidays used for business day convention. `NHOLIDAYS`-by-`1` of MATLAB date numbers.

**Default:** If no dates are specified, `holidays.m` is used.

### CapRate

`NINST`-by-`1` decimal annual rate or `NINST`-by-`1` cell array, where each element is a `NumDates`-by-`2` cell array, and the cell array first column is dates, and the second column is associated cap rates. The date indicates the last day that the cap rate is valid.

### FloorRate

`NINST`-by-`1` decimal annual rate or `NINST`-by-`1` cell array, where each element is a `NumDates`-by-`2` cell array, and the cell array first column is dates, and the second column is associated floor rates. The date indicates the last day that the floor rate is valid.

## Description

`[Price, PriceTree] = floatbybk(HJMTree, Spread, Settle, Maturity)` computes the price of a floating-rate note from an HJM tree.

`[Price, PriceTree] = floatbybk(HJMTree, Spread, Settle, Maturity, Reset, Basis, Principal, Options,` `EndMonthRule)` computes the price of a floating-rate note with optional inputs from an HJM tree.

`[Price, PriceTree] = floatbybk(HJMTree, Spread, Settle, Maturity,Name,Value)` computes the price of a floating-rate note from an HJM tree with additional options specified by one or more `Name,Value` pair arguments.

`Price` is an `NINST`-by-`1` vector of expected prices of the floating-rate note at time 0.

`PriceTree` is a structure of trees containing vectors of instrument prices and accrued interest, and a vector of observation times for each node.

`PriceTree.PBush` contains the clean prices.

`PriceTree.AIBush` contains the accrued interest.

`PriceTree.tObs` contains the observation times.

The `Settle` date for every floating-rate note is set to the `ValuationDate` of the HJM tree. The floating-rate note argument `Settle` is ignored.

## Examples

### Price a Floating-Rate Note Using an HJM Tree

Price a 20-basis point floating-rate note using an HJM forward-rate tree.

Load the file `deriv.mat`, which provides `HJMTree`. The `HJMTree` structure contains the time and interest-rate information needed to price the note.

```
load deriv.mat;
```

Define the floating-rate note using the required arguments. Other arguments use defaults.

```
Spread = 20;
Settle = '01-Jan-2000';
Maturity = '01-Jan-2003';
```

Use `floatbyhjm` to compute the price of the note.

```
Price = floatbyhw(HJMTree, Spread, Settle, Maturity)

Price =

  100.5529
```

### Price an Amortizing Floating-Rate Note

Price an amortizing floating-rate note using the `Principal` input argument to define the amortization schedule.

Create the `RateSpec`.

```
Rates = [0.03583; 0.042147; 0.047345; 0.052707; 0.054302];
ValuationDate = '15-Nov-2011';
StartDates = ValuationDate;
EndDates = {'15-Nov-2012';'15-Nov-2013';'15-Nov-2014' ;'15-Nov-2015';'15-Nov-2016'};
Compounding = 1;
RateSpec = intenvset('ValuationDate', ValuationDate,'StartDates', StartDates,...
'EndDates', EndDates,'Rates', Rates, 'Compounding', Compounding)
```

```
RateSpec =

           FinObj: 'RateSpec'
      Compounding: 1
             Disc: [5x1 double]
            Rates: [5x1 double]
         EndTimes: [5x1 double]
       StartTimes: [5x1 double]
         EndDates: [5x1 double]
       StartDates: 734822
    ValuationDate: 734822
            Basis: 0
     EndMonthRule: 1
```

Create the floating-rate instrument using the following data:

```
Settle ='15-Nov-2011';
Maturity = '15-Nov-2015';
```

```
Spread = 15;
```

Define the floating-rate note amortizing schedule.

```
Principal ={{'15-Nov-2012' 100;'15-Nov-2013' 70;'15-Nov-2014' 40;'15-Nov-2015' 10}};
```

Build the HJM tree using the following data:

```
MatDates = {'15-Nov-2012'; '15-Nov-2013';'15-Nov-2014';'15-Nov-2015';'15-Nov-2016';'15-Nov-2017'};
HJMTimeSpec = hjmtimespec(RateSpec.ValuationDate, MatDates);
Volatility = [.10; .08; .06; .04];
CurveTerm = [ 1; 2; 3; 4];
HJMVolSpec = hjmvolspec('Proportional', Volatility, CurveTerm, 1e6);
HJMT = hjmtree(HJMVolSpec,RateSpec,HJMTimeSpec);
```

Compute the price of the amortizing floating-rate note.

```
Price = floatbyhjm(HJMT, Spread, Settle, Maturity, 'Principal', Principal)
```

```
Price =

  100.3059
```

### Price a Collar with a Floating-Rate Note

Price a collar with a floating-rate note using the `CapRate` and `FloorRate` input argument to define the collar pricing.

Price a portfolio of collared floating-rate notes using the following data:

```
Rates = [0.0287; 0.03024; 0.03345; 0.03861; 0.04033];
ValuationDate = '1-April-2012';
StartDates = ValuationDate;
EndDates = {'1-April-2013';'1-April-2014';'1-April-2015' ;...
'1-April-2016';'1-April-2017'};
Compounding = 1;
```

Create the `RateSpec`.

```
RateSpec = intenvset('ValuationDate', ValuationDate,'StartDates', StartDates,...
```

```
'EndDates', EndDates,'Rates', Rates, 'Compounding', Compounding);
```

Build the HJM tree with the following data:

```
MatDates = {'1-April-2013'; '1-April-2014';'1-April-2015';...
'1-April-2016';'1-April-2017';'1-April-2018'};
HJMTimeSpec = hjmtimespec(RateSpec.ValuationDate, MatDates);
Volatility = [.10; .08; .06; .04];
CurveTerm = [ 1; 2; 3; 4];
HJMVolSpec = hjmvolspec('Proportional', Volatility, CurveTerm, 1e6);
HJMT = hjmtree(HJMVolSpec,RateSpec,HJMTimeSpec);
```

Create the floating-rate note instrument.

```
Settle ='1-April-2012';
Maturity = '1-April-2016';
Spread = 10;
Principal = 100;
```

Compute the price of two capped collared floating-rate notes.

```
CapStrike = [0.04;0.055];
PriceCapped = floatbyhjm(HJMT, Spread, Settle, Maturity,...
'CapRate', CapStrike)
```

```
PriceCapped =

   98.9986
  100.2051
```

Compute the price of two collared floating-rate notes.

```
FloorStrike = [0.035;0.040];
PriceCollared = floatbyhjm(HJMT, Spread, Settle, Maturity,...
'CapRate', CapStrike, 'FloorRate', FloorStrike)
```

```
PriceCollared =

   99.9246
```

```
102.2321
```

## See Also

`bondbyhjm` | `capbyhjm` | `cfbyhjm` | `fixedbyhjm` | `floorbyhjm` | `hjmtree` | `swapbyhjm`

**Purpose** Price floating-rate note from Hull-White interest-rate tree

**Syntax**

```
[Price, PriceTree] = floatbybk(HWTree, Spread,
Settle, Maturity)
[Price, PriceTree] = floatbybk(HWTree, Spread,
Settle, Maturity, Reset, Basis, Principal, Options,
EndMonthRule)
[Price, PriceTree] = floatbybk(HWTree, Spread, Settle,
Maturity,Name,Value)
```

## Input Arguments

| | |
|---|---|
| `HWTree` | Interest-rate tree structure created by `hwtree`. |
| `Spread` | Number of instruments (`NINST`)-by-`1` vector of number of basis points over the reference rate. |
| `Settle` | Settlement dates. `NINST`-by-`1` vector of dates representing the settlement dates of the floating-rate note.<br>**Note** The `Settle` date for every floating-rate note is set to the `ValuationDate` of the HW tree. The floating-rate note argument `Settle` is ignored. |
| `Maturity` | `NINST`-by-`1` vector of dates representing the maturity dates of the floating-rate note. |

### Ordered Input or Name-Value Pair Arguments

Enter the following optional inputs using an ordered syntax or as name-value pair arguments. You cannot mix ordered syntax with name-value pair arguments.

**Reset**

`NINST`-by-`1` vector representing the frequency of payments per year.

---

**Note** Payments on floating-rate notes (FRNs) are determined by the effective interest-rate between reset dates. If the reset period for a FRN spans more than one tree level, calculating the payment becomes impossible due to the recombining nature of the tree. That is, the tree path connecting the two consecutive reset dates can not be uniquely determined because there will be more than one possible path for connecting the two payment dates.

---

**Default:** `1`

### Basis

Day-count basis of the instrument. A vector of integers.

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (PSA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ISMA)
- 9 = actual/360 (ISMA)
- 10 = actual/365 (ISMA)
- 11 = 30/360E (ISMA)
- 12 = actual/365 (ISDA)

- 13 = BUS/252

For more information, see basis.

**Default:** `0` (actual/actual)

### Principal

`NINST`-by-`1` vector of notional principal amounts or `NINST`-by-`1` cell array. For the latter case, each element of the cell array is a `NumDates`-by-`2` matrix where the first column is dates and the second column is associated principal amount. The date indicates the last day that the principal value is valid.

**Default:** `100`

### Options

Derivatives pricing options structure created with `derivset`.

### EndMonthRule

End-of-month rule. `NINST`-by-`1` vector. This rule applies only when `Maturity` is an end-of-month date for a month having 30 or fewer days.

- `0` = Ignore rule, meaning that a bond coupon payment date is always the same numerical day of the month.
- `1` = Set rule on, meaning that a bond coupon payment date is always the last actual day of the month.

**Default:** `1`

## Name-Value Pair Arguments

Specify optional comma-separated pairs of `Name,Value` arguments, where `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside single quotes (`' '`). You can specify several name and value pair arguments in any order as `Name1,Value1,...,NameN,ValueN`.

**AdjustCashFlowsBasis**

Adjust the cash flows based on the actual period day count. `NINST`-by-`1` of logicals.

**Default:** `false`

**BusinessDayConvention**

Require payment dates to be business dates. `NINST`-by-`1` cell array with possible choices of business day convention:

- `actual`
- `follow`
- `modifiedfollow`
- `previous`
- `modifiedprevious`

**Default:** `actual`

**Holidays**

Holidays used for business day convention. `NHOLIDAYS`-by-`1` of MATLAB date numbers.

**Default:** If no dates are specified, `holidays.m` is used.

**CapRate**

`NINST`-by-`1` decimal annual rate or `NINST`-by-`1` cell array, where each element is a `NumDates`-by-`2` cell array, and the cell array first column is dates, and the second column is associated cap rates. The date indicates the last day that the cap rate is valid.

**FloorRate**

`NINST`-by-`1` decimal annual rate or `NINST`-by-`1` cell array, where each element is a `NumDates`-by-`2` cell array, and the cell array first column is dates, and the second column is associated floor rates. The date indicates the last day that the floor rate is valid.

## Description

`[Price, PriceTree] = floatbybk(HWTree, Spread, Settle, Maturity)` computes the price of a floating-rate note from a Hull-White tree.

`[Price, PriceTree] = floatbybk(HWTree, Spread, Settle, Maturity, Reset, Basis, Principal, Options, EndMonthRule)` computes the price of a floating-rate note with optional inputs from a Hull-White tree.

`[Price, PriceTree] = floatbybk(HWTree, Spread, Settle, Maturity,Name,Value)` computes the price of a floating-rate note from a Hull-White tree with additional options specified by one or more `Name,Value` pair arguments.

`Price` is an `NINST`-by-`1` vector of expected prices of the floating-rate note at time 0.

`PriceTree` is a structure of trees containing vectors of instrument prices and accrued interest, and a vector of observation times for each node.

`PriceTree.PTree` contains the clean prices.

`PriceTree.AITree` contains the accrued interest.

`PriceTree.tObs` contains the observation times.

The `Settle` date for every floating-rate note is set to the `ValuationDate` of the HW tree. The floating-rate note argument `Settle` is ignored.

## Examples

### Price a Floating-Rate Note Using a Hull-White Tree

Price a 20-basis point floating-rate note using a Hull-White interest-rate tree.

Load the file `deriv.mat`, which provides `HWTree`. The `HWTree` structure contains the time and interest-rate information needed to price the note.

```
load deriv.mat;
```

Define the floating-rate note using the required arguments. Other arguments use defaults.

```
Spread = 20;
Settle = '01-Jan-2005';
Maturity = '01-Jan-2006';
```

Use `floatbyhw` to compute the price of the note.

```
Price = floatbyhw(HWTree, Spread, Settle, Maturity)
```

```
Price =

  100.3825
```

### Price an Amortizing Floating-Rate Note

Price an amortizing floating-rate note using the `Principal` input argument to define the amortization schedule.

Create the `RateSpec`.

```
Rates = [0.03583; 0.042147; 0.047345; 0.052707; 0.054302];
ValuationDate = '15-Nov-2011';
StartDates = ValuationDate;
EndDates = {'15-Nov-2012';'15-Nov-2013';'15-Nov-2014' ;'15-Nov-2015';'15-Nov-2016'};
Compounding = 1;
RateSpec = intenvset('ValuationDate', ValuationDate,'StartDates', StartDates,...
'EndDates', EndDates,'Rates', Rates, 'Compounding', Compounding)
```

```
RateSpec =

          FinObj: 'RateSpec'
     Compounding: 1
            Disc: [5x1 double]
           Rates: [5x1 double]
        EndTimes: [5x1 double]
```

```
       StartTimes: [5x1 double]
         EndDates: [5x1 double]
       StartDates: 734822
    ValuationDate: 734822
            Basis: 0
     EndMonthRule: 1
```

Create the floating-rate instrument using the following data:

```
Settle ='15-Nov-2011';
Maturity = '15-Nov-2015';
Spread = 15;
```

Define the floating-rate note amortizing schedule.

```
Principal ={{'15-Nov-2012' 100;'15-Nov-2013' 70;'15-Nov-2014' 40;'15-Nov-2015' 10}};
```

Build the HW tree and assume the volatility is 10%.

```
VolDates = ['15-Nov-2012'; '15-Nov-2013';'15-Nov-2014';'15-Nov-2015';'15-Nov-2016';'15-Nov-2017'];
VolCurve = 0.1;
AlphaDates = '15-Nov-2017';
AlphaCurve = 0.1;

HWVolSpec = hwvolspec(RateSpec.ValuationDate, VolDates, VolCurve,...
AlphaDates, AlphaCurve);
HWTimeSpec = hwtimespec(RateSpec.ValuationDate, VolDates, Compounding);
HWT = hwtree(HWVolSpec, RateSpec, HWTimeSpec);
```

Compute the price of the amortizing floating-rate note.

```
Price = floatbyhw(HWT, Spread, Settle, Maturity, 'Principal', Principal)
```

```
Price =

  100.3059
```

### Price a Collar with a Floating-Rate Note

Price a collar with a floating-rate note using the `CapRate` and `FloorRate` input argument to define the collar pricing.

Price two collared floating-rate notes using the following data:

```
Rates = [0.0287; 0.03024; 0.03345; 0.03861; 0.04033];
ValuationDate = '1-April-2012';
StartDates = ValuationDate;
EndDates = {'1-April-2013';'1-April-2014';'1-April-2015' ;...
'1-April-2016';'1-April-2017'};
Compounding = 1;
```

Create the `RateSpec`.

```
RateSpec = intenvset('ValuationDate', ValuationDate,'StartDates', StartDates,...
'EndDates', EndDates,'Rates', Rates, 'Compounding', Compounding);
```

Build the HW tree and assume the volatility to be 5%.

```
VolDates = ['1-April-2013';'1-April-2014';'1-April-2015';...
'1-April-2016';'1-April-2017';'1-April-2018'];
VolCurve = 0.05;
AlphaDates = '15-Nov-2018';
AlphaCurve = 0.1;

HWVolSpec = hwvolspec(RateSpec.ValuationDate, VolDates, VolCurve,...
AlphaDates, AlphaCurve);
HWTimeSpec = hwtimespec(RateSpec.ValuationDate, VolDates, Compounding);
HWT = hwtree(HWVolSpec, RateSpec, HWTimeSpec);
```

Create the floating-rate note instrument.

```
Settle ='1-April-2012';
Maturity = '1-April-2016';
Spread = 10;
Principal = 100;
```

Compute the price of a vanilla floater.

```
Price = floatbyhw(HWT, Spread, Settle, Maturity)
```

```
Price =

  100.3680
```

Compute the price of the collared floating-rate notes.

```
CapStrike = {{'1-April-2014' 0.045; '1-April-2015' 0.05;...
 '1-April-2016' 0.06}; 0.06};

FloorStrike = {{'1-April-2014' 0.035; '1-April-2015' 0.04;...
 '1-April-2016' 0.05}; 0.03};
PriceCollared = floatbyhw(HWT, Spread, Settle, Maturity,....
'CapRate', CapStrike,'FloorRate', FloorStrike)
```

```
PriceCollared =

  102.0317
  100.9189
```

## See Also

`bondbyhw` | `capbyhw` | `cfbyhw` | `fixedbyhw` | `floorbyhw` | `hwtree` | `swapbyhw`

## Purpose

Price floating-rate note from set of zero curves

## Syntax

```
[Price, PriceNoAI, OutputCashFlows, CFlowDates] =
floatbyzero(RateSpec, Spread, Settle, Maturity)
[Price, PriceNoAI, OutputCashFlows, CFlowDates] =
floatbyzero(RateSpec, Spread, Settle, Maturity,
Reset,  Basis,  Principal, EndMonthRule,
LatestFloatingRate, ForwardRateSpec)
[Price PriceNoAI, OutputCashFlows, CFlowDates] =
floatbyzero(RateSpec, Spread, Settle, Maturity,
Name, Value)
```

## Description

`[Price, PriceNoAI, OutputCashFlows, CFlowDates] = floatbyzero(RateSpec, Spread, Settle, Maturity)` computes the price of a floating-rate note from a set of zero curves.

`[Price, PriceNoAI, OutputCashFlows, CFlowDates] = floatbyzero(RateSpec, Spread, Settle, Maturity, Reset, Basis, Principal, EndMonthRule, LatestFloatingRate, ForwardRateSpec)` computes the price of a floating-rate note from a set of zero curves using optional input arguments.

`[Price PriceNoAI, OutputCashFlows, CFlowDates] = floatbyzero(RateSpec, Spread, Settle, Maturity, Name, Value)` computes the price of a floating-rate note from a set of zero curves with additional options specified by one or more `Name, Value` pair arguments.

## Input Arguments

### RateSpec

Structure containing the properties of an interest-rate structure. See `intenvset` for information on creating `RateSpec`.

### Spread

Number of basis points over the reference rate.

### Settle

Settlement date. `Settle` must be either a scalar or `NINST`-by-`1` vector of serial date numbers or date strings of the same value which represent the settlement date for each bond. `Settle` must be earlier than `Maturity`.

### Maturity

Maturity date.

## Ordered Input or Name-Value Pair Arguments

Enter the following optional inputs using an ordered syntax or as name-value pair arguments. You cannot mix ordered syntax with name-value pair arguments.

### Reset

`NINST`-by-`1` vector representing the frequency of payments per year.

**Default:** `1`

### Basis

Day-count basis of the instrument. A vector of integers.

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (PSA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ISMA)

- 9 = actual/360 (ISMA)
- 10 = actual/365 (ISMA)
- 11 = 30/360E (ISMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see basis.

**Default:** `0` (actual/actual)

### Principal

`NINST`-by-`1` vector of notional principal amounts or `NINST`-by-`1` cell array. For the latter case, each element of the cell array is a `NumDates`-by-`2` matrix where the first column is dates and the second column is associated principal amount. The date indicates the last day that the principal value is valid.

**Default:** `100`

### EndMonthRule

End-of-month rule. `NINST`-by-`1` vector. This rule applies only when `Maturity` is an end-of-month date for a month having 30 or fewer days.

- `0` = Ignore rule, meaning that a bond coupon payment date is always the same numerical day of the month.
- `1` = Set rule on, meaning that a bond coupon payment date is always the last actual day of the month.

**Default:** `1`

### LatestFloatingRate

Rate for the next floating payment set at the last reset date. `NINST`-by-`1` of scalars. If this is not specified, the floating rate at the previous reset date must be computed from the `RateSpec`.

### ForwardRateSpec

The `RateSpec` to be used in generating floating cash flows. If no `ForwardRateSpec` is specified then the `RateSpec` is used to generate floating cash flows.

## Name-Value Pair Arguments

Specify optional comma-separated pairs of `Name,Value` arguments, where `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside single quotes (`' '`). You can specify several name and value pair arguments in any order as `Name1,Value1,...,NameN,ValueN`.

### AdjustCashFlowsBasis

Adjust the cash flows based on the actual period day count. `NINST`-by-`1` of logicals.

**Default:** `false`

### BusinessDayConvention

Require payment dates to be business dates. `NINST`-by-`1` cell array with possible choices of business day convention:

- `actual`
- `follow`
- `modifiedfollow`
- `previous`
- `modifiedprevious`

**Default:** `actual`

### Holidays

Holidays used for business day convention. `NHOLIDAYS`-by-`1` of MATLAB date numbers.

> **Default:** If no dates are specified, `holidays.m` is used.

## Output Arguments

### Price

Number of instruments (`NINST`) by number of curves (`NUMCURVES`) matrix of floating-rate note prices. Each column arises from one of the zero curves.

### PriceNoAI

`NINST`-by-`NUMCURVES` matrix of dirty bond price (clean + accrued interest). Each column arises from one of the zero curves.

### OutputCashFlows

`NINST`-by-`NUMCFS` matrix of cash flows for each bond.

---

**Note** If there is more than one curve specified in the `RateSpec` input, then the first `NCURVES` rows correspond to the first bond, the second `NCURVES` rows correspond to the second bond, and so on.

---

### CFlowDates

`NINST`-by-`NUMCFS` matrix of payment dates for each bond.

## Examples

### Price a Floating-Rate Note Using a Set of Zero Curves

Price a 20-basis point floating-rate note using a set of zero curves.

Load `deriv.mat`, which provides `ZeroRateSpec`, the interest-rate term structure, needed to price the bond.

```
load deriv.mat;
```

Define the floating-rate note using the required arguments. Other arguments use defaults.

```
Spread = 20;
Settle = '01-Jan-2000';
Maturity = '01-Jan-2003';
```

Use `floatbyzero` to compute the price of the note.

```
Price = floatbyzero(ZeroRateSpec, Spread, Settle, Maturity)
```

```
Price =

  100.5529
```

### Price an Amortizing Floating-Rate Note

Price an amortizing floating-rate note using the `Principal` input argument to define the amortization schedule.

Create the `RateSpec`.

```
Rates = [0.03583; 0.042147; 0.047345; 0.052707; 0.054302];
ValuationDate = '15-Nov-2011';
StartDates = ValuationDate;
EndDates = {'15-Nov-2012';'15-Nov-2013';'15-Nov-2014' ;'15-Nov-2015';'15-Nov-2016'};
Compounding = 1;
RateSpec = intenvset('ValuationDate', ValuationDate,'StartDates', StartDates,...
'EndDates', EndDates,'Rates', Rates, 'Compounding', Compounding)
```

```
RateSpec =

         FinObj: 'RateSpec'
    Compounding: 1
           Disc: [5x1 double]
          Rates: [5x1 double]
       EndTimes: [5x1 double]
     StartTimes: [5x1 double]
```

```
        EndDates: [5x1 double]
      StartDates: 734822
   ValuationDate: 734822
           Basis: 0
    EndMonthRule: 1
```

Create the floating-rate instrument using the following data:

```
Settle ='15-Nov-2011';
Maturity = '15-Nov-2015';
Spread = 15;
```

Define the floating-rate note amortizing schedule.

```
Principal ={{'15-Nov-2012' 100;'15-Nov-2013' 70;'15-Nov-2014' 40;'15-Nov-2015' 10}};
```

Compute the price of the amortizing floating-rate note.

```
Price  = floatbyzero(RateSpec, Spread, Settle, Maturity, 'Principal', Principal)
```

```
Price =

  100.3059
```

## See Also

| `bondbyzero` | `cfbyzero` | `fixedbyzero` | `swapbyzero`

# floorbybdt

**Purpose** Price floor instrument from Black-Derman-Toy interest-rate tree

**Syntax**

```
[Price, PriceTree] = floorbybdt(BDTTree, Strike, Settle, Maturity,
Reset, Basis, Principal, Options)
```

**Arguments**

| | |
|---|---|
| `BDTTree` | Interest-rate tree structure created by `bdttree`. |
| `Strike` | Number of instruments (`NINST`)-by-`1` vector of rates at which the floor is exercised. |
| `Settle` | Settlement date. `NINST`-by-`1` vector of dates representing the settlement dates of the floor. The `Settle` date for every floor is set to the `ValuationDate` of the BDT tree. The floor argument `Settle` is ignored. |
| `Maturity` | `NINST`-by-`1` vector of dates representing the maturity dates of the floor. |
| `Reset` | (Optional) `NINST`-by-`1` vector representing the frequency of payments per year. Default = `1`. |
| `Basis` | (Optional) Day-count basis of the instrument. A vector of integers.<br>• 0 = actual/actual (default)<br>• 1 = 30/360 (SIA)<br>• 2 = actual/360<br>• 3 = actual/365<br>• 4 = 30/360 (BMA)<br>• 5 = 30/360 (ISDA)<br>• 6 = 30/360 (European)<br>• 7 = actual/365 (Japanese)<br>• 8 = actual/actual (ICMA) |

| | |
|---|---|
| | - 9 = actual/360 (ICMA)<br>- 10 = actual/365 (ICMA)<br>- 11 = 30/360E (ICMA)<br>- 12 = actual/actual (ISDA)<br>- 13 = BUS/252<br>For more information, see basis. |
| `Principal` | (Optional) The notional principal amount. Default = `100`. |
| `Options` | (Optional) Derivatives pricing options structure created with `derivset`. |

## Description

`[Price, PriceTree] = floorbybdt(BDTTree, Strike, Settlement, Maturity, Reset, Basis, Principal, Options)` computes the price of a floor instrument from a BDT interest-rate tree.

`Price` is an `NINST`-by-`1` vector of the expected prices of the floor at time 0.

`PriceTree` is the tree structure with values of the floor at each node.

## Examples

**Example 1**. Price a 10% floor instrument using a BDT interest-rate tree.

Load the file `deriv.mat`, which provides `BDTTree`. `BDTTree` contains the time and interest-rate information needed to price the floor instrument.

```
load deriv.mat;
```

Set the required values. Other arguments will use defaults.

```
Strike = 0.10;
Settle = '01-Jan-2000';
Maturity = '01-Jan-2004';
```

Use `floorbybdt` to compute the price of the floor instrument.

```
Price = floorbybdt(BDTTree, Strike, Settle, Maturity)

Price =

   0.2428
```

**Example 2**. Here is a second example, showing the pricing of a 10% floor instrument using a newly created BDT tree.

First set the required arguments for the three needed specifications.

```
Compounding = 1;
ValuationDate = '01-01-2000';
StartDate = ValuationDate;
EndDates = ['01-01-2001'; '01-01-2002'; '01-01-2003';
'01-01-2004'; '01-01-2005'];
Rates = [.1; .11; .12; .125; .13];
Volatility = [.2; .19; .18; .17; .16];
```

Next create the specifications.

```
RateSpec = intenvset('Compounding', Compounding,...
'ValuationDate', ValuationDate,...
'StartDates', StartDate,...
'EndDates', EndDates,...
'Rates', Rates);
BDTTimeSpec = bdttimespec(ValuationDate, EndDates, Compounding);
BDTVolSpec = bdtvolspec(ValuationDate, EndDates, Volatility);
```

Now create the BDT tree from the specifications.

```
BDTTree = bdttree(BDTVolSpec, RateSpec, BDTTimeSpec);
```

Set the floor arguments. Remaining arguments will use defaults.

```
FloorStrike = 0.10;
```

```
Settlement = ValuationDate;
Maturity = '01-01-2002';
FloorReset = 1;
```

Finally, use `floorbybdt` to find the price of the floor instrument.

```
Price= floorbybdt(BDTTree, FloorStrike, Settlement, Maturity,...
FloorReset)

Price =

    0.0863
```

## See Also

`bdttree` | `capbybdt` | `cfbybdt` | `swapbybdt`

# floorbybk

## Purpose

Price floor instrument from Black-Karasinski interest-rate tree

## Syntax

```
[Price, PriceTree] = floorbybk(BKTree, Strike, Settle, Maturity,
Reset, Basis, Principal, Options)
```

## Arguments

| | |
|---|---|
| `BKTree` | Interest-rate tree structure created by `bktree`. |
| `Strike` | Number of instruments (`NINST`)-by-`1` vector of rates at which the floor is exercised. |
| `Settle` | Settlement date. `NINST`-by-`1` vector of dates representing the settlement dates of the floor. The `Settle` date for every floor is set to the `ValuationDate` of the BK tree. The floor argument `Settle` is ignored. |
| `Maturity` | `NINST`-by-`1` vector of dates representing the maturity dates of the floor. |
| `Reset` | (Optional) `NINST`-by-`1` vector representing the frequency of payments per year. Default = `1`. |
| `Basis` | (Optional) Day-count basis of the instrument. A vector of integers.<br>• 0 = actual/actual (default)<br>• 1 = 30/360 (SIA)<br>• 2 = actual/360<br>• 3 = actual/365<br>• 4 = 30/360 (BMA)<br>• 5 = 30/360 (ISDA)<br>• 6 = 30/360 (European)<br>• 7 = actual/365 (Japanese)<br>• 8 = actual/actual (ICMA) |

| | |
|---|---|
| | - 9 = actual/360 (ICMA)<br>- 10 = actual/365 (ICMA)<br>- 11 = 30/360E (ICMA)<br>- 12 = actual/actual (ISDA)<br>- 13 = BUS/252<br>For more information, see basis. |
| `Principal` | (Optional) The notional principal amount. Default = `100`. |
| `Options` | (Optional) Derivatives pricing options structure created with `derivset`. |

## Description

`[Price, PriceTree] = floorbybk(BKTree, Strike, Settlement, Maturity, Reset, Basis, Principal, Options)` computes the price of a floor instrument from a Black-Karasinski tree.

`Price` is an `NINST`-by-`1` vector of the expected prices of the floor at time 0.

`PriceTree` is the tree structure with values of the floor at each node.

## Examples

Price a 3% floor instrument using a Black-Karasinski interest-rate tree.

Load the file `deriv.mat`, which provides `BKTree`. The `BKTree` structure contains the time and interest rate information needed to price the floor instrument.

```
load deriv.mat;
```

Set the required values. Other arguments will use defaults.

```
Strike = 0.03;
Settle = '01-Jan-2005';
Maturity = '01-Jan-2009';
```

Use `floorbyhw` to compute the price of the floor instrument.

```
Price = floorbybk(BKTree, Strike, Settle, Maturity)

Price =

  0.2061
```

**See Also** `bktree` | `capbybk` | `cfbybk` | `swapbybk`

## Purpose

Price floors using Black option pricing model

## Syntax

```
[FloorPrice, Floorlets] = floorbyblk(RateSpec, Strike, Settle,
Maturity, Volatility)
[FloorPrice, Floorlets] = floorbyblk(RateSpec, Strike, Settle,
Maturity, Volatility, 'Name1', Value1...)
```

## Arguments

| | |
|---|---|
| `RateSpec` | The annualized, continuously compounded rate term structure. For more information, see `intenvset`. |
| `Strike` | `NINST`-by-`1` vector of rates at which the floor is exercised, as a decimal number. |
| `Settle` | Scalar representing the settle date of the floor. |
| `Maturity` | Scalar representing the maturity date of the floor. |
| `Volatility` | `NINST`-by-`1` vector of volatilities. |
| `Reset` | (Optional) `NINST`-by-`1` vector representing the frequency of payments per year. Default is `1`. |
| `Principal` | (Optional) `NINST`-by-`1` vector representing the notional principal amount. Default is `100`. |
| `Basis` | `NINST`-by-`1` vector representing the basis used when annualizing the input forward rate.<br>• 0 = actual/actual (default)<br>• 1 = 30/360 (SIA)<br>• 2 = actual/360<br>• 3 = actual/365<br>• 4 = 30/360 (BMA)<br>• 5 = 30/360 (ISDA)<br>• 6 = 30/360 (European) |

| | |
|---|---|
| | - 7 = actual/365 (Japanese)<br>- 8 = actual/actual (ICMA)<br>- 9 = actual/360 (ICMA)<br>- 10 = actual/365 (ICMA)<br>- 11 = 30/360E (ICMA)<br>- 12 = actual/actual (ISDA)<br>- 13 = BUS/252<br>For more information, see basis. |
| `ValuationDate` | (Optional) Scalar representing the observation date of the investment horizons. The default is the `Settle` date. |

---

**Note** All optional inputs are specified as matching parameter name/value pairs. The parameter name is specified as a character string, followed by the corresponding parameter value. You can specify parameter name/value pairs in any order. Names are case-insensitive and partial string matches are allowed provided no ambiguities exist.

---

## Description

```
[FloorPrice, Floorlets] = floorbyblk(RateSpec, Strike,
Settle,
Maturity, Volatility)
```

```
[FloorPrice, Floorlets] = floorbyblk(RateSpec, Strike,
Settle,
Maturity, Volatility, 'Name1', Value1...)
```

Use`floorbyblk` to price floors using the Black option pricing model.

The outputs are:

- `FloorPrice` — `NINST`-by-`1` expected prices of the floor.
- `Floorlets` — `NINST`-by-`NCF` array of floorlets, padded with `NaN`s.

## Examples

Consider an investor who gets into a contract that floors the interest rate on a $100,000 loan at 6% quarterly compounded for 3 months, starting on January 1, 2009’. Assuming that on January 1, 2008 the zero rate is 6.9394% continuously compounded and the volatility is 20%, use this data to compute the floor price.

Calculate the `RateSpec`:

```
ValuationDate = 'Jan-01-2008';
EndDates ='April-01-2010';
Rates = 0.069394;
Compounding = -1;
Basis = 1;

RateSpec = intenvset('ValuationDate', ValuationDate, ...
'StartDates', ValuationDate,'EndDates', EndDates, ...
'Rates', Rates,'Compounding', Compounding,'Basis', Basis);
```

Compute the price of the cap:

```
Settle = 'Jan-01-2009'; % floor starts in a year
Maturity = 'April-01-2009';
Volatility = 0.20;
FloorRate = 0.06;
FloorReset = 4;
Principal=100000;

FloorPrice = floorbyblk(RateSpec, FloorRate, Settle, Maturity, Volatility,...
'Reset',FloorReset,'ValuationDate',ValuationDate,'Principal', Principal,...
'Basis', Basis)

FloorPrice =

   37.4864
```

**See Also** `capbyblk`

## Purpose

Price floor instrument from Heath-Jarrow-Morton interest-rate tree

## Syntax

```
[Price, PriceTree] = floorbyhjm(HJMTree, Strike, Settle, Maturity,
Reset, Basis, Principal, Options)
```

## Arguments

| | |
|---|---|
| `HJMTree` | Forward-rate tree structure created by `hjmtree`. |
| `Strike` | Number of instruments (`NINST`)-by-`1` vector of rates at which the floor is exercised. |
| `Settle` | Settlement date. `NINST`-by-`1` vector of dates representing the settlement dates of the floor. The `Settle` date for every floor is set to the `ValuationDate` of the HJM tree. The floor argument `Settle` is ignored. |
| `Maturity` | `NINST`-by-`1` vector of dates representing the maturity dates of the floor. |
| `Reset` | (Optional) `NINST`-by-`1` vector representing the frequency of payments per year. Default = `1`. |
| `Basis` | (Optional) Day-count basis of the instrument. A vector of integers.<br>• 0 = actual/actual (default)<br>• 1 = 30/360 (SIA)<br>• 2 = actual/360<br>• 3 = actual/365<br>• 4 = 30/360 (BMA)<br>• 5 = 30/360 (ISDA)<br>• 6 = 30/360 (European)<br>• 7 = actual/365 (Japanese)<br>• 8 = actual/actual (ICMA) |

- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/actual (ISDA)
- 13 = BUS/252

For more information, see basis.

| | |
|---|---|
| `Principal` | (Optional) The notional principal amount. Default = `100`. |
| `Options` | (Optional) Derivatives pricing options structure created with `derivset`. |

## Description

`[Price, PriceTree] = floorbyhjm(HJMTree, Strike, Settlement, Maturity, Reset, Basis, Principal, Options)` computes the price of a floor instrument from an HJM tree.

`Price` is an `NINST`-by-`1` vector of the expected prices of the floor at time 0.

`PriceTree` is the tree structure with values of the floor at each node.

## Examples

Price a 3% floor instrument using an HJM forward-rate tree.

Load the file `deriv.mat`, which provides `HJMTree`. The `HJMTree` structure contains the time and forward-rate information needed to price the floor instrument.

```
load deriv.mat;
```

Set the required values. Other arguments will use defaults.

```
Strike = 0.03;
Settle = '01-Jan-2000';
Maturity = '01-Jan-2004';
```

Use `floorbyhjm` to compute the price of the floor instrument.

```
Price = floorbyhjm(HJMTree, Strike, Settle, Maturity)

Price =

  0.0486
```

## See Also

`capbyhjm` | `cfbyhjm` | `hjmtree` | `swapbyhjm`

# floorbyhw

## Purpose

Price floor instrument from Hull-White interest-rate tree

## Syntax

```
[Price, PriceTree] = floorbyhw(HWTree, Strike, Settle, Maturity,
Reset, Basis, Principal, Options)
```

## Arguments

| | |
|---|---|
| `HWTree` | Interest-rate tree structure created by `hwtree`. |
| `Strike` | Number of instruments (`NINST`)-by-`1` vector of rates at which the floor is exercised. |
| `Settle` | Settlement date. `NINST`-by-`1` vector of dates representing the settlement dates of the floor. The `Settle` date for every floor is set to the `ValuationDate` of the HW tree. The floor argument `Settle` is ignored. |
| `Maturity` | `NINST`-by-`1` vector of dates representing the maturity dates of the floor. |
| `Reset` | (Optional) `NINST`-by-`1` vector representing the frequency of payments per year. Default = `1`. |
| `Basis` | (Optional) Day-count basis of the instrument. A vector of integers.<br>• 0 = actual/actual (default)<br>• 1 = 30/360 (SIA)<br>• 2 = actual/360<br>• 3 = actual/365<br>• 4 = 30/360 (BMA)<br>• 5 = 30/360 (ISDA)<br>• 6 = 30/360 (European)<br>• 7 = actual/365 (Japanese)<br>• 8 = actual/actual (ICMA) |

| | |
|---|---|
| | - 9 = actual/360 (ICMA)<br>- 10 = actual/365 (ICMA)<br>- 11 = 30/360E (ICMA)<br>- 12 = actual/actual (ISDA)<br>- 13 = BUS/252<br>For more information, see basis. |
| `Principal` | (Optional) The notional principal amount. Default = `100`. |
| `Options` | (Optional) Derivatives pricing options structure created with `derivset`. |

## Description

`[Price, PriceTree] = floorbyhw(HWTree, Strike, Settlement, Maturity, Reset, Basis, Principal, Options)` computes the price of a floor instrument from an HW tree.

`Price` is an `NINST`-by-`1` vector of the expected prices of the floor at time 0.

`PriceTree` is the tree structure with values of the floor at each node.

## Examples

Price a 3% floor instrument using a Hull-White interest-rate tree.

Load the file `deriv.mat`, which provides `HWTree`. The `HWTree` structure contains the time and interest rate information needed to price the floor instrument.

```
load deriv.mat;
```

Set the required values. Other arguments will use defaults.

```
Strike = 0.03;
Settle = '01-Jan-2005';
Maturity = '01-Jan-2009';
```

Use `floorbyhw` to compute the price of the floor instrument.

```
Price = floorbyhw(HWTree, Strike, Settle, Maturity)

Price =

  0.4616
```

## See Also

`capbyhw` | `cfbyhw` | `hwtree` | `swapbyhw`

## Purpose

Determine price of gap digital options using Black-Scholes model

## Syntax

```
Price = gapbybls(RateSpec, StockSpec, Settle, Maturity,
OptSpec, Strike, StrikeThreshold)
```

## Arguments

| | |
|---|---|
| `RateSpec` | The annualized, continuously compounded rate term structure. For information on the interest rate specification, see `intenvset`. |
| `StockSpec` | Stock specification. See `stockspec`. |
| `Settle` | `NINST`-by-`1` vector of settlement or trade dates. |
| `Maturity` | `NINST`-by-`1` vector of maturity dates. |
| `OptSpec` | `NINST`-by-`1` cell array of strings `'call'` or `'put'`. |
| `Strike` | `NINST`-by-`1` vector of payoff strike price values. |
| `StrikeThreshold` | `NINST`-by-`1` vector of strike values that determine if the option pays off. |

## Description

`Price = gapbybls(RateSpec, StockSpec, Settle, Maturity, OptSpec, Strike, StrikeThreshold)` computes gap option prices using the Black-Scholes option pricing model.

`Price` is a `NINST`-by-`1` vector of expected option prices.

## Examples

Consider a gap call and put options on a nondividend paying stock with a strike of 57 and expiring on January 1, 2008. On July 1, 2008 the stock is trading at 50. Using this data, compute the price of the option if the risk-free rate is 9%, the strike threshold is 50, and the volatility is 20%.

Create the `RateSpec`:

```
Settle = 'Jan-1-2008';
Maturity = 'Jul-1-2008';
```

```
Compounding = -1;
Rates = 0.09;
RateSpec = intenvset('ValuationDate', Settle, 'StartDates', Settle,...
'EndDates', Maturity, 'Rates', Rates, 'Compounding', Compounding, 'Basis', 1);
```

Define the `StockSpec`:

```
AssetPrice = 50;
Sigma = .2;
StockSpec = stockspec(Sigma, AssetPrice);
```

Define the call and put options:

```
OptSpec = {'call'; 'put'};
Strike = 57;
StrikeThreshold = 50;
```

Calculate the price:

```
Pgap = gapbybls(RateSpec, StockSpec, Settle, Maturity, OptSpec,...
Strike, StrikeThreshold)

Pgap =

   -0.0053
    4.4866
```

## See Also

`assetbybls` | `cashbybls` | `gapsensbybls` | `supersharebybls`

## Purpose

Determine price and sensitivities of gap digital options using Black-Scholes model

## Syntax

```
PriceSens = gapsensbybls(RateSpec, StockSpec, Settle,
Maturity, OptSpec, Strike, StrikeThreshold)
PriceSens = gapsensbybls(RateSpec, StockSpec, Settle,
Maturity, OptSpec, Strike, StrikeThreshold, OutSpec)
```

## Arguments

| | |
|---|---|
| `RateSpec` | The annualized, continuously compounded rate term structure. For information on the interest rate specification, see `intenvset`. |
| `StockSpec` | Stock specification. See `stockspec`. |
| `Settle` | `NINST`-by-`1` vector of settlement or trade dates. |
| `Maturity` | `NINST`-by-`1` vector of maturity dates. |
| `OptSpec` | `NINST`-by-`1` cell array of strings `'call'` or `'put'`. |
| `Strike` | `NINST`-by-`1` vector of strike price values. |
| `StrikeThreshold` | `NINST`-by-`1` vector of strike values that determine if the option pays off. |
| `OutSpec` | (Optional) All optional inputs are specified as matching parameter name/value pairs. The parameter name is specified as a character string, followed by the corresponding parameter value. You can specify parameter name/value pairs may in any order. Names are case-insensitive and partial string matches are allowed provided no ambiguities exist. Valid parameter names are:<br>• `NOUT`-by-`1` or `1`-by-`NOUT` cell array of strings indicating the nature and order of the outputs for the function. Possible values are `'Price'`, `'Delta'`, `'Gamma'`, `'Vega'`, `'Lambda'`, `'Rho'`, `'Theta'`, or `'All'`. |

For example, `OutSpec = {'Price'; 'Lambda'; 'Rho'}` specifies that the output should be `Price`, `Lambda`, and `Rho`, in that order.

To invoke from a function: `[Price, Lambda, Rho] = gapsensbybls(..., 'OutSpec', {'Price', 'Lambda', 'Rho'})`

`OutSpec = {'All'}` specifies that the output should be `Delta`, `Gamma`, `Vega`, `Lambda`, `Rho`, `Theta`, and `Price`, in that order. This is the same as specifying `OutSpec` as `OutSpec = {'Delta', 'Gamma', 'Vega', 'Lambda', 'Rho', 'Theta', 'Price'};`.

- Default is `OutSpec = {'Price'}`.

## Description

`PriceSens = gapsensbybls(RateSpec, StockSpec, Settle, Maturity, OptSpec, Strike, StrikeThreshold)` computes gap option prices using the Black-Scholes option pricing model.

`PriceSens = gapsensbybls(RateSpec, StockSpec, Settle, Maturity, OptSpec, Strike, StrikeThreshold, OutSpec)` includes an `OutSpec` argument defined as parameter/value pairs, and computes gap option prices and sensitivities using the Black-Scholes option pricing model.

`PriceSens` is a `NINST-by-1` vector of expected option prices and sensitivities.

## Examples

Consider a gap call and put options on a nondividend paying stock with a strike of 57 and expiring on January 1, 2008. On July 1, 2008 the stock is trading at 50. Using this data, compute the price and sensitivity of the option if the risk-free rate is 9%, the strike threshold is 50, and the volatility is 20%.

Create the `RateSpec`:

```
Settle = 'Jan-1-2008';
Maturity = 'Jul-1-2008';
Compounding = -1;
Rates = 0.09;
RateSpec = intenvset('ValuationDate', Settle, 'StartDates', Settle,...
'EndDates', Maturity, 'Rates', Rates, 'Compounding', Compounding, 'Basis', 1);
```

Define the `StockSpec`:

```
AssetPrice = 50;
Sigma = .2;
StockSpec = stockspec(Sigma, AssetPrice);
```

Define the call and put options:

```
OptSpec = {'call'; 'put'};
Strike = 57;
StrikeThreshold = 50;
```

Calculate the price:

```
Pgap = gapbybls(RateSpec, StockSpec, Settle, Maturity, OptSpec,...
Strike, StrikeThreshold)

Pgap =

   -0.0053
    4.4866
```

Compute the gamma and delta:

```
OutSpec = {'gamma'; 'delta'};
[Gamma ,Delta] = gapsensbybls(RateSpec, StockSpec, Settle, Maturity,...
OptSpec, Strike, StrikeThreshold, 'OutSpec', OutSpec)

Gamma =

    0.0724
```

```
    0.0724


Delta =

    0.2852
   -0.7148
```

**See Also** `gapbybls`

## Purpose

Allocate optimal hedge for target costs or sensitivities

## Syntax

```
[PortSens, PortCost, PortHolds] = hedgeopt(Sensitivities,
Price,  CurrentHolds, FixedInd, NumCosts, TargetCost,
TargetSens, ConSet)
```

## Arguments

| | |
|---|---|
| `Sensitivities` | Number of instruments (`NINST`) by number of sensitivities (`NSENS`) matrix of dollar sensitivities of each instrument. Each row represents a different instrument. Each column represents a different sensitivity. |
| `Price` | `NINST`-by-`1` vector of portfolio instrument unit prices. |
| `CurrentHolds` | `NINST`-by-`1` vector of contracts allocated to each instrument. |
| `FixedInd` | (Optional) Number of fixed instruments (`NFIXED`)-by-`1` vector of indices of instruments to hold fixed. For example, to hold the first and third instruments of a 10 instrument portfolio unchanged, set `FixedInd = [1 3]`. Default = `[]`, no instruments held fixed. |
| `NumCosts` | (Optional) Number of points generated along the cost frontier when a vector of target costs (`TargetCost`) is not specified. The default is 10 equally spaced points between the point of minimum cost and the point of minimum exposure. When specifying `TargetCost`, enter `NumCosts` as an empty matrix `[]`. |

| | |
|---|---|
| `TargetCost` | (Optional) Vector of target cost values along the cost frontier. If `TargetCost` is empty, or not entered, `hedgeopt` evaluates `NumCosts` equally spaced target costs between the minimum cost and minimum exposure. When specified, the elements of `TargetCost` should be positive numbers that represent the maximum amount of money the owner is willing to spend to rebalance the portfolio. |
| `TargetSens` | (Optional) `1`-by-`NSENS` vector containing the target sensitivity values of the portfolio. When specifying `TargetSens`, enter `NumCosts` and `TargetCost` as empty matrices `[]`. |
| `ConSet` | (Optional) Number of constraints (`NCONS`) by number of instruments (`NINST`) matrix of additional conditions on the portfolio reallocations. An eligible `NINST`-by-`1` vector of contract holdings, `PortWts`, satisfies all the inequalities `A*PortWts <= b`, where `A = ConSet(:,1:end-1)` and `b = ConSet(:,end)`. |

---

### Notes

The user-specified constraints included in `ConSet` may be created with the functions `pcalims` or `portcons`. However, the `portcons` default `PortHolds` positivity constraints are typically inappropriate for hedging problems since short-selling is usually required.

`NPOINTS`, the number of rows in `PortSens` and `PortHolds` and the length of `PortCost`, is inferred from the inputs. When the target sensitivities, `TargetSens`, is entered, `NPOINTS = 1`; otherwise `NPOINTS = NumCosts`, or is equal to the length of the `TargetCost` vector.

Not all problems are solvable (for example, the solution space may be infeasible or unbounded, or the solution may fail to converge). When a valid solution is not found, the corresponding rows of `PortSens` and `PortHolds` and the elements of `PortCost` are padded with `NaN`s as placeholders.

---

## Description

`[PortSens, PortCost, PortHolds] = hedgeopt(Sensitivities, Price, CurrentHolds, FixedInd, NumCosts, TargetCost, TargetSens, ConSet)` allocates an optimal hedge by one of two criteria:

- Minimize portfolio sensitivities (exposure) for a given set of target costs.
- Minimize the cost of hedging a portfolio given a set of target sensitivities.

Hedging involves the fundamental tradeoff between portfolio insurance and the cost of insurance coverage. This function lets investors modify portfolio allocations among instruments to achieve either of the criteria. The chosen criterion is inferred from the input argument list. The problem is cast as a constrained linear least-squares problem.

`PortSens` is a number of points (`NPOINTS`)-by-`NSENS` matrix of portfolio sensitivities. When a perfect hedge exists, `PortSens` is zeros. Otherwise, the best hedge possible is chosen.

`PortCost` is a `1`-by-`NPOINTS` vector of total portfolio costs.

`PortHolds` is an `NPOINTS`-by-`NINST` matrix of contracts allocated to each instrument. These are the reallocated portfolios.

## See Also

`hedgeslf` | `pcalims` | `portcons` | `portopt` | `lsqlin`

## Purpose

Self-financing hedge

## Syntax

```
[PortSens, PortValue, PortHolds] = hedgeslf(Sensitivities,
Price, CurrentHolds, FixedInd, ConSet)
```

## Arguments

| | |
|---|---|
| `Sensitivities` | Number of instruments (`NINST`) by number of sensitivities (`NSENS`) matrix of dollar sensitivities of each instrument. Each row represents a different instrument. Each column represents a different sensitivity. |
| `Price` | `NINST`-by-`1` vector of instrument unit prices. |
| `CurrentHolds` | `NINST`-by-`1` vector of contracts allocated in each instrument. |
| `FixedInd` | (Optional) Empty or number of fixed instruments (`NFIXED`)-by-`1` vector of indices of instruments to hold fixed. The default is `FixedInd = 1`; the holdings in the first instrument are held fixed. If `NFIXED` instruments will not be changed, enter all their locations in the portfolio in a vector. If no instruments are to be held fixed, enter `FixedInd = []`. |
| `ConSet` | (Optional) Number of constraints (`NCONS`)-by-`NINST` matrix of additional conditions on the portfolio reallocations. An eligible `NINST`-by-`1` vector of contract holdings, `PortHolds`, satisfies all the inequalities `A*PortHolds <= b`, where `A = ConSet(:,1:end-1)` and `b = ConSet(:,end)`. |

## Description

`[PortSens, PortValue, PortHolds] = hedgeslf(Sensitivities, Price, CurrentHolds, FixedInd, ConSet)` allocates a self-financing hedge among a collection of instruments. hedgeslf finds the reallocation in a portfolio of financial instruments that hedges the portfolio against market moves and that is closest to being self-financing (maintaining constant portfolio value). By default the first instrument entered is hedged with the other instruments.

`PortSens` is a `1`-by-`NSENS` vector of portfolio dollar sensitivities. When a perfect hedge exists, `PortSens` is zeros. Otherwise, the best possible hedge is chosen.

`PortValue` is the total portfolio value (scalar). When a perfectly self-financing hedge exists, `PortValue` is equal to `dot(Price, CurrentWts)` of the initial portfolio.

`PortHolds` is an `NINST`-by-`1` vector of contracts allocated to each instrument. This is the reallocated portfolio.

---

### Notes

- The constraints `PortHolds(FixedInd) = CurrentHolds(FixedInd)` are appended to any constraints passed in `ConSet`. Pass `FixedInd = []` to specify all constraints through `ConSet`.
- The default constraints generated by `portcons` are inappropriate, since they require the sum of all holdings to be positive and equal to one.
- `hedgeself` first tries to find the allocations of the portfolio that make it closest to being self-financing, while reducing the sensitivities to 0. If no solution is found, it finds the allocations that minimize the sensitivities. If the resulting portfolio is self-financing, `PortValue` is equal to the value of the original portfolio.

## Examples

**Example 1**. Perfect sensitivity cannot be reached.

```
Sens = [0.44  0.32; 1.0 0.0];
Price = [1.2; 1.0];
W0 = [1; 1];
[PortSens, PortValue, PortHolds]= hedgeslf(Sens, Price, W0)

PortSens =

    0.0000
    0.3200

PortValue =

    0.7600

PortHolds =

    1.0000
   -0.4400
```

**Example 2**. Constraints are in conflict.

```
Sens = [0.44  0.32; 1.0 0.0];
Price = [1.2; 1.0];
W0 = [1; 1];
ConSet = pcalims([2 2])

% O.K. if nothing fixed.

[PortSens, PortValue, PortHolds]= hedgeslf(Sens, Price, W0,...
[], ConSet)

PortSens =

    2.8800
    0.6400
```

```
PortValue =

    4.4000

PortHolds =

     2
     2

% WO(1) is not greater than 2.

[PortSens, PortValue, PortHolds] = hedgeslf(Sens, Price, WO,...
1, ConSet)

??? Error using ==> hedgeslf
Overly restrictive allocation constraints implied by ConSet and
by fixing the weight of instruments(s): 1
```

**Example 3**. Constraints are impossible to meet.

```
Sens = [0.44  0.32; 1.0 0.0];
Price = [1.2; 1.0];
WO = [1; 1];
ConSet = pcalims([2 2],[1 1]);

[PortSens, PortValue, PortHolds] = hedgeslf(Sens, Price, WO,...
[],ConSet)

??? Error using ==> hedgeslf
Overly restrictive allocation constraints specified in ConSet
```

## See Also

`hedgeopt` | `lsqlin` | `portcons`

## Purpose

Instrument prices from Heath-Jarrow-Morton interest-rate tree

## Syntax

`Price = hjmprice(HJMTree, InstSet, Options)`

## Arguments

| | |
|---|---|
| `HJMTree` | Heath-Jarrow-Morton tree sampling a forward-rate process. See `hjmtree` for information on creating `HJMTree`. |
| `InstSet` | Variable containing a collection of instruments. Instruments are categorized by type. Each type can have different data fields. The stored data field is a row vector or string for each instrument. |
| `Options` | (Optional) Derivatives pricing options structure created with `derivset`. |

## Description

`Price = hjmprice(HJMTree, InstSet, Options)` computes arbitrage-free prices for instruments using an interest-rate tree created with `hjmtree`. A subset of `NINST` instruments from a financial instrument variable, `InstSet`, are priced.

`Price` is a `NINST`-by-1 vector of prices for each instrument. The prices are computed by backward dynamic programming on the interest-rate tree. If an instrument cannot be priced, `NaN` is returned.

`PriceTree` is a MATLAB structure of trees containing vectors of instrument prices and accrued interest, and a vector of observation times for each node.

`PriceTree.PBush` contains the clean prices.

`PriceTree.AIBush` contains the accrued interest.

`PriceTree.tObs` contains the observation times.

`hjmprice` handles instrument types: `'Bond'`, `'CashFlow'`, `'OptBond'`, `'OptEmBond'`, `'Fixed'`, `'Float'`, `'Cap'`, `'Floor'`, `'RangeFloat'`, `'Swap'`. See `instadd` to construct defined types.

Related single-type pricing functions are:

- `bondbyhjm`: Price a bond from an HJM tree.
- `capbyhjm`: Price a cap from an HJM tree.
- `cfbyhjm`: Price an arbitrary set of cash flows from an HJM tree.
- `fixedbyhjm`: Price a fixed-rate note from an HJM tree.
- `floatbyhjm`: Price a floating-rate note from an HJM tree.
- `floorbyhjm`: Price a floor from an HJM tree.
- `optbndbyhjm`: Price a bond option from an HJM tree.
- `optembndbyhjm`: Price a bond with embedded option by an HJM tree.
- `rangefloatbyhjm`: Price range floating note using a HJM tree.
- `swapbyhjm`: Price a swap from an HJM tree.
- `swaptionbyhjm`: Price a swaption from an HJM tree.

## Examples

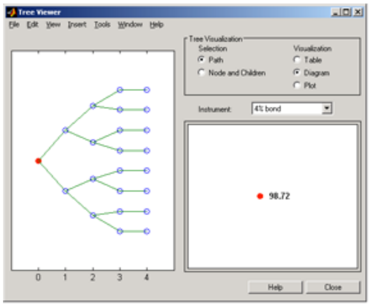
Load the HJM tree and instruments from the data file `deriv.mat`. Price the cap and bond instruments contained in the instrument set.

```
load deriv.mat;
HJMSubSet = instselect(HJMInstSet,'Type', {'Bond', 'Cap'});

instdisp(HJMSubSet)

%Table of instrument portfolio partially displayed:
Index  Type  CouponRate Settle      Maturity     Period  Basis ... Name     Quantity
1      Bond  0.04       01-Jan-2000 01-Jan-2003  1        NaN  ... 4% bond  100
2      Bond  0.04       01-Jan-2000 01-Jan-2004  2        NaN  ... 4% bond   50

Index Type Strike Settle      Maturity     CapReset  Basis ...  Name      Quantity
3     Cap  0.03   01-Jan-2000 01-Jan-2004  1         NaN   ...   3% Cap   30
```

```
[Price, PriceTree] = hjmprice(HJMTree, HJMSubSet)

Warning: Not all cash flows are aligned with the tree. Result will
be approximated.

Price =

   98.7159
   97.5280
    6.2831

PriceTree =

    FinObj: 'HJMPriceTree'
     PBush: {[3x1 double]  [3x1x2 double]  [3x2x2 double]  [3x4x2 double]  [3x8 double]}
    AIBush: {[3x1 double]  [3x1x2 double]  [3x2x2 double]  [3x4x2 double]  [3x8 double]}
      tObs: [0 1 2 3 4]
```

You can use `treeviewer` to see the prices of these three instruments along the price tree.

```
treeviewer(PriceTree, HJMSubSet)
```

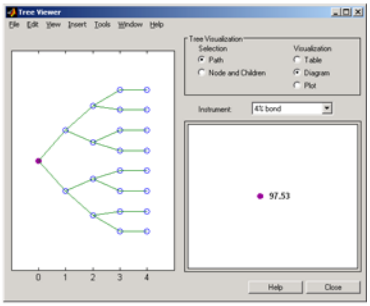


Selection: ● Path ○ Node and Children

Visualization: ○ Table ● Diagram ○ Plot

Instrument: 4% bond

First 4% Bond (Maturity 2003)

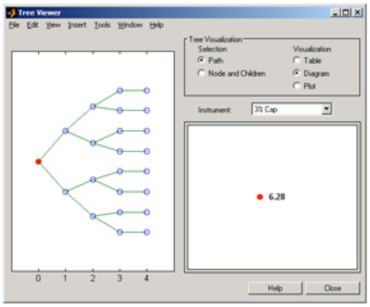


Selection: ● Path ○ Node and Children

Visualization: ○ Table ● Diagram ○ Plot

Instrument: 4% bond

Second 4% Bond (Maturity 2004)



Selection: ● Path ○ Node and Children

Visualization: ○ Table ● Diagram ○ Plot

Instrument: 3% Cap

3% Cap

Price the following multi-stepped coupon bonds using the following data:

```
% The data for the interest rate term structure is as follows:
Rates = [0.035; 0.042147; 0.047345; 0.052707];
ValuationDate = 'Jan-1-2010';
StartDates = ValuationDate;
EndDates = {'Jan-1-2011'; 'Jan-1-2012'; 'Jan-1-2013'; 'Jan-1-2014'};
Compounding = 1;

% Create RateSpec
RS = intenvset('ValuationDate', ValuationDate, 'StartDates', StartDates,...
'EndDates', EndDates,'Rates', Rates, 'Compounding', Compounding);
```

```
% Create a portfolio of stepped coupon bonds with different maturities
Settle = '01-Jan-2010';
Maturity = {'01-Jan-2011';'01-Jan-2012';'01-Jan-2013';'01-Jan-2014'};
CouponRate = {{'01-Jan-2011' .042;'01-Jan-2012' .05; '01-Jan-2013' .06; '01-Jan-2014' .07}};

ISet = instbond(CouponRate, Settle, Maturity, 1);
instdisp(ISet)

%Table of instrument portfolio partially displayed:
Index Type CouponRate Settle         Maturity   Period Basis EndMonthRule ... Face
1   Bond [Cell]     01-Jan-2010    01-Jan-2011    1      0     1          ... 100
2   Bond [Cell]     01-Jan-2010    01-Jan-2012    1      0     1          ... 100
3   Bond [Cell]     01-Jan-2010    01-Jan-2013    1      0     1          ... 100
4   Bond [Cell]     01-Jan-2010    01-Jan-2014    1      0     1          ... 100


% Build the tree with the following data
Volatility = [.2; .19; .18; .17];
CurveTerm = [ 1;  2;   3;   4];
HJMTimeSpec = hjmtimespec(ValuationDate, EndDates);
HJMVolSpec = hjmvolspec('Proportional', Volatility, CurveTerm, 1e6);
HJMT = hjmtree(HJMVolSpec,RS,HJMTimeSpec);

% Compute the price of the  stepped coupon bonds
PHJM = hjmprice(HJMT, ISet)

%Table of instrument portfolio partially displayed:
Index Type CouponRate Settle Maturity Period Basis EndMonthRule ... Face
1  Bond [Cell] 01-Jan-2010  01-Jan-2011    1   0     1          ... 100
2  Bond [Cell] 01-Jan-2010  01-Jan-2012    1   0     1          ... 100
3  Bond [Cell] 01-Jan-2010  01-Jan-2013    1   0     1          ... 100
4  Bond [Cell] 01-Jan-2010  01-Jan-2014    1   0     1          ... 100

PHJM =

  100.6763
  100.7368
```

```
  100.9266
  101.0115
```

Price a portfolio of stepped callable bonds and stepped vanilla bonds using the following data:

```
% Price a portfolio of stepped callable bonds and stepped vanilla bonds
% using the following data.

% The data for the interest rate term structure is as follows:
Rates = [0.035; 0.042147; 0.047345; 0.052707];
ValuationDate = 'Jan-1-2010';
StartDates = ValuationDate;
EndDates = {'Jan-1-2011'; 'Jan-1-2012'; 'Jan-1-2013'; 'Jan-1-2014'};
Compounding = 1;

%Create RateSpec
RS = intenvset('ValuationDate', ValuationDate, 'StartDates', StartDates,...
'EndDates', EndDates,'Rates', Rates, 'Compounding', Compounding);

% Create an instrument portfolio of 3 stepped callable bonds and three
% stepped vanilla bonds
Settle = '01-Jan-2010';
Maturity = {'01-Jan-2012';'01-Jan-2013';'01-Jan-2014'};
CouponRate = {{'01-Jan-2011' .042;'01-Jan-2012' .05; '01-Jan-2013' .06; '01-Jan-2014' .07}};
OptSpec='call';
Strike=100;
ExerciseDates='01-Jan-2011'; %Callable in one year

% Bonds with embedded option
ISet = instoptembnd(CouponRate, Settle, Maturity, OptSpec, Strike,...
ExerciseDates, 'Period', 1);

% Vanilla bonds
ISet = instbond(ISet, CouponRate, Settle, Maturity, 1);
```

```
% Display the instrument portfolio
instdisp(ISet)

%Table of instrument portfolio partially displayed:
Index Type      CouponRate Settle         Maturity   OptSpec Strike ExerciseDates ... AmericanOpt
1   OptEmBond [Cell]     01-Jan-2010    01-Jan-2012    call    100    01-Jan-2011 ... 0
2   OptEmBond [Cell]     01-Jan-2010    01-Jan-2013    call    100    01-Jan-2011 ... 0
3   OptEmBond [Cell]     01-Jan-2010    01-Jan-2014    call    100    01-Jan-2011 ... 0

Index Type CouponRate Settle         Maturity    Period Basis EndMonthRule ... Face
4   Bond [Cell]     01-Jan-2010    01-Jan-2012    1      0     1           ... 100
5   Bond [Cell]     01-Jan-2010    01-Jan-2013    1      0     1           ... 100
6   Bond [Cell]     01-Jan-2010    01-Jan-2014    1      0     1           ... 100


% Build the tree with the following data
Volatility = [.2; .19; .18; .17];
CurveTerm = [ 1;  2;   3;   4];
HJMTimeSpec = hjmtimespec(ValuationDate, EndDates);
HJMVolSpec = hjmvolspec('Proportional', Volatility, CurveTerm, 1e6);
HJMT = hjmtree(HJMVolSpec,RS,HJMTimeSpec);

%The first three rows corresponds to the price of the stepped callable bonds
% and the last three rows corresponds to the price of the stepped vanilla bonds.
PHJM = hjmprice(HJMT, ISet)

PHJM =

  100.3682
  100.1557
   99.9232
  100.7368
  100.9266
  101.0115
```

Compute the price of a portfolio using the following data:

```
% The data for the interest rate term structure is as follows:
Rates = [0.035; 0.042147; 0.047345; 0.052707];
ValuationDate = 'Jan-1-2011';
StartDates = ValuationDate;
EndDates = {'Jan-1-2012'; 'Jan-1-2013'; 'Jan-1-2014'; 'Jan-1-2015'};
Compounding = 1;

%  Create RateSpec
RS = intenvset('ValuationDate', ValuationDate, 'StartDates',...
StartDates, 'EndDates', EndDates,'Rates', Rates, 'Compounding', Compounding);

% Create an instrument portfolio with two range notes and a floating rate
% note with the following data:
Spread = 200;
Settle = 'Jan-1-2011';
Maturity = 'Jan-1-2014';

% First Range Note:
RateSched(1).Dates = {'Jan-1-2012'; 'Jan-1-2013'  ; 'Jan-1-2014'};
RateSched(1).Rates  = [0.045 0.055; 0.0525  0.0675; 0.06 0.08];

% Second Range Note:
RateSched(2).Dates = {'Jan-1-2012'; 'Jan-1-2013' ; 'Jan-1-2014'};
RateSched(2).Rates  = [0.048 0.059; 0.055  0.068 ; 0.07 0.09];

% Create InstSet
InstSet = instadd('RangeFloat', Spread, Settle, Maturity, RateSched);

% Add a floating-rate note
InstSet = instadd(InstSet, 'Float', Spread, Settle, Maturity);

% Display the portfolio instrument
instdisp(InstSet)
```

```
Index Type      Spread Settle       Maturity    RateSched FloatReset Basis Principal EndMonthRule
1     RangeFloat 200    01-Jan-2011  01-Jan-2014 [Struct]  1          0     100       1
2     RangeFloat 200    01-Jan-2011  01-Jan-2014 [Struct]  1          0     100       1

Index Type  Spread Settle         Maturity       FloatReset Basis Principal EndMonthRule
3     Float 200    01-Jan-2011    01-Jan-2014    1          0     100       1

% The data to build the tree is as follows:
Volatility = [.2; .19; .18; .17];
CurveTerm = [ 1;  2;   3;   4];
MaTree = {'Jan-1-2012'; 'Jan-1-2013'; 'Jan-1-2014'; 'Jan-1-2015'};
HJMTS = hjmtimespec(ValuationDate, MaTree);
HJMVS = hjmvolspec('Proportional', Volatility, CurveTerm, 1e6);
HJMT = hjmtree(HJMVS, RS, HJMTS);

% Price the portfolio
Price = hjmprice(HJMT, InstSet)

Price =

   91.1555
   90.6656
  105.5147
```

## See Also

`hjmsens` | `hjmtree` | `hjmvolspec` | `instadd` | `intenvprice` | `intenvsens`

# hjmsens

## Purpose

Instrument prices and sensitivities from Heath-Jarrow-Morton interest-rate tree

## Syntax

```
[Delta, Gamma, Vega, Price] = hjmsens(HJMTree, InstSet,
Options)
```

## Arguments

| | |
|---|---|
| `HJMTree` | Heath-Jarrow-Morton tree sampling a forward-rate process. See `hjmtree` for information on creating `HJMTree`. |
| `InstSet` | Variable containing a collection of instruments. Instruments are categorized by type. Each type can have different data fields. The stored data field is a row vector or string for each instrument. |
| `Options` | (Optional) Derivatives pricing options structure created with `derivset`. |

## Description

`[Delta, Gamma, Vega, Price] = hjmsens(HJMTree, InstSet, Options)` computes instrument sensitivities and prices for instruments using an interest-rate tree created with `hjmtree`. `NINST` instruments from a financial instrument variable, `InstSet`, are priced. `hjmsens` handles instrument types: `'Bond'`, `'CashFlow'`, `'OptBond'`, `'OptEmBond'`, `'Fixed'`, `'Float'`, `'Cap'`, `'Floor'`, `'RangeFloat'`, `'Swap'`. See `instadd` for information on instrument types.

`Delta` is an `NINST`-by-`1` vector of deltas, representing the rate of change of instrument prices with respect to changes in the interest rate. `Delta` is computed by finite differences in calls to `hjmtree`. See `hjmtree` for information on the observed yield curve.

`Gamma` is an `NINST`-by-`1` vector of gammas, representing the rate of change of instrument deltas with respect to the changes in the interest rate. `Gamma` is computed by finite differences in calls to `hjmtree`.

`Vega` is an `NINST`-by-`1` vector of vegas, representing the rate of change of instrument prices with respect to the changes in the volatility $\sigma(t,T)$. `Vega` is computed by finite differences in calls to `hjmtree`. See `hjmvolspec` for information on the volatility process.

---

**Note** All sensitivities are returned as dollar sensitivities. To find the per-dollar sensitivities, divide by the respective instrument price.

---

`Price` is an `NINST`-by-`1` vector of prices of each instrument. The prices are computed by backward dynamic programming on the interest-rate tree. If an instrument cannot be priced, `NaN` is returned.

`Delta` and `Gamma` are calculated based on yield shifts of 100 basis points. `Vega` is calculated based on a 1% shift in the volatility process.

## Examples

Load the tree and instruments from a data file. Compute `Delta` and `Gamma` for the cap and bond instruments contained in the instrument set.

```
load deriv.mat;
HJMSubSet = instselect(HJMInstSet,'Type', {'Bond', 'Cap'});
instdisp(HJMSubSet)

%Table of instrument portfolio partially displayed:
Index Type   CouponRate Settle      Maturity     Period ...   Name
1     Bond   0.04       01-Jan-2000 01-Jan-2003  1      ...   4% bond
2     Bond   0.04       01-Jan-2000 01-Jan-2004  2      ...   4% bond

Index Type Strike Settle      Maturity     CapReset ...    Name ...
3     Cap  0.03   01-Jan-2000 01-Jan-2004  1        ...  3% Cap ...

[Delta, Gamma] = hjmsens(HJMTree, HJMSubSet)

Warning: Not all cash flows are aligned with the tree. Result will
be approximated.
```

```
Delta =

 -272.6462
 -347.4315
  294.9700

Gamma =

  1.0e+003 *

    1.0299
    1.6227
    6.8526
```

**See Also** hjmprice | hjmtree | hjmvolspec | instadd

## Purpose

Specify time structure for Heath-Jarrow-Morton interest-rate tree

## Syntax

```
TimeSpec = hjmtimespec(ValuationDate, Maturity, Compounding)
```

## Arguments

| | |
|---|---|
| `ValuationDate` | Scalar date marking the pricing date and first observation in the tree. Specify as serial date number or date string. |
| `Maturity` | Number of levels (depth) of the tree. A number of levels (`NLEVELS`)-by-`1` vector of dates marking the cash flow dates of the tree. Cash flows with these maturities fall on tree nodes. Maturity should be in increasing order. |
| `Compounding` | (Optional) Scalar value representing the rate at which the input zero rates were compounded when annualized. Default = `1`. This argument determines the formula for the discount factors:<br>`Compounding` = `1`, `2`, `3`, `4`, `6`, `12`<br>`Disc = (1 + Z/F)^(-T)`, where `F` is the compounding frequency, `Z` is the zero rate, and `T` is the time in periodic units; for example, `T = F` is 1 year.<br>`Compounding` = `365`<br>`Disc = (1 + Z/F)^(-T)`, where `F` is the number of days in the basis year and `T` is a number of days elapsed computed by basis.<br>`Compounding` = `-1`<br>`Disc = exp(-T*Z)`, where `T` is time in years. |

# hjmtimespec

## Description

`TimeSpec = hjmtimespec(ValuationDate, Maturity, Compounding)` sets the number of levels and node times for an HJM tree and determines the mapping between dates and time for rate quoting.

`TimeSpec` is a structure specifying the time layout for `hjmtree`. The state observation dates are `[Settle; Maturity(1:end-1)]`. Because a forward rate is stored at the last observation, the tree can value cash flows out to `Maturity`.

## Examples

Specify an eight-period tree with semiannual nodes (every six months). Use exponential compounding to report rates.

```
Compounding = -1;
ValuationDate = '15-Jan-1999';
Maturity = datemnth(ValuationDate, 6*(1:8)');
TimeSpec = hjmtimespec(ValuationDate, Maturity, Compounding)

TimeSpec =

          FinObj: 'HJMTimeSpec'
   ValuationDate: 730135
        Maturity: [8x1 double]
     Compounding: -1
           Basis: 0
    EndMonthRule: 1
```

## See Also

`hjmtree` | `hjmvolspec`

## Purpose

Construct Heath-Jarrow-Morton interest-rate tree

## Syntax

```
HJMTree = hjmtree(VolSpec, RateSpec, TimeSpec)
```

## Arguments

| | |
|---|---|
| `VolSpec` | Volatility process specification. Sets the number of factors and the rules for computing the volatility $\sigma(t,T)$ for each factor. See `hjmvolspec` for information on the volatility process. |
| `RateSpec` | Interest-rate specification for the initial rate curve. See `intenvset` for information on declaring an interest-rate variable. |
| `TimeSpec` | Tree time layout specification. Defines the observation dates of the HJM tree and the compounding rule for date to time mapping and price-yield formulas. See `hjmtimespec` for information on the tree structure. |

## Description

`HJMTree = hjmtree(VolSpec, RateSpec, TimeSpec)` creates a structure containing time and forward-rate information on a bushy tree.

## Examples

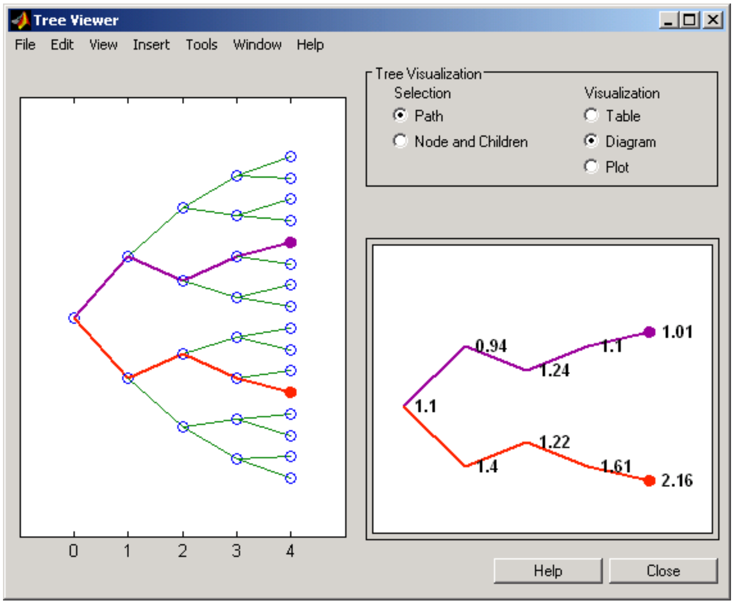
Using the data provided, create an HJM volatility specification (`VolSpec`), rate specification (`RateSpec`), and tree time layout specification (`TimeSpec`). Then use these specifications to create an HJM tree using `hjmtree`.

```
Compounding = 1;
ValuationDate = '01-01-2000';
StartDate = ['01-01-2000'; '01-01-2001'; '01-01-2002'; '01-01-2003'; '01-01-2004'];
EndDates = ['01-01-2001'; '01-01-2002'; '01-01-2003'; '01-01-2004'; '01-01-2005'];
Rates = [.1; .11; .12; .125; .13];
Volatility = [.2; .19; .18; .17; .16];
CurveTerm = [1; 2; 3; 4; 5];
```

```
HJMVolSpec = hjmvolspec('Stationary', Volatility , CurveTerm);

RateSpec = intenvset('Compounding', Compounding,...
           'ValuationDate', ValuationDate,...
           'StartDates', StartDate,...
           'EndDates', EndDates,...
           'Rates', Rates);

HJMTimeSpec = hjmtimespec(ValuationDate, EndDates, Compounding);
HJMTree = hjmtree(HJMVolSpec, RateSpec, HJMTimeSpec)
```

Use `treeviewer` to observe the tree you have created.

```
treeviewer(HJMTree)
```



Tree Visualization

Selection
- ● Path
- ○ Node and Children

Visualization
- ○ Table
- ● Diagram
- ○ Plot

## See Also

`hjmprice` | `hjmtimespec` | `hjmvolspec` | `intenvset`

## Purpose

Specify Heath-Jarrow-Morton interest-rate volatility process

## Syntax

```
Volspec = hjmvolspec(varargin)
```

## Arguments

The arguments to `hjmvolspec` vary according to the type and number of volatility factors specified when calling the function. Factors are specified by pairs of names and parameter sets. Factor names can be `'Constant'`, `'Stationary'`, `'Exponential'`, `'Vasicek'`, or `'Proportional'`. The parameter set is specific for each of these factor types:

- Constant volatility (Ho-Lee):
  `VolSpec = hjmvolspec('Constant', Sigma_0)`
- Stationary volatility:
  `VolSpec = hjmvolspec('Stationary', CurveVol, CurveTerm)`
- Exponential volatility:
  `VolSpec = hjmvolspec('Exponential', Sigma_0, Lambda)`
- Vasicek, Hull-White:
  `VolSpec = hjmvolspec('Vasicek', Sigma_0, CurveDecay, CurveTerm)`
- Nearly proportional stationary:
  `VolSpec = hjmvolspec('Proportional', CurveProp, CurveTerm, MaxSpot)`

You can specify more than one factor by concatenating names and parameter sets.

The following table defines the various arguments to `hjmvolspec`.

| Argument | Description |
|---|---|
| `Sigma_0` | Scalar base volatility over a unit time. |
| `Lambda` | Scalar decay factor. |
| `CurveVol` | Number of curves (`NCURVES`)-by`1` vector of `Vol` values at sample points. |

| Argument | Description |
|---|---|
| `CurveDecay` | `NCURVES`-by-`1` vector of `Decay` values at sample points. |
| `CurveProp` | `NCURVES`-by-`1` vector of `Prop` values at sample points. |
| `CurveTerm` | `NCURVES`-by-`1` vector of `Term` sample points. |

**Note** See the volatility specifications formulas below for a description of `Vol`, `Decay`, `Prop`, and `Term`.

## Description

`Volspec = hjmvolspec(varargin)` computes `VolSpec`, a structure that specifies the volatility model for `hjmtree`.

`hjmvolspec` specifies an HJM forward-rate volatility process. Each factor is specified with one of the functional forms.

| Volatility Specification | Formula |
|---|---|
| `Constant` | $\sigma(t,T)$ = Sigma_0 |
| `Stationary` | $\sigma(t,T)$ = Vol(T-t) = Vol(Term) |
| `Exponential` | $\sigma(t,T)$ = Sigma_0*exp(-Lambda*(T-t)) |
| `Vasicek, Hull-White` | $\sigma(t,T)$ = Sigma_0*exp(-Decay(T-t)) |
| `Proportional` | $\sigma(t,T)$ = Prop(T-t)*max(SpotRate(t), MaxSpot) |

The volatility process is $\sigma(t,T)$, where $t$ is the observation time and $T$ is the starting time of a forward rate. In a stationary process, the volatility term is $T$–$t$. Multiple factors can be specified sequentially.

The time values $T$, $t$, and `Term` are in coupon interval units specified by the `Compounding` input of `hjmtimespec`. For instance if `Compounding` = `2`, `Term = 1` is a semiannual period (six months).

## Examples

**Example 1**. Volatility is single-factor proportional.

```
CurveProp = [0.11765; 0.08825; 0.06865];
CurveTerm = [1; 2; 3];
VolSpec = hjmvolspec('Proportional', CurveProp, CurveTerm, 1e6)

 VolSpec =
          FinObj: 'HJMVolSpec'
    FactorModels: {'Proportional'}
      FactorArgs: {{1x3 cell}}
      SigmaShift: 0
      NumFactors: 1
       NumBranch: 2
         PBranch: [0.5000 0.5000]
     Fact2Branch: [-1 1]
```

**Example 2**. Volatility is two-factor exponential and constant.

```
VolSpec = hjmvolspec('Exponential', 0.1, 1, 'Constant', 0.2)

VolSpec =
          FinObj: 'HJMVolSpec'
    FactorModels: {'Exponential'  'Constant'}
      FactorArgs: {{1x2 cell}  {1x1 cell}}
      SigmaShift: 0
      NumFactors: 2
       NumBranch: 3
         PBranch: [0.2500 0.2500 0.5000]
     Fact2Branch: [2x3 double]
```

## See Also

`hjmtimespec` | `hjmtree`

## Purpose

Calibrate Hull-White tree using caps

## Syntax

```
[Alpha, Sigma, OptimOut] = hwcalbycap(RateSpec, MarketStrike,
MarketMaturity, MarketVolatility, Strike, Settle, Maturity)
[Alpha, Sigma, OptimOut] = hwcalbycap(RateSpec, MarketStrike,
MarketMaturity, MarketVolatility, Strike, Settle, Maturity,
'Name1', Value1...)
```

## Arguments

| | |
|---|---|
| `RateSpec` | The annualized, continuously compounded rate term structure. For more information, see `intenvset`. |
| `MarketStrike` | `NSTRIKES`-by-`1` vector of market cap strikes, as a decimal number. |
| `MarketMaturity` | `NMATS`-by-`1` vector of market cap maturity dates. |
| `MarketVolatility` | `NSTRIKES`-by-`NMATS` matrix of market flat volatilities, where `NSTRIKES` is the number of caplet strikes from `MarketStrike` and `NMATS` is the caplet maturity dates from `MarketMaturity`. |
| `Strike` | Scalar representing the rate at which the cap is exercised, as a decimal number. |
| `Settle` | Scalar representing the settle date of the cap. |
| `Maturity` | Scalar representing the maturity date of the cap. |
| `Reset` | (Optional) Scalar representing the frequency of payments per year. Default is `1`. |

| | |
|---|---|
| `Principal` | (Optional) Scalar representing the notional principal amount. Default is `100`. |
| `Basis` | (Optional) `NINST`-by-`1` vector representing the basis used when annualizing the input forward rate.<br>• 0 = actual/actual (default)<br>• 1 = 30/360 (SIA)<br>• 2 = actual/360<br>• 3 = actual/365<br>• 4 = 30/360 (BMA)<br>• 5 = 30/360 (ISDA)<br>• 6 = 30/360 (European)<br>• 7 = actual/365 (Japanese)<br>• 8 = actual/actual (ICMA)<br>• 9 = actual/360 (ICMA)<br>• 10 = actual/365 (ICMA)<br>• 11 = 30/360E (ICMA)<br>• 12 = actual/actual (ISDA)<br>• 13 = BUS/252<br>For more information, see basis. |
| `ValuationDate` | (Optional) Scalar representing the observation date of the investment horizons. The default is the `Settle` date. |

| | |
|---|---|
| `LB` | (Optional) `2`-by-`1` vector of the lower bounds, defined as `[LBSigma; LBAlpha]`, used in the search algorithm function. Default is `LB =[0;0]`. For more information, see `lsqnonlin`. |
| `UB` | (Optional) `2`-by-`1` vector of the upper bounds, defined as `[UBSigma; LBAlpha]`, used in the search algorithm function. Default is `UB =[ ]`(unbound). For more information, see `lsqnonlin`. |
| `X0` | (Optional) `2`-by-`1` vector of the initial values, defined as `[Sigma0; Alpha0]`, used in the search algorithm function. Default is `X0 = [0.5;0.5]`. For more information, see `lsqnonlin`. |
| `OptimOptions` | (Optional) Structure with optimization parameters. For more information, see `optimset`. |

---

**Note** All optional inputs are specified as matching parameter name/value pairs. The parameter name is specified as a character string, followed by the corresponding parameter value. You can specify parameter name/value pairs in any order. Names are case-insensitive and partial string matches are allowed provided no ambiguities exist.

---

## Description

```
[Alpha, Sigma, OptimOut] = hwcalbycap(RateSpec,
MarketStrike,
MarketMaturity, MarketVolatility, Strike, Settle, Maturity)
```

```
[Alpha, Sigma, OptimOut] = hwcalbycap(RateSpec,
MarketStrike,
```

```
MarketMaturity, MarketVolatility, Strike, Settle, Maturity,
'Name1', Value1...)
```

Use `hwcalbycap` to estimate the `Alpha` (mean reversion) and `Sigma` (volatility) using cap market data and the Hull-White model.

The outputs are:

- `Alpha` — Scalar representing the mean reversion value obtained from calibrating the cap using market information.
- `Sigma` — Scalar representing the volatility value obtained from calibrating the cap using market information.
- `OptimOut` — Structure with optimization results.

## Examples

For an example, see “Calibrating Hull-White Model Using Market Data” on page 2-79.

## See Also

`capbyblk` | `hwcalbyfloor` | `hwtree` | `lsqnonlin`

## Purpose

Calibrate Hull-White tree using floors

## Syntax

```
[Alpha, Sigma, OptimOut] = hwcalbyfloor(RateSpec,
MarketStrike, MarketMaturity, MarketVolatility, Strike,
Settle, Maturity)
[Alpha, Sigma, OptimOut] = hwcalbyfloor(RateSpec,
MarketStrike, MarketMaturity, MarketVolatility, Strike,
Settle, Maturity, 'Name1', Value1...)
```

## Arguments

| | |
|---|---|
| `RateSpec` | The annualized, continuously compounded rate term structure. For more information, see `intenvset`. |
| `MarketStrike` | `NSTRIKES`-by-`1` vector of market floor strikes as a decimal number. |
| `MarketMaturity` | `NMATS`-by-`1` vector of market floor maturity dates. |
| `MarketVolatility` | `NSTRIKES`-by-`NMATS` matrix of market flat volatilities. |
| `Strike` | Scalar representing the rate at which the floor is exercised, as a decimal number. |
| `Settle` | Scalar representing the settle date of the floor. |
| `Maturity` | Scalar representing the maturity date of the floor. |
| `Reset` | (Optional) Scalar representing the frequency of payments per year. Default is `1`. |

| | |
|---|---|
| `Principal` | (Optional) Scalar representing the notional principal amount. Default is `100`. |
| `Basis` | (Optional) `NINST`-by-`1` vector representing the basis used when annualizing the input forward rate.<br>• 0 = actual/actual (default)<br>• 1 = 30/360 (SIA)<br>• 2 = actual/360<br>• 3 = actual/365<br>• 4 = 30/360 (BMA)<br>• 5 = 30/360 (ISDA)<br>• 6 = 30/360 (European)<br>• 7 = actual/365 (Japanese)<br>• 8 = actual/actual (ICMA)<br>• 9 = actual/360 (ICMA)<br>• 10 = actual/365 (ICMA)<br>• 11 = 30/360E (ICMA)<br>• 12 = actual/actual (ISDA)<br>• 13 = BUS/252<br>For more information, see basis. |
| `ValuationDate` | (Optional) Scalar representing the observation date of the investment horizons. The default is the `Settle` date. |

| | |
|---|---|
| `LB` | (Optional) 2-by-1 vector of the lower bounds, defined as `[LBSigma; LBAlpha]`, used in the search algorithm function. Default is `LB =[0;0]`. For more information, see `lsqnonlin`. |
| `UB` | (Optional) 2-by-1 vector of the upper bounds, defined as `[UBSigma; LBAlpha]`, used in the search algorithm function. Default is `UB =[ ]`(unbound). For more information, see `lsqnonlin`. |
| `X0` | (Optional) 2-by-1 vector of the initial values, defined as `[Sigma0; Alpha0]`, used in the search algorithm function. Default is `X0 = [0.5;0.5]`. For more information, see `lsqnonlin`. |
| `OptimOptions` | (Optional) Structure with optimization parameters. For more information, see `optimset`. |

---

**Note** All optional inputs are specified as matching parameter name/value pairs. The parameter name is specified as a character string, followed by the corresponding parameter value. You can specify parameter name/value pairs in any order. Names are case-insensitive and partial string matches are allowed provided no ambiguities exist.

---

## Description

```
[Alpha, Sigma, OptimOut] = hwcalbyfloor(RateSpec,
MarketStrike, MarketMaturity, MarketVolatility, Strike,
Settle, Maturity)
```

```
[Alpha, Sigma, OptimOut] = hwcalbyfloor(RateSpec,
MarketStrike, MarketMaturity, MarketVolatility, Strike,
Settle, Maturity, 'Name1', Value1...)
```

Use `hwcalbyfloor` to estimate the `Alpha` (mean reversion) and `Sigma` (volatility) using floor market data and the Hull-White model.

The outputs are:

- `Alpha` — Scalar representing the mean reversion value obtained from calibrating the floor using market information.
- `Sigma` — Scalar representing the volatility value obtained from calibrating the floor using market information.
- `OptimOut` — Structure with optimization results.

## Examples

For an example, see “Calibrating Hull-White Model Using Market Data” on page 2-79.

## See Also

`floorbyblk` | `hwcalbycap` | `hwtree` | `lsqnonlin`

## Purpose

Instrument prices from Hull-White interest-rate tree

## Syntax

`[Price, PriceTree] = hwprice(HWTree, InstSet, Options)`

## Arguments

| | |
|---|---|
| `HWTree` | Interest-rate tree structure created by `hwtree`. |
| `InstSet` | Variable containing a collection of `NINST` instruments. Instruments are categorized by type; each type can have different data fields. The stored data field is a row vector or string for each instrument. |
| `Options` | (Optional) Derivatives pricing options structure created with `derivset`. |

## Description

`[Price, PriceTree] = hwprice(HWTree, InstSet, Options)` computes arbitrage-free prices for instruments using an interest-rate tree created with `hwtree`. All instruments contained in a financial instrument variable, `InstSet`, are priced.

`Price` is a number of instruments `(NINST)`-by-`1` vector of prices for each instrument. The prices are computed by backward dynamic programming on the interest-rate tree. If an instrument cannot be priced, `NaN` is returned.

`PriceTree` is a MATLAB structure of trees containing vectors of instrument prices and accrued interest, and a vector of observation times for each node.

`PriceTree.PTree` contains the clean prices.

`PriceTree.AITree` contains the accrued interest.

`PriceTree.tObs` contains the observation times.

`hwprice` handles instrument types: `'Bond'`, `'CashFlow'`, `'OptBond'`, `'OptEmBond'`, `'Fixed'`, `'Float'`, `'Cap'`, `'Floor'`, `'RangeFloat'`, `'Swap'`. See `instadd` to construct defined types.

Related single-type pricing functions are:

- `bondbyhw`: Price a bond from a Hull-White tree.
- `capbyhw`: Price a cap from a Hull-White tree.
- `cfbyhw`: Price an arbitrary set of cash flows from a Hull-White tree.
- `fixedbyhw`: Price a fixed-rate note from a Hull-White tree.
- `floatbyhw`: Price a floating-rate note from a Hull-White tree.
- `floorbyhw`: Price a floor from a Hull-White tree.
- `optbndbyhw`: Price a bond option from a Hull-White tree.
- `optembndbyhw`: Price a bond with embedded option by a Hull-White tree.
- `rangefloatbyhw`: Price range floating note using a Hull-White tree.
- `swapbyhw`: Price a swap from a Hull-White tree.
- `swaptionbyhw`: Price a swaption from a Hull-White tree.

## Examples

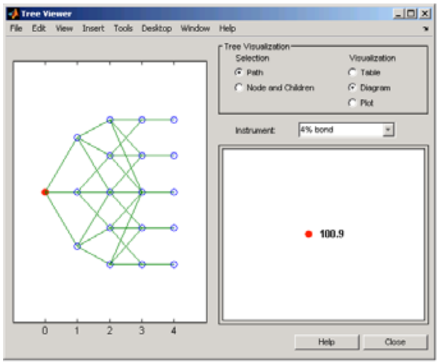
Load the HW tree and instruments from the data file `deriv.mat`. Price the cap and bond instruments contained in the instrument set.

```
load deriv.mat;
HWSubSet = instselect(HWInstSet,'Type', {'Bond', 'Cap'});

instdisp(HWSubSet)

%Table of instrument portfolio partially displayed:
Index Type   CouponRate Settle      Maturity    Period ...   Name ...
1     Bond   0.04       01-Jan-2004 01-Jan-2007 1      ...  4% bond
2     Bond   0.04       01-Jan-2004 01-Jan-2008 1      ...  4% bond

Index Type Strike Settle      Maturity     CapReset ...  Name ...
3     Cap  0.06   01-Jan-2004 01-Jan-2008  1        ...  6% Cap

[Price, PriceTree] = hwprice(HWTree, HWSubSet);
```

```
Price =

  100.9188
   99.3296
    0.5837
```

You can use `treeviewer` to see the prices of these three instruments along the price tree.

```
treeviewer(PriceTree, HWSubSet)
```



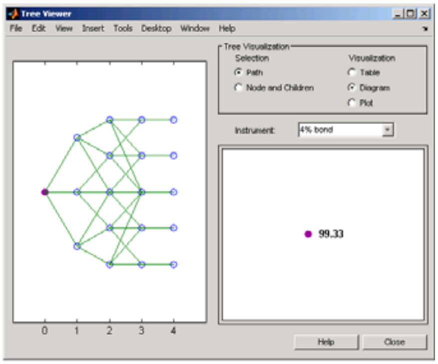
Selection: ● Path ○ Node and Children

Visualization: ○ Table ● Diagram ○ Plot

Instrument: 4% bond

100.9

First 4% Bond (Maturity 2007)



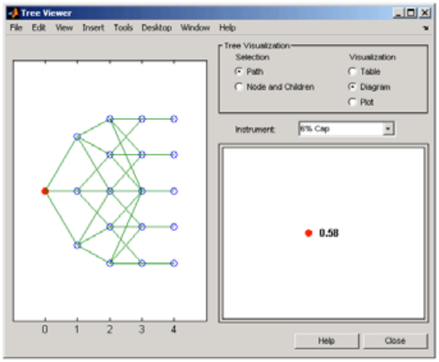
Selection: ● Path ○ Node and Children

Visualization: ○ Table ● Diagram ○ Plot

Instrument: 4% bond

99.33

Second 4% Bond (Maturity 2008)



Selection: ● Path ○ Node and Children

Visualization: ○ Table ● Diagram ○ Plot

Instrument: 6% Cap

0.58

6% Cap

---

Price the following multi-stepped coupon bonds using the following data:

```
% The data for the interest rate term structure is as follows:
Rates = [0.035; 0.042147; 0.047345; 0.052707];
ValuationDate = 'Jan-1-2010';
StartDates = ValuationDate;
EndDates = {'Jan-1-2011'; 'Jan-1-2012'; 'Jan-1-2013'; 'Jan-1-2014'};
Compounding = 1;

% Create RateSpec
RS = intenvset('ValuationDate', ValuationDate, 'StartDates', StartDates,...
'EndDates', EndDates,'Rates', Rates, 'Compounding', Compounding);

% Create a portfolio of stepped coupon bonds with different maturities
Settle = '01-Jan-2010';
Maturity = {'01-Jan-2011';'01-Jan-2012';'01-Jan-2013';'01-Jan-2014'};
CouponRate = {{'01-Jan-2011' .042;'01-Jan-2012' .05; '01-Jan-2013' .06; '01-Jan-2014' .07}};

ISet = instbond(CouponRate, Settle, Maturity, 1);
instdisp(ISet)

%Table of instrument portfolio partially displayed:
Index Type CouponRate Settle         Maturity    Period Basis EndMonthRule ... Face
1    Bond [Cell]     01-Jan-2010    01-Jan-2011    1      0     1           ... 100
2    Bond [Cell]     01-Jan-2010    01-Jan-2012    1      0     1           ... 100
3    Bond [Cell]     01-Jan-2010    01-Jan-2013    1      0     1           ... 100
4    Bond [Cell]     01-Jan-2010    01-Jan-2014    1      0     1           ... 100


% Build the tree with the following data
VolDates = ['1-Jan-2011'; '1-Jan-2012'; '1-Jan-2013'; '1-Jan-2014'];
VolCurve = 0.01;
AlphaDates = '01-01-2014';
AlphaCurve = 0.1;

HWVolSpec = hwvolspec(RS.ValuationDate, VolDates, VolCurve,...
AlphaDates, AlphaCurve);
HWTimeSpec = hwtimespec(RS.ValuationDate, VolDates, Compounding);
```

```
HWT = hwtree(HWVolSpec, RS, HWTimeSpec);

% Compute the price of the  stepped coupon bonds
PHW = hwprice(HWT, ISet)

%Table of instrument portfolio partially displayed:
Index Type CouponRate Settle  Maturity  Period Basis EndMonthRule ... Face
1 Bond [Cell]  01-Jan-2010    01-Jan-2011    1      0     1       ... 100
2 Bond [Cell]  01-Jan-2010    01-Jan-2012    1      0     1       ... 100
3 Bond [Cell]  01-Jan-2010    01-Jan-2013    1      0     1       ... 100
4 Bond [Cell]  01-Jan-2010    01-Jan-2014    1      0     1       ... 100


PHJM =

  100.6763
  100.7368
  100.9266
  101.0115
```

---

Price a portfolio of stepped callable bonds and stepped vanilla bonds using the following data:

```
% The data for the interest rate term structure is as follows:
Rates = [0.035; 0.042147; 0.047345; 0.052707];
ValuationDate = 'Jan-1-2010';
StartDates = ValuationDate;
EndDates = {'Jan-1-2011'; 'Jan-1-2012'; 'Jan-1-2013'; 'Jan-1-2014'};
Compounding = 1;

%Create RateSpec
RS = intenvset('ValuationDate', ValuationDate, 'StartDates', StartDates,...
'EndDates', EndDates,'Rates', Rates, 'Compounding', Compounding);

% Create an instrument portfolio of 3 stepped callable bonds and three
```

```
% stepped vanilla bonds
Settle = '01-Jan-2010';
Maturity = {'01-Jan-2012';'01-Jan-2013';'01-Jan-2014'};
CouponRate = {{'01-Jan-2011' .042;'01-Jan-2012' .05; '01-Jan-2013' .06; '01-Jan-2014' .07}};
OptSpec='call';
Strike=100;
ExerciseDates='01-Jan-2011'; %Callable in one year

% Bonds with embedded option
ISet = instoptembnd(CouponRate, Settle, Maturity, OptSpec, Strike,...
ExerciseDates, 'Period', 1);

% Vanilla bonds
ISet = instbond(ISet, CouponRate, Settle, Maturity, 1);

% Display the instrument portfolio
instdisp(ISet)

%Table of instrument portfolio partially displayed:
Index Type   CouponRate Settle         Maturity       OptSpec Strike ExerciseDates ... AmericanOpt
1   OptEmBond [Cell]     01-Jan-2010    01-Jan-2012    call    100    01-Jan-2011  ... 0
2   OptEmBond [Cell]     01-Jan-2010    01-Jan-2013    call    100    01-Jan-2011  ... 0
3   OptEmBond [Cell]     01-Jan-2010    01-Jan-2014    call    100    01-Jan-2011  ... 0

Index Type CouponRate Settle         Maturity       Period Basis EndMonthRule ... Face
4     Bond [Cell]     01-Jan-2010    01-Jan-2012    1      0     1            ... 100
5     Bond [Cell]     01-Jan-2010    01-Jan-2013    1      0     1            ... 100
6     Bond [Cell]     01-Jan-2010    01-Jan-2014    1      0     1            ... 100

% Build the tree with the following data
VolDates = ['1-Jan-2011'; '1-Jan-2012'; '1-Jan-2013'; '1-Jan-2014'];
VolCurve = 0.01;
AlphaDates = '01-01-2014';
AlphaCurve = 0.1;

HWVolSpec = hwvolspec(RS.ValuationDate, VolDates, VolCurve,...
AlphaDates, AlphaCurve);
```

```
HWTimeSpec = hwtimespec(RS.ValuationDate, VolDates, Compounding);
HWT = hwtree(HWVolSpec, RS, HWTimeSpec);

%The first three rows corresponds to the price of the stepped callable bonds
% and the last three rows corresponds to the price of the stepped vanilla bonds.

PHW = hwprice(HWT, ISet)

PHW =

  100.4222
  100.2275
  100.0502
  100.7368
  100.9266
  101.0115
```

---

Compute the price of a portfolio using the following data:

```
% The data for the interest rate term structure is as follows:
Rates = [0.035; 0.042147; 0.047345; 0.052707];
ValuationDate = 'Jan-1-2011';
StartDates = ValuationDate;
EndDates = {'Jan-1-2012'; 'Jan-1-2013'; 'Jan-1-2014'; 'Jan-1-2015'};
Compounding = 1;

%  Create RateSpec
RS = intenvset('ValuationDate', ValuationDate, 'StartDates',...
StartDates, 'EndDates', EndDates,'Rates', Rates, 'Compounding', Compounding);

% Create an instrument portfolio with two range notes and a floating rate
% note with the following data:
Spread = 200;
Settle = 'Jan-1-2011';
Maturity = 'Jan-1-2014';
```

```
% First Range Note:
RateSched(1).Dates = {'Jan-1-2012'; 'Jan-1-2013'  ; 'Jan-1-2014'};
RateSched(1).Rates  = [0.045 0.055; 0.0525  0.0675; 0.06 0.08];

% Second Range Note:
RateSched(2).Dates = {'Jan-1-2012'; 'Jan-1-2013' ; 'Jan-1-2014'};
RateSched(2).Rates  = [0.048 0.059; 0.055  0.068 ; 0.07 0.09];


% Create InstSet
InstSet = instadd('RangeFloat', Spread, Settle, Maturity, RateSched);

% Add a floating-rate note
InstSet = instadd(InstSet, 'Float', Spread, Settle, Maturity);

% Display the portfolio instrument
instdisp(InstSet)

Index Type      Spread Settle       Maturity   RateSched FloatReset Basis Principal EndMonthRule
1     RangeFloat 200    01-Jan-2011  01-Jan-2014 [Struct]  1          0     100       1
2     RangeFloat 200    01-Jan-2011  01-Jan-2014 [Struct]  1          0     100       1

Index Type  Spread Settle         Maturity       FloatReset Basis Principal EndMonthRule
3     Float 200    01-Jan-2011    01-Jan-2014    1          0     100       1

% The data to build the tree is as follows:
VolDates = ['1-Jan-2012'; '1-Jan-2013'; '1-Jan-2014';'1-Jan-2015'];
VolCurve = 0.01;
AlphaDates = '01-01-2015';
AlphaCurve = 0.1;

HWVS = hwvolspec(RS.ValuationDate, VolDates, VolCurve,...
AlphaDates, AlphaCurve);
HWTS = hwtimespec(RS.ValuationDate, VolDates, Compounding);
HWT = hwtree(HWVS, RS, HWTS);
```

```
% Price the portfolio
Price = hwprice(HWT, InstSet)

Price =

   99.4075
   98.1003
  105.5147
```

## See Also

hwsens | hwtree | instadd | intenvprice | intenvsens

## Purpose

Instrument prices and sensitivities from Hull-White interest-rate tree

## Syntax

`[Delta, Gamma, Vega, Price] = hwsens(HWTree, InstSet, Options)`

## Arguments

| | |
|---|---|
| `HWTree` | Interest-rate tree structure created by `hwtree`. |
| `InstSet` | Variable containing a collection of `NINST` instruments. Instruments are categorized by type; each type can have different data fields. The stored data field is a row vector or string for each instrument. |
| `Options` | (Optional) Derivatives pricing options structure created with `derivset`. |

## Description

`[Delta, Gamma, Vega, Price] = hwsens(HWTree, InstSet, Options)` computes instrument sensitivities and prices for instruments using an interest-rate tree created with the `hwtree` function. `NINST` instruments from a financial instrument variable, `InstSet`, are priced. `hwsens` handles instrument types: `'Bond'`, `'CashFlow'`, `'OptBond'`, `'OptEmBond'`, `'Fixed'`, `'Float'`, `'Cap'`, `'Floor'`, `'RangeFloat'`, `'Swap'`. See `instadd` for information on instrument types.

`Delta` is an `NINST`-by-`1` vector of deltas, representing the rate of change of instrument prices with respect to changes in the interest rate. `Delta` is computed by finite differences in calls to `hwtree`. See `hwtree` for information on the observed yield curve.

`Gamma` is an `NINST`-by-`1` vector of gammas, representing the rate of change of instrument deltas with respect to the changes in the interest rate. `Gamma` is computed by finite differences in calls to `hwtree`.

`Vega` is an `NINST`-by-`1` vector of vegas, representing the rate of change of instrument prices with respect to the changes in the volatility $\sigma(t,T)$.

`Vega` is computed by finite differences in calls to `hwtree`. See `hwvolspec` for information on the volatility process.

---

**Note** All sensitivities are returned as dollar sensitivities. To find the per-dollar sensitivities, divide by the respective instrument price.

---

`Price` is an `NINST`-by-`1` vector of prices of each instrument. The prices are computed by backward dynamic programming on the interest-rate tree. If an instrument cannot be priced, `NaN` is returned.

`Delta` and `Gamma` are calculated based on yield shifts of 100 basis points. `Vega` is calculated based on a 1% shift in the volatility process.

## Examples

Load the tree and instruments from a data file. Compute `Delta` and `Gamma` for the cap and bond instruments contained in the instrument set.

```
load deriv.mat;
HWSubSet = instselect(HWInstSet,'Type', {'Bond', 'Cap'});

instdisp(HWSubSet)

%Table of instrument portfolio partially displayed:
Index Type CouponRate Settle        Maturity     Period ...   Name ...
1     Bond 0.04       01-Jan-2004   01-Jan-2007  1      ...  4% Bond
2     Bond 0.04       01-Jan-2004   01-Jan-2008  1      ...  4% Bond

Index Type Strike Settle      Maturity     CapReset ...  Name ...
3     Cap  0.06   01-Jan-2004 01-Jan-2008  1        ...  6% Cap

[Delta, Gamma] = hwsens(HWTree, HWSubSet)

Delta =

       -291.26
       -374.64
```

```
         59.55

Gamma =

        858.41
       1460.88
       4843.65
```

## See Also

`hwprice` | `hwtree` | `hwvolspec` | `instadd`

# hwtimespec

## Purpose

Specify time structure for Hull-White interest-rate tree

## Syntax

```
TimeSpec = hwtimespec(ValuationDate, Maturity, Compounding)
```

## Arguments

| | |
|---|---|
| `ValuationDate` | Scalar date marking the pricing date and first observation in the tree. Specify as a serial date number or date string |
| `Maturity` | Number of levels (depth) of the tree. A number of levels (`NLEVELS`)-by-`1` vector of dates marking the cash flow dates of the tree. Cash flows with these maturities fall on tree nodes. Maturity should be in increasing order. |
| `Compounding` | (Optional) Scalar value representing the rate at which the input zero rates were compounded when annualized. Default = `-1` (continuous compounding). This argument determines the formula for the discount factors:<br>`Compounding` = `1`, `2`, `3`, `4`, `6`, `12`<br>`Disc = (1 + Z/F)^(-T)`, where `F` is the compounding frequency, `Z` is the zero rate, and `T` is the time in periodic units; for example, `T = F` is 1 year.<br>`Compounding` = `365`<br>`Disc = (1 + Z/F)^(-T)`, where `F` is the number of days in the basis year and `T` is a number of days elapsed computed by basis.<br>`Compounding` = `-1`<br>`Disc = exp(-T*Z)`, where `T` is time in years. |

## Description

`TimeSpec = hwtimespec(ValuationDate, Maturity, Compounding)` sets the number of levels and node times for a Hull-White tree and determines the mapping between dates and time for rate quoting.

`TimeSpec` is a structure specifying the time layout for `hwtree`. The state observation dates are `[Settle; Maturity(1:end-1)]`. Because a forward rate is stored at the last observation, the tree can value cash flows out to `Maturity`.

## Examples

Specify a four-period tree with annual nodes. Use annual compounding to report rates.

```
ValuationDate = 'Jan-1-2004';
Maturity = ['12-31-2004'; '12-31-2005'; '12-31-2006';
'12-31-2007'];
Compounding = 1;
TimeSpec = hwtimespec(ValuationDate, Maturity, Compounding)

TimeSpec =

           FinObj: 'HWTimeSpec'
    ValuationDate: 731947
         Maturity: [4x1 double]
      Compounding: 1
            Basis: 0
     EndMonthRule: 1
```

## See Also

`bktree` | `hwtree` | `hwvolspec`

## Purpose

Construct Hull-White interest-rate tree

## Syntax

```
HWTree = hwtree(VolSpec, RateSpec, TimeSpec)
HWTree = hwtree(VolSpec, RateSpec, TimeSpec, Name,Value)
```

## Description

`HWTree = hwtree(VolSpec, RateSpec, TimeSpec)` creates a structure containing time and interest-rate information on a recombining tree.

`HWTree = hwtree(VolSpec, RateSpec, TimeSpec, Name,Value)` creates a structure containing time and interest-rate information on a recombining tree with additional options specified by one or more `Name,Value` pair arguments.

## Input Arguments

**VolSpec**

Volatility process specification. See `hwvolspec` for information on the volatility process.

**RateSpec**

Interest-rate specification for the initial rate curve. See `intenvset` for information on declaring an interest-rate variable.

**TimeSpec**

Tree time layout specification. Defines the observation dates of the HW tree and the compounding rule for date to time mapping and price-yield formulas. See `hwtimespec` for information on the tree structure.

### Name-Value Pair Arguments

Specify optional comma-separated pairs of `Name,Value` arguments, where `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside single quotes (`' '`). You can specify several name and value pair arguments in any order as `Name1,Value1,...,NameN,ValueN`.

**Method**

String specifies the Hull-White method upon which the tree-node connectivity algorithm is based. Possible values are `HW1996` and `HW2000`.

---

**Note** `hwtree` supports two tree-node connectivity algorithms. `HW1996` is based on the original paper published in the *Journal of Derivatives*, and `HW2000` is the general version of the algorithm, as specified in the paper published in August 2000.

---

**Default:** `HW1996`

## Output Arguments

### HWTree

Structure containing time and interest rate information of a trinomial recombining tree.

## Examples

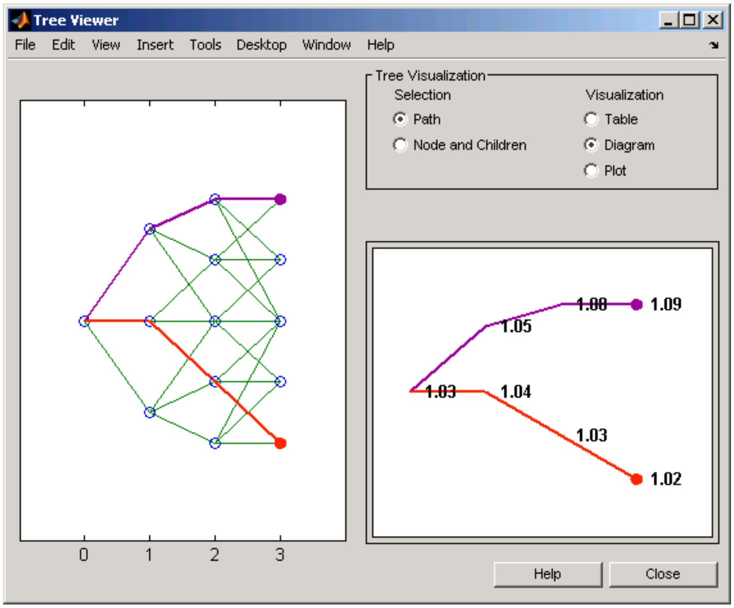
Using the data provided, create a Hull-White volatility specification (`VolSpec`), rate specification (`RateSpec`), and tree time layout specification (`TimeSpec`). Then use these specifications to create a Hull-White tree using `hwtree`.

```
Compounding = -1;
ValuationDate = '01-01-2004';
StartDate = ValuationDate;
VolDates = ['12-31-2004'; '12-31-2005'; '12-31-2006';
'12-31-2007'];
VolCurve = 0.01;
AlphaDates = '01-01-2008';
AlphaCurve = 0.1;
Rates = [0.0275; 0.0312; 0.0363; 0.0415];

HWVolSpec = hwvolspec(ValuationDate, VolDates, VolCurve,...
AlphaDates, AlphaCurve);

RateSpec = intenvset('Compounding', Compounding,...
      'ValuationDate', ValuationDate,...
```

```
      'StartDates', ValuationDate,...
      'EndDates', VolDates,...
      'Rates', Rates);

HWTimeSpec = hwtimespec(ValuationDate, VolDates, Compounding);
HWTree = hwtree(HWVolSpec, RateSpec, HWTimeSpec)

HWTree =

      FinObj: 'HWFwdTree'
     VolSpec: [1x1 struct]
    TimeSpec: [1x1 struct]
    RateSpec: [1x1 struct]
        tObs: [0 0.9973 1.9973 2.9973]
        dObs: [731947 732312 732677 733042]
      CFlowT: {[4x1 double]  [3x1 double]  [2x1 double]  [3.9973]}
       Probs: {[3x1 double]  [3x3 double]  [3x5 double]}
     Connect: {[2]  [2 3 4]  [2 2 3 4 4]}
     FwdTree: {1x4 cell}
```

Use `treeviewer` to observe the tree you have created.

```
treeviewer(HWTree)
```



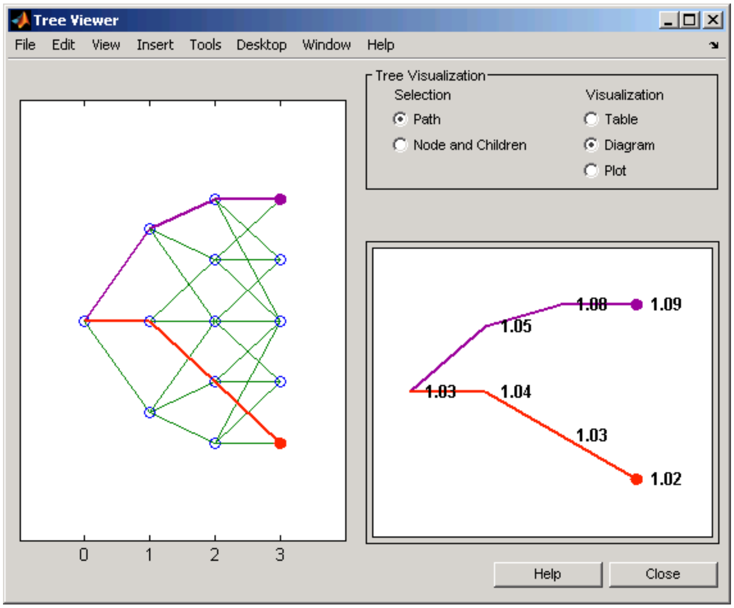
Using the data provided, create a Hull-White volatility specification (`VolSpec`), rate specification (`RateSpec`), and tree time layout specification (`TimeSpec`). Then use these specifications to create a Hull-White tree using `hwtree`.

```
Compounding = -1;
ValuationDate = '01-01-2004';
StartDate = ValuationDate;
VolDates = ['12-31-2004'; '12-31-2005'; '12-31-2006';
'12-31-2007'];
VolCurve = 0.01;
AlphaDates = '01-01-2008';
AlphaCurve = 0.1;
Rates = [0.0275; 0.0312; 0.0363; 0.0415];

HWVolSpec = hwvolspec(ValuationDate, VolDates, VolCurve,...
AlphaDates, AlphaCurve);
```

```
RateSpec = intenvset('Compounding', Compounding,...
      'ValuationDate', ValuationDate,...
      'StartDates', ValuationDate,...
      'EndDates', VolDates,...
      'Rates', Rates);

HWTimeSpec = hwtimespec(ValuationDate, VolDates, Compounding);
HWTree = hwtree(HWVolSpec, RateSpec, HWTimeSpec)

HWTree =

      FinObj: 'HWFwdTree'
     VolSpec: [1x1 struct]
    TimeSpec: [1x1 struct]
    RateSpec: [1x1 struct]
        tObs: [0 0.9973 1.9973 2.9973]
        dObs: [731947 732312 732677 733042]
      CFlowT: {[4x1 double]  [3x1 double]  [2x1 double]  [3.9973]}
       Probs: {[3x1 double]  [3x3 double]  [3x5 double]}
     Connect: {[2]  [2 3 4]  [2 2 3 4 4]}
     FwdTree: {1x4 cell}
```

Use `treeviewer` to observe the tree you have created.

```
treeviewer(HWTree)
```



Tree Visualization

Selection

● Path

○ Node and Children

Visualization

○ Table

● Diagram

○ Plot

## References

Hull, J., and A. White, "Using Hull-White Interest Rate Trees", *Journal of Derivatives*, 1996.

Hull, J., and A. White, "The General Hull-White Model and Super Calibration", August 2000.

## See Also

| `hwcalbycap` | `hwcalbyfloor` | `hwprice` | `hwtimespec` | `hwvolspec` | `intenvset` |

## Tutorials

- "Calibrating Hull-White Model Using Market Data" on page 2-79

## Purpose

Specify Hull-White interest-rate volatility process

## Syntax

`Volspec = hwvolspec(ValuationDate, VolDates, VolCurve, AlphaDates, AlphaCurve, InterpMethod)`

## Arguments

| | |
|---|---|
| `ValuationDate` | Scalar value representing the observation date of the investment horizon. |
| `VolDates` | Number of points `(NPOINTS)-by-1` vector of yield volatility end dates. |
| `VolCurve` | `NPOINTS-by-1` vector or scalar of yield volatility values in decimal form. |
| `AlphaDates` | `MPOINTS-by-1` vector of mean reversion end dates. |
| `AlphaCurve` | `MPOINTS-by-1` vector of positive mean reversion values or scalar in decimal form. |
| `InterpMethod` | (Optional) Interpolation method. Default is `'linear'`. See `interp1` for more information. |

**Note** The number of points in `VolCurve` and `AlphaCurve` do not have to be the same.

## Description

`Volspec = hwvolspec(ValuationDate, VolDates, VolCurve, AlphaDates, AlphaCurve, InterpMethod)` creates a structure specifying the volatility for `hwtree`.

The volatility process is such that the variance of r(t + dt) - r(t) is defined as follows: V = (Volatility.^2 .* (1 - exp(-2*Alpha .* dt))) ./ (2 * Alpha). For more information on using Hull-White interest rate trees, see “Hull-White (HW) and Black-Karasinski (BK) Modeling” on page C-4.

## Examples

Using the data provided, create a Hull-White volatility specification (`VolSpec`).

```
ValuationDate = '01-01-2004';
StartDate = ValuationDate;
VolDates = ['12-31-2004'; '12-31-2005'; '12-31-2006';
'12-31-2007'];
VolCurve = 0.01;
AlphaDates = '01-01-2008';
AlphaCurve = 0.1;

HWVolSpec = hwvolspec(ValuationDate, VolDates, VolCurve,...
AlphaDates, AlphaCurve)

HWVolSpec =

             FinObj: 'HWVolSpec'
      ValuationDate: 731947
           VolDates: [4x1 double]
           VolCurve: [4x1 double]
         AlphaCurve: 0.1000
         AlphaDates: 733408
    VolInterpMethod: 'linear'
```

## See Also

`bktree` | `hwcalbycap` | `hwcalbyfloor` | `interp1`

# impvbybjs

## Purpose

Determine implied volatility using Bjerksund-Stensland 2002 option pricing model

## Syntax

```
Volatility = impvbybjs(RateSpec, StockSpec, Settle,
Maturity, OptSpec, Strike, OptPrice, 'Name1', Value1...)
```

## Arguments

| | |
|---|---|
| `RateSpec` | The annualized continuously compounded rate term structure. For information on the interest rate specification, see `intenvset`. |
| `StockSpec` | Stock specification. See `stockspec`. |
| `Settle` | `NINST`-by-`1` vector of settlement or trade dates. |
| `Maturity` | `NINST`-by-`1` vector of maturity dates. |
| `OptSpec` | `NINST`-by-`1` cell array of strings `'call'` or `'put'`. |
| `Strike` | `NINST`-by-`1` vector of strike price values. |
| `OptPrice` | `NINST`-by-`1` vector of American option prices from which the implied volatility of the underlying asset are derived. |

**Note** All optional inputs are specified as matching parameter name/parameter value pairs. The parameter name is specified as a character string, followed by the corresponding parameter value. You can specify parameter name/parameter value pairs in any order; names are case-insensitive and partial string matches are allowed provided no ambiguities exist.

| | |
|---|---|
| `Limit` | (Optional) `1`-by-`2` positive vector representing the lower and upper bound of the implied volatility search interval. Default is `[0.1 10]`, or 10% to 1000% per annum. |
| `Tolerance` | (Optional) Positive scalar implied volatility termination tolerance. Default is `1e-6`. |

## Description

`Volatility = impvbybjs(RateSpec, StockSpec, Settle, Maturity, OptSpec, Strike, OptPrice, 'Name1', Value1...)` computes implied volatility using the Bjerksund-Stensland 2002 option pricing model.

`Volatility` is a `NINST`-by-`1` vector of expected implied volatility values. If no solution is found, a `NaN` is returned.

---

**Note** `impvbybjs` computes implied volatility of American options with continuous dividend yield using the Bjerksund-Stensland option pricing model.

---

## Examples

Consider three American call options with exercise prices of $100 that expire on July 1, 2008. The underlying stock is trading at $100 on January 1, 2008 and pays a continuous dividend yield of 10%. The annualized continuously compounded risk-free rate is 10% per annum and the option prices are $4.063, $6.77 and $9.46. Using this data, calculate the implied volatility of the stock using the Bjerksund-Stensland 2002 option pricing model:

```
AssetPrice = 100;
Settle = 'Jan-1-2008';
Maturity = 'Jul-1-2008';
Strike = 100;
DivAmount = 0.1;
Rate = 0.1;
```

Define `RateSpec` and `StockSpec`:

```
RateSpec = intenvset('ValuationDate', Settle, 'StartDates', Settle,...
'EndDates', Maturity, 'Rates', Rate, 'Compounding', -1, 'Basis', 1);

StockSpec = stockspec(NaN, AssetPrice, {'continuous'}, DivAmount);
```

Calculate the implied volatility of the call options:

```
OptSpec = {'call'};
OptionPrice = [4.063;6.77;9.46];

ImpVol =  impvbybjs(RateSpec, StockSpec, Settle, Maturity, OptSpec,...
Strike, OptionPrice)

ImpvVol =

    0.1500
    0.2501
    0.3500
```

The implied volatility is 15% for the first call, and 25% and 35% for the second and third call options.

## References

Bjerksund, P. and G. Stensland, *Closed-Form Approximation of American Options*, Scandinavian Journal of Management, 1993, Vol. 9, Suppl., pp. S88-S99.

Bjerksund, P. and G. Stensland, *Closed Form Valuation of American Options*, Discussion paper 2002 (http://brage.bibsys.no/nhh/bitstream/URN:NBN:no-bibsys_brage_22301/1/bjerksund%2

## See Also

`optstockbybjs` | `optstocksensbybjs`

## Purpose

Determine implied volatility using Black option pricing model

## Syntax

```
Volatility = impvbyblk(RateSpec, StockSpec, Settle,
Maturity, Strike, OptPrice, 'Name1', Value1...)
```

## Arguments

| | |
|---|---|
| `RateSpec` | The annualized continuously compounded rate term structure. For information on the interest rate specification, see `intenvset`. |
| `StockSpec` | Stock specification. See `stockspec`. |
| `Settle` | `NINST`-by-`1` vector of settlement or trade dates. |
| `Maturity` | `NINST`-by-`1` vector of maturity dates. |
| `OptSpec` | `NINST`-by-`1` cell array of strings `'call'` or `'put'`. |
| `Strike` | `NINST`-by-`1` vector of strike price values. |
| `OptPrice` | `NINST`-by-`1` vector of European option prices from which the implied volatility of the underlying asset are derived. |

---

**Note** All optional inputs are specified as matching parameter name/parameter value pairs. The parameter name is specified as a character string, followed by the corresponding parameter value. You can specify parameter name/parameter value pairs in any order; names are case-insensitive and partial string matches are allowed provided no ambiguities exist.

---

| | |
|---|---|
| `Limit` | (Optional) Positive scalar representing the upper bound of the implied volatility search interval. Default is `10`, or 1000% per annum. |
| `Tolerance` | (Optional) Positive scalar implied volatility termination tolerance. Default is `1e-6`. |

## Description

`Volatility = impvbyblk(RateSpec, StockSpec, Settle, Maturity, Strike, OptPrice, 'Name1', Value1...)` computes implied volatility using the Black option pricing model.

`Volatility` is a `NINST`-by-1 vector of expected implied volatility values. If no solution is found, a `NaN` is returned.

## Examples

Consider a European call and put options on a futures contract with exercise prices of $30 for the put and $40 for the call that expire on September 1, 2008. Assume that on May 1, 2008 the contract is trading at $35. The annualized continuously compounded risk-free rate is 5% per annum. What are the implied volatilities of the stock, if on that date, the call price is $1.14 and the put price is $0.82

```
AssetPrice = 35;
Strike = [30; 40];
Rates = 0.05;
Settle = 'May-01-08';
Maturity = 'Sep-01-08';
```

Create `RateSpec` and `StockSpec`:

```
RateSpec = intenvset('ValuationDate', Settle, 'StartDates', Settle,...
'EndDates', Maturity, 'Rates', Rates, 'Compounding', -1);

StockSpec = stockspec(NaN, AssetPrice);
```

Define the options:

```
OptSpec = {'put';'call'};
```

Calculate the implied volatility of the options:

```
Price = [1.14;0.82];
Volatility = impvbyblk(RateSpec, StockSpec, Settle, Maturity, OptSpec,...
Strike, Price)
```

```
Volatility =

    0.4052
    0.3021
```

The implied volatility would be 41% and 30%.

## See Also

`optstockbyblk` | `optstocksensbyblk`

# impvbybls

## Purpose

Determine implied volatility using Black-Scholes option pricing model

## Syntax

```
Volatility = impvbybls(RateSpec, StockSpec, Settle,
Maturity, Strike, OptPrice, 'Name1', Value1...)
```

## Arguments

| | |
|---|---|
| `RateSpec` | The annualized continuously compounded rate term structure. For information on the interest rate specification, see `intenvset`. |
| `StockSpec` | Stock specification. See `stockspec`. |
| `Settle` | `NINST`-by-`1` vector of settlement or trade dates. |
| `Maturity` | `NINST`-by-`1` vector of maturity dates. |
| `OptSpec` | `NINST`-by-`1` cell array of strings `'call'` or `'put'`. |
| `Strike` | `NINST`-by-`1` vector of strike price values. |
| `OptPrice` | `NINST`-by-`1` vector of European option prices from which the implied volatility of the underlying asset are derived. |

---

**Note** All optional inputs are specified as matching parameter name/parameter value pairs. The parameter name is specified as a character string, followed by the corresponding parameter value. You can specify parameter name/parameter value pairs in any order; names are case-insensitive and partial string matches are allowed provided no ambiguities exist.

---

| | |
|---|---|
| `Limit` | (Optional) Positive scalar representing the upper bound of the implied volatility search interval. Default is `10`, or 1000% per annum. |
| `Tolerance` | (Optional) Positive scalar implied volatility termination tolerance. Default is `1e-6`. |

## Description

`Volatility = impvbybls(RateSpec, StockSpec, Settle, Maturity, Strike, OptPrice, 'Name1', Value1...)` computes implied volatility using the Black-Scholes option pricing model.

`Volatility` is a `NINST`-by-1 vector of expected implied volatility values. If no solution is found, a `NaN` is returned.

## Examples

Consider a European call and put options with an exercise price of $40 that expires on June 1, 2008. The underlying stock is trading at $45 on January 1, 2008 and the risk-free rate is 5% per annum. The option price is $7.10 for the call and $2.85 for the put. Using this data, calculate the implied volatility of the European call and put using the Black-Scholes option pricing model:

```
AssetPrice = 45;
Settlement = 'Jan-01-2008';
Maturity = 'June-01-2008';
Strike = 40;
Rates = 0.05;
OptionPrice = [7.10; 2.85];
OptSpec = {'call';'put'};
```

Define `RateSpec` and `StockSpec` :

```
RateSpec = intenvset('ValuationDate', Settlement, 'StartDates', Settlement,...
'EndDates', Maturity, 'Rates', Rates, 'Compounding', -1, 'Basis', 1);

StockSpec = stockspec(NaN, AssetPrice);
```

Calculate the implied volatility of the options:

```
ImpvVol =  impvbybls(RateSpec, StockSpec, Settlement, Maturity, OptSpec,...
Strike, OptionPrice)

ImpvVol =
```

```
    0.3175
    0.4878
```

The implied volatility is 31.75% for the call and 48.78% for the put.

## See Also

`optstockbybls` | `optstocksensbybls`

## Purpose

Determine implied volatility using Roll-Geske-Whaley option pricing model for American call option

## Syntax

```
Volatility = impvbyrgw(RateSpec, StockSpec, Settle,
Maturity, Strike, OptPrice, 'Name1', Value1...)
```

## Arguments

| | |
|---|---|
| `RateSpec` | The annualized continuously compounded rate term structure. For information on the interest rate specification, see `intenvset`. |
| `StockSpec` | Stock specification. See `stockspec`. |
| `Settle` | `NINST`-by-`1` vector of settlement or trade dates. |
| `Maturity` | `NINST`-by-`1` vector of maturity dates. |
| `Strike` | `NINST`-by-`1` vector of strike price values. |
| `OptPrice` | `NINST`-by-`1` vector of American call option prices from which the implied volatility of the underlying asset are derived. |

---

**Note** All optional inputs are specified as matching parameter name/parameter value pairs. The parameter name is specified as a character string, followed by the corresponding parameter value. You can specify parameter name/parameter value pairs in any order; names are case-insensitive and partial string matches are allowed provided no ambiguities exist.

---

| | |
|---|---|
| `Limit` | (Optional) Positive scalar representing the upper bound of the implied volatility search interval. Default is `10`, or 1000% per annum. |
| `Tolerance` | (Optional) Positive scalar implied volatility termination tolerance. Default is `1e-6`. |

## Description

`Volatility = impvbyrgw(RateSpec, StockSpec, Settle, Maturity, Strike, OptPrice, 'Name1', Value1...)` computes implied volatility using the Roll-Geske-Whaley option pricing model.

`Volatility` is a `NINST`-by-1 vector of expected implied volatility values. If no solution is found, a `NaN` is returned.

**Note** `impvbyrgw` computes implied volatility of American calls with a single cash dividend using the Roll-Geske-Whaley option pricing model.

## Examples

Assume that on July 1, 2008 a stock is trading at $13 and pays a single cash dividend of $0.25 on November 1, 2008. The American call option with a strike price of $15 expires on July 1, 2009 and is trading at $1.346. The annualized continuously compounded risk-free rate is 5% per annum. Calculate the implied volatility of the stock using the Roll-Geske-Whaley option pricing model:

```
AssetPrice = 13;
Strike = 15;
Rates = 0.05;
Settle = 'July-01-08';
Maturity = 'July-01-09';
```

Define `StockSpec` and `RateSpec`:

```
RateSpec = intenvset('ValuationDate', Settle, 'StartDates', Settle,...
 'EndDates', Maturity, 'Rates', Rates, 'Compounding', -1);

StockSpec = stockspec(NaN, AssetPrice, {'cash'}, 0.25, {'Nov 1,2008'});
```

Calculate the implied volatility of the option:

```
Price = [1.346];
Volatility = impvbyrgw(RateSpec, StockSpec, Settle, Maturity, Strike, Price)

Volatility =
```

```
0.3539
```

## See Also

`optstockbyrgw` | `optstocksensbyrgw`

**Purpose**

Add types to instrument collection

**Syntax**

```
InstSet = instadd(InstSetOld, TypeString, Data1, Data2, ...)
InstSet = instadd('CashFlow', CFlowAmounts, CFlowDates, Settle,
Basis)
InstSet = instadd('CashFlow', CFlowAmounts, CFlowDates, Settle,
Basis)
InstSet = instadd('Barrier', OptSpec, Strike, Settle,
ExerciseDates, AmericanOpt, BarrierType, Barrier, Rebate)
InstSet = instadd('Bond', CouponRate, Settle, Maturity, Period,
Basis, EndMonthRule, IssueDate, FirstCouponDate, LastCouponDate,
StartDate, Face)
InstSet = instadd('OptEmBond', CouponRate, Settle, Maturity,
OptSpec, Strike, ExerciseDates, 'AmericanOpt', AmericanOpt, 'Period',
Period, 'Basis', Basis, 'EndMonthRule', EndMonthRule,'Face',Face,
'IssueDate', IssueDate, 'FirstCouponDate', FirstCouponDate,
'LastCouponDate', LastCouponDate, 'StartDate', StartDate)
InstSet = instadd('OptBond', BondIndex, OptSpec, Strike,
ExerciseDates, AmericanOpt)
InstSet = instadd('Cap', Strike, Settle, Maturity, Reset, Basis,
Principal)
InstSet = instadd('Compound', UOptSpec, UStrike, USettle,
UExerciseDates, UAmericanOpt,COptSpec, CStrike, CSettle,
CExerciseDates, CAmericanOpt)
InstSet = instadd('Fixed', CouponRate, Settle, Maturity, Reset,
Basis, Principal, EndMonthRule)
InstSet = instadd('Float', Spread, Settle, Maturity, Reset, Basis,
Principal, EndMonthRule)
InstSet = instadd('Floor', Strike, Settle, Maturity, Reset, Basis,
Principal)
InstSet = instadd('Lookback', OptSpec, Strike, Settle,
ExerciseDates, AmericanOpt)
InstSet = instadd('RangeFloat', Spread, Settle, Maturity,
RateSched, Reset, Basis, Principal, EndMonthRule)
InstSet = instadd('OptStock', OptSpec, Strike, Settle, Maturity,
AmericanOpt)
InstSet = instadd('Swap', LegRate, Settle, Maturity, LegReset,
```

`Basis, Principal, LegType, EndMonthRule, StartDate)`
`InstSet = instadd('Swaption', OptSpec, Strike, ExerciseDates,`
`Spread, Settle, Maturity, AmericanOpt, SwapReset,`
`Basis, Principal)`

## Description

`InstSet = instadd(InstSetOld, TypeString, Data1, Data2, ...)` adds an instruments to an existing collection.

`InstSet = instadd('CashFlow', CFlowAmounts, CFlowDates, Settle, Basis)` adds an arbitrary cash flow instrument. (See also `instcf`.)

`InstSet = instadd('CashFlow', CFlowAmounts, CFlowDates, Settle, Basis)` adds an asian instrument. (See also `instasian`.)

`InstSet = instadd('Barrier', OptSpec, Strike, Settle, ExerciseDates, AmericanOpt, BarrierType, Barrier, Rebate)` adds a barrier instrument. (See also `instbarrier`.)

`InstSet = instadd('Bond', CouponRate, Settle, Maturity, Period, Basis, EndMonthRule, IssueDate, FirstCouponDate, LastCouponDate, StartDate, Face)` adds a bond instrument. (See also `instbond`.)

`InstSet = instadd('OptEmBond', CouponRate, Settle, Maturity, OptSpec, Strike, ExerciseDates, 'AmericanOpt', AmericanOpt, 'Period', Period, 'Basis', Basis, 'EndMonthRule', EndMonthRule,'Face',Face, 'IssueDate', IssueDate, 'FirstCouponDate', FirstCouponDate, 'LastCouponDate', LastCouponDate, 'StartDate', StartDate)` adds a bond with embedded option instrument. (See also `instoptembnd`.)

`InstSet = instadd('OptBond', BondIndex, OptSpec, Strike, ExerciseDates, AmericanOpt)` adds a bond option instrument. (See also `instoptbnd`.)

`InstSet = instadd('Cap', Strike, Settle, Maturity, Reset, Basis,`
`Principal)` adds a cap instrument. (See also `instcap`.)

`InstSet = instadd('Compound', UOptSpec, UStrike, USettle, UExerciseDates, UAmericanOpt,COptSpec, CStrike, CSettle, CExerciseDates, CAmericanOpt)` adds a compound instrument. (See also `instcompound`.)

`InstSet = instadd('Fixed', CouponRate, Settle, Maturity, Reset,`
`Basis, Principal, EndMonthRule)` adds a fixed-rate note instrument. (See also `instfixed`.)

`InstSet = instadd('Float', Spread, Settle, Maturity, Reset, Basis,`
`Principal, EndMonthRule)` adds a floating-rate note instrument. (See also `instfloat`.)

`InstSet = instadd('Floor', Strike, Settle, Maturity, Reset, Basis,`
`Principal)` adds a floor instrument. (See also `instfloor`.)

`InstSet = instadd('Lookback', OptSpec, Strike, Settle, ExerciseDates, AmericanOpt)` adds a lookback instrument. (See also `instlookback`.)

`InstSet = instadd('RangeFloat', Spread, Settle, Maturity, RateSched, Reset, Basis, Principal, EndMonthRule)` adds a range floating note instrument. (See also `instrangefloat`.)

`InstSet = instadd('OptStock', OptSpec, Strike, Settle, Maturity,`
`AmericanOpt)` adds a stock option instrument. (See also `instoptstock`.)

`InstSet = instadd('Swap', LegRate, Settle, Maturity, LegReset,`
`Basis, Principal, LegType, EndMonthRule, StartDate)` adds a swap instrument. (See also `instswap`.)

`InstSet = instadd('Swaption', OptSpec, Strike, ExerciseDates, Spread, Settle, Maturity, AmericanOpt, SwapReset, Basis, Principal)` adds a swaption instrument. (See also `instswaption`.)

`instadd` stores instruments of types `'Asian'`, `'Barrier'`, `'Bond'`, `'Cap'`, `'CashFlow'`, `'Compound'`, `'Fixed'`, `'Float'`, `'Floor'`, `'Lookback'`, `'OptBond'`, `'OptStock'`, `'Swap'`, or `'Swaption'`. Financial Instruments Toolbox provides pricing and sensitivity routines for these instruments.

## Input Arguments

**InstSetOld**

Variable containing a collection of instruments. Instruments are classified by type; each type can have different data fields. The stored data field is a row vector or string for each instrument. For more information on instrument data parameters, see the reference entries for individual instrument types. For example, see `instcap` for additional information on the cap instrument.

## Output Arguments

**InstSet**

`InstSet` is an instrument set variable containing the new input data.

## Examples

### Create a Portfolio with Two Cap Instruments and a 4% Bond

Define the bond:

```
Strike = [0.06; 0.07];
CouponRate = 0.04;
Settle = '06-Feb-2000';
Maturity = '15-Jan-2003';
```

Create a portfolio with two cap instruments and a 4% bond and then display the portfolio:

```
InstSet = instadd('Cap', Strike, Settle, Maturity);
InstSet = instadd(InstSet, 'Bond', CouponRate, Settle, Maturity);
instdisp(InstSet)
```

```
Index Type Strike Settle         Maturity       CapReset Basis Principal
1     Cap  0.06   06-Feb-2000    15-Jan-2003    1        0     100
2     Cap  0.07   06-Feb-2000    15-Jan-2003    1        0     100

Index Type CouponRate Settle  Maturity  Period Basis EndMonthRule IssueDate FirstCouponDate LastCouponDat
3     Bond 0.04   06-Feb-2000 15-Jan-2003 2      0     1           NaN       NaN             NaN
```

## See Also

`instasian` | `instbarrier` | `instbond` | `instcap` | `instcf` | `instcompound` | `instfixed` | `instfloat` | `instfloor` | `instlookback` | `instoptbnd` | `instoptembnd` | `instoptstock` | `instswap` | `instswaption` | `instaddfield` | `instdisp`

## Related Examples

- “Portfolio Creation” on page 1-7
- “Creating Instruments or Properties” on page 1-18

## Concepts

- “Instrument Constructors” on page 1-17

## Purpose

Add new instruments to instrument collection

## Syntax

```
InstSet = instaddfield('FieldName', FieldList, 'Data',
DataList, 'Type', TypeString)
InstSet = instaddfield('FieldName', FieldList, 'FieldClass',
ClassList, 'Data', DataList, 'Type', TypeString)
InstSetNew = instaddfield(InstSet, 'FieldName', FieldList,
'Data', DataList,'Type',TypeString)
```

## Arguments

| | |
|---|---|
| `FieldList` | String or number of fields (`NFIELDS`)-by-`1` cell array of strings listing the name of each data field. `FieldList` cannot be named with the reserved name `Type` or `Index`. |
| `DataList` | Number of instruments (`NINST`)-by-`M` array or `NFIELDS`-by-`1` cell array of data contents for each field. Each row in a data array corresponds to a separate instrument. Single rows are copied to apply to all instruments to be worked on. The number of columns is arbitrary, and data is padded along columns. |
| `ClassList` | (Optional) String or `NFIELDS`-by-`1` cell array of strings listing the data class of each field. The class determines how `DataList` is parsed. Valid strings are `'dble'`, `'date'`, and `'char'`. The `'FieldClass', ClassList` pair is always optional. `ClassList` is inferred from existing field names or from the data if not entered. |

| | |
|---|---|
| `TypeString` | String specifying the type of instrument added. Instruments of different types can have different `Fieldname` collections. |
| `InstSet` | Variable containing a collection of instruments. Instruments are classified by type; each type can have different `Data` fields. The stored `Data` field is a row vector or string for each instrument. |

## Description

Use `instaddfield` to create your own types of instruments or to append new instruments to an existing collection. Argument value pairs can be entered in any order.

`InstSet = instaddfield('FieldName', FieldList, 'Data', DataList, 'Type', TypeString)`

`InstSet = instaddfield('FieldName', FieldList, 'FieldClass', ClassList, 'Data', DataList, 'Type', TypeString)` creates an instrument variable.

`InstSetNew = instaddfield(InstSet, 'FieldName', FieldList, 'Data', DataList,'Type',TypeString)` adds instruments to an existing instrument set, `InstSet`. The output `InstSetNew` is a new instrument set containing the input data.

## Examples

Build a portfolio around July options.

```
Strike  Call    Put
 95     12.2    2.9
100      9.2    4.9
105      6.8    7.4

Strike = (95:5:105)'
CallP = [12.2; 9.2; 6.8]
```

Enter three call options with data fields `Strike`, `Price`, and `Opt`.

```
InstSet = instaddfield('Type','Option','FieldName',...
{'Strike','Price','Opt'}, 'Data',{ Strike, CallP, 'Call'});
 instdisp(InstSet)

Index Type   Strike Price Opt
1     Option  95    12.2  Call
2     Option 100     9.2  Call
3     Option 105     6.8  Call
```

Add a futures contract and set the input parsing class.

```
InstSet = instaddfield(InstSet,'Type','Futures',...
'FieldName',{'Delivery','F'},'FieldClass',{'date','dble'},...
'Data' ,{'01-Jul-99',104.4 });
instdisp(InstSet)
Index Type   Strike Price Opt
1     Option  95    12.2  Call
2     Option 100     9.2  Call
3     Option 105     6.8  Call

Index Type    Delivery       F
4     Futures 01-Jul-1999    104.4
```

Add a put option.

```
FN = instfields(InstSet,'Type','Option')
InstSet = instaddfield(InstSet,'Type','Option',...
'FieldName',FN, 'Data',{105, 7.4, 'Put'});
instdisp(InstSet)

Index Type   Strike Price Opt
1     Option  95    12.2  Call
2     Option 100     9.2  Call
3     Option 105     6.8  Call

Index Type    Delivery       F
4     Futures 01-Jul-1999    104.4
```

```
Index Type   Strike Price Opt
5     Option 105     7.4  Put
```

Make a placeholder for another put.

```
InstSet = instaddfield(InstSet,'Type','Option',...
'FieldName','Opt','Data','Put')
instdisp(InstSet)

Index Type   Strike Price Opt
1     Option  95    12.2  Call
2     Option 100     9.2  Call
3     Option 105     6.8  Call

Index Type    Delivery       F
4     Futures 01-Jul-1999    104.4

Index Type   Strike Price Opt
5     Option 105     7.4  Put
6     Option NaN     NaN  Put
```

Add a cash instrument.

```
InstSet = instaddfield(InstSet, 'Type', 'TBill',...
'FieldName','Price','Data',99)
instdisp(InstSet)

Index Type   Strike Price Opt
1     Option  95    12.2  Call
2     Option 100     9.2  Call
3     Option 105     6.8  Call

Index Type    Delivery       F
4     Futures 01-Jul-1999    104.4

Index Type   Strike Price Opt
```

```
5     Option 105     7.4  Put
6     Option NaN     NaN  Put

Index Type  Price
7     TBill 99
```

## See Also

`instdisp` | `instget` | `instgetcell` | `instsetfield` | `instadd`

**Purpose** Construct Asian option

**Syntax**

```
InstSet = instasian(InstSet, OptSpec, Strike, Settle,
 ExerciseDates, AmericanOpt, AvgType, AvgPrice, AvgDate)
[FieldList, ClassList, TypeString] = instasian
```

**Arguments**

| | |
|---|---|
| `InstSet` | Instrument variable. This argument is specified only when adding asian instruments to an existing instrument set. See `instget` for more information on the `InstSet` variable. |
| `OptSpec` | `NINST`-by-`1` list of string values `'Call'` or `'Put'`. |
| `Strike` | `NINST`-by-`1` vector of strike price values. Each row is the schedule for one option. |
| `Settle` | `NINST`-by-`1` vector of `Settle` dates. |
| `ExerciseDates` | For a European option (`AmericanOpt = 0`):<br>`NINST`-by-`1` vector of exercise dates. Each row is the schedule for one option. For a European option, there is only one exercise date, the option expiry date.<br>For an American option (`AmericanOpt = 1`):<br>`NINST`-by-`2` vector of exercise date boundaries. For each instrument, the option can be exercised on any tree date between or including the pair of dates on that row. If only one non-`NaN` date is listed, or if `ExerciseDates` is `NINST`-by-`1`, the option can be exercised between the valuation date of the stock tree and the single listed exercise date. |

| | |
|---|---|
| `AmericanOpt` | (Optional) If `AmericanOpt = 0`, `NaN`, or is unspecified, the option is a European option. If `AmericanOpt = 1`, the option is an American option. |
| `AvgType` | (Optional) String = `'arithmetic'` for arithmetic average (default) or `'geometric'` for geometric average. |
| `AvgPrice` | (Optional) Scalar representing the average price of the underlying asset at `Settle`. This argument is used when `AvgDate < Settle`. Default is the current stock price. |
| `AvgDate` | (Optional) Scalar representing the date on which the averaging period begins. Default = `Settle`. |

Data arguments are `NINST`-by-`1` vectors, scalar, or empty. Fill unspecified entries in vectors with `NaN`. Only one data argument is required to create the instrument. The others may be omitted or passed as empty matrices `[]`.

## Description

`InstSet = instasian(InstSet, OptSpec, Strike, Settle, ExerciseDates, AmericanOpt, AvgType, AvgPrice, AvgDate)` specifies an Asian option.

`[FieldList, ClassList, TypeString] = instasian` displays the classes.

`FieldList` is a number of fields (`NFIELDS`)-by-`1` cell array of strings listing the name of each data field for this instrument type.

`ClassList` is an `NFIELDS`-by-`1` cell array of strings listing the data class of each field. The class determines how arguments are parsed. Valid strings are `'dble'`, `'date'`, and `'char'`.

`TypeString` is a string specifying the type of instrument added. For an Asian option instrument, `TypeString = 'Asian'`.

## Examples

### Create an Asian Option Instrument

Load the example instrument set, `deriv.mat`, and set the required values for an asian option instrument.

```
load deriv.mat
```

Create a subportfolio with barrier and lookback options.

```
CRRSubSet = instselect(CRRInstSet,'Type',{'Barrier', 'Lookback'});
```

Define the asian instrument.

```
OptSpec = 'put';
Strike = NaN;
Settle = '01-Jan-2003';
ExerciseDates = '01-Jan-2004';
```

Add a floating strike asian option to the instrument set.

```
InstSet = instasian(CRRSubSet, OptSpec, Strike, Settle, ExerciseDates);
instdisp(InstSet)
```

```
Index Type OptSpec Strike Settle  ExerciseDates  AmericanOpt BarrierSpec Barrier Rebate Name  Quantity
1     Barrier call    105    01-Jan-2003    01-Jan-2006    1           ui         102    0  Barrier1 1

Index Type   OptSpec Strike Settle         ExerciseDates  AmericanOpt Name      Quantity
2     Lookback call    115    01-Jan-2003    01-Jan-2006    0       Lookback1        7
3     Lookback call    115    01-Jan-2003    01-Jan-2007    0       Lookback2        9

Index Type OptSpec Strike Settle  ExerciseDates  AmericanOpt AvgType  AvgPrice AvgDate Name Quantity
4     Asian put     NaN    01-Jan-2003    01-Jan-2004    0    arithmetic NaN      NaN         NaN
```

## See Also

`instadd` | `instdisp` | `instget`

## Purpose

Construct barrier option

## Syntax

```
InstSet = instbarrier(InstSet, OptSpec, Strike, Settle,
ExerciseDates, AmericanOpt, BarrierSpec, Barrier, Rebate)
[FieldList, ClassList, TypeString] = instbarrier
```

## Arguments

| | |
|---|---|
| `InstSet` | Instrument variable. This argument is specified only when adding barrier instruments to an existing instrument set. See `instget` for more information on the `InstSet` variable. |
| `OptSpec` | `NINST`-by-`1` list of string values `'Call'` or `'Put'`. |
| `Strike` | `NINST`-by-`1` vector of strike price values. Each row is the schedule for one option. |
| `Settle` | `NINST`-by-`1` vector of `Settle` dates. |
| `ExerciseDates` | For a European option (`AmericanOpt = 0`):<br>`NINST`-by-`1` vector of exercise dates. Each row is the schedule for one option. For a European option, there is only one exercise date, the option expiry date.<br>For an American option (`AmericanOpt = 1`):<br>`NINST`-by-`2` vector of exercise date boundaries. For each instrument, the option can be exercised on any tree date between or including the pair of dates on that row. If only one non-`NaN` date is listed, or if `ExerciseDates` is `NINST`-by-`1`, the option can be exercised between the valuation date of the stock tree and the single listed exercise date. |

| | |
|---|---|
| `AmericanOpt` | If `AmericanOpt = 0`, `NaN`, or is unspecified, the option is a European option. If `AmericanOpt = 1`, the option is an American option. |
| `BarrierSpec` | List of string values:<br>`'UI'`: Up Knock In<br>`'UO'`: Up Knock Out<br>`'DI'`: Down Knock In<br>`'DO'`: Down Knock Out |
| `Barrier` | Vector of barrier values. |
| `Rebate` | (Optional) Vector of rebate values. |

Data arguments are `NINST`-by-`1` vectors, scalar, or empty. Fill unspecified entries in vectors with `NaN`. Only one data argument is required to create the instrument. The others may be omitted or passed as empty matrices `[]`.

## Description

`InstSet = instbarrier(InstSet, OptSpec, Strike, Settle, ExerciseDates, AmericanOpt, BarrierSpec, Barrier, Rebate)` specifies a barrier option.

`[FieldList, ClassList, TypeString] = instbarrier` displays the classes.

`FieldList` is a number of fields (`NFIELDS`)-by-`1` cell array of strings listing the name of each data field for this instrument type.

`ClassList` is an `NFIELDS`-by-`1` cell array of strings listing the data class of each field. The class determines how arguments are parsed. Valid strings are `'dble'`, `'date'`, and `'char'`.

`TypeString` is a string specifying the type of instrument added. For a barrier option instrument, `TypeString = 'Barrier'`.

## Examples

### Create Two Barrier Option Instruments

Create an instrument set of two barrier options with the following data:

```
OptSpec = {'put';'call'};
Strike = 112;
Settle = '01-Jan-2012';
ExerciseDates = '01-Jan-2015';
BarrierSpec = {'do';'ui'};
Barrier = [101;102];
AmericanOpt = 0;
```

Create the instrument set (`InstSet`) for the two barrier options.

```
InstSet = instbarrier(OptSpec, Strike, Settle, ExerciseDates,AmericanOpt, BarrierSpec, Barrier);
```

Display the instrument set.

```
instdisp(InstSet)
```

```
Index Type    OptSpec Strike Settle         ExerciseDates  AmericanOpt BarrierSpec Barrier Rebate
1     Barrier put     112    01-Jan-2012    01-Jan-2015    0           do          101     0
2     Barrier call    112    01-Jan-2012    01-Jan-2015    0           ui          102     0
```

## See Also

`instadd` | `instdisp` | `instget`

# instbond

## Purpose

Construct bond instrument

## Syntax

```
InstSet = instbond(InstSet, CouponRate, Settle, Maturity,
Period, Basis, EndMonthRule, IssueDate, FirstCouponDate,
LastCouponDate, StartDate, Face)
[FieldList, ClassList, TypeString] = instbond
```

## Arguments

| | |
|---|---|
| `InstSet` | Instrument variable. This argument is specified only when adding bond instruments to an existing instrument set. See `instget` for more information on the `InstSet` variable. |
| `CouponRate` | Decimal annual rate indicating the annual percentage rate used to determine the coupons payable on a bond. `CouponRate` is a `NINST`-by-`1` vector or `NINST`-by-`1` cell array of decimal annual rates, or decimal annual rate schedules. For the latter case of a variable coupon schedule, each element of the cell array is a `NumDates`-by-`2` cell array, where the first column is dates and the second column is its associated rate. The date indicates the last day that the coupon rate is valid. |
| `Settle` | Settlement date. A vector of serial date numbers or date strings. `Settle` must be earlier than `Maturity`. |
| `Maturity` | Maturity date. A vector of serial date numbers or date strings. |
| `Period` | (Optional) Coupons per year of the bond. A vector of integers. Allowed values are `1`, `2`, `3`, `4`, `6`, and `12`. Default = `2`. |

| | |
|---|---|
| `Basis` | (Optional) Day-count basis of the instrument. A vector of integers.<br>• 0 = actual/actual (default)<br>• 1 = 30/360 (SIA)<br>• 2 = actual/360<br>• 3 = actual/365<br>• 4 = 30/360 (BMA)<br>• 5 = 30/360 (ISDA)<br>• 6 = 30/360 (European)<br>• 7 = actual/365 (Japanese)<br>• 8 = actual/actual (ICMA)<br>• 9 = actual/360 (ICMA)<br>• 10 = actual/365 (ICMA)<br>• 11 = 30/360E (ICMA)<br>• 12 = actual/actual (ISDA)<br>• 13 = BUS/252<br>For more information, see basis. |
| `EndMonthRule` | (Optional) End-of-month rule. A vector. This rule applies only when `Maturity` is an end-of-month date for a month having 30 or fewer days. `0` = ignore rule, meaning that a bond's coupon payment date is always the same numerical day of the month. `1` = set rule on (default), meaning that a bond's coupon payment date is always the last actual day of the month. |
| `IssueDate` | (Optional) Date when a bond was issued. |

| | |
|---|---|
| `FirstCouponDate` | (Optional) Last coupon date of a bond before the maturity date; used when bond has an irregular last coupon period. In the absence of a specified `FirstCouponDate`, a specified `LastCouponDate` determines the coupon structure of the bond. The coupon structure of a bond is truncated at the `LastCouponDate`, regardless of where it falls, and is followed only by the bond's maturity cash flow date. If you do not specify a `LastCouponDate`, the cash flow payment dates are determined from other inputs. |
| `LastCouponDate` | (Optional) Last coupon date of a bond before the maturity date; used when bond has an irregular last coupon period. In the absence of a specified `FirstCouponDate`, a specified `LastCouponDate` determines the coupon structure of the bond. The coupon structure of a bond is truncated at the `LastCouponDate`, regardless of where it falls, and is followed only by the bond's maturity cash flow date. If you do not specify a `LastCouponDate`, the cash flow payment dates are determined from other inputs. |
| `StartDate` | (Optional) Date when a bond actually starts (i.e. the date when a bond's cash flows can be considered). To make an instrument forward starting, specify this date as a future date. If `StartDate` is not explicitly specified, the effective start date is the `Settle` date. |
| `Face` | (Optional) Face or par value. `Face` is a `NINST`-by-`1` vector or `NINST`-by-`1` cell array of face values, or face value schedules. For the latter case, each element of the cell array is a `NumDates`-by-`2` cell array, where the first |

column is dates and the second column is its associated face value. The date indicates the last day that the face value is valid. Default = `100`.

Data arguments are number of instruments (`NINST`)-by-`1` vectors, scalar, or empty. Fill unspecified entries in vectors with `NaN`. Only one data argument is required to create the instrument. The others may be omitted or passed as empty matrices `[]`.

## Description

`InstSet = instbond(InstSet, CouponRate, Settle, Maturity, Period, Basis, EndMonthRule, IssueDate, FirstCouponDate, LastCouponDate, StartDate, Face)` creates a new instrument set containing bond instruments or adds bond instruments to a existing instrument set.

`[FieldList, ClassList, TypeString] = instbond` displays the classes.

`FieldList` is a number of fields (`NFIELDS`)-by-`1` cell array of strings listing the name of each data field for this instrument type.

`ClassList` is an `NFIELDS`-by-`1` cell array of strings listing the data class of each field. The class determines how arguments are parsed. Valid strings are `'dble'`, `'date'`, and `'char'`.

`TypeString` is a string specifying the type of instrument added. For a bond instrument, `TypeString = 'Bond'`.

## See Also

`hjmprice` | `instaddfield` | `instdisp` | `instget` | `intenvprice`

## Purpose

Construct cap instrument

## Syntax

```
InstSet = instcap(InstSet, Strike, Settle, Maturity, Reset,
Basis, Principal)
[FieldList, ClassList, TypeString] = instcap
```

## Arguments

| | |
|---|---|
| `InstSet` | Instrument variable. This argument is specified only when adding cap instruments to an existing instrument set. See `instget` for more information on the `InstSet` variable. |
| `Strike` | Rate at which the cap is exercised, as a decimal number. |
| `Settle` | Settlement date. Serial date number representing the settlement date of the cap. |
| `Maturity` | Serial date number representing the maturity date of the cap. |
| `Reset` | (Optional) `NINST`-by-`1` vector representing the frequency of payments per year. Default = `1`. |
| `Basis` | (Optional) Day-count basis of the instrument. A vector of integers.<br>• 0 = actual/actual (default)<br>• 1 = 30/360 (SIA)<br>• 2 = actual/360<br>• 3 = actual/365<br>• 4 = 30/360 (BMA)<br>• 5 = 30/360 (ISDA)<br>• 6 = 30/360 (European)<br>• 7 = actual/365 (Japanese) |

| | |
|---|---|
| | - 8 = actual/actual (ICMA)<br>- 9 = actual/360 (ICMA)<br>- 10 = actual/365 (ICMA)<br>- 11 = 30/360E (ICMA)<br>- 12 = actual/actual (ISDA)<br>- 13 = BUS/252<br>For more information, see basis. |
| `Principal` | (Optional) The notional principal amount. Default = `100`. |

## Description

`InstSet = instcap(InstSet, Strike, Settle, Maturity, Reset, Basis, Principal)` creates a new instrument set containing cap instruments or adds cap instruments to an existing instrument set.

`[FieldList, ClassList, TypeString] = instcap` displays the classes.

`FieldList` is a number of fields (`NFIELDS`)-by-`1` cell array of strings listing the name of each data field for this instrument type.

`ClassList` is an `NFIELDS`-by-`1` cell array of strings listing the data class of each field. The class determines how arguments are parsed. Valid strings are `'dble'`, `'date'`, and `'char'`.

`TypeString` is a string specifying the type of instrument added. For a cap instrument, `TypeString = 'Cap'`.

## See Also

`hjmprice` | `instaddfield` | `instbond` | `instdisp` | `instfloor` | `instswap` | `intenvprice`

## Purpose

Construct cash flow instrument

## Syntax

```
InstSet = instcf(InstSet, CFlowAmounts, CFlowDates, Settle,
Basis)
[FieldList, ClassList, TypeString] = instcf
```

## Arguments

| | |
|---|---|
| `InstSet` | Instrument variable. This argument is specified only when adding cash flow instruments to an existing instrument set. See `instget` for more information on the `InstSet` variable. |
| `CFlowAmounts` | Number of instruments (`NINST`) by maximum number of cash flows (`MOSTCFS`) matrix of cash flow amounts. Each row is a list of cash flow values for one instrument. If an instrument has fewer than `MOSTCFS` cash flows, the end of the row is padded with `NaN`s. |
| `CFlowDates` | `NINST`-by-`MOSTCFS` matrix of cash flow dates. Each entry contains the date of the corresponding cash flow in `CFlowAmounts`. |
| `Settle` | Settlement date on which the cash flows are priced. |
| `Basis` | (Optional) Day-count basis of the instrument. A vector of integers.<br>• 0 = actual/actual (default)<br>• 1 = 30/360 (SIA)<br>• 2 = actual/360<br>• 3 = actual/365<br>• 4 = 30/360 (BMA)<br>• 5 = 30/360 (ISDA) |

- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/actual (ISDA)
- 13 = BUS/252

For more information, see basis.

Only one data argument is required to create an instrument. Other arguments can be omitted or passed as empty matrices `[]`. Dates can be input as serial date numbers or date strings.

## Description

`InstSet = instcf(InstSet, CFlowAmounts, CFlowDates, Settle, Basis)` creates a new instrument set from data arrays or adds instruments of type `CashFlow` to an instrument set.

`[FieldList, ClassList, TypeString] = instcf` lists field metadata for an instrument of type `CashFlow`.

`FieldList` is a number of fields (`NFIELDS`)-by-`1` cell array of strings listing the name of each data field for this instrument type.

`ClassList` is an `NFIELDS`-by-`1` cell array of strings listing the data class of each field. The class determines how arguments are parsed. Valid strings are `'dble'`, `'date'`, and `'char'`.

`TypeString` specifies the type of instrument added; for example, `TypeString = 'CashFlow'`.

## See Also

`instadd` | `instdisp` | `instget` | `intenvprice`

## Purpose

Construct compound option

## Syntax

```
InstSet = instcompound(InstSet, UOptSpec, UStrike, USettle,
UExerciseDates, UAmericanOpt, COptSpec, CStrike, CSettle,
CExerciseDates, CAmericanOpt)
[FieldList, ClassList, TypeString] = instcompound
```

## Arguments

| | |
|---|---|
| `InstSet` | Instrument variable. This argument is specified only when adding compound instruments to an existing instrument set. See `instget` for more information on the `InstSet` variable. |
| `UOptSpec` | String = `'Call'` or `'Put'`. |
| `UStrike` | `1`-by-`1` vector of strike price values. |
| `USettle` | `1`-by-`1` vector of `Settle` dates. |
| `UExerciseDates` | For a European option (`UAmericanOpt = 0`):<br>`1`-by-`1` vector of exercise dates. For a European option, there is only one exercise date, the option expiry date.<br>For an American option (`UAmericanOpt = 1`):<br>`1`-by-`2` vector of exercise date boundaries. The option can be exercised on any tree date. If only one non-`NaN` date is listed, or if `ExerciseDates` is `1`-by-`1`, the option can be exercised between the valuation date of the stock tree and the single listed exercise date. |
| `UAmericanOpt` | If `UAmericanOpt = 0`, `NaN`, or is unspecified, the option is a European option. If `UAmericanOpt = 1`, the option is an American option. |
| `COptSpec` | `NINST`-by-`1` list of string values `'Call'` or `'Put'` of the compound option. |

| | |
|---|---|
| `CStrike` | `NINST`-by-`1` vector of strike price values. Each row is the schedule for one option. |
| `CSettle` | `1`-by-`1` vector containing the settlement or trade date. |
| `CExerciseDates` | For a European option (`CAmericanOpt = 0`):<br>`NINST`-by-`1` vector of exercise dates. Each row is the schedule for one option. For a European option, there is only one exercise date, the option expiry date.<br>For an American option (`CAmericanOpt = 1`):<br>`NINST`-by-`2` vector of exercise date boundaries. For each instrument, the option can be exercised on any tree date between or including the pair of dates on that row. If only one non-`NaN` date is listed, or if `ExerciseDates` is `NINST`-by-`1`, the option can be exercised between the valuation date of the stock tree and the single listed exercise date. |
| `CAmericanOpt` | If `CAmericanOpt = 0`, `NaN`, or is unspecified, the option is a European option. If `CAmericanOpt = 1`, the option is an American option. |

## Description

`InstSet = instcompound(InstSet, UOptSpec, UStrike, USettle, UExerciseDates, UAmericanOpt, COptSpec, CStrike, CSettle, CExerciseDates, CAmericanOpt)` specifies a compound option.

`[FieldList, ClassList, TypeString] = instcompound` displays the classes.

`FieldList` is a number of fields (`NFIELDS`)-by-`1` cell array of strings listing the name of each data field for this instrument type.

`ClassList` is an `NFIELDS`-by-1 cell array of strings listing the data class of each field. The class determines how arguments are parsed. Valid strings are `'dble'`, `'date'`, and `'char'`.

`TypeString` is a string specifying the type of instrument added. For a compound option instrument, `TypeString = 'Compound'`.

## Examples

### Create a Compound Option Instrument

Define a compound option instrument with the following data:

```
UOptSpec = 'Call';
UStrike = 130;
USettle = '01-Jan-2012';
UExerciseDates = '01-Jan-2015';
UAmericanOpt = 0;
COptSpec = 'Put';
CStrike = 5;
CSettle = '01-Jan-2012';
CExerciseDates = '01-Jan-2014';
CAmericanOpt = 0;

InstSet = instcompound(UOptSpec, UStrike, USettle,UExerciseDates, ...
UAmericanOpt, COptSpec, CStrike, CSettle,CExerciseDates, CAmericanOpt)

InstSet = instcompound(UOptSpec, UStrike, USettle,UExerciseDates, ...
UAmericanOpt, COptSpec, CStrike, CSettle,CExerciseDates)
```

```
InstSet =

        FinObj: 'Instruments'
    IndexTable: [1x1 struct]
          Type: {'Compound'}
     FieldName: {{10x1 cell}}
    FieldClass: {{10x1 cell}}
     FieldData: {{10x1 cell}}
```

```
InstSet =

        FinObj: 'Instruments'
    IndexTable: [1x1 struct]
          Type: {'Compound'}
     FieldName: {{10x1 cell}}
    FieldClass: {{10x1 cell}}
     FieldData: {{10x1 cell}}
```

Display the instrument set.

```
instdisp(InstSet)
```

```
Index Type     UOptSpec UStrike USettle   UExerciseDates UAmericanOpt COptSpec CStrike CSettle    CE
1     Compound Call     130    01-Jan-2012    01-Jan-2015    0         Put      5       01-Jan-2012
```

## See Also

`instadd` | `instdisp` | `instget`

## Purpose

Complement of instrument set by matching conditions

## Syntax

```
ISubSet = instdelete(InstSet, 'FieldName', FieldList, 'Data',
DataList, 'Index', IndexSet, 'Type', TypeList)
```

## Arguments

| | |
|---|---|
| `InstSet` | Variable containing a collection of instruments. Instruments are classified by type; each type can have different data fields. The stored data field is a row vector or string for each instrument. |
| `FieldList` | String or number of fields (`NFIELDS`)-by-`1` cell array of strings listing the name of each data field to match with data values. |
| `DataList` | Number of values (`NVALUES`)-by-`M` array or `NFIELDS`-by-`1` cell array of acceptable data values for each field. Each row lists a data row value to search for in the corresponding `FieldList`. The number of columns is arbitrary and matching will ignore trailing `NaN`s or spaces. |
| `IndexSet` | (Optional) Number of instruments (`NINST`)-by-`1` vector restricting positions of instruments to check for matches. The default is all indices available in the instrument variable. |
| `TypeList` | (Optional) String or number of types (`NTYPES`)-by-`1` cell array of strings restricting instruments to match one of `TypeList` types. The default is all types in the instrument variable. |

---

**Note** Argument value pairs can be entered in any order. The `InstSet` variable must be the first argument. `'FieldName'` and `'Data'` arguments must appear together or not at all.

---

## Description

The output argument `ISubSet` contains instruments *not* matching the input criteria. Instruments are deleted from `ISubSet` if all the `Field`, `Index`, and `Type` conditions are met. An instrument meets an individual `Field` condition if the stored `FieldName` data matches any of the rows listed in the `DataList` for that `FieldName`. See `instfind` for more examples on matching criteria.

## Examples

Retrieve the instrument set variable `ExampleInst` from the data file `InstSetExamples.mat`. The variable contains three types of instruments: `Option`, `Futures`, and `TBill`.

```
load InstSetExamples;
instdisp(ExampleInst)

Index Type   Strike Price Opt  Contracts
1     Option  95    12.2  Call     0
2     Option 100     9.2  Call     0
3     Option 105     6.8  Call  1000

Index Type    Delivery       F     Contracts
4     Futures 01-Jul-1999    104.4 -1000

Index Type   Strike Price Opt  Contracts
5     Option 105     7.4  Put  -1000
6     Option  95     2.9  Put      0

Index Type  Price  Maturity       Contracts
7     TBill 99     01-Jul-1999    6
```

Create a new variable, `ISet`, with all options deleted.

```
ISet = instdelete(ExampleInst, 'Type','Option');
instdisp(ISet)

Index Type    Delivery       F     Contracts
1     Futures 01-Jul-1999    104.4 -1000

Index Type  Price  Maturity       Contracts
2     TBill 99     01-Jul-1999    6
```

## See Also

`instaddfield` | `instfind` | `instget` | `instselect`

## Purpose

Display instruments

## Syntax

```
CharTable = instdisp(InstSet)
```

## Arguments

| | |
|---|---|
| `InstSet` | Variable containing a collection of instruments. See `instaddfield` for examples on constructing the variable. |

## Description

`CharTable = instdisp(InstSet)` creates a character array displaying the contents of an instrument collection, `InstSet`. If instdisp is called without output arguments, the table is displayed in the Command Window.

---

**Note** When using `instdisp`, a value of `NaN` in one of the columns for an instrument indicates that the default value for that parameter will be used in the instrument's pricing function.

---

`CharTable` is a character array with a table of instruments in `InstSet`. For each instrument row, the `Index` and `Type` are printed along with the field contents. Field headers are printed at the tops of the columns.

## Examples

Retrieve the instrument set `ExampleInst` from the data file `InstSetExamples.mat`. `ExampleInst` contains three types of instruments: `Option`, `Futures`, and `TBill`.

```
load InstSetExamples;
instdisp(ExampleInst)

Index Type   Strike Price Opt  Contracts
1     Option  95    12.2  Call     0
2     Option 100     9.2  Call     0
```

```
3     Option 105     6.8  Call  1000

Index Type    Delivery       F     Contracts
4     Futures 01-Jul-1999    104.4 -1000

Index Type   Strike Price Opt  Contracts
5     Option 105      7.4 Put  -1000
6     Option  95      2.9 Put      0

Index Type Price Maturity       Contracts
7     TBill 99    01-Jul-1999   6
```

Create a swap instrument and use `instdisp` to display the instrument. Notice that value of `NaN` in two columns for this instrument indicates that the default values for `LegReset` and `LegType` parameters will be used in the swap instrument's pricing function.

```
LegRate1 = [0.065, 0];
Settle1 = datenum('jan-1-2007');
Maturity1 = datenum('jan-1-2012');

ISet = instswap(LegRate1, Settle1, Maturity1);
instdisp(ISet)

Index Type LegRate    Settle        Maturity     LegReset Basis Principal LegType EndMonthRule
1     Swap [0.065  0] 01-Jan-2007   01-Jan-2012  [NaN]    0     100       [NaN]   1
```

## See Also

`datestr` | `num2str` | `instaddfield` | `instget`

## Purpose

List field names

## Syntax

```
FieldList = instfields(InstSet, 'Type', TypeList)
```

## Arguments

| | |
|---|---|
| `InstSet` | Variable containing a collection of instruments. Instruments are classified by type; each type can have different data fields. The stored data field is a row vector or string for each instrument. |
| `TypeList` | (Optional) String or number of types (`NTYPES`)-by-1 cell array of strings listing the instrument types to query. |

## Description

`FieldList = instfields(InstSet, 'Type', TypeList)` retrieves the list of fields stored in an instrument variable.

`FieldList` is a number of fields (`NFIELDS`)-by-1 cell array of strings listing the name of each data field corresponding to the listed types.

## Examples

Retrieve the instrument set `ExampleInst` from the data file `InstSetExamples.mat`. `ExampleInst` contains three types of instruments: `Option`, `Futures`, and `TBill`.

```
load InstSetExamples;
instdisp(ExampleInst)

Index Type   Strike Price Opt  Contracts
1     Option  95    12.2  Call     0
2     Option 100     9.2  Call     0
3     Option 105     6.8  Call  1000

Index Type    Delivery       F      Contracts
4     Futures 01-Jul-1999    104.4  -1000
```

```
Index Type   Strike Price Opt  Contracts
5     Option 105      7.4 Put  -1000
6     Option  95      2.9 Put      0

Index Type Price Maturity       Contracts
7     TBill 99    01-Jul-1999   6
```

Get the fields listed for type `'Option'`.

```
[FieldList, ClassList] = instfields(ExampleInst, 'Type',...
'Option')

FieldList =

    'Strike'
    'Price'
    'Opt'
    'Contracts'

ClassList =

    'dble'
    'dble'
    'char'
    'dble'
```

Get the fields listed for types `'Option'` and `'TBill'`.

```
FieldList = instfields(ExampleInst, 'Type', {'Option', 'TBill'})

FieldList =

    'Strike'
    'Opt'
    'Price'
    'Maturity'
    'Contracts'
```

Get all the fields listed in any type in the variable.

```
FieldList = instfields(ExampleInst)
FieldList =

    'Delivery'
    'F'
    'Strike'
    'Opt'
    'Price'
    'Maturity'
    'Contracts'
```

## See Also

`instdisp` | `instlength` | `insttypes`

## Purpose

Search instruments for matching conditions

## Syntax

```
IndexMatch = instfind(InstSet, 'FieldName', FieldList, 'Data',
DataList,'Index', IndexSet, 'Type', TypeList)
```

## Arguments

| | |
|---|---|
| `InstSet` | Variable containing a collection of instruments. Instruments are classified by type; each type can have different data fields. The stored data field is a row vector or string for each instrument. |
| `FieldList` | String or number of fields (`NFIELDS`)-by-`1` cell array of strings listing the name of each data field to match with data values. |
| `DataList` | Number of values (`NVALUES`)-by-`M` array or `NFIELDS`-by-`1` cell array of acceptable data values for each field. Each row lists a data row value to search for in the corresponding `FieldList`. The number of columns is arbitrary, and matching will ignore trailing `NaN`s or spaces. |
| `IndexSet` | (Optional) Number of instruments (`NINST`)-by-`1` vector restricting positions of instruments to check for matches. The default is all indices available in the instrument variable. |
| `TypeList` | (Optional) String or number of types (`NTYPES`)-by-`1` cell array of strings restricting instruments to match one of `TypeList` types. The default is all types in the instrument variable. |

Argument value pairs can be entered in any order. The `InstSet` variable must be the first argument. `'FieldName'` and `'Data'` arguments must appear together or not at all.

## Description

`IndexMatch = instfind(InstSet, 'FieldName', FieldList, 'Data', DataList,'Index', IndexSet, 'Type', TypeList)` returns indices of instruments matching `Type`, `Field`, or `Index` values.

`IndexMatch` is an `NINST`-by-1 vector of positions of instruments matching the input criteria. Instruments are returned in `IndexMatch` if all the `Field`, `Index`, and `Type` conditions are met. An instrument meets an individual `Field` condition if the stored `FieldName` data matches any of the rows listed in the `DataList` for that `FieldName`.

## Examples

Retrieve the instrument set `ExampleInst` from the data file `InstSetExamples.mat`. `ExampleInst` contains three types of instruments: `Option`, `Futures`, and `TBill`.

```
load InstSetExamples;
instdisp(ExampleInst)

Index Type   Strike Price Opt  Contracts
1     Option  95    12.2  Call     0
2     Option 100     9.2  Call     0
3     Option 105     6.8  Call  1000

Index Type    Delivery       F     Contracts
4     Futures 01-Jul-1999    104.4 -1000

Index Type   Strike   Price Opt  Contracts
5     Option 105      7.4   Put  -1000
6     Option  95      2.9   Put      0

Index Type  Price Maturity       Contracts
7     TBill 99    01-Jul-1999    6
```

Make a vector, `Opt95`, containing the indexes within `ExampleInst` of the options struck at 95.

```
Opt95 = instfind(ExampleInst, 'FieldName','Strike','Data','95')
```

```
Opt95 =

     1
     6
```

Locate the futures and Treasury bill instruments within `ExampleInst`.

```
Types = instfind(ExampleInst,'Type',{'Futures';'TBill'})

Types =

     4
     7
```

## See Also

`instaddfield` | `instget` | `instgetcell` | `instselect`

## Purpose

Construct fixed-rate instrument

## Syntax

```
InstSet = instfixed(InstSet, CouponRate, Settle, Maturity,
Reset, Basis, Principal, EndMonthRule)
[FieldList, ClassList, TypeString] = instfixed
```

## Arguments

| | |
|---|---|
| `InstSet` | Instrument variable. This argument is specified only when adding fixed-rate note instruments to an existing instrument set. See `instget` for more information on the `InstSet` variable. |
| `CouponRate` | Decimal annual rate. |
| `Settle` | Settlement date. Date string or serial date number representing the settlement date of the fixed-rate note. |
| `Maturity` | Date string or serial date number representing the maturity date of the fixed-rate note. |
| `Reset` | (Optional) `NINST`-by-`1` vector representing the frequency of payments per year. Default = `1`. |
| `Basis` | (Optional) Day-count basis of the instrument. A vector of integers.<br>• 0 = actual/actual (default)<br>• 1 = 30/360 (SIA)<br>• 2 = actual/360<br>• 3 = actual/365<br>• 4 = 30/360 (BMA)<br>• 5 = 30/360 (ISDA)<br>• 6 = 30/360 (European)<br>• 7 = actual/365 (Japanese) |

| | |
|---|---|
| | • 8 = actual/actual (ICMA)<br>• 9 = actual/360 (ICMA)<br>• 10 = actual/365 (ICMA)<br>• 11 = 30/360E (ICMA)<br>• 12 = actual/actual (ISDA)<br>• 13 = BUS/252<br>For more information, see basis. |
| `Principal` | (Optional) The notional principal amount. Default = `100`. |
| `EndMonthRule` | (Optional) `NINST`-by-`1` vector representing the End-of-month rule. Default = `1`. |

Data arguments are number of instruments (`NINST`)-by-`1` vectors, scalar, or empty. Fill unspecified entries in vectors with `NaN`. Only one data argument is required to create the instrument. The others may be omitted or passed as empty matrices `[]`.

## Description

`InstSet = instfixed(InstSet, CouponRate, Settle, Maturity, Reset, Basis, Principal, EndMonthRule)` creates a new instrument set containing fixed-rate instruments or adds fixed-rate instruments to an existing instrument set.

`[FieldList, ClassList, TypeString] = instfixed` displays the classes.

`FieldList` is a number of fields (`NFIELDS`)-by-`1` cell array of strings listing the name of each data field for this instrument type.

`ClassList` is an `NFIELDS`-by-`1` cell array of strings listing the data class of each field. The class determines how arguments are parsed. Valid strings are `'dble'`, `'date'`, and `'char'`.

`TypeString` is a string specifying the type of instrument added. For a fixed-rate instrument, `TypeString = 'Fixed'`.

## See Also

`hjmprice` | `instaddfield` | `instbond` | `instcap` | `instdisp` | `instswap` | `intenvprice`

# instfloat

## Purpose

Construct floating-rate instrument

## Syntax

```
InstSet = instfloat(InstSet, Spread, Settle, Maturity, Reset,
Basis, Principal, EndMonthRule)
InstSet = instfloat(InstSet, Spread, Settle, Maturity, Reset,
Basis, Principal, EndMonthRule, CapRate, FloorRate)
[FieldList, ClassList, TypeString] = instfloat
```

## Arguments

| | |
|---|---|
| `InstSet` | Instrument variable. This argument is specified only when adding floating-rate note instruments to an existing instrument set. See `instget` for more information on the `InstSet` variable. |
| `Spread` | Number of basis points over the reference rate. |
| `Settle` | Settlement date. Date string or serial date number representing the settlement date of the floating-rate note. |
| `Maturity` | Date string or serial date number representing the maturity date of the floating-rate note. |
| `Reset` | (Optional) `NINST`-by-`1` vector representing the frequency of payments per year. Default = `1`. |
| `Basis` | (Optional) Day-count basis of the instrument. A vector of integers.<br>• 0 = actual/actual (default)<br>• 1 = 30/360 (SIA)<br>• 2 = actual/360<br>• 3 = actual/365<br>• 4 = 30/360 (BMA)<br>• 5 = 30/360 (ISDA)<br>• 6 = 30/360 (European) |

| | |
|---|---|
| | • 7 = actual/365 (Japanese)<br>• 8 = actual/actual (ICMA)<br>• 9 = actual/360 (ICMA)<br>• 10 = actual/365 (ICMA)<br>• 11 = 30/360E (ICMA)<br>• 12 = actual/actual (ISDA)<br>• 13 = BUS/252<br>For more information, see basis. |
| `Principal` | (Optional) The notional principal amount. Default = `100`. |
| `EndMonthRule` | (Optional) `NINST`-by-`1` vector representing the End-of-month rule. Default = `1`. |
| `CapRate` | (Optional) `NINST`-by-`1` decimal annual rate or `NINST`-by-`1` cell array, where each element is a `NumDates`-by-`2` cell array, and the cell array first column is dates, and the second column is associated cap rates. The date indicates the last day that the cap rate is valid. |
| `FloorRate` | (Optional) `NINST`-by-`1` decimal annual rate or `NINST`-by-`1` cell array, where each element is a `NumDates`-by-`2` cell array, and the cell array first column is dates, and the second column is associated floor rates. The date indicates the last day that the floor rate is valid. |

Data arguments are number of instruments (`NINST`)-by-`1` vectors, scalar, or empty. Fill unspecified entries in vectors with `NaN`. Only one data argument is required to create the instrument. The others may be omitted or passed as empty matrices `[]`.

## Description

`InstSet = instfloat(InstSet, Spread, Settle, Maturity,`
`Reset,`
`Basis, Principal, EndMonthRule)` creates a new instrument set containing floating-rate instruments or adds floating-rate instruments to an existing instrument set.

`InstSet = instfloat(InstSet, Spread, Settle, Maturity,`
`Reset,`
`Basis, Principal, EndMonthRule, CapRate, FloorRate)` creates a new instrument set containing capped floating-rate instruments or adds capped floating-rate instruments to an existing instrument set.

`[FieldList, ClassList, TypeString] = instfloat` displays the classes.

`FieldList` is a number of fields (`NFIELDS`)-by-`1` cell array of strings listing the name of each data field for this instrument type.

`ClassList` is an `NFIELDS`-by-`1` cell array of strings listing the data class of each field. The class determines how arguments are parsed. Valid strings are `'dble'`, `'date'`, and `'char'`.

`TypeString` is a string specifying the type of instrument added. For a floating-rate instrument, `TypeString = 'Float'`.

## See Also

`hjmprice` | `instaddfield` | `instbond` | `instcap` | `instdisp` | `instswap` | `intenvprice`

## Purpose

Construct floor instrument

## Syntax

```
InstSet = instfloor(InstSet, Strike, Settle, Maturity, Reset,
Basis, Principal)
[FieldList, ClassList, TypeString] = instfloor
```

## Arguments

| | |
|---|---|
| `InstSet` | Instrument variable. This argument is specified only when adding floor instruments to an existing instrument set. See `instget` for more information on the `InstSet` variable. |
| `Strike` | Rate at which the floor is exercised, as a decimal number. |
| `Settle` | Settlement date. A vector of serial date numbers or date strings. `Settle` must be earlier than `Maturity`. |
| `Maturity` | Maturity date. A vector of serial date numbers or date strings. |
| `Reset` | (Optional) `NINST`-by-`1` vector representing the frequency of payments per year. Default = `1`. |
| `Basis` | (Optional) Day-count basis of the instrument. A vector of integers.<br>• 0 = actual/actual (default)<br>• 1 = 30/360 (SIA)<br>• 2 = actual/360<br>• 3 = actual/365<br>• 4 = 30/360 (BMA)<br>• 5 = 30/360 (ISDA)<br>• 6 = 30/360 (European)<br>• 7 = actual/365 (Japanese) |

| | |
|---|---|
| | - 8 = actual/actual (ICMA)<br>- 9 = actual/360 (ICMA)<br>- 10 = actual/365 (ICMA)<br>- 11 = 30/360E (ICMA)<br>- 12 = actual/actual (ISDA)<br>- 13 = BUS/252<br>For more information, see basis. |
| `Principal` | (Optional) The notional principal amount. Default = `100`. |

## Description

`InstSet = instfloor(InstSet, Strike, Settle, Maturity, Reset, Basis, Principal)` creates a new instrument set containing floor instruments or adds floor instruments to an existing instrument set.

`[FieldList, ClassList, TypeString] = instfloor` displays the classes.

`FieldList` is a number of fields (`NFIELDS`)-by-`1` cell array of strings listing the name of each data field for this instrument type.

`ClassList` is an `NFIELDS`-by-`1` cell array of strings listing the data class of each field. The class determines how arguments are parsed. Valid strings are `'dble'`, `'date'`, and `'char'`.

`TypeString` is a string specifying the type of instrument added. For a floor instrument, `TypeString = 'Floor'`.

## See Also

`hjmprice` | `instaddfield` | `instbond` | `instcap` | `instdisp` | `instswap` | `intenvprice`

## Purpose

Data from instrument variable

## Syntax

```
[Data_1, Data_2,...,Data_n] = instget(InstSet, 'FieldName',
FieldList, 'Index', IndexSet, 'Type', TypeList)
```

## Arguments

| | |
|---|---|
| `InstSet` | Variable containing a collection of instruments. Instruments are classified by type; each type can have different data fields. The stored data field is a row vector or string for each instrument. |
| `FieldList` | (Optional) String or number of fields (`NFIELDS`)-by-`1` cell array of strings listing the name of each data field to match with data values. `FieldList` entries can also be either `'Type'` or `'Index'`; these return type strings and index numbers respectively. The default is all fields available for the returned set of instruments. |
| `IndexSet` | (Optional) Number of instruments (`NINST`)-by-`1` vector of positions of instruments to work on. If `TypeList` is also entered, instruments referenced must be one of `TypeList` types and contained in `IndexSet`. The default is all indices available in the instrument variable. |
| `TypeList` | (Optional) String or number of types (`NTYPES`)-by-`1` cell array of strings restricting instruments to match one of `TypeList` types. The default is all types in the instrument variable. |

Argument value pairs can be entered in any order. The `InstSet` variable must be the first argument.

## Description

`[Data_1, Data_2,...,Data_n] = instget(InstSet, 'FieldName', FieldList, 'Index', IndexSet, 'Type', TypeList)` retrieves data arrays from an instrument variable.

`Data_1` is an `NINST`-by-`M` array of data contents for the first field in `FieldList`. Each row corresponds to a separate instrument in `IndexSet`. Unavailable data is returned as `NaN` or as spaces.

`Data_n` is an `NINST`-by-`M` array of data contents for the last field in `FieldList`.

## Examples

Retrieve the instrument set `ExampleInst` from the data file. `InstSetExamples.mat`. `ExampleInst` contains three types of instruments: `Option`, `Futures`, and `TBill`.

```
load InstSetExamples;
instdisp(ExampleInst)

Index Type   Strike Price Opt  Contracts
1     Option  95    12.2  Call     0
2     Option 100     9.2  Call     0
3     Option 105     6.8  Call  1000

Index Type    Delivery       F     Contracts
4     Futures 01-Jul-1999    104.4 -1000

Index Type   Strike Price Opt  Contracts
5     Option 105      7.4 Put  -1000
6     Option  95      2.9 Put      0

Index Type Price Maturity       Contracts
7     TBill 99    01-Jul-1999   6
```

Extract the price from all instruments.

```
P = instget(ExampleInst,'FieldName','Price')

P =
```

```
   12.2000
    9.2000
    6.8000
       NaN
    7.4000
    2.9000
   99.0000
```

Get all the prices and the number of contracts held.

```
[P,C] = instget(ExampleInst, 'FieldName', {'Price', 'Contracts'})

P =

   12.2000
    9.2000
    6.8000
       Nan
    7.4000
    2.9000
   99.0000

C =

      0
      0
   1000
  -1000
  -1000
      0
      6
```

Compute a value `V`. Create a new variable `ISet` that appends `V` to `ExampleInst`.

```
V = P.*C
```

```
ISet = instsetfield(ExampleInst, 'FieldName', 'Value', 'Data',...
V);
instdisp(ISet)

Index Type   Strike Price Opt  Contracts Value
1     Option  95     12.2 Call    0           0
2     Option 100      9.2 Call    0           0
3     Option 105      6.8 Call  1000       6800

Index Type    Delivery       F     Contracts Value
4     Futures 01-Jul-1999    104.4 -1000     NaN

Index Type   Strike Price Opt  Contracts Value
5     Option 105     7.4 Put  -1000      -7400
6     Option  95     2.9 Put     0           0

Index Type Price Maturity       Contracts Value
7     TBill 99    01-Jul-1999   6         594
```

Look at only the instruments that have nonzero `Contracts`.

```
Ind = find(C ~= 0)

Ind =

     3
     4
     5
     7
```

Get the `Type` and `Opt` parameters from those instruments. (Only options have a stored `'Opt'` field.)

```
[T,O] = instget(ExampleInst, 'Index', Ind, 'FieldName',...
{'Type', 'Opt'})

T =
```

```
Option
Futures
Option
TBill

O =

Call

Put
```

Create a string report of holdings `Type`, `Opt`, and `Value`.

```
rstring = [T, O, num2str(V(Ind))]

rstring =

Option Call   6800
Futures        NaN
Option Put   -7400
TBill          594
```

## See Also

`instaddfield` | `instdisp` | `instgetcell`

# instgetcell

## Purpose

Data and context from instrument variable

## Syntax

```
[DataList, FieldList, ClassList] =
instgetcell(InstSet,  'FieldName', FieldList, 'Index',
IndexSet, 'Type', TypeList)
```

## Arguments

| | |
|---|---|
| `InstSet` | Variable containing a collection of instruments. Instruments are classified by type; each type can have different data fields. The stored data field is a row vector or string for each instrument. |
| `FieldList` | (Optional) String or number of fields (`NFIELDS`)-by-`1` cell array of strings listing the name of each data field to match with data values. `FieldList` should not be either `Type` or `Index`; these field names are reserved. The default is all fields available for the returned set of instruments. |
| `IndexSet` | (Optional) Number of instruments (`NINST`)-by-`1` vector of positions of instruments to work on. If `TypeList` is also entered, instruments referenced must be one of `TypeList` types and contained in `IndexSet`. The default is all indices available in the instrument variable. |
| `TypeList` | (Optional) String or number of types (`NTYPES`)-by-`1` cell array of strings restricting instruments to match one of `TypeList` types. The default is all types in the instrument variable. |

Argument value pairs can be entered in any order. The `InstSet` variable must be the first argument.

## Description

```
[DataList, FieldList, ClassList] =
instgetcell(InstSet, 'FieldName', FieldList, 'Index',
```

`IndexSet, 'Type', TypeList)` retrieves data and context from an instrument variable.

`DataList` is an `NFIELDS`-by-`1` cell array of data contents for each field. Each cell is an `NINST`-by-`M` array, where each row corresponds to a separate instrument in `IndexSet`. Any data which is not available is returned as `NaN` or as spaces.

`FieldList` is an `NFIELDS`-by-`1` cell array of strings listing the name of each field in `DataList`.

`ClassList` is an `NFIELDS`-by-`1` cell array of strings listing the data class of each field. The class determines how arguments are parsed. Valid strings are `'dble'`, `'date'`, and `'char'`.

`IndexSet` is an `NINST`-by-`1` vector of positions of instruments returned in `DataList`.

`TypeSet` is an `NINST`-by-`1` cell array of strings listing the type of each instrument row returned in `DataList`.

## Examples

Retrieve the instrument set `ExampleInst` from the data file `InstSetExamples.mat`. `ExampleInst` contains three types of instruments: `Option`, `Futures`, and `TBill`.

```
load InstSetExamples;
instdisp(ExampleInst)

Index Type   Strike Price Opt  Contracts
1     Option  95    12.2  Call     0
2     Option 100     9.2  Call     0
3     Option 105     6.8  Call  1000

Index Type    Delivery       F     Contracts
4     Futures 01-Jul-1999    104.4 -1000

Index Type   Strike Price Opt  Contracts
5     Option 105      7.4 Put  -1000
6     Option  95      2.9 Put      0
```

```
Index Type Price Maturity       Contracts
7     TBill 99    01-Jul-1999   6
```

Get the prices and contracts from all instruments.

```
FieldList = {'Price'; 'Contracts'}
DataList = instgetcell(ExampleInst, 'FieldName', FieldList )
P = DataList{1}
C = DataList{2}

P =

   12.2000
    9.2000
    6.8000
       NaN
    7.4000
    2.9000
   99.0000

C =

      0
      0
   1000
  -1000
  -1000
      0
      6
```

Get all the option data: `Strike`, `Price`, `Opt`, `Contracts`.

```
[DataList, FieldList, ClassList] = instgetcell(ExampleInst,...
'Type','Option')

DataList =
```

```
    [5x1 double]
    [5x1 double]
    [5x4 char  ]
    [5x1 double]

FieldList =

    'Strike'
    'Price'
    'Opt'
    'Contracts'

ClassList =

    'dble'
    'dble'
    'char'
    'dble'
```

Look at the data as a comma-separated list. Type `help lists` for more information on cell array lists.

```
DataList{:}

ans =

     95
    100
    105
    105
     95

ans =

    12.2100
     9.2000
```

```
    6.8000
    7.3900
    2.9000

ans =

   Call
   Call
   Call
   Put
   Put

ans =

     0
     0
   100
  -100
     0
```

## See Also

`instaddfield` | `instdisp` | `instget`

# instlength

## Purpose

Count instruments

## Syntax

`NInst = instlength(InstSet)`

## Arguments

| | |
|---|---|
| `InstSet` | Variable containing a collection of instruments. Instruments are classified by type; each type can have different data fields. The stored data field is a row vector or string for each instrument. |

## Description

`NInst = instlength(InstSet)` computes `NInst`, the number of instruments contained in the variable, `InstSet`.

## See Also

`instdisp` | `instfields` | `insttypes`

# instlookback

## Purpose

Construct lookback option

## Syntax

```
InstSet = instlookback(InstSet, OptSpec, Strike, Settle,
ExerciseDates, AmericanOpt)
[FieldList, ClassList, TypeString] = instlookback
```

## Arguments

| | |
|---|---|
| `InstSet` | Instrument variable. This argument is specified only when adding lookback instruments to an existing instrument set. See `instget` for more information on the `InstSet` variable. |
| `OptSpec` | `NINST`-by-`1` list of string values `'Call'` or `'Put'`. |
| `Strike` | `NINST`-by-`1` vector of strike price values. Each row is the schedule for one option. |
| `Settle` | `NINST`-by-`1` vector of `Settle` dates. |
| `ExerciseDates` | For a European option (`AmericanOpt = 0`):<br>`NINST`-by-`1` vector of exercise dates. Each row is the schedule for one option. For a European option, there is only one exercise date, the option expiry date.<br>For an American option (`AmericanOpt = 1`):<br>`NINST`-by-`2` vector of exercise date boundaries. For each instrument, the option can be exercised on any tree date between or including the pair of dates on that row. If only one non-`NaN` date is listed, or if `ExerciseDates` is `NINST`-by-`1`, the option can be exercised between the valuation date of the stock tree and the single listed exercise date. |
| `AmericanOpt` | (Optional) If `AmericanOpt = 0`, `NaN`, or is unspecified, the option is a European option. If |

`AmericanOpt = 1`, the option is an American option.

Data arguments are number of instruments (`NINST`)-by-`1` vectors, scalar, or empty. Fill unspecified entries in vectors with `NaN`. Only one data argument is required to create the instrument. The others may be omitted or passed as empty matrices `[]`.

## Description

`InstSet = instlookback(InstSet, OptSpec, Strike, Settle, ExerciseDates, AmericanOpt)` specifies a lookback option.

`[FieldList, ClassList, TypeString] = instlookback` displays the classes.

`FieldList` is a number of fields (`NFIELDS`)-by-`1` cell array of strings listing the name of each data field for this instrument type.

`ClassList` is an `NFIELDS`-by-`1` cell array of strings listing the data class of each field. The class determines how arguments are parsed. Valid strings are `'dble'`, `'date'`, and `'char'`.

`TypeString` is a string specifying the type of instrument added. For a lookback option instrument, `TypeString = 'Lookback'`.

## Examples

### Create a Lookback Option Instrument

Define a floating strike lookback instrument with the following data:

```
OptSpec = 'call';
Strike = NaN;
Settle = '01-Jan-2012';
ExerciseDates = '01-Jan-2015';
```

Create the instrument set.

```
InstSet = instlookback(OptSpec, Strike, Settle, ExerciseDates);
```

Display the lookback instrument.

```
instdisp(InstSet)
```

```
Index Type     OptSpec Strike Settle         ExerciseDates  AmericanOpt
1     Lookback call    NaN    01-Jan-2012    01-Jan-2015    0
```

**See Also** `instadd` | `instdisp` | `instget`

## Purpose

Construct bond option

## Syntax

```
InstSet = instoptbnd(InstSet, BondIndex, OptSpec, Strike,
ExerciseDates)
InstSet = instoptbnd(InstSet, BondIndex, OptSpec, Strike,
ExerciseDates, AmericanOpt)
[FieldList, ClassList, TypeString] = instoptbnd
```

## Arguments

| | |
|---|---|
| `InstSet` | Variable containing a collection of instruments. Instruments are classified by type; each type can have different data fields. The stored data field is a row vector or string for each instrument. |
| `BondIndex` | Number of instruments (`NINST`)-by-`1` vector of indices pointing to underlying instruments of Type `'Bond'` which are also stored in `InstSet`. See `instbond` for information on specifying the bond data. |
| `OptSpec` | `NINST`-by-`1` list of string values `'Call'` or `'Put'`. |

---

**Note** The interpretation of the `Strike` and `ExerciseDates` arguments depends upon the setting of the `AmericanOpt` argument. If `AmericanOpt = 0`, `NaN`, or is unspecified, the option is a European or Bermuda option. If `AmericanOpt = 1`, the option is an American option.

---

| | |
|---|---|
| `Strike` | European option: `NINST`-by-`1` vector of strike price values.<br>Bermuda option: `NINST` by number of strikes (`NSTRIKES`) matrix of strike price values.<br>Each row is the schedule for one option. If an option has fewer than `NSTRIKES` exercise opportunities, the end of the row is padded with `NaN`s.<br>For an American option:<br>`NINST`-by-`1` vector of strike price values for each option. |
| `ExerciseDates` | `NINST`-by-`1` (European option) or `NINST`-by-`NSTRIKES` (Bermuda option) matrix of exercise dates. Each row is the schedule for one option. For a European option, there is only one exercise date, the option expiry date.<br>For an American option:<br>`NINST`-by-`2` vector of exercise date boundaries. For each instrument, the option can be exercised on any coupon date between or including the pair of dates on that row. If only one non-`NaN` date is listed, or if `ExerciseDates` is `NINST`-by-`1`, the option can be exercised between the underlying bond `Settle` and the single listed exercise date. |

Data arguments are `NINST`-by-`1` vectors, scalar, or empty. Fill unspecified entries in vectors with `NaN`. Only one data argument is required to create the instrument. The others may be omitted or passed as empty matrices `[]`.

## Description

`InstSet = instoptbnd(InstSet, BondIndex, OptSpec, Strike, ExerciseDates)` specifies a European or Bermuda option.

`InstSet = instoptbnd(InstSet, BondIndex, OptSpec, Strike, ExerciseDates, AmericanOpt)` specifies an American option if `AmericanOpt` is set to `1`. If `AmericanOpt` is not set to `1`, the function specifies a European or Bermuda option.

`[FieldList, ClassList, TypeString] = instoptbnd` displays the classes.

`FieldList` is a number of fields (`NFIELDS`)-by-`1` cell array of strings listing the name of each data field for this instrument type.

`ClassList` is an `NFIELDS`-by-`1` cell array of strings listing the data class of each field. The class determines how arguments are parsed. Valid strings are `'dble'`, `'date'`, and `'char'`.

`TypeString` is a string specifying the type of instrument added. For a bond option instrument, `TypeString = 'OptBond'`.

## See Also

`hjmprice` | `instadd` | `instdisp` | `instget`

# instoptembnd

## Purpose

Construct bond with embedded option

## Syntax

```
InstSet = instoptembnd (CouponRate, Settle, Maturity,
OptSpec, Strike, ExerciseDates, 'AmericanOpt',
AmericanOpt, 'Period', Period, 'Basis', Basis,
'EndMonthRule', EndMonthRule,'Face', Face, 'IssueDate',
IssueDate, 'FirstCouponDate', FirstCouponDate,
'LastCouponDate', LastCouponDate,'StartDate',StartDate)
InstSet = instoptembnd(InstSetOld, CouponRate,...)
[FieldList, ClassList, TypeString] = instoptembnd
```

## Arguments

| | |
|---|---|
| `CouponRate` | Decimal annual rate indicating the annual percentage rate used to determine the coupons payable on a bond. `CouponRate` is a `NINST`-by-`1` vector or `NINST`-by-`1` cell array of decimal annual rates, or decimal annual rate schedules. For the latter case of a variable coupon schedule, each element of the cell array is a `NumDates`-by-`2` cell array, where the first column is dates and the second column is its associated rate. The date indicates the last day that the coupon rate is valid. |
| `Settle` | `NINST`-by-`1` vector of settlement dates. |
| `Maturity` | `NINST`-by-`1` vector of maturity dates. |
| `OptSpec` | `NINST`-by-`1` vector of string values `'Call'` or `'Put'`. |

For a European or Bermuda option

| | |
|---|---|
| `Strike` | `NINST`-by-`NSTRIKES` matrix of strike price values. Each row is the schedule for one option. If an option has fewer than `NSTRIKES` exercise opportunities, the end of the row is padded with `NaN`'s. |

| | |
|---|---|
| `ExerciseDates` | `NINST`-by-`NSTRIKES` matrix of exercise dates. Each row is the schedule for one option. For a European option, there is only one `ExerciseDate` on the option expiry date. |
| `AmericanOpt` | (Optional) `NINST`-by-`1` vector of flags. `AmericanOpt` is `0` for each European or Bermuda option. The default is `0` if `AmericanOpt` is `NaN` or not entered. |

For an American option

| | |
|---|---|
| `Strike` | `NINST`-by-`1` vector of strike price values for each option. |
| `ExerciseDates` | `NINST`-by-`2` vector of exercise date boundaries. For each instrument, the option can be exercised on any coupon date between or including the pair of dates on that row. If only one non-`NaN` date is listed, or if `ExerciseDates` is `NINST`-by-`1`, the option can be exercised between the underlying bond `Settle` and the single listed `ExerciseDate`. |
| `AmericanOpt` | `NINST`-by-`1` vector of flags. `AmericanOpt` is `1` for each American option. The `AmericanOpt` argument is required to invoke American exercise rules. |
| `Period` | (Optional) `NINST`-by-`1` matrix for coupons per year. The default value is `2`. |

| | |
|---|---|
| `Basis` | (Optional) Day-count basis of the instrument. `Basis` is a vector of integers with the following possible values:<br>• 0 = actual/actual (default)<br>• 1 = 30/360 (SIA)<br>• 2 = actual/360<br>• 3 = actual/365<br>• 4 = 30/360 (BMA)<br>• 5 = 30/360 (ISDA)<br>• 6 = 30/360 (European)<br>• 7 = actual/365 (Japanese)<br>• 8 = actual/actual (ICMA)<br>• 9 = actual/360 (ICMA)<br>• 10 = actual/365 (ICMA)<br>• 11 = 30/360E (ICMA)<br>• 12 = actual/actual (ISDA)<br>• 13 = BUS/252<br>For more information, see basis. |
| `EndMonthRule` | (Optional) `NINST`-by-`1` matrix for the end-of-month rule. This rule applies only when `Maturity` is an end-of-month date for a month having 30 or fewer days. When the value is `0`, the end-of-month rule is ignored, meaning that a bond's coupon payment date is always the same numerical day of the month. When the value is `1`, the end-of-month rule is set rule on (default), meaning that a bond's coupon payment date is always the last actual day of the month. |

| | |
|---|---|
| `IssueDate` | (Optional) `NINST`-by-`1` matrix for the bond issue date. |
| `FirstCouponDate` | (Optional) Date when a bond makes its first coupon payment; used when bond has an irregular first coupon period. When `FirstCouponDate` and `LastCouponDate` are both specified, `FirstCouponDate` takes precedence in determining the coupon payment structure. If you do not specify a `FirstCouponDate`, the cash flow payment dates are determined from other inputs. |
| `LastCouponDate` | (Optional) Last coupon date of a bond before the maturity date; used when bond has an irregular last coupon period. In the absence of a specified `FirstCouponDate`, a specified `LastCouponDate` determines the coupon structure of the bond. The coupon structure of a bond is truncated at the `LastCouponDate`, regardless of where it falls, and is followed only by the bond's maturity cash flow date. If you do not specify a `LastCouponDate`, the cash flow payment dates are determined from other inputs. |
| `StartDate` | (Optional) `NINST`-by-`1` matrix for date when a bond actually starts (i.e. the date from which a bond's cash flows can be considered). To make an instrument forward starting, specify this date as a future date. If `StartDate` is not explicitly specified, the effective start date is the `Settle` date. |
| `Face` | (Optional) Face value. `Face` is a `NINST`-by-`1` vector or `NINST`-by-`1` cell array of face values, or face value schedules. For the latter case, each element of the cell array is a `NumDates`-by-`2` cell array, where the first column is dates and the second column is its associated face value. The |

date indicates the last day that the face value is valid. Default is 100.

---

**Note** Data arguments are `NINST`-by-`1` vectors, scalar, or empty. Fill unspecified entries in vectors with `NaN`. Only one data argument is required to create the instrument. The others may be omitted or passed as empty matrices `[]`.

---

## Description

`InstSet = instoptembnd (CouponRate, Settle, Maturity, OptSpec, Strike, ExerciseDates, 'AmericanOpt', AmericanOpt, 'Period', Period, 'Basis', Basis, 'EndMonthRule', EndMonthRule,'Face', Face, 'IssueDate', IssueDate, 'FirstCouponDate', FirstCouponDate, 'LastCouponDate', LastCouponDate,'StartDate',StartDate)` creates `InstSet`, a variable containing a collection of instruments.

---

**Note** `instopembnd` uses optional parameter name/value pairs such that, `'Name1'`, `Value1`, `'Name2'`, `Value2`, and so on, are a variable length list of name/value pairs.

---

Instruments are broken down by type and each type can have different data fields. Each stored data field has a row vector or string for each instrument. See `instget` for more information on the `InstSet` variable.

`InstSet = instoptembnd(InstSetOld, CouponRate,...)` adds `'OptEmBond'` instruments to an instrument variable.

`[FieldList, ClassList, TypeString] = instoptembnd` lists field metadata for the `'OptEmBond'` instrument.

`FieldList` is a number of fields (`NFIELDS`)-by-`1` cell array of strings listing the name of each data field for this instrument type.

`ClassList` is an `NFIELDS`-by-`1` cell array of strings listing the data class of each field. The class determines how arguments are parsed. Valid strings are `'dble'`, `'date'`, and `'char'`.

`TypeString` is a string specifying the type of instrument added. For a bond option instrument, `TypeString = 'OptEmBond'`.

## Examples

To create a bond with embedded options with the following data:

```
Settle = 'jan-1-2007';
Maturity   = 'jan-1-2010';
CouponRate = 0.07;
OptSpec = 'call';
Strike= 100;
ExerciseDates= {'jan-1-2008' '01-Jan-2010'};
AmericanOpt=1;
Period = 1;

InstSet = instoptembnd(CouponRate, ...
Settle, Maturity, OptSpec, Strike,  ExerciseDates,'AmericanOpt', AmericanOpt, ...
'Period', Period);
```

To display the instrument:

```
instdisp(InstSet)
```

## See Also

`instadd` | `instdisp` | `instget`

# instoptstock

## Purpose

Construct stock option

## Syntax

```
InstSet = instoptstock(InstSet, OptSpec, Strike,
Settle, ExerciseDates)
InstSet = instoptstock(InstSet, OptSpec, Strike,
Settle, ExerciseDates, AmericanOpt)
[FieldList, ClassList, TypeString] = instoptstock
```

## Arguments

| | |
|---|---|
| `InstSet` | Instrument variable. This argument is specified only when adding stock option instruments to an existing instrument set. See `instget` for more information on the `InstSet` variable. |
| `OptSpec` | `NINST`-by-`1` list of string values `'Call'` or `'Put'`. |

**Note** The interpretation of the `Strike` and `ExerciseDates` arguments depends upon the setting of the `AmericanOpt` argument. If `AmericanOpt = 0`, `NaN`, or is unspecified, the option is a European or Bermuda option. If `AmericanOpt = 1`, the option is an American option.

| | |
|---|---|
| `Strike` | European option: `NINST`-by-`1` vector of strike price values.<br>Bermuda option: `NINST` by number of strikes (`NSTRIKES`) matrix of strike price values.<br>Each row is the schedule for one option. If an option has fewer than `NSTRIKES` exercise opportunities, the end of the row is padded with `NaN`s.<br>American option: `NINST`-by-`1` vector of strike price values for each option. |

| | |
|---|---|
| `Settle` | `NINST`-by-`1` vector of settlement dates. |
| `ExerciseDates` | `NINST`-by-`1` (European option) or `NINST`-by-`NSTRIKES` (Bermuda option) matrix of exercise dates. Each row is the schedule for one option. For a European option, there is only one exercise date, the option expiry date.<br>For an American option:<br>`NINST`-by-`2` vector of exercise date boundaries. For each instrument, the option can be exercised on any coupon date between or including the pair of dates on that row. If only one non-`NaN` date is listed, or if `ExerciseDates` is `NINST`-by-`1`, the option can be exercised between the underlying bond `Settle` and the single listed exercise date. |

Data arguments are `NINST`-by-`1` vectors, scalar, or empty. Fill unspecified entries in vectors with `NaN`. Only one data argument is required to create the instrument. The others may be omitted or passed as empty matrices `[]`.

## Description

`InstSet = instoptstock(InstSet, OptSpec, Strike, Settle, ExerciseDates)` specifies a European or Bermuda option.

`InstSet = instoptstock(InstSet, OptSpec, Strike, Settle, ExerciseDates, AmericanOpt)` specifies an American option if `AmericanOpt` is set to `1`. If `AmericanOpt` is not set to `1`, the function specifies a European or Bermuda option.

`[FieldList, ClassList, TypeString] = instoptstock` displays the classes.

`FieldList` is a number of fields (`NFIELDS`)-by-`1` cell array of strings listing the name of each data field for this instrument type.

`ClassList` is an `NFIELDS`-by-1 cell array of strings listing the data class of each field. The class determines how arguments are parsed. Valid strings are `'dble'`, `'date'`, and `'char'`.

`TypeString` is a string specifying the type of instrument added. For a stock option instrument, `TypeString = 'OptStock'`.

## Examples

### Create a Stock Option Instrument

Create an instrument set of two stock options with the following data:

```
OptSpec = {'put';'call'};
Strike = [95;98];
Settle = '01-May-2012';
ExerciseDates = {'01-May-2014';'01-May-2015'};
AmericanOpt = [0;1];
```

Create the stock option instruments.

```
InstSet = instoptstock(OptSpec, Strike,Settle, ExerciseDates, AmericanOpt
```

Display the instrument set.

```
instdisp(InstSet)
```

```
Index Type     OptSpec Strike Settle         ExerciseDates  AmericanOpt
1     OptStock put     95     01-May-2012    01-May-2014    0
2     OptStock call    98     01-May-2012    01-May-2015    1
```

## See Also

`instadd` | `instdisp` | `instget`

## Purpose

Construct range note instrument

## Syntax

```
ISet = instrangefloat(Spread,Settle,Maturity,RateSched,
 Reset,Basis,Principal,EndMonthRule)
```

## Description

`ISet = instrangefloat(Spread,Settle,Maturity,RateSched, Reset,Basis,Principal,EndMonthRule)` creates a new range instrument from data arrays.

## Input Arguments

### Spread

Number of basis points over the reference rate.

### Settle

`NINST`-by-`1` vector of dates representing the settle date of the floating-rate note.

### Maturity

`NINST`-by-`1` vector of dates representing the maturity date of the floating-rate note.

### RateSched

`NINST`-by-`1` vector of structures representing the range of rates within which cash flows are nonzero. Each element of the structure array contains two fields:

- `RateSched.Dates` — `NDates`-by-`1` cell array of dates corresponding to the range schedule.
- `RateSched.Rates` —`NDates`-by-`2` array with the first column containing the lower bound of the range and the second column containing the upper bound of the range. Cash flow for date `RateSched.Dates`($n$) is nonzero for rates in the range `RateSched.Rates`($n$,1) < `Rate` < `RateSched.Rate` ($n$,2).

### Reset

(Optional) `NINST`-by-`1` vector representing the frequency of payments per year.

**Default:** `1`

### Basis

(Optional) Day-count basis of the instrument. A vector of integers.

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (BMA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/actual (ISDA)
- 13 = BUS/252

For more information, see basis.

**Default:** `0` (actual/actual)

### Principal

(Optional) `NINST`-by-`1` vector of the notional principal amount.

**Default:** `100`

**EndMonthRule**

(Optional) `NINST`-by-`1` vector for end-of-month rule. Values are `1` (in effect) and `0` (not in effect).

> **Default:** `1` (in effect)

---

**Note** Data arguments are number of instruments `NINST`-by-`1` vectors, scalar, or empty. Fill unspecified entries in vectors with `NaN.` Only one data argument is required to create the instrument. You can omit or pass the others as empty matrices `[].` However, you cannot price the instrument when using the range note pricing function if you are missing any of the required input arguments.

---

## Output Arguments

**ISet**

Variable containing a collection of instruments. Instruments are divided by type and each type can have different data fields. Each stored data field has a row vector or string for each instrument. Values are:

- `FieldList` — `NFIELDS`-by-`1` cell array of strings listing the name of each data field for this instrument type.
- `ClassList` — `NFIELDS`-by-`1` cell array of strings listing the data class of each field. The class determines how arguments are parsed. Valid strings are`'dble'`, `'date'`, and `'char'`.
- `TypeString` — String specifying the type of instrument added. `TypeString = 'RangeFloat'`.

For more information, on `ISet` see `instget`.

## Definitions

### Range Note Instrument

A range note is a structured (market-linked) security whose coupon rate is equal to the reference rate as long as the reference rate is within a certain range. If the reference rate is outside of the range, the coupon rate is 0 for that period. This type of instrument entitles the holder to cash flows that depend on the level of some reference interest rate and are floored to be positive. The note holder gets directs exposure to the reference rate. In return for the drawback that no interest will be paid for the time the range is left, they offer higher coupon rates than comparable standard products, like vanilla floating notes.

## Examples

Create a range note instrument:

```
% Create an instrument portfolio with a range note:
Spread = 100;
Settle = 'Jan-1-2011';
Maturity = 'Jan-1-2014';

RateSched.Dates = {'Jan-1-2012'; 'Jan-1-2013'  ; 'Jan-1-2014'};
RateSched.Rates  = [0.045 0.055; 0.0525  0.0675; 0.06 0.08];

% Create InstSet
InstSet = instrangefloat(Spread, Settle, Maturity, RateSched);

% Display the portfolio instrument
instdisp(InstSet)
Index Type       Spread Settle      Maturity  RateSched FloatReset Basis Principal EndMonthRule
1     RangeFloat 100    01-Jan-2011 01-Jan-2014 [Struct]  1           0     100       1

% Add another range note
% Second Range Note:
Spread2 = 200;
Settle2 = 'Jan-1-2011';
Maturity2 = 'Jan-1-2013';
RateSched2.Dates = {'Jan-1-2012'; 'Jan-1-2013'};
RateSched2.Rates  = [0.048 0.059; 0.055  0.068];

InstSet = instrangefloat(InstSet, Spread2, Settle2, Maturity2, RateSched2);

% Display the portfolio instrument
instdisp(InstSet)
```

```
Index Type       Spread Settle     Maturity      RateSched FloatReset Basis Principal EndMonthRule
1   RangeFloat 100   01-Jan-2011   01-Jan-2014   [Struct]  1          0     100       1
2   RangeFloat 200   01-Jan-2011   01-Jan-2013   [Struct]  1          0     100       1
```

## References

Jarrow, Robert, *Modelling Fixed Income Securities and Interest Rate Options*, Stanford Economics and Finance, 2nd edition, 2002.

## See Also

| `instbond` | `instcap` | `instswap` | `instaddfield` | `instdisp` | `intenvprice` | `rangefloatbybk` | `rangefloatbybdt` | `rangefloatbyhw` | `rangefloatbyhjm` |

# instselect

## Purpose

Create instrument subset by matching conditions

## Syntax

```
InstSubSet = instselect(InstSet, 'FieldName', FieldList,
'Data', DataList, 'Index', IndexSet,'Type', TypeList)
```

## Arguments

| | |
|---|---|
| `InstSet` | Variable containing a collection of instruments. Instruments are classified by type; each type can have different data fields. The stored data field is a row vector or string for each instrument. |
| `FieldList` | String or number of fields (`NFIELDS`)-by-`1` cell array of strings listing the name of each data field to match with data values. |
| `DataList` | Number of values (`NVALUES`)-by-`M` array or `NFIELDS`-by-`1` cell array of acceptable data values for each field. Each row lists a data row value to search for in the corresponding `FieldList`. The number of columns is arbitrary and matching will ignore trailing `NaN`s or spaces. |
| `IndexSet` | (Optional) Number of instruments (`NINST`)-by-`1` vector restricting positions of instruments to check for matches. The default is all indices available in the instrument variable. |
| `TypeList` | (Optional) String or number of types (`NTYPES`)-by-`1` cell array of strings restricting instruments to match one of `TypeList` types. The default is all types in the instrument variable. |

Argument value pairs can be entered in any order. The `InstSet` variable must be the first argument. `'FieldName'` and `'Data'` arguments must appear together or not at all. `'Index'` and `'Type'` arguments are each optional.

## Description

`InstSubSet = instselect(InstSet, 'FieldName', FieldList, 'Data', DataList, 'Index', IndexSet,'Type', TypeList)` creates an instrument subset (`InstSubSet`) from an existing set of instruments (`InstSet`).

`InstSubSet` is a variable containing instruments matching the input criteria. Instruments are returned in `InstSubSet` if all the `Field`, `Index`, and `Type` conditions are met. An instrument meets an individual `Field` condition if the stored `FieldName` data matches any of the rows listed in the `DataList` for that `FieldName`. See `instfind` for examples on matching criteria.

## Examples

Retrieve the instrument set `ExampleInst` from the data file `InstSetExamples.mat`. The variable contains three types of instruments: `Option`, `Futures`, and `TBill`.

```
load InstSetExamples
instdisp(ExampleInst)

Index Type   Strike Price Opt  Contracts
1     Option  95    12.2  Call     0
2     Option 100     9.2  Call     0
3     Option 105     6.8  Call  1000

Index Type    Delivery       F     Contracts
4     Futures 01-Jul-1999    104.4 -1000

Index Type   Strike   Price Opt  Contracts
5     Option 105      7.4   Put  -1000
6     Option  95      2.9   Put      0

Index Type  Price Maturity       Contracts
7     TBill 99    01-Jul-1999    6
```

Make a new portfolio containing only options struck at 95.

```
Opt95 = instselect(ExampleInst, 'FieldName', 'Strike',...
'Data', '95')
```

```
instdisp(Opt95)

Opt95 =

Index Type   Strike Price Opt  Contracts
1     Option  95     12.2 Call    0
2     Option  95      2.9 Put     0
```

Make a new portfolio containing only futures and Treasury bills.

```
FutTBill = instselect(ExampleInst,'Type',{'Futures';'TBill'})

instdisp(FutTBill) =

Index Type    Delivery       F     Contracts
1     Futures 01-Jul-1999    104.4 -1000

Index Type  Price Maturity       Contracts
2     TBill 99    01-Jul-1999    6
```

## See Also

`instaddfield` | `instdelete` | `instfind` | `instget` | `instgetcell`

## Purpose

Add or reset data for existing instruments

## Syntax

```
InstSet = instsetfield(InstSet, 'FieldName', FieldList,
'Data', DataList)
InstSet = instsetfield(InstSet, 'FieldName', FieldList,
'Data', DataList, 'Index', IndexSet, 'Type', TypeList)
```

## Arguments

| | |
|---|---|
| `InstSet` | Variable containing a collection of instruments. Instruments are classified by type; each type can have different data fields. The stored data field is a row vector or string for each instrument. `InstSet` must be the first argument in the list. |
| `FieldList` | String or number of fields (`NFIELDS`)-by-`1` cell array of strings listing the name of each data field. `FieldList` cannot be named with the reserved names `Type` or `Index`. |
| `DataList` | Number of instruments (`NINST`)-by-`M` array or `NFIELDS`-by-`1` cell array of data contents for each field. Each row in a data array corresponds to a separate instrument. Single rows are copied to apply to all instruments to be worked on. The number of columns is arbitrary, and data is padded along columns. |
| `IndexSet` | `NINST`-by-`1` vector of positions of instruments to work on. If `TypeList` is also entered, instruments referenced must be one of `TypeList` types and contained in `IndexSet`. |
| `TypeList` | String or number of types (`NTYPES`)-by-`1` cell array of strings restricting instruments worked on to match one of `TypeList` types. |

Argument value pairs can be entered in any order.

## Description

`instsetfield` sets data for existing instruments in a collection variable.

`InstSet = instsetfield(InstSet, 'FieldName', FieldList, 'Data', DataList)` resets or adds fields to every instrument.

`InstSet = instsetfield(InstSet, 'FieldName', FieldList, 'Data', DataList, 'Index', IndexSet, 'Type', TypeList)` resets or adds fields to a subset of instruments.

The output `InstSet` is a new instrument set variable containing the input data.

## Examples

Retrieve the instrument set `ExampleInstSF` from the data file `InstSetExamples.mat`. `ExampleInstSF` contains three types of instruments: `Option`, `Futures`, and `TBill`.

```
load InstSetExamples;
ISet = ExampleInstSF;
instdisp(ISet)

Index Type   Strike Price Opt
1     Option  95    12.2  Call
2     Option 100     9.2  Call
3     Option 105     6.8  Call

Index Type    Delivery       F
4     Futures 01-Jul-1999    104.4

Index Type   Strike Price Opt
5     Option 105     7.4  Put
6     Option NaN     NaN  Put

Index Type  Price
7     TBill 99
```

Enter data for the option in `Index 6`: `Price 2.9` for a `Strike` of `95`.

```
ISet = instsetfield(ISet, 'Index',6,...
'FieldName',{'Strike','Price'}, 'Data',{ 95 , 2.9 });
```

```
instdisp(ISet)

Index Type   Strike Price Opt
1     Option  95    12.2  Call
2     Option 100     9.2  Call
3     Option 105     6.8  Call
Index Type    Delivery       F
4     Futures 01-Jul-1999    104.4
Index Type   Strike Price Opt
5     Option 105     7.4  Put
6     Option  95     2.9  Put

Index Type  Price
7     TBill 99
```

Create a new field `Maturity` for the cash instrument.

```
MDate = datenum('7/1/99');
ISet = instsetfield(ISet, 'Type', 'TBill', 'FieldName',...
'Maturity','FieldClass', 'date', 'Data', MDate);
instdisp(ISet)
Index Type  Price  Maturity
7     TBill 99     01-Jul-1999
```

Create a new field `Contracts` for all instruments.

```
ISet = instsetfield(ISet, 'FieldName', 'Contracts', 'Data', 0);
instdisp(ISet)
Index Type   Strike Price Opt  Contracts
1     Option  95    12.2  Call 0
2     Option 100     9.2  Call 0
3     Option 105     6.8  Call 0

Index Type    Delivery       F     Contracts
4     Futures 01-Jul-1999    104.4 0

Index Type   Strike Price Opt  Contracts
```

```
5     Option 105     7.4  Put  0
6     Option  95     2.9  Put  0

Index Type  Price  Maturity     Contracts
7     TBill 99     01-Jul-1999  0
```

Set the `Contracts` fields for some instruments.

```
ISet = instsetfield(ISet,'Index',[3; 5; 4; 7],...
'FieldName','Contracts',  'Data', [1000; -1000; -1000; 6]);

instdisp(ISet)

Index Type   Strike Price Opt  Contracts
1     Option  95    12.2  Call     0
2     Option 100     9.2  Call     0
3     Option 105     6.8  Call  1000

Index Type    Delivery       F     Contracts
4     Futures 01-Jul-1999    104.4 -1000

Index Type   Strike Price Opt  Contracts
5     Option 105     7.4  Put  -1000
6     Option  95     2.9  Put      0

Index Type  Price  Maturity     Contracts
7     TBill 99     01-Jul-1999  6
```

## See Also

`instaddfield` | `instdisp` | `instget` | `instgetcell`

**Purpose** Construct swap instrument

**Syntax**

```
InstSet = instswap(InstSet, LegRate, Settle, Maturity)
InstSet = instswap(InstSet, LegRate, Settle, Maturity,
InstSet, LegReset, Basis, Principal, LegType,
EndMonthRule)
InstSet = instswap(InstSet, LegRate, Settle, Maturity, LegReset, Basi
Principal, LegType, EndMonthRule, StartDate)
[FieldList, ClassList, TypeString] = instswap
```

## Arguments

| | |
|---|---|
| `InstSet` | Instrument variable. This argument is specified only when adding a swap to an existing instrument set. See `instget` for more information on the `InstSet` variable. |
| `LegRate` | Number of instruments (`NINST`)-by-`2` matrix, with each row defined as:<br>`[CouponRate Spread]` or `[Spread CouponRate]`<br>`CouponRate` is the decimal annual rate. `Spread` is the number of basis points over the reference rate. The first column represents the receiving leg, while the second column represents the paying leg. |
| `Settle` | Settlement date. `NINST`-by-`1` vector of serial date numbers or date strings. `Settle` must be earlier than `Maturity`. |
| `Maturity` | Maturity date. `NINST`-by-`1` vector of dates representing the maturity date for each swap. |
| `LegReset` | (Optional) `NINST`-by-`2` matrix representing the reset frequency per year for each swap. Default = `[1 1]`. |

| | |
|---|---|
| `Basis` | (Optional) Day-count basis of the instrument. A vector of integers.<br>• 0 = actual/actual (default)<br>• 1 = 30/360 (SIA)<br>• 2 = actual/360<br>• 3 = actual/365<br>• 4 = 30/360 (BMA)<br>• 5 = 30/360 (ISDA)<br>• 6 = 30/360 (European)<br>• 7 = actual/365 (Japanese)<br>• 8 = actual/actual (ICMA)<br>• 9 = actual/360 (ICMA)<br>• 10 = actual/365 (ICMA)<br>• 11 = 30/360E (ICMA)<br>• 12 = actual/actual (ISDA)<br>• 13 = BUS/252<br>For more information, see basis. |
| `Principal` | (Optional) `NINST`-by-`1` vector or `NINST`-by-`1` cell array of the notional principal amounts or principal value schedules. For the latter case, each element of the cell array is a `NumDates`-by-`2` matrix where the first column is dates and the second column is its associated notional principal value. The date indicates the last day that the principal value is valid.. Default = `100`. |

| | |
|---|---|
| `LegType` | (Optional) `NINST`-by-`2` matrix. Each row represents an instrument. Each column indicates if the corresponding leg is fixed (`1`) or floating (`0`). This matrix defines the interpretation of the values entered in `LegRate`. Default is `[1,0]` for each instrument. |
| `EndMonthRule` | (Optional) `NINST`-by-`1` vector representing the End-of-month rule. Default = `1`. |
| `StartDate` | (Optional) `NINST`-by-`1` vector of dates when the swaps actually start. Default is `Settle`. |

Data arguments are number of instruments (`NINST`)-by-`1` vectors, scalar, or empty. Fill unspecified entries in vectors with `NaN`. Only one data argument is required to create the instrument; the others may be omitted or passed as empty matrices `[]`.

## Description

`InstSet = instswap(InstSet, LegRate, Settle, Maturity)` creates a new instrument set containing swap instruments or adds swap instruments to an existing instrument set.

`InstSet = instswap(InstSet, LegRate, Settle, Maturity, InstSet, LegReset, Basis, Principal, LegType, EndMonthRule)` uses optional input arguments to create a new instrument set containing swap instruments or adds swap instruments to an existing instrument set.

`InstSet = instswap(InstSet, LegRate, Settle, Maturity, LegReset, Basis, Principal, LegType, EndMonthRule, StartDate)` to create a new forward swap instrument or to add a forward swap instrument to an existing portfolio..

`[FieldList, ClassList, TypeString] = instswap` displays the classes.

`FieldList` is a number of fields (`NFIELDS`)-by-`1` cell array of strings listing the name of each data field for this instrument type.

`ClassList` is an `NFIELDS`-by-1 cell array of strings listing the data class of each field. The class determines how arguments are parsed. Valid strings are `'dble'`, `'date'`, and `'char'`.

`TypeString` is a string specifying the type of instrument added. For a swap instrument, `TypeString = 'Swap'`.

## Definitions

### Amortizing Swap

In an amortizing swap, the notional principal decreases periodically because it is tied to an underlying financial instrument with a declining (amortizing) principal balance, such as a mortgage.

### Forward Swap

Agreement to enter into an interest-rate swap arrangement on a fixed date in future.

## Examples

### Create a Vanilla Swap Instrument

Create a vanilla swap using market data.

Use the following market data to create a swap instrument.

```
LegRate = [0.065, 0]
Settle = 'jan-1-2007';
Maturity = 'jan-1-2012';
LegReset = [1, 1];
Basis = 0
Principal = 100
LegType = [1, 0]

InstSet = instswap(LegRate, Settle, Maturity, LegReset, Basis, Principal, LegType);
```

View the swap instrument using `instdisp`.

```
instdisp(InstSet)
```

```
Index Type LegRate     Settle         Maturity       LegReset Basis Principal LegType
1     Swap [0.065  0] 01-Jan-2007    01-Jan-2012    1  1     0     100       [1  0]
```

## See Also

`hjmprice` | `instaddfield` | `instbond` | `instcap` | `instdisp` | `instfloor` | `intenvprice`

# instswaption

## Purpose

Construct swaption instrument

## Syntax

```
InstSet = instswaption(OptSpec, Strike, ExerciseDates,
Spread, Settle, Maturity)
InstSet = instswaption(OptSpec, Strike, ExerciseDates,
Spread, Settle, Maturity, AmericanOpt,
SwapReset, Basis, Principal)
InstSet = instswaption(InstSetOld, OptSpec, Strike,
ExerciseDates, Spread, ...)
[FieldList, ClassList, TypeString] = instswaption;
```

## Arguments

Fill unspecified entries in vectors with the value `NaN`. Only one data argument is required to create the instruments; the others may be omitted or passed as empty matrices [ ]. Type `[FieldList, ClassList] = instswaption` to see the classes. Dates can be input as serial date numbers or date strings.

| | |
|---|---|
| `OptSpec` | `NINST`-by-`1` cell array of strings `'call'` or `'put'`. A `'call'` swaption entitles the buyer to pay the fixed rate. A `'put'` swaption entitles the buyer to receive the fixed rate. |
| `Strike` | `NINST`-by-`1` vector of strike swap rate values. |

For a European option:

| | |
|---|---|
| `ExerciseDates` | `NINST`-by-`1` vector of exercise dates. Each row is the schedule for one option. For a European option, there is only one `ExerciseDate` on the option expiry date. |
| `AmericanOpt` | `NINST`-by-`1` vector of flags. `AmericanOpt` is 0 for each European option. The default is 0 if `AmericanOpt` is `NaN` or not entered. |

For an American option:

| | |
|---|---|
| `ExerciseDates` | `NINST`-by-`2` vector of exercise date boundaries. For each instrument, the option can be exercised on any coupon date between or including the pair of dates on that row. If only one non-`NaN` date is listed, or if `ExerciseDates` is `NINST`-by-`1`, the option can be exercised between the underlying swap `Settle` and the single listed `ExerciseDate`. |
| `AmericanOpt` | `NINST`-by-`1` vector of flags. `AmericanOpt` is 1 for each American option. The `AmericanOpt` argument is required to invoke American exercise rules. |

For an American or a European option:

| | |
|---|---|
| `Spread` | `NINST`-by-`1` vector representing the number of basis points over the reference rate. |
| `Settle` | `NINST`-by-`1` vector of dates representing the settle date for each swap. |
| `Maturity` | `NINST`-by-`1` vector of dates representing the maturity date for each swap. |
| `SwapReset` | (Optional) `NINST`-by-`1` vector representing the reset frequency per year for the underlying swap. Default is 1. |
| `Basis` | (Optional) Day-count basis of the instrument. A vector of integers.<br>• 0 = actual/actual (default)<br>• 1 = 30/360 (SIA)<br>• 2 = actual/360<br>• 3 = actual/365<br>• 4 = 30/360 (BMA)<br>• 5 = 30/360 (ISDA)<br>• 6 = 30/360 (European) |

- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/actual (ISDA)
- 13 = BUS/252

For more information, see basis.

| | |
|---|---|
| `Principal` | (Optional) `NINST`-by-`1` vector of the notional principal amounts. Default is 100. |

## Description

To specify a European option: `InstSet = instswaption(OptSpec, Strike, ExerciseDates, Spread, Settle, Maturity)`

To specify an American option: `InstSet = instswaption(OptSpec, Strike, ExerciseDates, Spread, Settle, Maturity, AmericanOpt, SwapReset, Basis, Principal)`

To add swaption instruments to an instrument variable: `InstSet = instswaption(InstSetOld, OptSpec, Strike, ExerciseDates, Spread, ...)`

To list field metadata for the swaption instrument: `[FieldList, ClassList, TypeString] = instswaption;`

Outputs:

| | |
|---|---|
| `InstSet` | Variable containing a collection of instruments. Instruments are broken down by type and each type can have different data fields. Each stored data field has a row vector or string for each instrument. For more information on the `ISet` variable, see `instget`. |
| `FieldList` | `NFIELDS`-by-`1` cell array of strings listing the name of each data field for this instrument type. |
| `ClassList` | `NFIELDS`-by-`1` cell array of strings listing the data class of each field. The class determines how arguments will be parsed. Valid strings are `'dble'`, `'date'`, and `'char'`. |
| `TypeString` | String specifying the type of instrument added. `TypeString = 'Swaption'`. |

## Examples

Create two European swaption instruments with the following data:

```
OptSpec = {'Call'; 'Put'}
Strike = .05;
ExerciseDates = 'jan-1-2011';
Spread=0;
Settle = 'jan-1-2007';
Maturity = 'jan-1-2012';
AmericanOpt = 0;

OptSpec =


    'Call'
    'Put'

InstSet = instswaption(OptSpec, Strike, ExerciseDates, Spread, Settle, Maturity, ...
 AmericanOpt);
```

View the two European swaption instruments by using `instdisp`:

```
instdisp(InstSet)

Indx Type OptSpec Stke ExerDates Spread Settle Maturity AmerOpt SwpReset Basis Prinpal
1   Swaption Call  0.05  01-Jan-2011   0   01-Jan-2007   01-Jan-2012   0   1    0   100
2   Swaption Put   0.05  01-Jan-2011   0   01-Jan-2007   01-Jan-2012   0   1    0   100
```

## See Also

`instadd` | `instget` | `instdisp`

## Purpose

List types

## Syntax

```
TypeList = insttypes(InstSet)
```

## Arguments

| | |
|---|---|
| `InstSet` | Variable containing a collection of instruments. Instruments are classified by type; each type can have different data fields. The stored data field is a row vector or string for each instrument. |

## Description

`TypeList = insttypes(InstSet)` retrieves a list of types stored in an instrument variable.

`TypeList` is a number of types (`NTYPES`)-by-`1` cell array of strings listing the `Type` of instruments contained in the variable.

## Examples

Retrieve the instrument set variable `ExampleInst` from the data file `InstSetExamples.mat`. `ExampleInst` contains three types of instruments: `Option`, `Futures`, and `TBill`.

```
load InstSetExamples;
instdisp(ExampleInst)

Index Type   Strike Price Opt  Contracts
1     Option  95    12.2  Call     0
2     Option 100     9.2  Call     0
3     Option 105     6.8  Call  1000

Index Type    Delivery       F     Contracts
4     Futures 01-Jul-1999    104.4 -1000

Index Type   Strike Price Opt  Contracts
5     Option 105      7.4 Put  -1000
6     Option  95      2.9 Put      0
```

```
Index Type Price Maturity       Contracts
7     TBill 99    01-Jul-1999   6
```

List all of the types included in `ExampleInst`.

```
TypeList = insttypes(ExampleInst)
TypeList =
         'Futures'
         'Option'
         'TBill'
```

## See Also

`instdisp` | `instfields` | `instlength`

## Purpose

Properties of interest-rate structure

## Syntax

```
ParameterValue = intenvget(RateSpec,'ParameterName')
```

## Arguments

| | |
|---|---|
| `RateSpec` | A structure containing the properties of an interest-rate structure. See `intenvset` for information on creating `RateSpec`. |
| `ParameterName` | String indicating the parameter name to be accessed. The value of the named parameter is extracted from the structure `RateSpec`. It is sufficient to type only the leading characters that uniquely identify the parameter. Case is ignored for parameter names. |

## Description

`ParameterValue = intenvget(RateSpec,'ParameterName')` obtains the value of the named parameter `ParameterName` extracted from `RateSpec`.

## Examples

Use `intenvset` to set the interest-rate structure.

```
RateSpec = intenvset('Rates', 0.05, 'StartDates',...
'20-Jan-2000', 'EndDates', '20-Jan-2001')
```

Now use `intenvget` to extract the values from `RateSpec`.

```
[R, RateSpec] = intenvget(RateSpec, 'Rates')

R =

    0.0500
RateSpec =

  FinObj: 'RateSpec'
```

```
Compounding: 2
Disc: 0.9518
Rates: 0.0500
EndTimes: 2
StartTimes: 0
EndDates: 730871
StartDates: 730505
ValuationDate: 730505
Basis: 0
EndMonthRule: 1
```

**See Also** `intenvset`

## Purpose

Price instruments from set of zero curves

## Syntax

```
Price = intenvprice(RateSpec, InstSet)
```

## Arguments

| | |
|---|---|
| `RateSpec` | A structure containing the properties of an interest-rate structure. See `intenvset` for information on creating `RateSpec`. |
| `InstSet` | Variable containing a collection of instruments. Instruments are categorized by type; each type can have different data fields. The stored data field is a row vector or string for each instrument. |

## Description

`Price = intenvprice(RateSpec, InstSet)` computes arbitrage-free prices for instruments against a set of zero coupon bond rate curves.

`Price` is a number of instruments (`NINST`) by number of curves (`NUMCURVES`) matrix of prices of each instrument. If an instrument cannot be priced, a `NaN` is returned in that entry.

`intenvprice` handles the following instrument types: `'Bond'`, `'CashFlow'`, `'Fixed'`, `'Float'`, `'Swap'`. See `instadd` for information about constructing defined types.

See single-type pricing functions to retrieve pricing information.

| | |
|---|---|
| `bondbyzero` | Price bonds from a set of zero curves. |
| `cfbyzero` | Price arbitrary cash flow instrument from a set of zero curves. |
| `fixedbyzero` | Fixed-rate note prices from a set of zero curves. |
| `floatbyzero` | Floating-rate note prices from a set of zero curves. |
| `swapbyzero` | Swap prices from a set of zero curves. |

## Examples

### Load Zero Curves and Instruments from Data File

Load the zero curves and instruments.

```
load deriv.mat
instdisp(ZeroInstSet)
```

```
Index Type CouponRate Settle         Maturity       Period ...
Name    Quantity
1     Bond 0.04      01-Jan-2000    01-Jan-2003    1                            4%
bond 100
2     Bond 0.04       01-Jan-2000    01-Jan-2004    2
4% bond  50

Index Type  CouponRate Settle       Maturity     FixedReset Basis Principal Name
Quantity
3    Fixed 0.04     01-Jan-2000   01-Jan-2003   1       NaN   NaN     4% Fixed 80
```

Price the instruments.

```
Price = intenvprice(ZeroRateSpec, ZeroInstSet)
```

```
Price =

  98.7159
  97.5334
  98.7159
 100.5529
   3.6923
```

### Price a Multi-Stepped Coupon Bond

Price the following multi-stepped coupon bonds using market data.

The data for the interest rate term structure is as follows:

```
Rates = [0.035; 0.042147; 0.047345; 0.052707];
```

```
ValuationDate = 'Jan-1-2010';
StartDates = ValuationDate;
EndDates = {'Jan-1-2011'; 'Jan-1-2012'; 'Jan-1-2013'; 'Jan-1-2014'};
Compounding = 1;
```

Create `RateSpec`.

```
RS = intenvset('ValuationDate', ValuationDate, 'StartDates', StartDates,...
'EndDates', EndDates,'Rates', Rates, 'Compounding', Compounding);
```

Create a portfolio of stepped coupon bonds with different maturities.

```
Settle = '01-Jan-2010';
Maturity = {'01-Jan-2011';'01-Jan-2012';'01-Jan-2013';'01-Jan-2014'};
CouponRate = {{'01-Jan-2011' .042;'01-Jan-2012' .05; '01-Jan-2013' .06; '01-Jan-2014' .07}};
ISet = instbond(CouponRate, Settle, Maturity, 1);
instdisp(ISet)
```

```
Index Type CouponRate Settle    Maturity     Period Basis EndMonthRule ... Face
1  Bond [Cell]   01-Jan-2010    01-Jan-2011    1      0     1          ... 100
2  Bond [Cell]   01-Jan-2010    01-Jan-2012    1      0     1          ... 100
3  Bond [Cell]   01-Jan-2010    01-Jan-2013    1      0     1          ... 100
4  Bond [Cell]   01-Jan-2010    01-Jan-2014    1      0     1          ... 100
```

Table of instrument portfolio is partially displayed.

Compute the price of the stepped coupon bonds.

```
PZero = intenvprice(RS, ISet)
```

```
Index Type CouponRate Settle Maturity  Period Basis EndMonthRule ... Face
1 Bond [Cell] 01-Jan-2010    01-Jan-2011    1      0     1       ... 100
2 Bond [Cell] 01-Jan-2010    01-Jan-2012    1      0     1       ... 100
3 Bond [Cell] 01-Jan-2010    01-Jan-2013    1      0     1       ... 100
4 Bond [Cell] 01-Jan-2010    01-Jan-2014    1      0     1       ... 100


PZero =
```

```
100.6763
100.7368
100.9266
101.0115
```

Table of instrument portfolio is partially displayed.

## See Also

`hjmprice` | `hjmsens` | `instadd` | `intenvsens` | `intenvset`

## Purpose

Instrument price and sensitivities from set of zero curves

## Syntax

```
[Delta, Gamma, Price] = intenvsens(RateSpec, InstSet)
```

## Arguments

| | |
|---|---|
| `RateSpec` | A structure containing the properties of an interest-rate structure. See `intenvset` for information on creating `RateSpec`. |
| `InstSet` | Variable containing a collection of instruments. Instruments are categorized by type; each type can have different data fields. The stored data field is a row vector or string for each instrument. |

## Description

`[Delta, Gamma, Price] = intenvsens(RateSpec, InstSet)` computes dollar prices and price sensitivities for instruments that use a zero coupon bond rate structure.

`Delta` is a number of instruments (`NINST`) by number of curves (`NUMCURVES`) matrix of deltas, representing the rate of change of instrument prices with respect to shifts in the observed forward yield curve. `Delta` is computed by finite differences.

`Gamma` is an `NINST`-by-`NUMCURVES` matrix of gammas, representing the rate of change of instrument deltas with respect to shifts in the observed forward yield curve. `Gamma` is computed by finite differences.

---

**Note** Both sensitivities are returned as dollar sensitivities. To find the per-dollar sensitivities, divide by the respective instrument price.

---

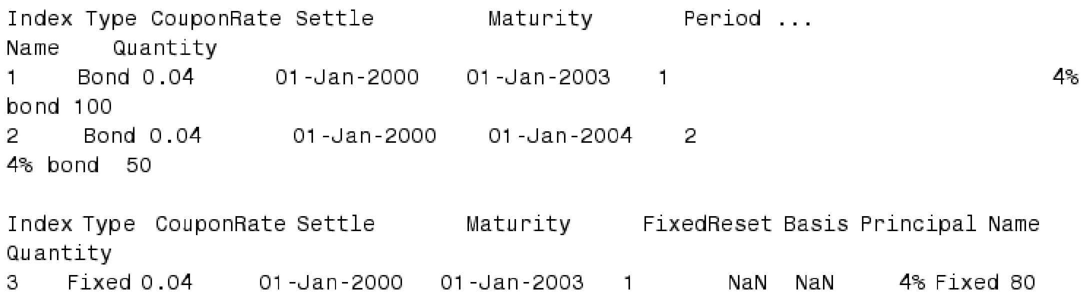
`Price` is an `NINST`-by-`NUMCURVES` matrix of prices of each instrument. If an instrument cannot be priced, a `NaN` is returned.

`intenvsens` handles the following instrument types: `'Bond'`, `'CashFlow'`, `'Fixed'`, `'Float'`, `'Swap'`. See `instadd` for information about constructing defined types.

**Examples**

Load the tree and instruments from a data file.

```
load deriv.mat
instdisp(ZeroInstSet)
```



```
Index Type CouponRate Settle         Maturity       Period ...
Name    Quantity
1     Bond 0.04      01-Jan-2000    01-Jan-2003    1                            4%
bond 100
2     Bond 0.04       01-Jan-2000    01-Jan-2004    2
4% bond  50

Index Type  CouponRate Settle       Maturity      FixedReset Basis Principal Name
Quantity
3    Fixed 0.04     01-Jan-2000   01-Jan-2003   1       NaN   NaN     4% Fixed 80
```

```
[Delta, Gamma] = intenvsens(ZeroRateSpec, ZeroInstSet)

Delta =

 -272.6403
 -347.4386
 -272.6403
   -1.0445
 -282.0405

Gamma =

   1.0e+003 *

     1.0298
     1.6227
     1.0298
```

```
          0.0033
          1.0596
```

## See Also

`hjmprice` | `hjmsens` | `instadd` | `intenvprice` | `intenvset`

## Purpose

Set properties of interest-rate structure

## Syntax

```
[RateSpec, RateSpecOld] = intenvset(RateSpec, 'Argument1',
Value1, 'Argument2', Value2, ...)
[RateSpec, RateSpecOld] = intenvset
intenvset
```

## Arguments

| | |
|---|---|
| `RateSpec` | (Optional) An existing interest-rate specification structure to be changed, probably created from a previous call to `intenvset`. |

Arguments may be chosen from the following table and specified in any order.

| | |
|---|---|
| `Compounding` | Scalar value representing the rate at which the input zero rates were compounded when annualized. Default = `2`. This argument determines the formula for the discount factors:<br>`Compounding` = `1`, `2`, `3`, `4`, `6`, `12`<br>`Disc = (1 + Z/F)^(-T)`, where `F` is the compounding frequency, `Z` is the zero rate, and `T` is the time in periodic units; for example, `T = F` is 1 year.<br>`Compounding` = `365`<br>`Disc = (1 + Z/F)^(-T)`, where `F` is the number of days in the basis year and `T` is a number of days elapsed computed by basis.<br>`Compounding` = `-1` |

| | |
|---|---|
| | `Disc = exp(-T*Z)`, where `T` is time in years. |
| `Disc` | Number of points (`NPOINTS`) by number of curves (`NCURVES`) matrix of unit bond prices over investment intervals from `StartDates`, when the cash flow is valued, to `EndDates`, when the cash flow is received. |
| `Rates` | Number of points (`NPOINTS`) by number of curves (`NCURVES`) matrix of rates in decimal form. For example, 5% is `0.05` in `Rates`. `Rates` are the yields over investment intervals from `StartDates`, when the cash flow is valued, to `EndDates`, when the cash flow is received. |
| `EndDates` | `NPOINTS`-by-`1` vector or scalar of serial maturity dates ending the interval to discount over. |
| `StartDates` | `NPOINTS`-by-`1` vector or scalar of serial dates starting the interval to discount over. Default = `ValuationDate`. `StartDates` must be earlier than `EndDates`. |
| `ValuationDate` | (Optional) Scalar value in serial date number form representing the observation date of the investment horizons entered in `StartDates` and `EndDates`. Default = `min(StartDates)`. |
| `Basis` | (Optional) Day-count basis of the instrument. A scalar of integers.<br>• 0 = actual/actual (default)<br>• 1 = 30/360 (SIA)<br>• 2 = actual/360<br>• 3 = actual/365<br>• 4 = 30/360 (BMA)<br>• 5 = 30/360 (ISDA) |

- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/actual (ISDA)
- 13 = BUS/252

For more information, see basis.

| | |
|---|---|
| `EndMonthRule` | (Optional) End-of-month rule. A scalar. This rule applies only when `EndDates` is an end-of-month date for a month having 30 or fewer days. `0` = ignore rule, meaning that a bond's coupon payment date is always the same numerical day of the month. `1` = set rule on (default), meaning that a bond's coupon payment date is always the last actual day of the month. |

It is sufficient to type only the leading characters that uniquely identify the parameter. Case is ignored for argument names.

When creating a new `RateSpec`, the set of arguments passed to `intenvset` must include `StartDates`, `EndDates`, and either `Rates` or `Disc`.

Call `intenvset` with no input or output arguments to display a list of argument names and possible values.

## Description

`[RateSpec, RateSpecOld] = intenvset(RateSpec, 'Argument1', Value1, 'Argument2', Value2, ...)` creates an interest term structure (`RateSpec`) in which the input argument list is specified as argument name /argument value pairs. The argument name portion of the pair must be recognized as a valid field of the output structure `RateSpec`; the argument value portion of the pair is then assigned to its paired field.

If the optional argument `RateSpec` is specified, `intenvset` modifies an existing interest term structure `RateSpec` by changing the named argument to the specified values and recalculating the arguments dependent on the new values.

`[RateSpec, RateSpecOld] = intenvset` creates an interest term structure `RateSpec` with all fields set to `[]`.

`intenvset` with no input or output arguments displays a list of argument names and possible values.

`RateSpecOld` is a structure containing the properties of an interest-rate structure before the changes introduced by the call to `intenvset`.

## Examples

### Create a RateSpec

Use `intenvset` to create a `RateSpec`.

Use `intenvset` to create a `RateSpec`.

```
RateSpec = intenvset('Rates', 0.05, 'StartDates',...
'20-Jan-2000', 'EndDates', '20-Jan-2001')
```

```
RateSpec =

           FinObj: 'RateSpec'
      Compounding: 2
             Disc: 0.9518
            Rates: 0.0500
         EndTimes: 2
       StartTimes: 0
         EndDates: 730871
       StartDates: 730505
    ValuationDate: 730505
            Basis: 0
```

```
     EndMonthRule: 1
```

Now change the `Compounding` argument to `1` (annual).

```
RateSpec = intenvset(RateSpec, 'Compounding', 1)

RateSpec = 

           FinObj: 'RateSpec'
      Compounding: 1
             Disc: 0.9518
            Rates: 0.0506
         EndTimes: 1
       StartTimes: 0
         EndDates: 730871
       StartDates: 730505
    ValuationDate: 730505
            Basis: 0
     EndMonthRule: 1
```

Calling `intenvset` with no input or output arguments displays a list of argument names and possible values.

```
intenvset

 Compounding: [ 1 | {2} | 3 | 4 | 6 | 12 | 365 | -1 ]
                  Disc: [ scalar | vector (NPOINTS x 1) ]
                 Rates: [ scalar | vector (NPOINTS x 1) ]
              EndDates: [ scalar | vector (NPOINTS x 1) ]
            StartDates: [ scalar | vector (NPOINTS x 1) ]
         ValuationDate: [ scalar ]
                 Basis: [ {0} | 1 | 2 | 3 ]
          EndMonthRule: [ 0 | {1} ]
```

### Create RateSpec Using Two Curves

Create a `RateSpec` for two interest-rate curves.

Define data for the interest-rate term structure and use `intenvset` to create a `RateSpec`.

```
StartDates = '01-Oct-2011';
EndDates = ['01-Oct-2012'; '01-Oct-2013';'01-Oct-2014';'01-Oct-2015'];
Rates = [[0.0356;0.041185;0.04489;0.047741],[0.0325;0.0423;0.0437;0.0465]];
RateSpec = intenvset('Rates', Rates, 'StartDates',StartDates,...
'EndDates', EndDates, 'Compounding', 1)
```

```
RateSpec =

           FinObj: 'RateSpec'
      Compounding: 1
             Disc: [4x2 double]
            Rates: [4x2 double]
         EndTimes: [4x1 double]
       StartTimes: [4x1 double]
         EndDates: [4x1 double]
       StartDates: 734777
    ValuationDate: 734777
            Basis: 0
     EndMonthRule: 1
```

To look at the `Rates` for the two interest-rate curves:

```
RateSpec.Rates
```

```
ans =

    0.0356    0.0325
    0.0412    0.0423
    0.0449    0.0437
    0.0477    0.0465
```

## See Also

`intenvget` | `intenvprice`

# isafin

## Purpose

True if input argument is financial structure type or financial object class

## Syntax

```
IsFinObj = isafin(Obj, ClassName)
```

## Arguments

| | |
|---|---|
| `Obj` | Name of a financial structure. |
| `ClassName` | String containing the name of a financial structure class. |

## Description

`IsFinObj = isafin(Obj, ClassName)` returns `True` if input argument is a financial structure type or financial object class, otherwise `False` is returned.

## Examples

```
load deriv.mat
IsFinObj = isafin(HJMTree, 'HJMFwdTree') returns True
```

## See Also

`classfin`

## Purpose

Price instruments using implied trinomial tree (ITT)

## Syntax

```
Price = ittprice(ITTTree, InstSet)
Price = ittprice(ITTTree, InstSet, Options)
[Price, PriceTree] = ittprice(ITTTree, InstSet, Options)
```

## Arguments

| | |
|---|---|
| `ITTTree` | Implied trinomial stock tree. See `itttree` for information on creating the variable `ITTTree`. |
| `InstSet` | Variable containing a collection of `NINST` instruments. Instruments are broken down by type and each type can have different data fields. |
| `Options` | (Optional) Structure created using `derivset` containing derivative pricing options. |

## Description

`Price = ittprice(ITTTree, InstSet)`

`Price = ittprice(ITTTree, InstSet, Options)`

`[Price, PriceTree] = ittprice(ITTTree, InstSet, Options)`

The outputs for `ittprice` are:

- `Price` is a `NINST`-by-`1` vector of prices of each instrument at time 0. The prices are computed by backward dynamic programming on the stock tree. If an instrument cannot be priced, a NaN is returned in that entry.
- `PriceTree` is a structure containing trees of vectors of instrument prices and a vector of observation times for each node.
  - `PriceTree.PTree` contains the prices.
  - `PriceTree.tObs` contains the observation times.
  - `PriceTree.dObs` contains the observation dates.

`ittprice` computes prices for instruments using an implied trinomial tree created with `itttree`.

---

**Note** `ittprice` handles the following instrument types: optstock, barrier, Asian, lookback, and compound. Use `instadd` to construct the defined types.

---

When using an implied trinomial tree, pricing of path-dependent options is done using Hull-White. Consequently, for these options there are no unique prices on the tree nodes with the exception of the root node. The corresponding nodes of the tree are populated with `NaN`s for these particular options. For information on single-type pricing functions to retrieve state-by-state pricing tree information, see the following:

- `barrierbyitt` for pricing barrier options using an ITT tree
- `optstockbyitt` for pricing American, European or Bermuda options using an ITT tree
- `asianbyitt` for pricing Asian options using an ITT tree
- `lookbackbyitt` for pricing lookback options using an ITT tree
- `compoundbyitt` for price compound options using an ITT tree

## Examples

Load the ITT tree and instruments from the data file `deriv.mat`.

```
load deriv.mat
```

Price the barrier and Asian options contained in the instrument set.

```
ITTSubSet = instselect(ITTInstSet,'Type', {'Barrier', 'Asian'});

instdisp(ITTSubSet)

instdisp(ITTSubSet)
Index Type OptSpec Strike Settle ExerDates AmerOpt BarrSpec Barr Rebate Name Quantity
```

```
1   Barrier call    85   01-Jan-2006  31-Dec-2008    1       ui    115     0  Barrier1 1

IndxType OptSpec Strike Settle ExerDates AmerOpt AvgType AvgPrice AvgDate Name Quantity
2   Asian call    55   01-Jan-2006   01-Jan-2008   0    arithmetic NaN    NaN   Asian1 5
3   Asian call    55   01-Jan-2006   01-Jan-2010   0    arithmetic NaN    NaN   Asian2 7

[Price, PriceTree] = ittprice(ITTTree, ITTSubSet)

Price =
    2.4074
    3.2052
    6.6074
PriceTree =
    FinObj: 'TrinPriceTree'
     PTree: {[3x1 double]  [3x3 double]  [3x5 double]  [3x7 double]  [3x9 double]}
      tObs: [0 1 2 3 4]
      dObs: [732678 733043 733408 733773 734139]
```

## See Also

`ittsens` | `itttree`

# ittsens

## Purpose

Instrument sensitivities and prices using implied trinomial tree (ITT)

## Syntax

```
[Delta, Gamma, Vega] = ittsens(ITTTree, InstSet)
[Delta, Gamma, Vega, Price] = ittsens(ITTTree, InstSet)
[Delta, Gamma, Vega, Price] = ittsens(ITTTree, InstSet,
Options)
```

## Arguments

| | |
|---|---|
| `ITTTree` | Implied trinomial stock tree. See `itttree` for information on creating the variable `ITTTree`. |
| `InstSet` | Variable containing a collection of `NINST` instruments. Instruments are broken down by type and each type can have different data fields. |
| `Options` | (Optional) Structure created using `derivset` containing derivative pricing options. |

## Description

`[Delta, Gamma, Vega] = ittsens(ITTTree, InstSet)`

`[Delta, Gamma, Vega, Price] = ittsens(ITTTree, InstSet)`

`[Delta, Gamma, Vega, Price] = ittsens(ITTTree, InstSet, Options)`

The outputs for `ittsens` are:

- `Delta` is a `NINST`-by-1 vector of deltas, representing the rate of change of instruments prices with respect to changes in the stock price.
- `Gamma` is a `NINST`-by-1 vector of gammas, representing the rate of change of instruments deltas with respect to changes in the stock price.
- `Vega` is a `NINST`-by-1 vector of vegas, representing the rate of change of instruments prices with respect to changes in the volatility of the stock. `Vega` is computed by finite differences in calls to `itttree`.

- `Price` is a `NINST`-by-`1` vector of prices of each instrument. The prices are computed by backward dynamic programming on the stock tree. If an instrument cannot be priced, a NaN is returned.

`ittsens` computes dollar sensitivities and prices for instruments using an ITT tree created with `itttree`.

---

**Note** `ittsens` handles the following instrument types: optstock, barrier, Asian, lookback, and compound. Use `instadd` to construct the defined types.

---

For path-dependent options (lookbacks and Asians), `Delta` and `Gamma` are computed by finite differences in calls to `ittprice`. For the rest of the options (optstock, barrier, and compound), `Delta` and `Gamma` are computed from the ITT tree and the corresponding option price tree.

All sensitivities are returned as dollar sensitivities. To find the per-dollar sensitivities, they must be divided by their respective instrument price.

## Examples

Load the ITT tree and instruments from the data file `deriv.mat`. Compute the `Delta` and `Gamma` sensitivities of vanilla options and barrier option contained in the instrument set.

```
load deriv.mat
ITTSubSet = instselect(ITTInstSet,'Type', {'OptStock', 'Barrier'});

instdisp(ITTSubSet)

Index Type  OptSpec Strike Settle  ExerciseDates  AmericanOpt Name  Quantity

1   OptStock call    95   01-Jan-2006    31-Dec-2008    1     Call1  10

2   OptStock put     80   01-Jan-2006    01-Jan-2010    0     Put1   4

Index Type OptSpec Strike Settle ExercDates AmerOpt BarrSpec Barr Rebate Name Quantity
```

```
3   Barrier call    85   01-Jan-2006  31-Dec-2008    1     ui    115    0    Barrier1 1

[Delta, Gamma] = ittsens(ITTTree, ITTSubSet)


Warning: The option set specified in StockOptSpec was too narrow for the generated tree.
This made extrapolation necessary. Below is a list of the options that were outside of
the range of those specified in StockOptSpec.

Option Type: 'call'   Maturity: 01-Jan-2007  Strike=67.2897
Option Type: 'put'   Maturity: 01-Jan-2007  Strike=37.1528
Option Type: 'put'   Maturity: 01-Jan-2008  Strike=27.6066
Option Type: 'put'   Maturity: 31-Dec-2008  Strike=20.5132
Option Type: 'call'   Maturity: 01-Jan-2010  Strike=164.0157
Option Type: 'put'   Maturity: 01-Jan-2010  Strike=15.2424

> In itttree>InterpOptPrices at 675
  In itttree at 277
  In stocktreesens>stocktreevega at 191
  In stocktreesens at 92
  In ittsens at 81

Delta =

    0.2387
   -0.4283
    0.3482

Gamma =

   0.0260
   0.0188
   0.0380
```

## References

Chriss, Neil. and I. Kawaller, *Black-Scholes and Beyond: Options Pricing Models*, McGraw-Hill, 1996, pp. 308-312.

**See Also** `ittprice` | `itttree`

## Purpose

Specify time structure using implied trinomial tree (ITT)

## Syntax

```
TimeSpec = itttimespec(ValuationDate, Maturity, NumPeriods)
```

## Arguments

| | |
|---|---|
| `ValuationDate` | Scalar date marking the pricing date and first observation in the tree. Specify `ValuationDate` as a serial date number or date string. |
| `Maturity` | Scalar date marking the depth of the tree. |
| `NumPeriods` | Scalar that determines how many time steps are in the tree. |

## Description

`TimeSpec = itttimespec(ValuationDate, Maturity, NumPeriods)` creates the structure specifying the time layout for an ITT tree.

## Examples

Specify a four-period tree with time steps of 1 year.

```
ValuationDate = '1-July-2006';
Maturity = '1-July-2010';
TimeSpec = itttimespec(ValuationDate, Maturity, 4);
```

## See Also

`itttree` | `stockspec`

## Purpose

Build implied trinomial stock tree

## Syntax

`itttree(StockSpec, RateSpec, TimeSpec, StockOptSpec)`

## Arguments

| | |
|---|---|
| `StockSpec` | Stock specification. For more information, see `stockspec`. |
| `RateSpec` | Interest rate specification of the initial risk-free rate curve. For more information on declaring an interest rate variable, see `intenvset`. |
| `TimeSpec` | Tree time layout specification. Defines the observation dates of the implied trinomial tree. For more information on the tree structure, see `itttimespec`. |
| `StockOptSpec` | Option stock specification. For more information, see `stockoptspec`. |

## Description

`itttree(StockSpec, RateSpec, TimeSpec, StockOptSpec)` creates the `itttree` structure specifying stock and time information for an implied trinomial tree.

## Examples

For this example, assume that the interest rate is fixed at 8% annually between the valuation date of the tree (January 1, 2006) until its maturity.

```
Rate = 0.08;
ValuationDate = '01-01-2006';
EndDate = '01-01-2008';

RateSpec = intenvset('StartDates', ValuationDate, 'EndDates', EndDate, ...
    'ValuationDate', ValuationDate, 'Rates', Rate, 'Compounding', -1);
```

To build an ITTTree, create `StockSpec`, `TimeSpec`, and `StockOptSpec` structures.

To create the`StockSpec` structure:

```
Sigma = 0.20;
AssetPrice = 50;
DividendType = 'cash';
DividendAmounts = [0.50; 0.50; 0.50; 0.50];
ExDividendDates = {'03-Jan-2007'; '01-Apr-2007'; '05-July-2007';'01-Oct-2007'}

StockSpec = stockspec(Sigma, AssetPrice, DividendType, ...
DividendAmounts, ExDividendDates)
StockSpec =

             FinObj: 'StockSpec'
              Sigma: 0.2000
         AssetPrice: 50
       DividendType: 'cash'
    DividendAmounts: [4x1 double]
    ExDividendDates: [4x1 double]
```

The syntax for building a `TimeSpec` structure is `TimeSpec = itttimespec(ValuationDate, Maturity, NumPeriods)`.

Consider building an ITT tree, with a valuation date of January 1, 2006; a maturity date of January 1, 2008; and four time steps.

```
ValuationDate = '01-01-2006';
EndDate = '01-01-2008';
NumPeriods = 4;

TimeSpec = itttimespec(ValuationDate, EndDate, NumPeriods)

TimeSpec =

           FinObj: 'ITTTimeSpec'
    ValuationDate: 732678
```

```
        Maturity: 733408
      NumPeriods: 4
           Basis: 0
    EndMonthRule: 1
            tObs: [0 0.5000 1 1.5000 2]
            dObs: [732678 732860 733043 733225 733408]
```

The syntax for building a `StockOptSpec` structure is `[StockOptSpec] = stockoptspec(OptPrice, Strike, Settle, Maturity, OptSpec)`.

```
Settle =   '01/01/06';

Maturity =    ['07/01/06';
    '07/01/06';
    '07/01/06';
    '07/01/06';
    '01/01/07';
    '01/01/07';
    '01/01/07';
    '01/01/07';
    '07/01/07';
    '07/01/07';
    '07/01/07';
    '07/01/07';
    '01/01/08';
    '01/01/08';
    '01/01/08';
    '01/01/08'];

Strike = [113;
   101;
   100;
    88;
   128;
   112;
   100;
    78;
```

```
   144;
   112;
   100;
    69;
   162;
   112;
   100;
    61];

OptPrice =[                 0;
   4.807905472659144;
   1.306321897011867;
   0.048039195057173;
                   0;
   2.310953054191461;
   1.421950392866235;
   0.020414826276740;
                   0;
   5.091986935627730;
   1.346534812295291;
   0.005101325584140;
                   0;
   8.047628153217246;
   1.219653432150932;
   0.001041436654748];


OptSpec = { 'call';
    'call';
    'put';
    'put';
    'call';
    'call';
    'put';
    'put';
    'call';
    'call';
```

```
    'put';
    'put';
    'call';
    'call';
    'put';
    'put'};

StockOptSpec = stockoptspec(OptPrice, Strike, Settle, Maturity, OptSpec)

StockOptSpec =

          FinObj: 'StockOptSpec'
        OptPrice: [16x1 double]
          Strike: [16x1 double]
          Settle: 732678
        Maturity: [16x1 double]
         OptSpec: {16x1 cell}
    InterpMethod: 'price'
```

---

**Note** In this example, the extrapolation warnings are turned on to display warnings on the Command Window. These warnings are a consequence of having to extrapolate to find the option price of the tree nodes. In this example, the set of inputs options was too narrow for the shift in the tree nodes introduced by the disturbance used to calculate the sensitivities. As a consequence extrapolation for some of the nodes was needed.

---

Use the following command to turn on extrapolation warnings:

```
warning('on', 'fininst:itttree:Extrapolation');
```

Use the `StockSpec`, `RateSpec`, `TimeSpec`, and `StockOptSpec` structure to create an `ITTTree`.

```
ITTTree = itttree(StockSpec, RateSpec, TimeSpec, StockOptSpec)
```

```
Warning: The option set specified in StockOptSpec was too narrow for the generated tree.
This made extrapolation necessary. Below is a list of the options that were outside of
the range of those specified in StockOptSpec.

Option Type: 'call'   Maturity: 02-Jul-2006  Strike=60.7466
Option Type: 'put'   Maturity: 02-Jul-2006  Strike=50.0731
Option Type: 'put'   Maturity: 02-Jul-2006  Strike=41.3344
Option Type: 'call'   Maturity: 01-Jan-2007  Strike=73.8592
Option Type: 'call'   Maturity: 01-Jan-2007  Strike=60.8227
Option Type: 'put'   Maturity: 01-Jan-2007  Strike=50.1492
Option Type: 'put'   Maturity: 01-Jan-2007  Strike=41.4105
Option Type: 'put'   Maturity: 01-Jan-2007  Strike=34.2559
Option Type: 'call'   Maturity: 02-Jul-2007  Strike=88.8310
Option Type: 'call'   Maturity: 02-Jul-2007  Strike=72.9081
Option Type: 'call'   Maturity: 02-Jul-2007  Strike=59.8715
Option Type: 'put'   Maturity: 02-Jul-2007  Strike=49.1980
Option Type: 'put'   Maturity: 02-Jul-2007  Strike=40.4594
Option Type: 'put'   Maturity: 02-Jul-2007  Strike=33.3047
Option Type: 'put'   Maturity: 02-Jul-2007  Strike=27.4470
Option Type: 'call'   Maturity: 01-Jan-2008  Strike=107.2895
Option Type: 'call'   Maturity: 01-Jan-2008  Strike=87.8412
Option Type: 'call'   Maturity: 01-Jan-2008  Strike=71.9183
Option Type: 'call'   Maturity: 01-Jan-2008  Strike=58.8817
Option Type: 'put'   Maturity: 01-Jan-2008  Strike=48.2083
Option Type: 'put'   Maturity: 01-Jan-2008  Strike=39.4696
Option Type: 'put'   Maturity: 01-Jan-2008  Strike=32.3150
Option Type: 'put'   Maturity: 01-Jan-2008  Strike=26.4573
Option Type: 'put'   Maturity: 01-Jan-2008  Strike=21.6614

> In itttree>InterpOptPrices at 675
  In itttree at 277

ITTTree =

          FinObj: 'ITStockTree'
       StockSpec: [1x1 struct]
    StockOptSpec: [1x1 struct]
```

```
TimeSpec: [1x1 struct]
RateSpec: [1x1 struct]
    tObs: [0 0.500000000000000 1 1.500000000000000 2]
    dObs: [732678 732860 733043 733225 733408]
   STree: {1x5 cell}
   Probs: {[3x1 double]  [3x3 double]  [3x5 double]  [3x7 double]}
```

## See Also

`intenvset` | `itttimespec` | `stockoptspec` | `stockspec`

## Purpose

Price lookback option from Cox-Ross-Rubinstein tree

## Syntax

```
PriceTree = lookbackbycrr(CRRTree, OptSpec, Strike,
Settle, ExerciseDates, AmericanOpt)
```

## Arguments

| | |
|---|---|
| `CRRTree` | Stock tree structure created by `crrtree`. |
| `OptSpec` | Number of instruments (`NINST`)-by-`1` cell array of strings `'call'` or `'put'`. |
| `Strike` | `NINST`-by-`1` vector of strike price values. Each row is the schedule for one option. To calculate the value of a floating-strike lookback option, specify `Strike` as `NaN`. |
| `Settle` | `NINST`-by-`1` vector of `Settle` dates. The settle date for every lookback is set to the valuation date of the stock tree. The lookback argument `Settle` is ignored. |
| `ExerciseDates` | For a European option (`AmericanOpt = 0`):<br>`NINST`-by-`1` vector of exercise dates. Each row is the schedule for one option. For a European option, there is only one exercise date, the option expiry date.<br>For an American option (`AmericanOpt = 1`):<br>`NINST`-by-`2` vector of exercise date boundaries. For each instrument, the option can be exercised on any tree date between or including the pair of dates on that row. If only one non-`NaN` date is listed, or if `ExerciseDates` is `NINST`-by-`1`, the option can be exercised |

| | |
|---|---|
| | between the valuation date of the stock tree and the single listed exercise date. |
| `AmericanOpt` | (Optional) If `AmericanOpt = 0`, `NaN`, or is unspecified, the option is a European option. If `AmericanOpt = 1`, the option is an American option. |

## Description

`PriceTree = lookbackbycrr(CRRTree, OptSpec, Strike, Settle, ExerciseDates, AmericanOpt)` calculates the value of fixed- and floating-strike lookback options. Data arguments are `NINST`-by-`1` vectors, scalar, or empty. Fill unspecified entries in vectors with `NaN`. Only one data argument is required to create the instrument. The others may be omitted or passed as empty matrices `[]`.

`Price` is a `NINST`-by-`1` vector of expected option prices at time 0.

---

**Note** `lookbackbycrr` calculates values of fixed and floating strike lookback options. To compute the value of a floating strike lookback option, strike should be specified as `NaN`. Pricing of lookback options is done using Hull-White (1993). Consequently, for these options there are not unique prices on the tree nodes with the exception of the root node.

---

## Examples

Price a lookback option using a CRR binomial tree.

Load the file `deriv.mat`, which provides `CRRTree`. The `CRRTree` structure contains the stock specification and time information needed to price the option.

```
load deriv.mat;
```

Set the required values. Other arguments will use defaults.

```
OptSpec = 'Call';
Strike = 115;
```

```
Settle = '01-Jan-2003';
ExerciseDates = '01-Jan-2006';

Price = lookbackbycrr(CRRTree, OptSpec, Strike, Settle, ...
ExerciseDates)

Price =

    7.6015
```

## References

Hull, J., and A. White, "Efficient Procedures for Valuing European and American Path-Dependent Options," *Journal of Derivatives*, Fall 1993, pp. 21-31.

## See Also

`crrtree` | `instlookback`

## Purpose

Price lookback option from Equal Probabilities binomial tree

## Syntax

```
[Price, PriceTree] = lookbackbyeqp(EQPTree, OptSpec, Strike,
Settle, ExerciseDates, AmericanOpt)
```

## Arguments

| | |
|---|---|
| `EQPTree` | Stock tree structure created by `eqptree`. |
| `OptSpec` | Number of instruments (`NINST`)-by-`1` cell array of strings `'call'` or `'put'`. |
| `Strike` | `NINST`-by-`1` vector of strike price values. Each row is the schedule for one option. To calculate the value of a floating-strike lookback option, specify `Strike` as `NaN`. |
| `Settle` | `NINST`-by-`1` vector of `Settle` dates. The settle date for every lookback is set to the valuation date of the stock tree. The lookback argument `Settle` is ignored. |
| `ExerciseDates` | For a European option (`AmericanOpt = 0`):<br>`NINST`-by-`1` vector of exercise dates. Each row is the schedule for one option. For a European option, there is only one exercise date, the option expiry date.<br>For an American option (`AmericanOpt = 1`):<br>`NINST`-by-`2` vector of exercise date boundaries. For each instrument, the option can be exercised on any tree date between or including the pair of dates on that row. If only one non-`NaN` date is listed, or if `ExerciseDates` is `NINST`-by-`1`, the option can be exercised |

| | |
|---|---|
| | between the valuation date of the stock tree and the single listed exercise date. |
| `AmericanOpt` | (Optional) If `AmericanOpt = 0`, `NaN`, or is unspecified, the option is a European option. If `AmericanOpt = 1`, the option is an American option. |

## Description

`Price = lookbackbyeqp(EQPTree, OptSpec, Strike, ExerciseDates, AmericanOpt)` calculates the value of fixed- and floating-strike lookback options. Data arguments are `NINST`-by-1 vectors, scalar, or empty. Fill unspecified entries in vectors with `NaN`. Only one data argument is required to create the instrument. The others may be omitted or passed as empty matrices `[]`.

`Price` is a `NINST`-by-1 vector of expected option prices at time 0.

**Note** `lookbackbyeqp` calculates values of fixed and floating strike lookback options. To compute the value of a floating strike lookback option, strike should be specified as `NaN`. Pricing of lookback options is done using Hull-White (1993). Consequently, for these options there are not unique prices on the tree nodes with the exception of the root node.

## Examples

Price a lookback option using an EQP equity tree.

Load the file `deriv.mat,` which provides `EQPTree`. The `EQPTree` structure contains the stock specification and time information needed to price the option.

```
load deriv.mat
```

Set the required values. Other arguments will use defaults.

```
OptSpec = 'Call';
Strike = 115;
```

```
Settle = '01-Jan-2003';
ExerciseDates = '01-Jan-2006';

Price = lookbackbyeqp(EQPTree, OptSpec, Strike, Settle, ...
ExerciseDates)

Price =

 8.7941
```

## References

Hull, J., and A. White, “Efficient Procedures for Valuing European and American Path-Dependent Options,” *Journal of Derivatives*, Fall 1993, pp. 21-31.

## See Also

`eqptree` | `instlookback`

## Purpose

Price lookback option using implied trinomial tree (ITT)

## Syntax

```
[Price, PriceTree] = lookbackbyitt(ITTTree, OptSpec, Strike,
Settle, ExerciseDates, AmericanOpt)
```

## Arguments

| | |
|---|---|
| `ITTTree` | Stock tree structure created by `itttree`. |
| `OptSpec` | Number of instruments (`NINST`)-by-`1` cell array of strings `'call'` or `'put'`. |
| `Strike` | `NINST`-by-`1` vector of strike price values. Each row is the schedule for one option. To calculate the value of a floating-strike lookback option, specify `Strike` as `NaN`. |
| `Settle` | `NINST`-by-`1` vector of `Settle` dates. The settle date for every lookback option is set to the `ValuationDate` of the stock tree. The lookback argument `Settle` is ignored. |
| `ExerciseDates` | For a European option (`AmericanOpt = 0`):<br>`NINST`-by-`1` vector of exercise dates. Each row is the schedule for one option. For a European option, there is only one exercise date, the option expiry date.<br>For an American option (`AmericanOpt = 1`):<br>`NINST`-by-`2` vector of exercise date boundaries. For each instrument, the option can be exercised on any tree date between or including the pair of dates on that row. If only one non-`NaN` date is listed, or if `ExerciseDates` is `NINST`-by-`1`, the option can be exercised |

| | |
|---|---|
| | between the valuation date of the stock tree and the single listed exercise date. |
| `AmericanOpt` | (Optional) If `AmericanOpt = 0`, `NaN`, or is unspecified, the option is a European option. If `AmericanOpt = 1`, the option is an American option. |

## Description

`Price = lookbackbyitt(ITTTree, OptSpec, Strike, Settle, ExerciseDates, AmericanOpt)` calculates the value of fixed- and floating-strike lookback options. Data arguments for `lookbackbyitt` are `NINST`-by-`1` vectors, scalar, or empty. Fill unspecified entries in vectors with `NaN`. Only one data argument is required to create the instrument; the others may be omitted or passed as empty matrices `[]`.

`Price` is a `NINST`-by-`1` vector of expected option prices at time 0.

---

**Note** `lookbackbyitt` calculates values of fixed and floating strike lookback options. To compute the value of a floating strike lookback option, strike should be specified as NaN. Pricing of lookback options is done using Hull-White (1993). Consequently, for these options there are not unique prices on the tree nodes with the exception of the root node.

---

## Examples

Price a lookback option using an ITT equity tree.

Load the file `deriv.mat` which provides the `ITTTree`. The `ITTTree` structure contains the stock specification and time information needed to price the option.

```
load deriv.mat
```

Set the required values. Other arguments will use defaults.

```
OptSpec = 'Call';
Strike = 85;
```

```
Settle = '01-Jan-2006';
ExerciseDates = '01-Jan-2008';

Price = lookbackbyitt(ITTTree, OptSpec, Strike, Settle, ExerciseDates)

Price =

    0.5426
```

## References

Hull, J., and A. White, "Efficient Procedures for Valuing European and American Path-Dependent Options," *Journal of Derivatives*, Fall 1993, pp. 21-31.

## See Also

`instlookback` | `itttree`

## Purpose

Specify time structure for Leisen-Reimer binomial tree

## Syntax

```
TimeSpec = lrtimespec(ValuationDate, Maturity, NumPeriods)
```

## Description

`TimeSpec = lrtimespec(ValuationDate, Maturity, NumPeriods)` specifies a time structure for a Leisen-Reimer stock tree.

## Input Arguments

### ValuationDate

Scalar date marking the pricing date and first observation in the Leisen-Reimer stock tree. Specify `ValuationDate` as a serial date number or date string.

### Maturity

Scalar date marking the depth of the Leisen-Reimer stock tree.

### NumPeriods

Scalar value determining how many time steps are in the Leisen-Reimer stock tree.

---

**Note** Leisen-Reimer requires the number of steps to be an odd number.

---

## Output Arguments

### TimeSpec

Structure specifying the time layout for a Leisen-Reimer stock tree.

## Examples

Specify a 5-period tree with time steps of 1 year:

```
ValuationDate = '1-July-2010';
Maturity = '1-July-2015';
TimeSpec = lrtimespec(ValuationDate, Maturity, 5);
TimeSpec =

 FinObj: 'BinTimeSpec'
```

```
    ValuationDate: 734320
         Maturity: 736146
       NumPeriods: 5
            Basis: 0
     EndMonthRule: 1
             tObs: [0 1 2 3 4 5]
             dObs: [734320 734685 735050 735415 735780 736146]
```

## References

Leisen D.P., M. Reimer, "Binomial Models for Option Valuation – Examining and Improving Convergence," *Applied Mathematical Finance*, Number 3, 1996, pp. 319-346.

## See Also

`| stockspec | lrtree`

## Purpose

Build Leisen-Reimer stock tree

## Syntax

```
LRTree = lrtree(StockSpec, RateSpec, TimeSpec, Strike)
LRTree = lrtree(StockSpec, RateSpec, TimeSpec,
Strike, Name,Value)
```

## Description

`LRTree = lrtree(StockSpec, RateSpec, TimeSpec, Strike)` constructs a Leisen-Reimer stock tree.

`LRTree = lrtree(StockSpec, RateSpec, TimeSpec, Strike, Name,Value)` constructs a Leisen-Reimer stock tree with additional options specified by one or more `Name,Value` pair arguments.

## Input Arguments

**StockSpec**

Stock specification. For more information, see `stockspec`.

**RateSpec**

Interest rate specification of the initial risk-free rate curve. For more information, see `intenvset`.

**TimeSpec**

Tree time layout specification. For more information, see `lrtimespec`.

**Strike**

Scalar defining the option strike.

### Name-Value Pair Arguments

Specify optional comma-separated pairs of `Name,Value` arguments, where `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside single quotes (`' '`). You can specify several name and value pair arguments in any order as `Name1,Value1,...,NameN,ValueN`.

**Method**

String value. Use `PP1` for Peizer-Pratt method 1 inversion and `PP2` for Peizer-Pratt method 2 inversion. For more information on `PP1` and `PP2` methods, see "Leisen-Reimer Tree (LR) Modeling" on page C-7.

**Default:** `PP1`

## Output Arguments

**LRTree**

Structure specifying stock and time information for a Leisen-Reimer tree.

## Examples

Consider a European put option with an exercise price of $30 that expires on June 1, 2010. The underlying stock is trading at $30 on January 1, 2010 and has a volatility of 30% per annum. The annualized continuously compounded risk-free rate is 5% per annum. Using this data, create a Leisen-Reimer tree with 101 steps using the `PP1` method.

```
AssetPrice = 30;
Strike = 30;

ValuationDate = 'Jan-1-2010';
Maturity = 'June-1-2010';

% Define StockSpec
Sigma = 0.3;
StockSpec = stockspec(Sigma, AssetPrice);

% Define RateSpec
Rates = 0.05;
Settle = ValuationDate;
Basis = 1;
Compounding = -1;

RateSpec = intenvset('ValuationDate', ValuationDate, 'StartDates', Settle, ...
'EndDates', Maturity, 'Rates', Rates, 'Compounding', Compounding, 'Basis', Basis);

% Build the Leisen-Reimer (LR) tree with 101 steps
```

```
LRTimeSpec = lrtimespec(ValuationDate, Maturity, 101);

% Use the PP1 method
LRMethod  = 'PP1';

LRTree = lrtree(StockSpec, RateSpec, LRTimeSpec, Strike, ...
'method', LRMethod)

LRTree =

       FinObj: 'BinStockTree'
       Method: 'LR'
    Submethod: 'PP1'
       Strike: 30
    StockSpec: [1x1 struct]
     TimeSpec: [1x1 struct]
     RateSpec: [1x1 struct]
         tObs: [1x102 double]
         dObs: [1x102 double]
        STree: {1x102 cell}
      UpProbs: [101x1 double]
```

## References

Leisen D.P., M. Reimer, "Binomial Models for Option Valuation – Examining and Improving Convergence," *Applied Mathematical Finance*, Number 3, 1996, pp. 319-346.

## See Also

`| stockspec | lrtimespec | intenvset | optstockbylr | optstocksensbylr`

# maxassetbystulz

## Purpose

Determine European rainbow option price on maximum of two risky assets using Stulz option pricing model

## Syntax

```
Price = maxassetbystulz(RateSpec, StockSpec1, StockSpec2,
Settle, Maturity, OptSpec, Strike, Corr)
```

## Arguments

| | |
|---|---|
| `RateSpec` | The annualized, continuously compounded rate term structure. For information on the interest rate specification, see `intenvset`. |
| `StockSpec1` | Stock specification for asset 1. See `stockspec`. |
| `StockSpec2` | Stock specification for asset 2. See `stockspec`. |
| `Settle` | `NINST`-by-`1` vector of settlement or trade dates. |
| `Maturity` | `NINST`-by-`1` vector of maturity dates. |
| `OptSpec` | `NINST`-by-`1` cell array of strings `'call'` or `'put'`. |
| `Strike` | `NINST`-by-`1` vector of strike price values. |
| `Corr` | `NINST`-by-`1` vector of correlation between the underlying asset prices. |

## Description

`Price = maxassetbystulz(RateSpec, StockSpec1, StockSpec2, Settle, Maturity, OptSpec, Strike, Corr)` computes rainbow option prices using the Stulz option pricing model.

`Price` is a `NINST`-by-`1` vector of expected option prices.

## Examples

Consider a European rainbow option that gives the holder the right to buy either $100,000 worth of an equity index at a strike price of 1000 (asset 1) or $100,000 of a government bond (asset 2) with a strike price of 100% of face value, whichever is worth more at the end of 12 months. On January 15, 2008, the equity index is trading at 950, pays a dividend of 2% annually and has a return volatility of 22%. Also on January 15, 2008, the government bond is trading at 98, pays a coupon yield of 6%, and has a return volatility of 15%. The risk-free rate is 5%. Using this data, if the correlation between the rates of return is -0.5, 0, and 0.5, calculate the price of the European rainbow option.

Since the asset prices in this example are in different units, it is necessary to work in either index points (asset 1) or in dollars (asset 2). The European rainbow option allows the holder to buy the following: 100 units of the equity index at $1000 each (for a total of $100,000) or 1000 units of the government bonds at $100 each (for a total of $100,000). To convert the bond price (asset 2) to index units (asset 1), you must make the following adjustments:

- Multiply the strike price and current price of the government bond by 10 (1000/100).
- Multiply the option price by 100, considering that there are 100 equity index units in the option.

Once these adjustments are introduced, the strike price is the same for both assets ($1000).

Create the `RateSpec`:

```
Settle = 'Jan-15-2008';
Maturity = 'Jan-15-2009';
Rates = 0.05;
Basis = 1;

RateSpec = intenvset('ValuationDate', Settle, 'StartDates', Settle,...
'EndDates', Maturity, 'Rates', Rates, 'Compounding', -1, 'Basis', Basis);
```

Create the two `StockSpec` definitions:

```
AssetPrice1 = 950;   % Asset 1 => Equity index
AssetPrice2 = 980;   % Asset 2 => Government bond
Sigma1 = 0.22;
Sigma2 = 0.15;
Div1 = 0.02;
```

```
Div2 = 0.06;

StockSpec1 = stockspec(Sigma1, AssetPrice1, 'continuous', Div1);
StockSpec2 = stockspec(Sigma2, AssetPrice2, 'continuous', Div2);
```

Calculate the price of the options for different correlation levels:

```
Strike = 1000 ;
Corr = [-0.5; 0; 0.5];
OptSpec = 'call';

Price = maxassetbystulz(RateSpec, StockSpec1, StockSpec2,...
Settle, Maturity, OptSpec, Strike, Corr)

Price =

  111.6683
  103.7715
   92.4412
```

These are the prices of one unit. This means that the premium is 11166.83, 10377.15, and 9244.12 (for 100 units).

## See Also

`intenvset` | `maxassetsensbystulz` | `minassetbystulz` | `stockspec`

## Purpose

Determine European rainbow option prices and sensitivities on maximum of two risky assets using Stulz pricing model

## Syntax

```
PriceSens = maxassetsensbystulz(RateSpec, StockSpec1,
StockSpec2, Settle, Maturity, OptSpec, Strike, Corr)
PriceSens = maxassetsensbystulz(RateSpec, StockSpec1,
 StockSpec2, Settle, Maturity, OptSpec, Strike,
Corr, OutSpec)
```

## Arguments

| | |
|---|---|
| `RateSpec` | The annualized, continuously compounded rate term structure. For information on the interest rate specification, see `intenvset`. |
| `StockSpec1` | Stock specification for asset 1. See `stockspec`. |
| `StockSpec2` | Stock specification for asset 2. See `stockspec`. |
| `Settle` | `NINST`-by-`1` vector of settlement or trade dates. |
| `Maturity` | `NINST`-by-`1` vector of maturity dates. |
| `OptSpec` | `NINST`-by-`1` cell array of strings `'call'` or `'put'`. |
| `Strike` | `NINST`-by-`1` vector of strike price values. |
| `Corr` | `NINST`-by-`1` vector of correlation between the underlying asset prices. |
| `OutSpec` | (Optional) All optional inputs are specified as matching parameter name/value pairs. The parameter name is specified as a character string, followed by the corresponding parameter value. You can specify parameter name/value pairs in any order. Names are case-insensitive and partial string matches are allowed provided no ambiguities exist. Valid parameter names are:<br>• `NOUT`-by-`1` or `1`-by-`NOUT` cell array of strings indicating the nature and order of the outputs |

for the function. Possible values are `'Price'`, `'Delta'`, `'Gamma'`, `'Vega'`, `'Lambda'`, `'Rho'`, `'Theta'`, or `'All'`.

For example, `OutSpec = {'Price'; 'Lambda'; 'Rho'}` specifies that the output should be `Price`, `Lambda`, and `Rho`, in that order.

To invoke from a function: `[Price, Lambda, Rho] = maxassetsensbystulz(..., 'OutSpec', {'Price', 'Lambda', 'Rho'})`

`OutSpec = {'All'}` specifies that the output should be `Delta`, `Gamma`, `Vega`, `Lambda`, `Rho`, `Theta`, and `Price`, in that order. This is the same as specifying `OutSpec` as `OutSpec = {'Delta', 'Gamma', 'Vega', 'Lambda', 'Rho', 'Theta', 'Price'};`.

- Default is `OutSpec = {'Price'}`.

## Description

`PriceSens = maxassetsensbystulz(RateSpec, StockSpec1, StockSpec2, Settle, Maturity, OptSpec, Strike, Corr)` computes rainbow option prices using the Stulz option pricing model.

`PriceSens = maxassetsensbystulz(RateSpec, StockSpec1, StockSpec2, Settle, Maturity, OptSpec, Strike, Corr, OutSpec)` computes rainbow option prices and sensitivities using the Stulz option pricing model.

`PriceSens` is a `NINST`-by-1 or `NINST`-by-2 vector of expected prices and sensitivities values.

## Examples

Consider a European rainbow option that gives the holder the right to buy either $100,000 of an equity index at a strike price of 1000 (asset 1) or $100,000 of a government bond (asset 2) with a strike price of 100% of face value, whichever is worth more at the end of 12 months. On

January 15, 2008, the equity index is trading at 950, pays a dividend of 2% annually, and has a return volatility of 22%. Also on January 15, 2008, the government bond is trading at 98, pays a coupon yield of 6%, and has a return volatility of 15%. The risk-free rate is 5%. Using this data, calculate the price and sensitivity of the European rainbow option if the correlation between the rates of return is -0.5, 0, and 0.5.

Since the asset prices in this example are in different units, it is necessary to work in either index points (for asset 1) or in dollars (for asset 2). The European rainbow option allows the holder to buy the following: 100 units of the equity index at $1000 each (for a total of $100,000) or 1000 units of the government bonds at $100 each (for a total of $100,000). To convert the bond price (asset 2) to index units (asset 1), uou must make the following adjustments:

- Multiply the strike price and current price of the government bond by 10 (1000/100).
- Multiply the option price by 100, considering that there are 100 equity index units in the option.

Once these adjustments are introduced, the strike price is the same for both assets ($1000).

Create the `RateSpec`:

```
Settle = 'Jan-15-2008';
Maturity = 'Jan-15-2009';
Rates = 0.05;
Basis = 1;

RateSpec = intenvset('ValuationDate', Settle, 'StartDates', Settle,...
'EndDates', Maturity, 'Rates', Rates, 'Compounding', -1, 'Basis', Basis);
```

Create the two `StockSpec` definitions:

```
AssetPrice1 = 950;   % Asset 1 => Equity index
AssetPrice2 = 980;   % Asset 2 => Government bond
Sigma1 = 0.22;
Sigma2 = 0.15;
```

```
Div1 = 0.02;
Div2 = 0.06;

StockSpec1 = stockspec(Sigma1, AssetPrice1, 'continuous', Div1);
StockSpec2 = stockspec(Sigma2, AssetPrice2, 'continuous', Div2);
```

Calculate the price and delta for different correlation levels:

```
Strike = 1000 ;
Corr = [-0.5; 0; 0.5];
OutSpec = {'price'; 'delta'};
[Price, Delta] = maxassetsensbystulz(RateSpec, StockSpec1, StockSpec2,...
Settle, Maturity, OptSpec, Strike, Corr,'OutSpec', OutSpec)

Price =

  111.6683
  103.7715
   92.4412

Delta =

    0.4594    0.3698
    0.4292    0.3166
    0.4053    0.2512
```

The output `Delta` has two columns: the first column represents the `Delta` with respect to the equity index (asset 1), and the second column represents the `Delta` with respect to the government bond (asset 2). The value 0.4595 represents `Delta` with respect to one unit of the equity index. Since there are 100 units of the equity index, the overall `Delta` would be 45.94 (100 * 0.4594 ) for a correlation level of -0.5. To calculate the `Delta` with respect to the government bond, remember that an adjusted price of 980 was used instead of 98. Therefore, for example, the `Delta` with respect to government bond, for a correlation of 0.5 would be 251.2 (0.2512 * 100 * 10 ).

**See Also** `intenvset` | `maxassetbystulz` | `stockspec`

# minassetbystulz

## Purpose

Determine European rainbow option prices on minimum of two risky assets using Stulz option pricing model

## Syntax

```
Price = minassetbystulz(RateSpec, StockSpec1, StockSpec2,
Settle, Maturity, OptSpec, Strike, Corr)
```

## Arguments

| | |
|---|---|
| `RateSpec` | The annualized, continuously compounded rate term structure. For information on the interest rate specification, see `intenvset`. |
| `StockSpec1` | Stock specification for asset 1. See `stockspec`. |
| `StockSpec2` | Stock specification for asset 2. See `stockspec`. |
| `Settle` | `NINST`-by-`1` vector of settlement or trade dates. |
| `Maturity` | `NINST`-by-`1` vector of maturity dates. |
| `OptSpec` | `NINST`-by-`1` cell array of strings `'call'` or `'put'`. |
| `Strike` | `NINST`-by-`1` vector of strike price values. |
| `Corr` | `NINST`-by-`1` vector of correlation between the underlying asset prices. |

## Description

`Price = minassetbystulz(RateSpec, StockSpec1, StockSpec2, Settle, Maturity, OptSpec, Strike, Corr)` computes option prices using the Stulz option pricing model.

`Price` is a `NINST`-by-`1` vector of expected option prices.

## Examples

Consider a European rainbow put option that gives the holder the right to sell either stock A or stock B at a strike of 50.25, whichever has the lower value on the expiration date May 15, 2009. On November 15, 2008, stock A is trading at 49.75 with a continuous annual dividend yield of 4.5% and has a return volatility of 11%. Stock B is trading at 51 with a continuous dividend yield of 5% and has a return volatility of 16%. The risk-free rate is 4.5%. Using this data, if the correlation between the rates of return is -0.5, 0, and 0.5, calculate the price of the minimum of two assets that are European rainbow put options.

Create the `RateSpec`:

```
Settle = 'Nov-15-2008';
Maturity = 'May-15-2009';
Rates = 0.045;
Basis = 1;

RateSpec = intenvset('ValuationDate', Settle, 'StartDates', Settle,...
'EndDates', Maturity, 'Rates', Rates, 'Compounding', -1, 'Basis', Basis);
```

Create the two `StockSpec` definitions:

```
AssetPriceA = 49.75;
AssetPriceB = 51;
SigmaA = 0.11;
SigmaB = 0.16;
DivA = 0.045;
DivB = 0.05;

StockSpecA = stockspec(SigmaA, AssetPriceA, 'continuous', DivA);
StockSpecB = stockspec(SigmaB, AssetPriceB, 'continuous', DivB);
```

Compute the price of the options for different correlation levels:

```
Strike = 50.25;
Corr = [-0.5;0;0.5];
OptSpec = 'put';

Price = minassetbystulz(RateSpec, StockSpecA, StockSpecB, Settle,...
Maturity, OptSpec, Strike, Corr)

Price =

    3.4320
```

```
    3.1384
    2.7694
```

The values 3.43, 3.14, and 2.77 are the price of the European rainbow put options with a correlation level of -0.5, 0, and 0.5 respectively.

## See Also

`intenvset` | `maxassetbystulz` | `minassetsensbystulz` | `stockspec`

## Purpose

Determine European rainbow option prices and sensitivities on minimum of two risky assets using Stulz pricing model

## Syntax

```
PriceSens = minassetsensbystulz(RateSpec, StockSpec1,
StockSpec2, Settle, Maturity, OptSpec, Strike, Corr)
PriceSens = minassetsensbystulz(RateSpec, StockSpec1,
StockSpec2, Settle, Maturity, OptSpec, Strike,
Corr, OutSpec)
```

## Arguments

| | |
|---|---|
| `RateSpec` | The annualized continuously compounded rate term structure. For information on the interest rate specification, see `intenvset`. |
| `StockSpec1` | Stock specification for asset 1. See `stockspec`. |
| `StockSpec2` | Stock specification for asset 2. See `stockspec`. |
| `Settle` | `NINST`-by-`1` vector of settlement or trade dates. |
| `Maturity` | `NINST`-by-`1` vector of maturity dates. |
| `OptSpec` | `NINST`-by-`1` cell array of strings `'call'` or `'put'`. |
| `Strike` | `NINST`-by-`1` vector of strike price values. |
| `Corr` | `NINST`-by-`1` vector of correlation between the underlying asset prices. |
| `OutSpec` | (Optional) All optional inputs are specified as matching parameter name/value pairs. The parameter name is specified as a character string, followed by the corresponding parameter value. You can specify parameter name/value pairs in any order. Names are case-insensitive and partial string matches are allowed provided no ambiguities exist. Valid parameter names are:<br>• `NOUT`-by-`1` or `1`-by-`NOUT` cell array of strings indicating the nature and order of the outputs |

for the function. Possible values are `'Price'`, `'Delta'`, `'Gamma'`, `'Vega'`, `'Lambda'`, `'Rho'`, `'Theta'`, or `'All'`.

For example, `OutSpec = {'Price'; 'Lambda'; 'Rho'}` specifies that the output should be `Price`, `Lambda`, and `Rho`, in that order.

To invoke from a function: `[Price, Lambda, Rho] = minassetsensbystulz(..., 'OutSpec', {'Price', 'Lambda', 'Rho'})`

`OutSpec = {'All'}` specifies that the output should be `Delta`, `Gamma`, `Vega`, `Lambda`, `Rho`, `Theta`, and `Price`, in that order. This is the same as specifying `OutSpec` as `OutSpec = {'Delta', 'Gamma', 'Vega', 'Lambda', 'Rho', 'Theta', 'Price'};`.

- Default is `OutSpec` = `{'Price'}`.

## Description

`PriceSens = minassetsensbystulz(RateSpec, StockSpec1, StockSpec2, Settle, Maturity, OptSpec, Strike, Corr)` computes rainbow option prices using the Stulz option pricing model.

`PriceSens = minassetsensbystulz(RateSpec, StockSpec1, StockSpec2, Settle, Maturity, OptSpec, Strike, Corr, OutSpec)` computes rainbow option prices and sensitivities using the Stulz option pricing model.

`PriceSens` is a `NINST`-by-`1` or `NINST`-by-`2` vector of expected prices and sensitivities.

## Examples

Consider a European rainbow put option that gives the holder the right to sell either stock A or stock B at a strike of 50.25, whichever has the lower value on the expiration date May 15, 2009. On November 15, 2008, stock A is trading at 49.75 with a continuous annual dividend yield of 4.5% and has a return volatility of 11%. Stock B is trading at 51 with a continuous dividend yield of 5% and has a return volatility of 16%. The risk-free rate is 4.5%. Using this data, if the correlation between the rates of return is -0.5, 0, and 0.5, calculate the price and sensitivity of the minimum of two assets that are European rainbow put options.

Create the `RateSpec`:

```
Settle = 'Nov-15-2008';
Maturity = 'May-15-2009';
Rates = 0.045;
Basis = 1;

RateSpec = intenvset('ValuationDate', Settle, 'StartDates', Settle,...
'EndDates', Maturity, 'Rates', Rates, 'Compounding', -1, 'Basis', Basis);
```

Create the two `StockSpec` definitions:

```
AssetPriceA = 49.75;
AssetPriceB = 51;
SigmaA = 0.11;
SigmaB = 0.16;
DivA = 0.045;
DivB = 0.05;

StockSpecA = stockspec(SigmaA, AssetPriceA, 'continuous', DivA);
StockSpecB = stockspec(SigmaB, AssetPriceB, 'continuous', DivB);
```

Calculate price and delta for different correlation levels:

```
Strike = 50.25;
Corr = [-0.5;0;0.5];
OutSpec = {'Price'; 'delta'};
[P, D] = minassetsensbystulz(RateSpec, StockSpecA, StockSpecB,...
Settle, Maturity, OptSpec, Strike, Corr, 'OutSpec', OutSpec)

P =
```

```
    3.4320
    3.1384
    2.7694

D =

   -0.4183   -0.3496
   -0.3746   -0.3189
   -0.3304   -0.2905
```

The output `Delta` has two columns: the first column represents the `Delta` with respect to the stock A (asset 1), and the second column represents the `Delta` with respect to the stock B (asset 2). The value 0.4183 represents `Delta` with respect to the stock A for a correlation level of -0.5. The `Delta` with respect to stock B, for a correlation of zero is -0.3189.

## See Also

`intenvset` | `minassetbystulz` | `stockspec`

## Purpose

Create bushy tree

## Syntax

```
[Tree, NumStates] = mkbush(NumLevels, NumChild, NumPos, Trim,
NodeVal)
```

## Arguments

| | |
|---|---|
| `NumLevels` | Number of time levels of the tree. |
| `NumChild` | `1`-by- number of levels (`NUMLEVELS`) vector with number of branches (children) of the nodes in each level. |
| `NumPos` | `1`-by-`NUMLEVELS` vector containing the length of the state vectors in each time level. |
| `Trim` | (Optional) Scalar `0` or `1`. If `Trim = 1`, `NumPos` decreases by `1` when moving from one time level to the next. Otherwise, if `Trim = 0` (Default), `NumPos` does not decrease. |
| `NodeVal` | (Optional) Initial value at each node of the tree. Default = `NaN`. |

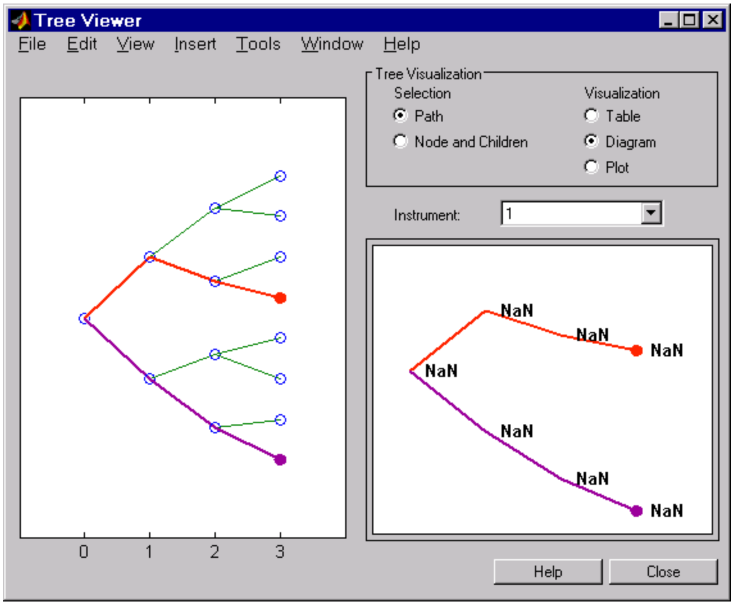
## Description

`[Tree, NumStates] = mkbush(NumLevels, NumChild, NumPos, Trim, NodeVal)` creates a bushy tree `Tree` with initial values `NodeVal` at each node. `NumStates` is a `1`-by-`NUMLEVELS` vector containing the number of state vectors in each level.

## Examples

Create a tree with four time levels, two branches per node, and a vector of three elements in each node with each element initialized to `NaN`.

```
Tree = mkbush(4, 2, 3);
treeviewer(Tree)
```



Tree Visualization

Selection

● Path

○ Node and Children

Visualization

○ Table

● Diagram

○ Plot

Instrument: 1

**See Also** `bushpath` | `bushshape`

## Purpose

Create recombining binomial tree

## Syntax

```
Tree = mktree(NumLevels, NumPos, NodeVal, IsPriceTree)
```

## Arguments

| | |
|---|---|
| `NumLevels` | Number of time levels of the tree. |
| `NumPos` | `1`-by-`NUMLEVELS` vector containing the length of the state vectors in each time level. |
| `NodeVal` | (Optional) Initial value at each node of the tree. Default = `NaN`. |
| `IsPriceTree` | (Optional) Boolean determining if a final horizontal branch is added to the tree. Default = `0`. |

## Description

`Tree = mktree(NumLevels, NumPos, NodeVal, IsPriceTree)` creates a recombining tree `Tree` with initial values `NodeVal` at each node.

## Examples

Create a recombining tree of four time levels with a vector of two elements in each node and each element initialized to `NaN`.

```
Tree = mktree(4, 2);
```

## See Also

`treepath` | `treeshape`

# mktrintree

## Purpose

Create recombining trinomial tree

## Syntax

```
TrinTree = mktrintree(NumLevels, NumPos, NumStates, NodeVal)
```

## Arguments

| | |
|---|---|
| `NumLevels` | Number of time levels of the tree. |
| `NumPos` | `1`-by-`NUMLEVELS` vector containing the length of the state vectors in each time level. |
| `NumStates` | `1`-by-`NUMLEVELS` vector containing the number of state vectors in each time level. |
| `NodeVal` | (Optional) Initial value at each node of the tree. Default = `NaN`. |

## Description

`TrinTree = mktrintree(NumLevels, NumPos, NumStates, NodeVal)` creates a recombining tree `Tree` with initial values `NodeVal` at each node.

## Examples

Create a recombining trinomial tree of four time levels with a vector of two elements in each node and each element initialized to `NaN`.

```
TrinTree = mktrintree(4, [2 2 2 2], [1 3 5 7]);
```

## See Also

`trintreepath` | `trintreeshape`

## Purpose

Create money-market tree from Black-Derman-Toy interest-rate tree

## Syntax

```
MMktTree = mmktbybdt(BDTTree)
```

## Arguments

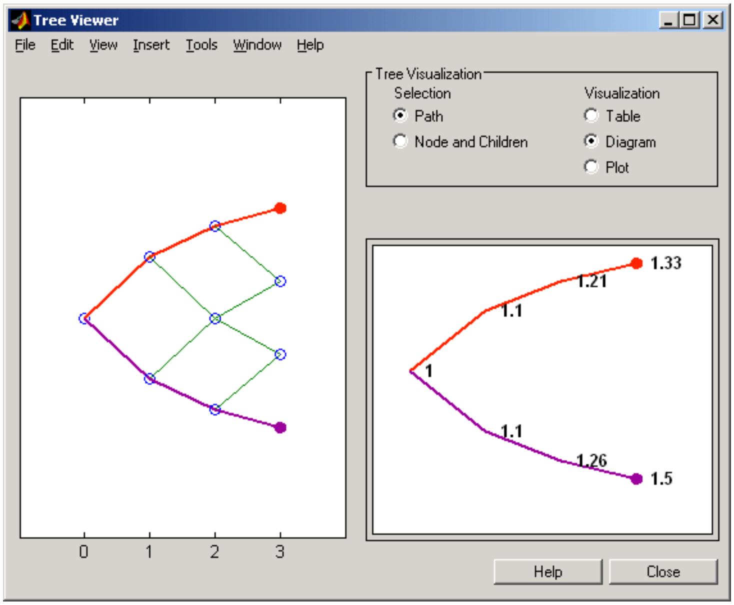
| | |
|---|---|
| `BDTTree` | Interest-rate tree structure created by `bdttree`. |

## Description

`MMktTree = mmktbybdt(BDTTree)` creates a money-market tree from an interest-rate tree structure created by `bdttree`.

## Examples

```
load deriv.mat;
MMktTree = mmktbybdt(BDTTree);
treeviewer(MMktTree)
```



**See Also** `bdttree`

## Purpose

Create money-market tree from Heath-Jarrow-Morton interest-rate tree

## Syntax

```
MMktTree = mmktbyhjm(HJMTree)
```

## Arguments

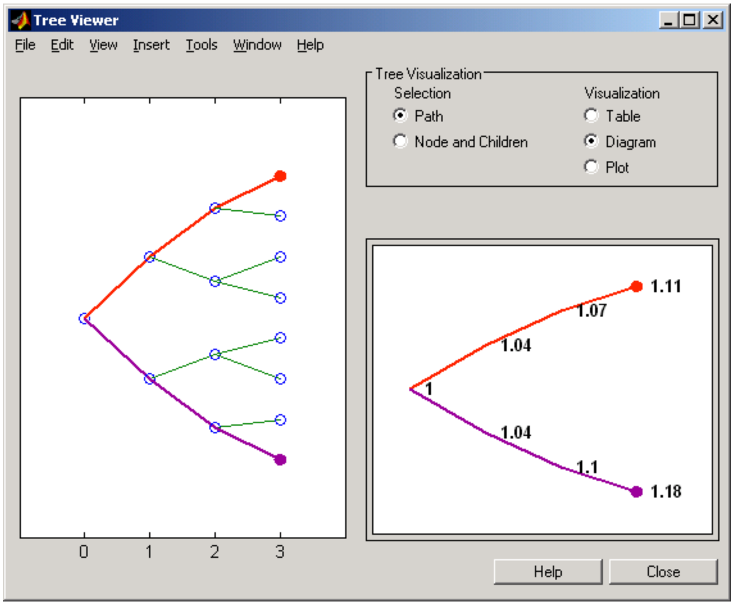
| | |
|---|---|
| `HJMTree` | Forward-rate tree structure created by `hjmtree`. |

## Description

`MMktTree = mmktbyhjm(HJMTree)` creates a money-market tree from a forward-rate tree structure created by `hjmtree`.

## Examples

```
load deriv.mat;
MMktTree = mmktbyhjm(HJMTree);
treeviewer(MMktTree)
```



Tree Visualization

Selection

● Path

○ Node and Children

Visualization

○ Table

● Diagram

○ Plot

**See Also** `hjmtree`

## Purpose

Determine option adjusted spread using Black-Derman-Toy model

## Syntax

```
[OAS, OAD, OAC] = oasbybdt(BDTTree, Price, CouponRate, Settle,
Maturity, OptSpec, Strike, ExerciseDates)
[OAS, OAD, OAC] = oasbybdt(BDTTree, Price, CouponRate, Settle,
Maturity, OptSpec, Strike, ExerciseDates,
Name,Value)
```

## Description

`[OAS, OAD, OAC] = oasbybdt(BDTTree, Price, CouponRate, Settle, Maturity, OptSpec, Strike, ExerciseDates)` calculates option adjusted spread (`OAS`), duration (`OAD`), and convexity (`OAC`) using a BDT model.

`[OAS, OAD, OAC] = oasbybdt(BDTTree, Price, CouponRate, Settle, Maturity, OptSpec, Strike, ExerciseDates, Name,Value)` calculates option adjusted spread (`OAS`), duration (`OAD`), and convexity (`OAC`) using a BDT model with additional options specified by one or more `Name,Value` pair arguments.

## Input Arguments

### BDTTree

Interest-rate tree structure created by `bdttree`.

### Price

`NINST`-by-`1` vector of market prices of bonds with embedded options.

### CouponRate

`NINST`-by-`1` vector for decimal annual rate.

### Settle

`NINST`-by-`1` vector for settlement date.

### Maturity

`NINST`-by-`1` vector for maturity date.

### OptSpec

`NINST`-by-`1` cell array of strings for `'call'` or `'put'`.

### Strike

Matrix of strike price values for supported option types:

- European option: `NINST`-by-`1` vector of strike price values.
- Bermuda option: `NINST` by number of strikes (`NSTRIKES`) matrix of strike price values.

  Each row is the schedule for one option. If an option has fewer than `NSTRIKES` exercise opportunities, the end of the row is padded with `NaN`s.
- American option: `NINST`-by-`1` vector of strike price values for each option.

### ExerciseDates

Matrix of exercise callable or puttable dates for supported option types:

- `NINST`-by-`1` (European option) or `NINST`-by-`NSTRIKES` (Bermuda option) matrix of exercise dates. Each row is the schedule for one option. For a European option, there is only one exercise date, the option expiry date.
- American option: `NINST`-by-`2` vector of exercise date boundaries. For each instrument, the option is exercised on any coupon date between or including the pair of dates on that row. If only one non-`NaN` date is listed, or if `ExerciseDates` is `NINST`-by-`1`, the option is exercised between the underlying bond `Settle` and the single listed exercise date.

## Name-Value Pair Arguments

Specify optional comma-separated pairs of `Name,Value` arguments, where `Name` is the argument name and `Value` is the corresponding

value. `Name` must appear inside single quotes (`' '`). You can specify several name and value pair arguments in any order as `Name1,Value1,...,NameN,ValueN`.

### AmericanOpt

`NINST`-by-`1` vector for option flags: `0` (European/Bermuda) or `1` (American).

**Default:** `0` (European/Bermuda)

### Basis

Day-count basis of the instrument. A vector of integers.

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (BMA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/actual (ISDA)
- 13 = BUS/252

For more information, see basis.

**Default:** `0` (actual/actual)

**EndMonthRule**

`NINST`-by-1 vector for end-of-month rule. Values are `1` (in effect) and `0` (not in effect).

> **Default:** `1` (in effect)

**IssueDate**

`NINST`-by-1 vector of bond issue date.

> **Default:** If you do not specify an `IssueDate`, the cash flow payment dates are determined from other inputs.

**Face**

`NINST`-by-1 vector for face value.

> **Default:** `100`

**FirstCouponDate**

`NINST`-by-1 vector. Date when a bond makes its first coupon payment; used when bond has an irregular first coupon period. When `FirstCouponDate` and `LastCouponDate` are both specified, `FirstCouponDate` takes precedence in determining the coupon payment structure.

> **Default:** If you do not specify a `FirstCouponDate`, the cash flow payment dates are determined from other inputs.

**LastCouponDate**

`NINST`-by-1 vector. Last coupon date of a bond before the maturity date; used when bond has an irregular last coupon period. In the absence of a specified `FirstCouponDate`, a specified `LastCouponDate` determines the coupon structure of the bond. The coupon structure of a bond is truncated at the `LastCouponDate`, regardless of where it falls, and is followed only by the bond's maturity cash flow date.

**Default:** If you do not specify a `LastCouponDate`, the cash flow payment dates are determined from other inputs.

### Period

`NINST`-by-`1` vector for coupons per year.

**Default:** `2` per year

### Options

Structure created with `derivset` containing derivatives pricing options.

**Default:** None

## Output Arguments

### OAS

`NINST`-by-`1` vector for option adjusted spread.

### OAD

`NINST`-by-`1` vector for option adjusted duration.

### OAC

`NINST`-by-`1` vector for option adjusted convexity.

## Definitions

### Bond with Embedded Options

A bond with embedded option allows the issuer to buy back (callable) or redeem (puttable) the bond at a predetermined price at specified future dates. Financial Instruments Toolbox software supports American, European, and Bermuda callable and puttable bonds.

The pricing for a bond with embedded options is as follows:

- Callable bond — The holder bought a bond and sold a call option to the issuer. For example, if interest rates go down by the time of the call date, the issuer is able to refinance its debt at a cheaper level and can call the bond. The price of a callable bond is:

`Price callable bond = Price Option free bond − Price call option`

- Puttable bond — The holder bought a bond and a put option. For example, if interest rates rise, the future value of coupon payments becomes less valuable. Therefore, the investor can sell the bond back to the issuer and then lend proceeds elsewhere at a higher rate. The price of a puttable bond is:

  `Price puttable bond = Price Option free bond + Price put option`

## Examples

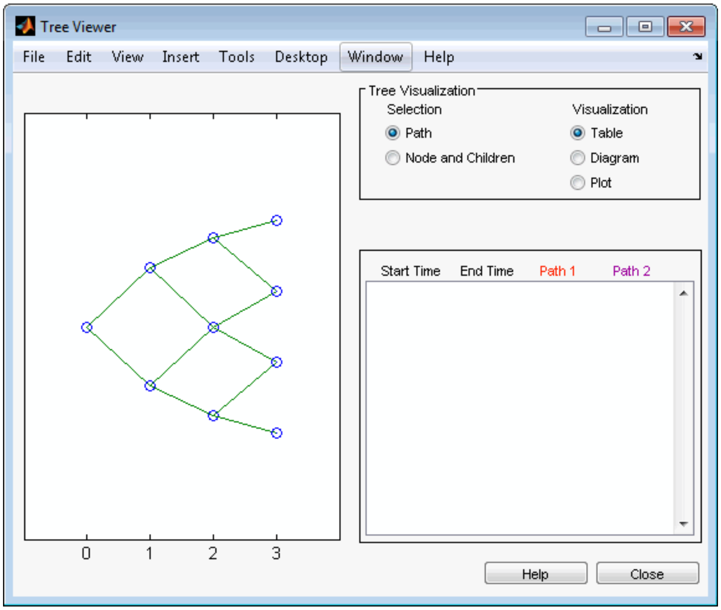
Compute OAS using the Black-Derman-Toy (BDT) model with:

```
ValuationDate = 'Oct-1-2010';
Rates = [0.035; 0.042; 0.047; 0.052];
StartDates = ValuationDate;
EndDates = datemnth(ValuationDate, 12:12:48)';
Compounding = 1;
% Define RateSpec
RateSpec = intenvset('ValuationDate', ValuationDate,...
'StartDates', StartDates, 'EndDates', EndDates, ...
'Rates', Rates, 'Compounding', Compounding);

% Specify VolsSpec and TimeSpec
Sigma = 0.20;
VS = bdtvolspec(ValuationDate, EndDates, Sigma*ones(size(EndDates)));
TS = bdttimespec(ValuationDate, EndDates, Compounding);

% Build the BDT tree
BDTTree = bdttree(VS, RateSpec, TS);
BDTTreenew = cvtree(BDTTree);

% Instrument information
CouponRate = 0.065;
Settle = ValuationDate;
Maturity = '01-Oct-2014';
```

```
OptSpec = 'call';
Strike = 100;
ExerciseDates ='01-Oct-2011';
Period = 1;
Price = 101.58;

% Compute the OAS
OAS = oasbybdt(BDTTree, Price, CouponRate, Settle, Maturity,...
OptSpec, Strike, ExerciseDates, 'Period', Period)

OAS =

   36.5591
```

Use `treeviewer` to observe the tree you created:

```
treeviewer(BDTTree)
```



## References

Fabozzi, F., *Handbook of Fixed Income Securities*, McGraw-Hill, 7th edition, 2005.

Windas, T., *Introduction to Option-Adjusted Spread Analysis*, Bloomberg Press, 3rd edition, 2007.

## See Also

| `bdttree` | `bdtprice` | `instoptembnd` | `optembndbybdt` | `oasbyhjm` | `oasbyhw` | `oasbybk`

## Purpose

Determine option adjusted spread using Black-Karasinski model

## Syntax

`[OAS, OAD, OAC] = oasbybk(BKTree, Price, CouponRate, Settle, Maturity, OptSpec, Strike, ExerciseDates)`
`[OAS, OAD, OAC] = oasbybk(BKTree, Price, CouponRate, Settle, Maturity, OptSpec, Strike, ExerciseDates, Name,Value)`

## Description

`[OAS, OAD, OAC] = oasbybk(BKTree, Price, CouponRate, Settle, Maturity, OptSpec, Strike, ExerciseDates)` calculates option adjusted spread (`OAS`), duration (`OAD`), and convexity (`OAC`) using a BK model.

`[OAS, OAD, OAC] = oasbybk(BKTree, Price, CouponRate, Settle, Maturity, OptSpec, Strike, ExerciseDates, Name,Value)` calculates option adjusted spread (`OAS`), duration (`OAD`), and convexity (`OAC`) using a BK model with additional options specified by one or more `Name,Value` pair arguments.

## Input Arguments

### BKTree

Interest-rate tree structure created by `bktree`.

### Price

`NINST`-by-`1` vector of market prices of bonds with embedded options.

### CouponRate

`NINST`-by-`1` vector for decimal annual rate.

### Settle

`NINST`-by-`1` vector for settlement date.

### Maturity

`NINST`-by-`1` vector for maturity date.

### OptSpec

`NINST`-by-`1` cell array of strings for `'call'` or `'put'`.

### Strike

Matrix of strike price values for supported option types:

- European option: `NINST`-by-`1` vector of strike price values.
- Bermuda option: `NINST` by number of strikes (`NSTRIKES`) matrix of strike price values.

  Each row is the schedule for one option. If an option has fewer than `NSTRIKES` exercise opportunities, the end of the row is padded with `NaN`s.
- American option: `NINST`-by-`1` vector of strike price values for each option.

### ExerciseDates

Matrix of exercise callable or puttable dates for supported option types:

- `NINST`-by-`1` (European option) or `NINST`-by-`NSTRIKES` (Bermuda option) matrix of exercise dates. Each row is the schedule for one option. For a European option, there is only one exercise date, the option expiry date.
- American option: `NINST`-by-`2` vector of exercise date boundaries. For each instrument, the option is exercised on any coupon date between or including the pair of dates on that row. If only one non-`NaN` date is listed, or if `ExerciseDates` is `NINST`-by-`1`, the option is exercised between the underlying bond `Settle` and the single listed exercise date.

## Name-Value Pair Arguments

Specify optional comma-separated pairs of `Name,Value` arguments, where `Name` is the argument name and `Value` is the corresponding

value. `Name` must appear inside single quotes (`' '`). You can specify several name and value pair arguments in any order as `Name1,Value1,...,NameN,ValueN`.

### AmericanOpt

`NINST`-by-`1` vector for option flags: `0` (European/Bermuda) or `1` (American).

**Default:** `0` (European/Bermuda)

### Basis

Day-count basis of the instrument. A vector of integers.

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (BMA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/actual (ISDA)
- 13 = BUS/252

For more information, see basis.

**Default:** `0` (actual/actual)

**EndMonthRule**

`NINST`-by-1 vector for end-of-month rule. Values are `1` (in effect) and `0` (not in effect).

> **Default:** `1` (in effect)

**IssueDate**

`NINST`-by-1 vector of bond issue date.

> **Default:** If you do not specify an `IssueDate`, the cash flow payment dates are determined from other inputs

**Face**

`NINST`-by-1 vector for face value.

> **Default:** `100`

**FirstCouponDate**

`NINST`-by-1 vector. Date when a bond makes its first coupon payment; used when bond has an irregular first coupon period. When `FirstCouponDate` and `LastCouponDate` are both specified, `FirstCouponDate` takes precedence in determining the coupon payment structure.

> **Default:** If you do not specify a `FirstCouponDate`, the cash flow payment dates are determined from other inputs.

**LastCouponDate**

`NINST`-by-1 vector. Last coupon date of a bond before the maturity date; used when bond has an irregular last coupon period. In the absence of a specified `FirstCouponDate`, a specified `LastCouponDate` determines the coupon structure of the bond. The coupon structure of a bond is truncated at the `LastCouponDate`, regardless of where it falls, and is followed only by the bond's maturity cash flow date.

**Default:** If you do not specify a `LastCouponDate`, the cash flow payment dates are determined from other inputs.

### Period

`NINST`-by-`1` vector for coupons per year.

**Default:** `2` per year

### Options

Structure created with `derivset` containing derivatives pricing options.

**Default:** None

## Output Arguments

### OAS

`NINST`-by-`1` vector for option adjusted spread.

### OAD

`NINST`-by-`1` vector for option adjusted duration.

### OAC

`NINST`-by-`1` vector for option adjusted convexity.

## Definitions

### Bond with Embedded Options

A bond with embedded option allows the issuer to buy back (callable) or redeem (puttable) the bond at a predetermined price at specified future dates. Financial Instruments Toolbox software supports American, European, and Bermuda callable and puttable bonds.

The pricing for a bond with embedded options is as follows:

- Callable bond — The holder bought a bond and sold a call option to the issuer. For example, if interest rates go down by the time of the call date, the issuer is able to refinance its debt at a cheaper level and can call the bond. The price of a callable bond is:

`Price callable bond = Price Option free bond − Price call option`

- Puttable bond — The holder bought a bond and a put option. For example, if interest rates rise, the future value of coupon payments becomes less valuable. Therefore, the investor can sell the bond back to the issuer and then lend proceeds elsewhere at a higher rate. The price of a puttable bond is:

  `Price puttable bond = Price Option free bond + Price put option`

## Examples

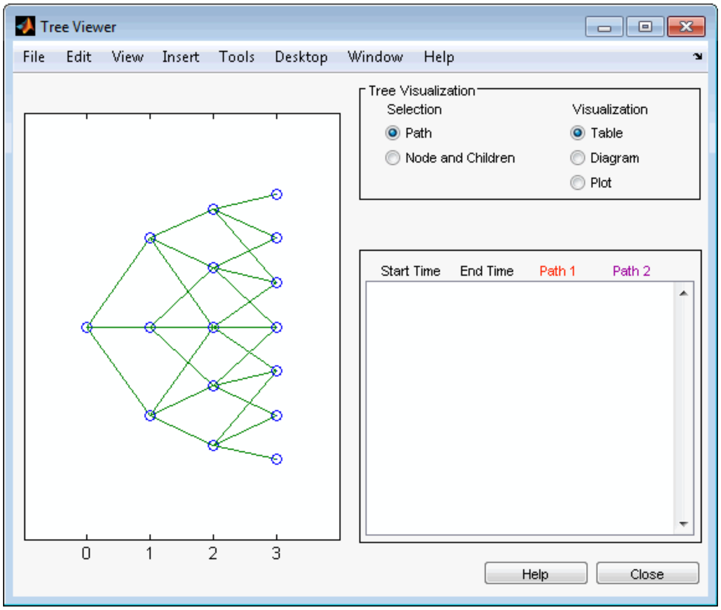
Compute OAS and OAD using the Black-Karasinski (BK) model with:

```
ValuationDate = 'Aug-2-2010';
Rates = [0.0355; 0.0382; 0.0427; 0.0489];
StartDates = ValuationDate;
EndDates = datemnth(ValuationDate, 12:12:48)';
Compounding = 1;

% Define RateSpec
RateSpec = intenvset('ValuationDate', ValuationDate,...
'StartDates', StartDates,'EndDates', EndDates, ...
'Rates', Rates,'Compounding', Compounding);

% Specify VolsSpec and TimeSpec
Sigma = 0.10;
Alpha = 0.01;
VS = bkvolspec(ValuationDate, EndDates, Sigma*ones(size(EndDates)),...
EndDates, Alpha*ones(size(EndDates)));
TS = bktimespec(ValuationDate, EndDates, Compounding);

% Build the BK tree
BKTree = bktree(VS, RateSpec, TS);

% Instrument information
CouponRate = 0.045;
```

```
Settle = ValuationDate;
Maturity = '02-Aug-2014';
OptSpec = 'put';
Strike = 100;
ExerciseDates ='02-Aug-2013';
Period = 1;
AmericanOpt = 1;
Price = 101;

% Compute OAS and OAD
[OAS, OAD] = oasbybk(BKTree, Price, CouponRate, Settle, Maturity,...
OptSpec, Strike, ExerciseDates, 'Period', Period, 'AmericanOpt', AmericanOpt)

OAS =

   21.0839

OAD =

    1.7833
```

Use `treeviewer` to observe the tree you created:

```
treeviewer(BKTree)
```



## References

Fabozzi, F., *Handbook of Fixed Income Securities*, McGraw-Hill, 7th edition, 2005.

Windas, T., *Introduction to Option-Adjusted Spread Analysis*, Bloomberg Press, 3rd edition, 2007.

## See Also

| `bktree` | `bkprice` | `instoptembnd` | `optembndbybk` | `oasbyhjm` | `oasbyhw` | `oasbybdt`

## Purpose

Determine option adjusted spread using Heath-Jarrow-Morton model

## Syntax

```
[OAS, OAD, OAC] = oasbyhjm(HJMTree, Price, CouponRate, Settle,
Maturity, OptSpec, Strike, ExerciseDates)
[OAS, OAD, OAC] = oasbyhjm(HJMTree, Price, CouponRate, Settle,
Maturity, OptSpec, Strike, ExerciseDates,
Name,Value)
```

## Description

`[OAS, OAD, OAC] = oasbyhjm(HJMTree, Price, CouponRate, Settle, Maturity, OptSpec, Strike, ExerciseDates)` calculates option adjusted spread (`OAS`), duration (`OAD`), and convexity (`OAC`) using an HJM model.

`[OAS, OAD, OAC] = oasbyhjm(HJMTree, Price, CouponRate, Settle, Maturity, OptSpec, Strike, ExerciseDates, Name,Value)` calculates option adjusted spread (`OAS`), duration (`OAD`), and convexity (`OAC`) using an HJM model with additional options specified by one or more `Name,Value` pair arguments.

## Input Arguments

### HJMTree

Interest-rate tree structure created by `hjmtree`.

### Price

`NINST`-by-`1` vector of market prices of bonds with embedded options.

### CouponRate

`NINST`-by-`1` vector for decimal annual rate.

### Settle

`NINST`-by-`1` vector for settlement date.

### Maturity

`NINST`-by-`1` vector for maturity date.

### OptSpec

`NINST`-by-`1` cell array of strings for `'call'` or `'put'`.

### Strike

Matrix of strike price values for supported option types:

- European option: `NINST`-by-`1` vector of strike price values.
- Bermuda option: `NINST` by number of strikes (`NSTRIKES`) matrix of strike price values.

  Each row is the schedule for one option. If an option has fewer than `NSTRIKES` exercise opportunities, the end of the row is padded with `NaN`s.
- American option: `NINST`-by-`1` vector of strike price values for each option.

### ExerciseDates

Matrix of exercise callable or puttable dates for supported option types:

- `NINST`-by-`1` (European option) or `NINST`-by-`NSTRIKES` (Bermuda option) matrix of exercise dates. Each row is the schedule for one option. For a European option, there is only one exercise date, the option expiry date.
- American option: `NINST`-by-`2` vector of exercise date boundaries. For each instrument, the option is exercised on any coupon date between or including the pair of dates on that row. If only one non-`NaN` date is listed, or if `ExerciseDates` is `NINST`-by-`1`, the option is exercised between the underlying bond `Settle` and the single listed exercise date.

## Name-Value Pair Arguments

Specify optional comma-separated pairs of `Name,Value` arguments, where `Name` is the argument name and `Value` is the corresponding

value. `Name` must appear inside single quotes (`' '`). You can specify several name and value pair arguments in any order as `Name1,Value1,...,NameN,ValueN`.

### AmericanOpt

`NINST`-by-`1` vector for option flags: `0` (European/Bermuda) or `1` (American).

**Default:** `0` (European/Bermuda)

### Basis

Day-count basis of the instrument. A vector of integers.

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (BMA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/actual (ISDA)
- 13 = BUS/252

For more information, see basis.

**Default:** `0` (actual/actual)

**EndMonthRule**

`NINST`-by-1 vector for end-of-month rule. Values are `1` (in effect) and `0` (not in effect).

> **Default:** `1` (in effect)

**IssueDate**

`NINST`-by-1 vector of bond issue date.

> **Default:** If you do not specify an `IssueDate`, the cash flow payment dates are determined from other inputs.

**Face**

`NINST`-by-1 vector for face value.

> **Default:** `100`

**FirstCouponDate**

`NINST`-by-1 vector. Date when a bond makes its first coupon payment; used when bond has an irregular first coupon period. When `FirstCouponDate` and `LastCouponDate` are both specified, `FirstCouponDate` takes precedence in determining the coupon payment structure.

> **Default:** If you do not specify a `FirstCouponDate`, the cash flow payment dates are determined from other inputs.

**LastCouponDate**

`NINST`-by-1 vector. Last coupon date of a bond before the maturity date; used when bond has an irregular last coupon period. In the absence of a specified `FirstCouponDate`, a specified `LastCouponDate` determines the coupon structure of the bond. The coupon structure of a bond is truncated at the `LastCouponDate`, regardless of where it falls, and is followed only by the bond's maturity cash flow date.

**Default:** If you do not specify a `LastCouponDate`, the cash flow payment dates are determined from other inputs.

### Period

`NINST`-by-`1` vector for coupons per year.

**Default:** `2` per year

### Options

Structure created with `derivset` containing derivatives pricing options.

**Default:** None

## Output Arguments

### OAS

`NINST`-by-`1` vector for option adjusted spread.

### OAD

`NINST`-by-`1` vector for option adjusted duration.

### OAC

`NINST`-by-`1` vector for option adjusted convexity.

## Definitions

### Bond with Embedded Options

A bond with embedded option allows the issuer to buy back (callable) or redeem (puttable) the bond at a predetermined price at specified future dates. Financial Instruments Toolbox software supports American, European, and Bermuda callable and puttable bonds.

The pricing for a bond with embedded options is as follows:

- Callable bond — The holder bought a bond and sold a call option to the issuer. For example, if interest rates go down by the time of the call date, the issuer is able to refinance its debt at a cheaper level and can call the bond. The price of a callable bond is:

`Price callable bond = Price Option free bond − Price call option`

- Puttable bond — The holder bought a bond and a put option. For example, if interest rates rise, the future value of coupon payments becomes less valuable. Therefore, the investor can sell the bond back to the issuer and then lend proceeds elsewhere at a higher rate. The price of a puttable bond is:

  `Price puttable bond = Price Option free bond + Price put option`

## Examples

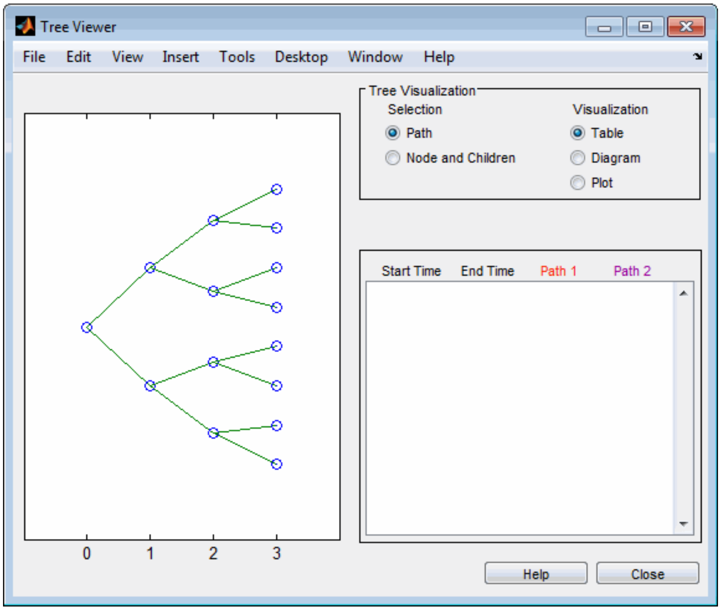
Compute OAS using the Heath-Jarrow-Morton (HJM) model with:

```
ValuationDate = 'Nov-1-2010';
Rates = [0.0356; 0.0427; 0.0478; 0.0529];
StartDates = ValuationDate;
EndDates = datemnth(ValuationDate, 12:12:48)';
Compounding = 1;

% Define RateSpec
RateSpec = intenvset('ValuationDate', ValuationDate,...
'StartDates', StartDates,'EndDates', EndDates, ...
'Rates', Rates,'Compounding', Compounding);

% Specify VolsSpec and TimeSpec
Sigma = 0.02;
VS = hjmvolspec('Constant', Sigma)
TS = hjmtimespec(ValuationDate, EndDates, Compounding);

% Build the HJM tree
HJMTree = hjmtree(VS, RateSpec, TS);
HJMTreenew = cvtree(HJMTree);

% Instrument information
CouponRate = 0.05;
Settle = ValuationDate;
```

```
Maturity = '01-Nov-2014';
OptSpec = 'call';
Strike = 100;
ExerciseDates ='01-Nov-2011';
Period = 1;
Price = 97.5;

% Compute the OAS
OAS = oasbyhjm(HJMTree, Price, CouponRate, Settle, Maturity, OptSpec, Strike,...
ExerciseDates, 'Period', Period)

VS =

          FinObj: 'HJMVolSpec'
    FactorModels: {'Constant'}
      FactorArgs: {{1x1 cell}}
      SigmaShift: 0
      NumFactors: 1
       NumBranch: 2
         PBranch: [0.5000 0.5000]
     Fact2Branch: [-1 1]

OAS =

    5.0601
```

Use `treeviewer` to observe the tree you created:

```
treeviewer(HJMTree)
```



Tree Visualization

Selection
● Path
○ Node and Children

Visualization
● Table
○ Diagram
○ Plot

## References

Fabozzi, F., *Handbook of Fixed Income Securities*, McGraw-Hill, 7th edition, 2005.

Windas, T., *Introduction to Option-Adjusted Spread Analysis*, Bloomberg Press, 3rd edition, 2007.

## See Also

| `hjmtree` | `hjmprice` | `instoptembnd` | `optembndbyhjm` | `oasbybdt` | `oasbyhw` | `oasbybk`

## Purpose

Determine option adjusted spread using Hull-White model

## Syntax

```
[OAS, OAD, OAC] = oasbyhw(HWTree, Price, CouponRate, Settle,
Maturity, OptSpec, Strike, ExerciseDates)
[OAS, OAD, OAC] = oasbyhw(HWTree, Price, CouponRate, Settle,
Maturity, OptSpec, Strike, ExerciseDates,
Name,Value)
```

## Description

`[OAS, OAD, OAC] = oasbyhw(HWTree, Price, CouponRate, Settle, Maturity, OptSpec, Strike, ExerciseDates)` calculates option adjusted spread (`OAS`), duration (`OAD`), and convexity (`OAC`) using an HW model.

`[OAS, OAD, OAC] = oasbyhw(HWTree, Price, CouponRate, Settle, Maturity, OptSpec, Strike, ExerciseDates, Name,Value)` calculates option adjusted spread (`OAS`), duration (`OAD`), and convexity (`OAC`) using an HW model with additional options specified by one or more `Name,Value` pair arguments.

## Input Arguments

### HWTree

Interest-rate tree structure created by `hwtree`.

### Price

`NINST`-by-`1` vector of market prices of bonds with embedded options.

### CouponRate

`NINST`-by-`1` vector for decimal annual rate.

### Settle

`NINST`-by-`1` vector for settlement date.

### Maturity

`NINST`-by-`1` vector for maturity date.

### OptSpec

`NINST`-by-`1` cell array of strings for `'call'` or `'put'`.

### Strike

Matrix of strike price values for supported option types:

- European option: `NINST`-by-`1` vector of strike price values.
- Bermuda option: `NINST` by number of strikes (`NSTRIKES`) matrix of strike price values.

  Each row is the schedule for one option. If an option has fewer than `NSTRIKES` exercise opportunities, the end of the row is padded with `NaN`s.
- American option: `NINST`-by-`1` vector of strike price values for each option.

### ExerciseDates

Matrix of exercise callable or puttable dates for supported option types:

- `NINST`-by-`1` (European option) or `NINST`-by-`NSTRIKES` (Bermuda option) matrix of exercise dates. Each row is the schedule for one option. For a European option, there is only one exercise date, the option expiry date.
- American option: `NINST`-by-`2` vector of exercise date boundaries. For each instrument, the option is exercised on any coupon date between or including the pair of dates on that row. If only one non-`NaN` date is listed, or if `ExerciseDates` is `NINST`-by-`1`, the option is exercised between the underlying bond `Settle` and the single listed exercise date.

## Name-Value Pair Arguments

Specify optional comma-separated pairs of `Name,Value` arguments, where `Name` is the argument name and `Value` is the corresponding

value. `Name` must appear inside single quotes (`' '`). You can specify several name and value pair arguments in any order as `Name1,Value1,...,NameN,ValueN`.

### AmericanOpt

`NINST`-by-`1` vector for option flags: `0` (European/Bermuda) or `1` (American).

**Default:** `0` (European/Bermuda)

### Basis

Day-count basis of the instrument. A vector of integers.

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (BMA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/actual (ISDA)
- 13 = BUS/252

For more information, see basis.

**Default:** `0` (actual/actual)

**EndMonthRule**

`NINST`-by-`1` vector for end-of-month rule. Values are `1` (in effect) and `0` (not in effect).

> **Default:** `1` (in effect)

**Face**

`NINST`-by-`1` vector for face value.

> **Default:** `100`

**IssueDate**

`NINST`-by-`1` vector of bond issue date.

> **Default:** If you do not specify an `IssueDate`, the cash flow payment dates are determined from other inputs.

**FirstCouponDate**

`NINST`-by-`1` vector. Date when a bond makes its first coupon payment; used when bond has an irregular first coupon period. When `FirstCouponDate` and `LastCouponDate` are both specified, `FirstCouponDate` takes precedence in determining the coupon payment structure.

> **Default:** If you do not specify a `FirstCouponDate`, the cash flow payment dates are determined from other inputs.

**LastCouponDate**

`NINST`-by-`1` vector. Last coupon date of a bond before the maturity date; used when bond has an irregular last coupon period. In the absence of a specified `FirstCouponDate`, a specified `LastCouponDate` determines the coupon structure of the bond. The coupon structure of a bond is truncated at the `LastCouponDate`, regardless of where it falls, and is followed only by the bond's maturity cash flow date.

**Default:** If you do not specify a `LastCouponDate`, the cash flow payment dates are determined from other inputs.

### Period

`NINST`-by-`1` vector for coupons per year.

**Default:** `2` per year

### Options

Structure created with `derivset` containing derivatives pricing options.

**Default:** None

## Output Arguments

### OAS

`NINST`-by-`1` vector for option adjusted spread.

### OAD

`NINST`-by-`1` vector for option adjusted duration.

### OAC

`NINST`-by-`1` vector for option adjusted convexity.

## Definitions

### Bond with Embedded Options

A bond with embedded option allows the issuer to buy back (callable) or redeem (puttable) the bond at a predetermined price at specified future dates. Financial Instruments Toolbox software supports American, European, and Bermuda callable and puttable bonds.

The pricing for a bond with embedded options is as follows:

- Callable bond — The holder bought a bond and sold a call option to the issuer. For example, if interest rates go down by the time of the call date, the issuer is able to refinance its debt at a cheaper level and can call the bond. The price of a callable bond is:

`Price callable bond = Price Option free bond − Price call option`

- Puttable bond — The holder bought a bond and a put option. For example, if interest rates rise, the future value of coupon payments becomes less valuable. Therefore, the investor can sell the bond back to the issuer and then lend proceeds elsewhere at a higher rate. The price of a puttable bond is:

  `Price puttable bond = Price Option free bond + Price put option`

## Examples

Compute OAS and OAD using the Hull-White (HW) model with:

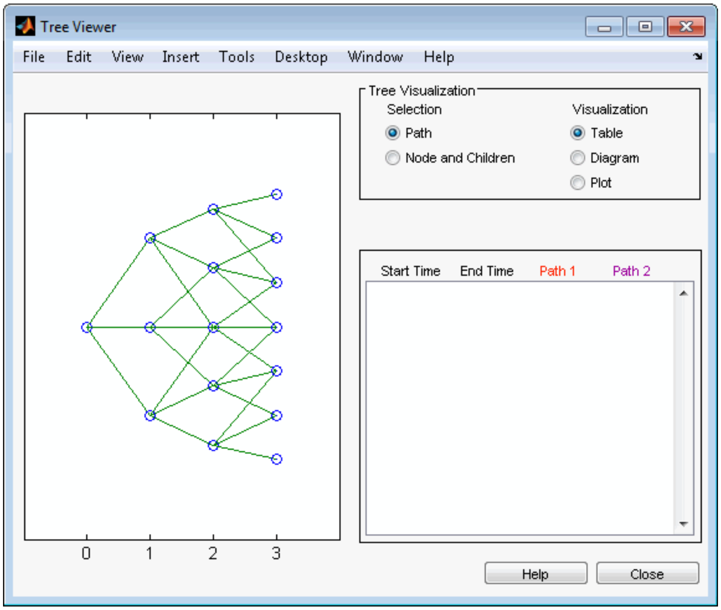
```
ValuationDate = 'October-25-2010';
Rates = [0.0355; 0.0382; 0.0427; 0.0489];
StartDates = ValuationDate;
EndDates = datemnth(ValuationDate, 12:12:48)';
Compounding = 1;

% Define RateSpec
RateSpec = intenvset('ValuationDate', ValuationDate,...
'StartDates', StartDates, 'EndDates', EndDates, ...
'Rates', Rates,'Compounding', Compounding);

% Specify VolsSpec and TimeSpec
Sigma = 0.05;
Alpha = 0.01;
VS = hwvolspec(ValuationDate, EndDates, Sigma*ones(size(EndDates)),...
EndDates, Alpha*ones(size(EndDates)));
TS = hwtimespec(ValuationDate, EndDates, Compounding);

% Build the HW tree
HWTree = hwtree(VS, RateSpec, TS);

% Instrument information
CouponRate = 0.045;
```

```
Settle = ValuationDate;
Maturity = '25-October-2014';
OptSpec = 'call';
Strike = 100;
ExerciseDates = {'25-October-2010','25-October-2013'};
Period = 1;
AmericanOpt = 0;
Price = 97;

% Compute the OAS
[OAS, OAD] = oasbyhw(HWTree, Price, CouponRate, Settle, Maturity,...
OptSpec, Strike, ExerciseDates, 'Period', Period, 'AmericanOpt', AmericanOpt)

OAS =

  -12.4436

OAD =

    3.3045
```

At a 5% volatility, the OAS is -12.44 basis points. A negative OAS means that the callable bond is expensive (overvalued) on a relative value basis. OAS depends on the assumed interest rate volatility, so, if a 1% interest rate volatility is assumed (`Sigma = 0.01`), the OAS is 51 basis points (positive), and in this case the bond is attractive (underpriced).

Use `treeviewer` to observe the tree you created:

```
treeviewer(HWTree)
```



Tree Visualization

Selection
● Path
○ Node and Children

Visualization
● Table
○ Diagram
○ Plot

## References

Fabozzi, F., *Handbook of Fixed Income Securities*, McGraw-Hill, 7th edition, 2005.

Windas, T., *Introduction to Option-Adjusted Spread Analysis*, Bloomberg Press, 3rd edition, 2007.

## See Also

| `hwtree` | `hwprice` | `instoptembnd` | `optembndbyhw` | `oasbybdt` | `oasbyhjm` | `oasbybk`

## Purpose

Price bond option from Black-Derman-Toy interest-rate tree

## Syntax

```
[Price, PriceTree] = optbndbybdt(BDTTree, OptSpec, Strike,
ExerciseDates, AmericanOpt, CouponRate, Settle, Maturity,
Period,  Basis, EndMonthRule, IssueDate, FirstCouponDate,
LastCouponDate, StartDate, Face, Options)
```

## Arguments

| | |
|---|---|
| `BDTTree` | Forward-rate tree structure created by `bdttree`. |
| `OptSpec` | Number of instruments (`NINST`)-by-`1` cell array of string values `'Call'` or `'Put'`. |
| `Strike` | European option: `NINST`-by-`1` vector of strike price values.<br>Bermuda option: `NINST` by number of strikes (`NSTRIKES`) matrix of strike price values.<br>Each row is the schedule for one option. If an option has fewer than `NSTRIKES` exercise opportunities, the end of the row is padded with `NaN`s.<br>For an American option:<br>`NINST`-by-`1` vector of strike price values for each option. |

| | |
|---|---|
| `ExerciseDates` | `NINST`-by-`1` (European option) or `NINST`-by-`NSTRIKES` (Bermuda option) matrix of exercise dates. Each row is the schedule for one option. For a European option, there is only one exercise date, the option expiry date.<br>For an American option:<br>`NINST`-by-`2` vector of exercise date boundaries. For each instrument, the option can be exercised on any coupon date between or including the pair of dates on that row. If only one non-`NaN` date is listed, or if `ExerciseDates` is `NINST`-by-`1`, the option can be exercised between the underlying bond `Settle` and the single listed exercise date. |
| `AmericanOpt` | `NINST`-by-`1` vector of flags: `0` (European/Bermuda) or `1` (American). |
| `CouponRate` | Decimal annual rate. `CouponRate` is a `NINST`-by-`1` vector. |
| `Settle` | Settlement date. A vector of serial date numbers or date strings. `Settle` must be earlier than `Maturity`. |
| `Maturity` | Maturity date. A vector of serial date numbers or date strings. |
| `Period` | (Optional) Coupons per year of the bond. A vector of integers. Allowed values are `1`, `2`, `3`, `4`, `6`, and `12`. Default = `2`. |

| | |
|---|---|
| `Basis` | (Optional) Day-count basis of the instrument. A vector of integers.<br>• 0 = actual/actual (default)<br>• 1 = 30/360 (SIA)<br>• 2 = actual/360<br>• 3 = actual/365<br>• 4 = 30/360 (BMA)<br>• 5 = 30/360 (ISDA)<br>• 6 = 30/360 (European)<br>• 7 = actual/365 (Japanese)<br>• 8 = actual/actual (ICMA)<br>• 9 = actual/360 (ICMA)<br>• 10 = actual/365 (ICMA)<br>• 11 = 30/360E (ICMA)<br>• 12 = actual/actual (ISDA)<br>• 13 = BUS/252<br>For more information, see basis. |
| `EndMonthRule` | (Optional) End-of-month rule. A vector. This rule applies only when `Maturity` is an end-of-month date for a month having 30 or fewer days. `0` = ignore rule, meaning that a bond's coupon payment date is always the same numerical day of the month. `1` = set rule on (default), meaning that a bond's coupon payment date is always the last actual day of the month. |
| `IssueDate` | (Optional) Date when a bond was issued. |

| | |
|---|---|
| `FirstCouponDate` | (Optional) Date when a bond makes its first coupon payment; used when bond has an irregular first coupon period. When `FirstCouponDate` and `LastCouponDate` are both specified, `FirstCouponDate` takes precedence in determining the coupon payment structure. If you do not specify a `FirstCouponDate`, the cash flow payment dates are determined from other inputs. |
| `LastCouponDate` | (Optional) Last coupon date of a bond before the maturity date; used when bond has an irregular last coupon period. In the absence of a specified `FirstCouponDate`, a specified `LastCouponDate` determines the coupon structure of the bond. The coupon structure of a bond is truncated at the `LastCouponDate`, regardless of where it falls, and is followed only by the bond's maturity cash flow date. If you do not specify a `LastCouponDate`, the cash flow payment dates are determined from other inputs. |
| `StartDate` | (Optional) Date when a bond actually starts (the date from which a bond cash flow is considered). To make an instrument forward-starting, specify this date as a future date. If you do not specify `StartDate`, the effective start date is the `Settle` date. |
| `Face` | (Optional) Face or par value. `Face` is a `NINST`-by-`1` vector. Default = `100`. |
| `Options` | (Optional) Derivatives pricing options structure created with `derivset`. |

The `Settle` date for every bond is set to the `ValuationDate` of the BDT tree. The bond argument `Settle` is ignored.

## Description

`[Price, PriceTree] = optbndbybdt(BDTTree, OptSpec, Strike, ExerciseDates, AmericanOpt, CouponRate, Settle, Maturity, Period, Basis, EndMonthRule, IssueDate, FirstCouponDate, LastCouponDate, StartDate, Face, Options)` computes the price of a bond option from a BDT interest-rate tree.

`Price` is an `NINST`-by-`1` matrix of expected prices at time 0.

`PriceTree` is a tree structure with a vector of instrument prices at each node.

## Examples

**Example 1**. Using the BDT interest-rate tree in the `deriv.mat` file, price a European call option on a 10% bond with a strike of 95. The exercise date for the option is Jan. 01, 2002. The settle date for the bond is Jan. 01, 2000, and the maturity date is Jan. 01, 2003.

Load the file `deriv.mat`, which provides `BDTTree`. The `BDTTree` structure contains the time and forward-rate information needed to price the bond.

```
load deriv.mat;
```

Use `optbondbybdt` to compute the price of the option.

```
Price = optbndbybdt(BDTTree,'Call',95,'01-Jan-2002',...
0,0.10,'01-Jan-2000','01-Jan-2003',1)

Price =

    1.7657
```

**Example 2**. Now use `optbndbybdt` to compute the price of a put option on the same bond.

```
Price = optbndbybdt(BDTTree,'Put',95,'01-Jan-2002',...
0,0.10,'01-Jan-2000','01-Jan-2003',1)
```

```
Price =

    0.5740
```

**See Also** `bdtprice` | `bdttree` | `instoptbnd`

## Purpose

Price bond option from Black-Karasinski interest-rate tree

## Syntax

```
[Price, PriceTree] = optbndbybk(BKTree, OptSpec, Strike,
ExerciseDates, AmericanOpt, CouponRate, Settle, Maturity,
Period,  Basis, EndMonthRule, IssueDate, FirstCouponDate,
LastCouponDate, StartDate, Face, Options)
```

## Arguments

| | |
|---|---|
| `BKTree` | Forward-rate tree structure created by `bktree`. |
| `OptSpec` | Number of instruments (`NINST`)-by-`1` cell array of string values `'Call'` or `'Put'`. |
| `Strike` | European option: `NINST`-by-`1` vector of strike price values.<br>Bermuda option: `NINST`-by-number of strikes (`NSTRIKES`) matrix of strike price values.<br>Each row is the schedule for one option. If an option has fewer than `NSTRIKES` exercise opportunities, the end of the row is padded with `NaN`s.<br>For an American option:<br>`NINST`-by-`1` vector of strike price values for each option. |

| | |
|---|---|
| `ExerciseDates` | `NINST`-by-`1` (European option) or `NINST`-by-`NSTRIKES` (Bermuda option) matrix of exercise dates. Each row is the schedule for one option. For a European option, there is only one exercise date, the option expiry date.<br>For an American option:<br>`NINST`-by-`2` vector of exercise date boundaries. For each instrument, the option can be exercised on any coupon date between or including the pair of dates on that row. If only one non-`NaN` date is listed, or if `ExerciseDates` is `NINST`-by-`1`, the option can be exercised between the underlying bond `Settle` and the single listed exercise date. |
| `AmericanOpt` | `NINST`-by-`1` vector of flags: `0` (European/Bermuda) or `1` (American). |
| `CouponRate` | Decimal annual rate. `CouponRate` is a `NINST`-by-`1` vector. |
| `Settle` | Settlement date. A vector of serial date numbers or date strings. `Settle` must be earlier than `Maturity`. |
| `Maturity` | Maturity date. A vector of serial date numbers or date strings. |
| `Period` | (Optional) Coupons per year of the bond. A vector of integers. Allowed values are `1`, `2`, `3`, `4`, `6`, and `12`. Default = `2`. |

| | |
|---|---|
| `Basis` | (Optional) Day-count basis of the instrument. A vector of integers.<br>• 0 = actual/actual (default)<br>• 1 = 30/360 (SIA)<br>• 2 = actual/360<br>• 3 = actual/365<br>• 4 = 30/360 (BMA)<br>• 5 = 30/360 (ISDA)<br>• 6 = 30/360 (European)<br>• 7 = actual/365 (Japanese)<br>• 8 = actual/actual (ICMA)<br>• 9 = actual/360 (ICMA)<br>• 10 = actual/365 (ICMA)<br>• 11 = 30/360E (ICMA)<br>• 12 = actual/actual (ISDA)<br>• 13 = BUS/252<br>For more information, see basis. |
| `EndMonthRule` | (Optional) End-of-month rule. A vector. This rule applies only when `Maturity` is an end-of-month date for a month having 30 or fewer days. `0` = ignore rule, meaning that a bond's coupon payment date is always the same numerical day of the month. `1` = set rule on (default), meaning that a bond's coupon payment date is always the last actual day of the month. |
| `IssueDate` | (Optional) Date when a bond was issued. |

| | |
|---|---|
| `FirstCouponDate` | (Optional) Date when a bond makes its first coupon payment; used when bond has an irregular first coupon period. When `FirstCouponDate` and `LastCouponDate` are both specified, `FirstCouponDate` takes precedence in determining the coupon payment structure. If you do not specify a `FirstCouponDate`, the cash flow payment dates are determined from other inputs. |
| `LastCouponDate` | (Optional) Last coupon date of a bond before the maturity date; used when bond has an irregular last coupon period. In the absence of a specified `FirstCouponDate`, a specified `LastCouponDate` determines the coupon structure of the bond. The coupon structure of a bond is truncated at the `LastCouponDate`, regardless of where it falls, and is followed only by the bond's maturity cash flow date. If you do not specify a `LastCouponDate`, the cash flow payment dates are determined from other inputs. |
| `StartDate` | (Optional) Date when a bond actually starts (the date from which a bond cash flow is considered). To make an instrument forward-starting, specify this date as a future date. If you do not specify `StartDate`, the effective start date is the `Settle` date. |
| `Face` | (Optional) Face or par value. `Face` is a `NINST`-by-`1` vector. Default = `100`. |
| `Options` | (Optional) Derivatives pricing options structure created with `derivset`. |

The `Settle` date for every bond is set to the `ValuationDate` of the BK tree. The bond argument `Settle` is ignored.

## Description

`[Price, PriceTree] = optbndbybk(BKTree, OptSpec, Strike, ExerciseDates, AmericanOpt, CouponRate, Settle, Maturity, Period, Basis, EndMonthRule, IssueDate, FirstCouponDate, LastCouponDate, StartDate, Face, Options)` computes the price of a bond option from a Black-Karasinski interest rate tree.

`Price` is an `NINST`-by-`1` matrix of expected prices at time 0.

`PriceTree` is a tree structure with a vector of instrument prices at each node.

## Examples

**Example 1**. Using the BK interest rate tree in the `deriv.mat` file, price a European call option on a 4% bond with a strike of 96. The exercise date for the option is Jan. 01, 2006. The settle date for the bond is Jan. 01, 2005, and the maturity date is Jan. 01, 2009.

Load the file `deriv.mat`, which provides `BKTree`. The `BKTree` structure contains the time and forward-rate information needed to price the bond.

```
load deriv.mat;
```

Use `optbondbybk` to compute the price of the option.

```
Price = optbndbybk(BKTree,'Call',96,'01-Jan-2006',...
0,0.04,'01-Jan-2005','01-Jan-2009')

Warning: OptBonds are valued at Tree ValuationDate rather than Settle
> In optbndbytrintree at 43
  In optbndbybk at 88
Warning: Not all cash flows are aligned with the tree. Result will be
approximated.
> In optbndbytrintree at 151
  In optbndbybk at 88
```

```
Price =

    0.1512
```

**Example 2**. Now use `optbndbybk` to compute the price of a put option on the same bond.

```
Price = optbndbybk(BKTree,'Put',96,'01-Jan-2006',...
0,0.04,'01-Jan-2005','01-Jan-2009')

Warning: OptBonds are valued at Tree ValuationDate rather than Settle
> In optbndbytrintree at 43
  In optbndbybk at 88
Warning: Not all cash flows are aligned with the tree. Result will be
approximated.
> In optbndbytrintree at 151
  In optbndbybk at 88

Price =

    0.0272
```

## See Also

`bkprice` | `bktree` | `instoptbnd`

## Purpose

Price bond option from Heath-Jarrow-Morton interest-rate tree

## Syntax

```
[Price, PriceTree] = optbndbyhjm(HJMTree, OptSpec, Strike,
ExerciseDates, AmericanOpt, CouponRate, Settle, Maturity,
Period,  Basis, EndMonthRule, IssueDate, FirstCouponDate,
LastCouponDate, StartDate, Face, Options)
```

## Arguments

| | |
|---|---|
| `HJMTree` | Forward-rate tree structure created by `hjmtree`. |
| `OptSpec` | Number of instruments (`NINST`)-by-`1` cell array of string values `'Call'` or `'Put'`. |
| `Strike` | European option: `NINST`-by-`1` vector of strike price values.<br>Bermuda option: `NINST` by number of strikes (`NSTRIKES`) matrix of strike price values.<br>Each row is the schedule for one option. If an option has fewer than `NSTRIKES` exercise opportunities, the end of the row is padded with `NaN`s.<br>For an American option:<br>`NINST`-by-`1` vector of strike price values for each option. |

| | |
|---|---|
| `ExerciseDates` | `NINST`-by-`1` (European option) or `NINST`-by-`NSTRIKES` (Bermuda option) matrix of exercise dates. Each row is the schedule for one option. For a European option, there is only one exercise date, the option expiry date.<br>For an American option:<br>`NINST`-by-`2` vector of exercise date boundaries. For each instrument, the option can be exercised on any coupon date between or including the pair of dates on that row. If only one non-`NaN` date is listed, or if `ExerciseDates` is `NINST`-by-`1`, the option can be exercised between the underlying bond `Settle` and the single listed exercise date. |
| `AmericanOpt` | `NINST`-by-`1` vector of flags: `0` (European/Bermuda) or `1` (American). |
| `CouponRate` | Decimal annual rate. `CouponRate` is a `NINST`-by-`1` vector. |
| `Settle` | Settlement date. A vector of serial date numbers or date strings. `Settle` must be earlier than `Maturity`. |
| `Maturity` | Maturity date. A vector of serial date numbers or date strings. |
| `Period` | (Optional) Coupons per year of the bond. A vector of integers. Allowed values are `1`, `2`, `3`, `4`, `6`, and `12`. Default = `2`. |

| | |
|---|---|
| `Basis` | (Optional) Day-count basis of the instrument. A vector of integers.<br>• 0 = actual/actual (default)<br>• 1 = 30/360 (SIA)<br>• 2 = actual/360<br>• 3 = actual/365<br>• 4 = 30/360 (BMA)<br>• 5 = 30/360 (ISDA)<br>• 6 = 30/360 (European)<br>• 7 = actual/365 (Japanese)<br>• 8 = actual/actual (ICMA)<br>• 9 = actual/360 (ICMA)<br>• 10 = actual/365 (ICMA)<br>• 11 = 30/360E (ICMA)<br>• 12 = actual/actual (ISDA)<br>• 13 = BUS/252<br>For more information, see basis. |
| `EndMonthRule` | (Optional) End-of-month rule. A vector. This rule applies only when `Maturity` is an end-of-month date for a month having 30 or fewer days. `0` = ignore rule, meaning that a bond's coupon payment date is always the same numerical day of the month. `1` = set rule on (default), meaning that a bond's coupon payment date is always the last actual day of the month. |
| `IssueDate` | (Optional) Date when a bond was issued. |

| | |
|---|---|
| `FirstCouponDate` | (Optional) Date when a bond makes its first coupon payment; used when bond has an irregular first coupon period. When `FirstCouponDate` and `LastCouponDate` are both specified, `FirstCouponDate` takes precedence in determining the coupon payment structure. If you do not specify a `FirstCouponDate`, the cash flow payment dates are determined from other inputs. |
| `LastCouponDate` | (Optional) Last coupon date of a bond before the maturity date; used when bond has an irregular last coupon period. In the absence of a specified `FirstCouponDate`, a specified `LastCouponDate` determines the coupon structure of the bond. The coupon structure of a bond is truncated at the `LastCouponDate`, regardless of where it falls, and is followed only by the bond's maturity cash flow date. If you do not specify a `LastCouponDate`, the cash flow payment dates are determined from other inputs. |
| `StartDate` | (Optional) Date when a bond actually starts (the date from which a bond cash flow is considered). To make an instrument forward-starting, specify this date as a future date. If you do not specify `StartDate`, the effective start date is the `Settle` date. |
| `Face` | (Optional) Face or par value. `Face` is a `NINST`-by-1 vector. Default = `100`. |
| `Options` | (Optional) Derivatives pricing options structure created with `derivset`. |

The `Settle` date for every bond is set to the `ValuationDate` of the HJM tree. The bond argument `Settle` is ignored.

## Description

`[Price, PriceTree] = optbndbyhjm(HJMTree, OptSpec, Strike, ExerciseDates, AmericanOpt, CouponRate, Settle, Maturity, Period, Basis, EndMonthRule, IssueDate, FirstCouponDate, LastCouponDate, StartDate, Face, Options)` computes the price of a bond option from an HJM forward-rate tree.

`Price` is an `NINST`-by-`1` matrix of expected prices at time 0.

`PriceTree` is a tree structure with a vector of instrument prices at each node.

## Examples

Using the HJM forward-rate tree in the `deriv.mat` file, price a European call option on a 4% bond with a strike of 96. The exercise date for the option is Jan. 01, 2003. The settle date for the bond is Jan. 01, 2000, and the maturity date is Jan. 01, 2004.

Load the file `deriv.mat`, which provides `HJMTree`. The `HJMTree` structure contains the time and forward-rate information needed to price the bond.

```
load deriv.mat;
```

Use `optbondbyhjm` to compute the price of the option.

```
Price = optbndbyhjm(HJMTree,'Call',96,'01-Jan-2003',...
0,0.04,'01-Jan-2000','01-Jan-2004')
Warning: Not all cash flows are aligned with the tree. Result will
be approximated.

Price =

    2.2410
```

## See Also

`hjmprice` | `hjmtree` | `instoptbnd`

# optbndbyhw

**Purpose** Price bond option from Hull-White interest-rate tree

**Syntax**

```
[Price, PriceTree] = optbndbyhw(HWTree, OptSpec, Strike,
ExerciseDates, AmericanOpt, CouponRate, Settle, Maturity,
Period,  Basis, EndMonthRule, IssueDate, FirstCouponDate,
LastCouponDate, StartDate, Face, Options)
```

**Arguments**

| | |
|---|---|
| `HWTree` | Forward-rate tree structure created by `hwtree`. |
| `OptSpec` | Number of instruments (`NINST`)-by-`1` cell array of string values `'Call'` or `'Put'`. |
| `Strike` | European option: `NINST`-by-`1` vector of strike price values.<br>Bermuda option: `NINST` by number of strikes (`NSTRIKES`) matrix of strike price values.<br>Each row is the schedule for one option. If an option has fewer than `NSTRIKES` exercise opportunities, the end of the row is padded with `NaN`s.<br>For an American option:<br>`NINST`-by-`1` vector of strike price values for each option. |

| | |
|---|---|
| `ExerciseDates` | `NINST`-by-`1` (European option) or `NINST`-by-`NSTRIKES` (Bermuda option) matrix of exercise dates. Each row is the schedule for one option. For a European option, there is only one exercise date, the option expiry date.<br>For an American option:<br>`NINST`-by-`2` vector of exercise date boundaries. For each instrument, the option can be exercised on any coupon date between or including the pair of dates on that row. If only one non-`NaN` date is listed, or if `ExerciseDates` is `NINST`-by-`1`, the option can be exercised between the underlying bond `Settle` and the single listed exercise date. |
| `AmericanOpt` | `NINST`-by-`1` vector of flags: `0` (European/Bermuda) or `1` (American). |
| `CouponRate` | Decimal annual rate. `CouponRate` is a `NINST`-by-`1` vector. |
| `Settle` | Settlement date. A vector of serial date numbers or date strings. `Settle` must be earlier than `Maturity`. |
| `Maturity` | Maturity date. A vector of serial date numbers or date strings. |
| `Period` | (Optional) Coupons per year of the bond. A vector of integers. Allowed values are `1`, `2`, `3`, `4`, `6`, and `12`. Default = `2`. |

| | |
|---|---|
| `Basis` | (Optional) Day-count basis of the instrument. A vector of integers.<br>• 0 = actual/actual (default)<br>• 1 = 30/360 (SIA)<br>• 2 = actual/360<br>• 3 = actual/365<br>• 4 = 30/360 (BMA)<br>• 5 = 30/360 (ISDA)<br>• 6 = 30/360 (European)<br>• 7 = actual/365 (Japanese)<br>• 8 = actual/actual (ICMA)<br>• 9 = actual/360 (ICMA)<br>• 10 = actual/365 (ICMA)<br>• 11 = 30/360E (ICMA)<br>• 12 = actual/actual (ISDA)<br>• 13 = BUS/252<br>For more information, see basis. |
| `EndMonthRule` | (Optional) End-of-month rule. A vector. This rule applies only when `Maturity` is an end-of-month date for a month having 30 or fewer days. `0` = ignore rule, meaning that a bond's coupon payment date is always the same numerical day of the month. `1` = set rule on (default), meaning that a bond's coupon payment date is always the last actual day of the month. |
| `IssueDate` | (Optional) Date when a bond was issued. |

| | |
|---|---|
| `FirstCouponDate` | (Optional) Date when a bond makes its first coupon payment; used when bond has an irregular first coupon period. When `FirstCouponDate` and `LastCouponDate` are both specified, `FirstCouponDate` takes precedence in determining the coupon payment structure. If you do not specify a `FirstCouponDate`, the cash flow payment dates are determined from other inputs. |
| `LastCouponDate` | (Optional) Last coupon date of a bond before the maturity date; used when bond has an irregular last coupon period. In the absence of a specified `FirstCouponDate`, a specified `LastCouponDate` determines the coupon structure of the bond. The coupon structure of a bond is truncated at the `LastCouponDate`, regardless of where it falls, and is followed only by the bond's maturity cash flow date. If you do not specify a `LastCouponDate`, the cash flow payment dates are determined from other inputs. |
| `StartDate` | (Optional) Date when a bond actually starts (the date from which a bond cash flow is considered). To make an instrument forward-starting, specify this date as a future date. If you do not specify `StartDate`, the effective start date is the `Settle` date. |
| `Face` | (Optional) Face or par value. `Face` is a `NINST`-by-1 vector. Default = `100`. |
| `Options` | (Optional) Derivatives pricing options structure created with `derivset`. |

The `Settle` date for every bond is set to the `ValuationDate` of the HW tree. The bond argument `Settle` is ignored.

## Description

`[Price, PriceTree] = optbndbyhw(HWTree, OptSpec, Strike, ExerciseDates, AmericanOpt, CouponRate, Settle, Maturity, Period, Basis, EndMonthRule, IssueDate, FirstCouponDate, LastCouponDate, StartDate, Face, Options)` computes the price of a bond option from a Hull-White interest rate tree.

`Price` is an `NINST`-by-`1` matrix of expected prices at time 0.

`PriceTree` is a tree structure with a vector of instrument prices at each node.

## Examples

**Example 1**. Using the HW interest rate tree in the `deriv.mat` file, price a European call option on a 4% bond with a strike of 96. The exercise date for the option is Jan. 01, 2006. The settle date for the bond is Jan. 01, 2005, and the maturity date is Jan. 01, 2009.

Load the file `deriv.mat`, which provides `HWTree`. The `HWTree` structure contains the time and forward-rate information needed to price the bond.

```
load deriv.mat;
```

Use `optbondbyhw` to compute the price of the option.

```
Price = optbndbyhw(HWTree,'Call',96,'01-Jan-2006',...
0,0.04,'01-Jan-2005','01-Jan-2009')
Warning: OptBonds are valued at Tree ValuationDate rather than Settle
Warning: Not all cash flows are aligned with the tree. Result will
be approximated.

Price =

    1.1556
```

**Example 2**. Now use `optbondbyhw` to compute the price of a put option on the same bond.

```
Price = optbndbyhw(HWTree,'Put',96,'01-Jan-2006',...
0,0.04,'01-Jan-2005','01-Jan-2009')
Warning: OptBonds are valued at Tree ValuationDate rather than Settle
Warning: Not all cash flows are aligned with the tree. Result will
be approximated.

Price =

    1.0150
```

## See Also

`hwprice` | `hwtree` | `instoptbnd`

## Purpose

Price bonds with embedded options by Black-Derman-Toy interest-rate tree

## Syntax

```
[Price, PriceTree] = optembndbybdt(BDTTree, CouponRate,
Settle, Maturity, OptSpec, Strike, ExerciseDates,
'Name1', Value1, 'Name2', Value2, ...)
```

## Arguments

| | |
|---|---|
| `BDTTree` | Interest-rate tree structure created by `bdttree`. |
| `CouponRate` | Decimal annual rate. `CouponRate` is a `NINST`-by-`1` vector or `NINST`-by-`1` cell array of decimal annual rates, or decimal annual rate schedules. For the latter case of a variable coupon schedule, each element of the cell array is a `NumDates`-by-`2` cell array, where the first column is dates and the second column is its associated rate. The date indicates the last day that the coupon rate is valid. |
| `Settle` | `NINST`-by-`1` matrix for the settlement date. A vector of serial date numbers or date strings. `Settle` must be earlier than `Maturity`. |
| `Maturity` | `NINST`-by-`1` matrix for the maturity date. A vector of serial date numbers or date strings. |
| `OptSpec` | `NINST`-by-`1` cell array of strings `'call'` or `'put'`. |

| | |
|---|---|
| `Strike` | European option: `NINST`-by-`1` vector of strike price values.<br>Bermuda option: `NINST` by number of strikes (`NSTRIKES`) matrix of strike price values.<br>Each row is the schedule for one option. If an option has fewer than `NSTRIKES` exercise opportunities, the end of the row is padded with `NaN`s.<br>For an American option:<br>`NINST`-by-`1` vector of strike price values for each option. |
| `ExerciseDates` | `NINST`-by-`1` (European option) or `NINST`-by-`NSTRIKES` (Bermuda option) matrix of exercise dates. Each row is the schedule for one option. For a European option, there is only one exercise date, the option expiry date.<br>For an American option:<br>`NINST`-by-`2` vector of exercise date boundaries. For each instrument, the option can be exercised on any coupon date between or including the pair of dates on that row. If only one non-`NaN` date is listed, or if `ExerciseDates` is `NINST`-by-`1`, the option can be exercised between the underlying bond `Settle` and the single listed exercise date. |
| `'Name1', 'Value1' 'Name2', 'Value2'...` | (Optional) The name/value pairs are a variable length list of parameters. All optional inputs are specified as matching parameter name/value pairs. The parameter name is specified as a character string, followed by the corresponding parameter value. Parameter name/value pairs may |

be specified in any order; names are case insensitive and partial string matches are allowed provided no ambiguities exist. Valid parameter names are as follows:

- `AmericanOpt` is a `NINST`-by-`1` matrix for flags options: `0` (European/Bermuda) or `1` (American). Default is `0`.
- `Period` is a `NINST`-by-`1` matrix for coupons per year. The default value is `2`.
- `Basis` is a day-count basis of the instrument. `Basis` is a vector of integers with the following possible values:
  - 0 = actual/actual (default)
  - 1 = 30/360 (SIA)
  - 2 = actual/360
  - 3 = actual/365
  - 4 = 30/360 (BMA)
  - 5 = 30/360 (ISDA)
  - 6 = 30/360 (European)
  - 7 = actual/365 (Japanese)
  - 8 = actual/actual (ICMA)
  - 9 = actual/360 (ICMA)
  - 10 = actual/365 (ICMA)
  - 11 = 30/360E (ICMA)
  - 12 = actual/actual (ISDA)
  - 13 = BUS/252

  For more information, see basis.

- `EndMonthRule` is a `NINST`-by-`1` matrix for the end-of-month rule. This rule applies only when `Maturity` is an end-of-month date for a month having 30 or fewer days. When the value is `0` the end-of-month rule is ignored; this means that a bond's coupon payment date is always the same numerical day of the month. Use `1` to set the rule on; this is the default value and means that a bond's coupon payment date is always the last actual day of the month.
- `IssueDate` is a `NINST`-by-`1` matrix for the bond issue date.
- `FirstCouponDate` is a `NINST`-by-`1` matrix for a date when a bond makes its first coupon payment; used when bond has an irregular first coupon period. When `FirstCouponDate` and `LastCouponDate` are both specified, `FirstCouponDate` takes precedence in determining the coupon payment structure. If you do not specify a `FirstCouponDate`, the cash flow payment dates are determined from other inputs.
- `LastCouponDate` is a `NINST`-by-`1` matrix for a last coupon date of a bond before the maturity date; used when bond has an irregular last coupon period. In the absence of a specified `FirstCouponDate`, a specified `LastCouponDate` determines the coupon structure of the bond. The coupon structure of a bond is truncated at the `LastCouponDate`, regardless of where it falls, and is followed only by the bond's maturity cash flow date. If you do not specify a `LastCouponDate`, the cash flow

payment dates are determined from other inputs.

- `StartDate` is a `NINST`-by-`1` matrix for date when a bond actually starts (the date from which a bond cash flow is considered). To make an instrument forward-starting, specify this date as a future date. If you do not specify `StartDate`, the effective start date is the `Settle` date.
- Face value. `Face` is a `NINST`-by-`1` vector or `NINST`-by-`1` cell array of face values, or face value schedules. For the latter case, each element of the cell array is a `NumDates`-by-`2` cell array, where the first column is dates and the second column is its associated face value. The date indicates the last day that the face value is valid. Default is 100.
- `Options` is a derivatives pricing options structure created with `derivset`.

---

**Note** The `Settle` date for every bond with an embedded option is set to the `ValuationDate` of the BDT tree; the bond's argument for `Settle` date is ignored.

---

## Description

`[Price, PriceTree] = optembndbybdt(BDTTree, CouponRate, Settle, Maturity, OptSpec, Strike, ExerciseDates, 'Name1', Value1, 'Name2', Value2, ...)` prices bonds with embedded options using a BDT interest-rate tree.

`Price` is a number of instruments (`NINST`)-by-`1` matrix of expected prices at time 0.

`PriceTree` is a MATLAB structure of trees containing vectors of instrument prices and observation times for each node. Within `PriceTree`

- `PriceTree.PTree` contains the clean prices.
- `PriceTree.tObs` contains the observation times.

`optembndbybdt` computes prices of vanilla bonds with embedded options, stepped coupon bonds with embedded options, and bonds with sinking fund option provisions.

## Definitions

### Vanilla Bond with Embedded Option

A vanilla coupon bond is a security representing an obligation to repay a borrowed amount at a designated time and to make periodic interest payments until that time. The issuer of a bond makes the periodic interest payments until the bond matures. At maturity, the issuer pays to the holder of the bond the principal amount owed (face value) and the last interest payment. A vanilla bond with an embedded option is where an option contract has an underlying asset of a vanilla bond.

### Stepped Coupon Bond with Callable and Puttable Features

A step-up and step-down bond is a debt security with a predetermined coupon structure over time. With these instruments, coupons increase (step up) or decrease (step down) at specific times during the life of the bond. Stepped coupon bonds can have options features (call and puts).

### Sinking Fund Bond with Embedded Option

A sinking fund bond is a coupon bond with a sinking fund provision. This provision obligates the issuer to amortize portions of the principal prior to maturity, affecting bond prices since the time of the principal repayment changes. This means that investors receive the coupon and a portion of the principal paid back over time. These types of bonds reduce credit risk, since it lowers the probability of investors not receiving their principal payment at maturity.

The bond may have a sinking fund option provision allowing the issuer to retire the sinking fund obligation either by purchasing the bonds to be redeemed from the market or by calling the bond via a sinking fund call, whichever is cheaper. If interest rates are high, then the issuer will buy back the requirement amount of bonds from the market since bonds will be cheap, but if interest rates are low (bond prices are high), then most likely the issuer will be buying the bonds at the call price. Unlike a call feature, however, if a bond has a sinking fund option provision, it is an obligation, not an option, for the issuer to buy back the increments of the issue as stated. Because of this, a sinking fund bond trades at a lower price than a non-sinking fund bond.

## Examples

To price a callable bond using the BDT model, create a BDTTree with the following data:

```
ZeroRates = [ 0.035;0.04;0.045];
Compounding = 1;
StartDates = ['jan-1-2007';'jan-1-2008';'jan-1-2009'];
EndDates   = ['jan-1-2008';'jan-1-2009';'jan-1-2010'];
ValuationDate = 'jan-1-2007';
```

Create a `RateSpec`:

```
RateSpec = intenvset('Rates', ZeroRates, 'StartDates', ValuationDate, 'EndDates', ...
EndDates, 'Compounding', Compounding, 'ValuationDate', ValuationDate);
```

Specify a `VolSpec`:

```
Volatility = 0.10 * ones (3,1);
VolSpec = bdtvolspec(ValuationDate, EndDates, Volatility);
```

Specify a `TimeSpec`:

```
TimeSpec = bdttimespec(ValuationDate, EndDates, Compounding);
```

Build the BDTTree:

```
BDTTree = bdttree(VolSpec, RateSpec, TimeSpec);
```

To compute the price of an American callable bond that pays a 5.25% annual coupon, matures in Jan-1-2010, and is callable on Jan-1-2008 and 01-Jan-2010:

```
BondSettlement = 'jan-1-2007';
BondMaturity   = 'jan-1-2010';
CouponRate = 0.0525;
Period = 1;
OptSpec = 'call';
Strike = [100];
ExerciseDates = {'jan-1-2008' '01-Jan-2010'};
AmericanOpt = 1;

PriceCallBond = optembndbybdt(BDTTree, CouponRate, BondSettlement, BondMaturity,...
OptSpec, Strike, ExerciseDates, 'Period', 1,'AmericanOp', 1)

PriceCallBond =

101.4750
```

---

Price the following single stepped callable bonds using the following data:

```
% The data for the interest rate term structure is as follows:
Rates = [0.035; 0.042147; 0.047345; 0.052707];
ValuationDate = 'Jan-1-2010';
StartDates = ValuationDate;
EndDates = {'Jan-1-2011'; 'Jan-1-2012'; 'Jan-1-2013'; 'Jan-1-2014'};
Compounding = 1;

% Create RateSpec
RS = intenvset('ValuationDate', ValuationDate, 'StartDates', StartDates,...
'EndDates', EndDates,'Rates', Rates, 'Compounding', Compounding);

% Instrument
Settle = '01-Jan-2010';
```

```
Maturity = {'01-Jan-2013';'01-Jan-2014'};
CouponRate = {{'01-Jan-2012' .0425;'01-Jan-2014' .0750}};
OptSpec='call';
Strike=100;
ExerciseDates='01-Jan-2012';  %Callable in two years

% Build the tree
% Assume the volatility to be 10%
Sigma = 0.1;
BDTTimeSpec = bdttimespec(ValuationDate, EndDates, Compounding);
BDTVolSpec = bdtvolspec(ValuationDate, EndDates, Sigma*ones(1, length(EndDates))');
BDTT = bdttree(BDTVolSpec, RS, BDTTimeSpec);

% The first row corresponds to the price of the callable bond with maturity
% of three years. The second row corresponds to the price of the callable bond
% with maturity of four years.
PBDT=  optembndbybdt(BDTT, CouponRate, Settle, Maturity ,OptSpec, Strike,...
ExerciseDates, 'Period', 1)

PBDT =

  100.0945
  100.0297
```

---

A corporation issues a three year bond with a sinking fund obligation requiring the company to sink 1/3 of face value after the first year and 1/3 after the second year. The company has the option to buy the bonds in the market or call them at $98. The following data describes the details needed for pricing the sinking fund bond:

```
% The data for the interest rate term structure is as follows:
Rates = [0.1;0.1;0.1;0.1];
ValuationDate = 'Jan-1-2011';
StartDates = ValuationDate;
EndDates = {'Jan-1-2012'; 'Jan-1-2013'; 'Jan-1-2014'; 'Jan-1-2015'};
Compounding = 1;
```

```
% Create RateSpec
RateSpec = intenvset('ValuationDate', ValuationDate, 'StartDates',...
StartDates, 'EndDates', EndDates, 'Rates', Rates, 'Compounding', Compounding);

% Build the BDT tree
% Assume the volatility to be 10%
Sigma = 0.1;
BDTTimeSpec = bdttimespec(ValuationDate, EndDates);
BDTVolSpec = bdtvolspec(ValuationDate, EndDates, Sigma*ones(1, length(EndDates))');
BDTT = bdttree(BDTVolSpec, RateSpec, BDTTimeSpec);

% Instrument
% The bond has a coupon rate of 9%, a period of one year and matures in
% 1-Jan-2014. Face decreases 1/3 after the first year and 1/3 after the
% second year.
CouponRate = 0.09;
Settle = 'Jan-1-2011';
Maturity =  'Jan-1-2014';
Period = 1;
Face = { ...
            {'Jan-1-2012'  100; ...
             'Jan-1-2013'   66.6666; ...
             'Jan-1-2014'   33.3333};
        };

% Option provision
OptSpec = 'call';
Strike = [98 98];
ExerciseDates ={'Jan-1-2012', 'Jan-1-2013'};



% Price of non-sinking fund bond.
PNSF = bondbybdt(BDTT, CouponRate, Settle, Maturity, Period)
PNSF =

   97.5131
```

```
% Price of the bond with the option sinking provision.
PriceSF = optembndbybdt(BDTT, CouponRate, Settle, Maturity,...
OptSpec, Strike, ExerciseDates,'Period', Period, 'Face', Face)

PriceSF =

   96.8364
```

## See Also

`bdtprice` | `bdttree` | `cfamounts` | `instoptembnd`

## Purpose

Price bonds with embedded options by Black-Karasinski interest-rate tree

## Syntax

```
[Price, PriceTree] = optembndybk(BKTree, CouponRate,
Settle, Maturity, OptSpec, Strike, Strike, ExerciseDates,
'Name1', Value1, 'Name2', Value2, ...)
```

## Arguments

| | |
|---|---|
| `BKTree` | Interest-rate tree structure created by `bktree`. |
| `CouponRate` | Decimal annual rate. `CouponRate` is a `NINST`-by-`1` vector or `NINST`-by-`1` cell array of decimal annual rates, or decimal annual rate schedules. For the latter case of a variable coupon schedule, each element of the cell array is a `NumDates`-by-`2` cell array, where the first column is dates and the second column is its associated rate. The date indicates the last day that the coupon rate is valid. |
| `Settle` | `NINST`-by-`1` matrix for the settlement date. A vector of serial date numbers or date strings. `Settle` must be earlier than `Maturity`. |
| `Maturity` | `NINST`-by-`1` matrix for the maturity date. A vector of serial date numbers or date strings. |
| `OptSpec` | `NINST`-by-`1` cell array of strings `'call'` or `'put'`. |

| | |
|---|---|
| `Strike` | European option: `NINST`-by-`1` vector of strike price values.<br>Bermuda option: `NINST` by number of strikes (`NSTRIKES`) matrix of strike price values.<br>Each row is the schedule for one option. If an option has fewer than `NSTRIKES` exercise opportunities, the end of the row is padded with `NaN`s.<br>For an American option:<br>`NINST`-by-`1` vector of strike price values for each option. |
| `ExerciseDates` | `NINST`-by-`1` (European option) or `NINST`-by-`NSTRIKES` (Bermuda option) matrix of exercise dates. Each row is the schedule for one option. For a European option, there is only one exercise date, the option expiry date.<br>For an American option:<br>`NINST`-by-`2` vector of exercise date boundaries. For each instrument, the option can be exercised on any coupon date between or including the pair of dates on that row. If only one non-`NaN` date is listed, or if `ExerciseDates` is `NINST`-by-`1`, the option can be exercised between the underlying bond `Settle` and the single listed exercise date. |
| `'Name1', Value1, 'Name2', Value2 ...` | (Optional) The name/value pairs are a variable length list of parameters. All optional inputs are specified as matching parameter name/value pairs. The parameter name is specified as a character string, followed by the corresponding parameter value. Parameter name/value pairs may |

be specified in any order; names are case insensitive and partial string matches are allowed provided no ambiguities exist. Valid parameter names are as follows:

- `AmericanOpt` is a `NINST`-by-`1` matrix for flags options: `0` (European/Bermuda) or `1` (American). Default is `0`.
- `Period` is a `NINST`-by-`1` matrix for coupons per year. Default is `2`.
- `Basis` is a day-count basis of the instrument. `Basis` is a vector of integers with the following supported values:
    - 0 = actual/actual (default)
    - 1 = 30/360 (SIA)
    - 2 = actual/360
    - 3 = actual/365
    - 4 = 30/360 (BMA)
    - 5 = 30/360 (ISDA)
    - 6 = 30/360 (European)
    - 7 = actual/365 (Japanese)
    - 8 = actual/actual (ICMA)
    - 9 = actual/360 (ICMA)
    - 10 = actual/365 (ICMA)
    - 11 = 30/360E (ICMA)
    - 12 = actual/actual (ISDA)
    - 13 = BUS/252

  For more information, see basis.

- `EndMonthRule` is a `NINST`-by-`1` matrix for the end-of-month rule. This rule applies only when `Maturity` is an end-of-month date for a month having 30 or fewer days. When the value is `0`, the end-of-month rule is ignored, meaning that a bond's coupon payment date is always the same numerical day of the month. When the value is `1`, the end-of-month rule is set on (default), meaning that a bond's coupon payment date is always the last actual day of the month.
- `IssueDate` is a `NINST`-by-`1` matrix for the bond issue date.
- `FirstCouponDate` is a `NINST`-by-`1` matrix for a date when a bond makes its first coupon payment; used when bond has an irregular first coupon period. When `FirstCouponDate` and `LastCouponDate` are both specified, `FirstCouponDate` takes precedence in determining the coupon payment structure. If you do not specify a `FirstCouponDate`, the cash flow payment dates are determined from other inputs.
- `LastCouponDate` is a `NINST`-by-`1` matrix for a last coupon date of a bond before the maturity date; used when bond has an irregular last coupon period. In the absence of a specified `FirstCouponDate`, a specified `LastCouponDate` determines the coupon structure of the bond. The coupon structure of a bond is truncated at the `LastCouponDate`, regardless of where it falls, and is followed only by the bond's maturity cash flow date. If you do not

specify a `LastCouponDate`, the cash flow payment dates are determined from other inputs.

- `StartDate` is a `NINST`-by-`1` matrix for date when a bond actually starts (the date from which a bond cash flow is considered). To make an instrument forward-starting, specify this date as a future date. If you do not specify `StartDate`, the effective start date is the `Settle` date.
- Face value. `Face` is a `NINST`-by-`1` vector or `NINST`-by-`1` cell array of face values, or face value schedules. For the latter case, each element of the cell array is a `NumDates`-by-`2` cell array, where the first column is dates and the second column is its associated face value. The date indicates the last day that the face value is valid. Default is 100.
- `Options` is a derivatives pricing options structure created with `derivset`.

---

**Note** The `Settle` date for every bond with embedded option is set to the `ValuationDate` of the BKTree; the bond's argument for `Settle` date is ignored.

---

## Description

`[Price, PriceTree] = optembndybk(BKTree, CouponRate, Settle, Maturity, OptSpec, Strike, Strike, ExerciseDates, 'Name1', Value1, 'Name2', Value2, ...)` prices bonds with embedded options by a BK interest-rate tree.

`Price` is a number of instruments (`NINST`)-by-`1` matrix of expected prices at time 0.

`PriceTree` is a structure of trees containing vectors of instrument prices and observation times for each node. Within `PriceTree`:

- `PriceTree.PTree` contains the clean prices.
- `PriceTree.tObs` contains the observation times.

`optembndbybk` computes prices of vanilla bonds with embedded options, stepped coupon bonds with embedded options, and bonds with sinking fund option provisions.

## Definitions

### Vanilla Bond with Embedded Option

A vanilla coupon bond is a security representing an obligation to repay a borrowed amount at a designated time and to make periodic interest payments until that time. The issuer of a bond makes the periodic interest payments until the bond matures. At maturity, the issuer pays to the holder of the bond the principal amount owed (face value) and the last interest payment. A vanilla bond with an embedded option is where an option contract has an underlying asset of a vanilla bond.

### Stepped Coupon Bond with Callable and Puttable Features

A step-up and step-down bond is a debt security with a predetermined coupon structure over time. With these instruments, coupons increase (step up) or decrease (step down) at specific times during the life of the bond. Stepped coupon bonds can have options features (call and puts).

### Sinking Fund Bond with Embedded Option

A sinking fund bond is a coupon bond with a sinking fund provision. This provision obligates the issuer to amortize portions of the principal prior to maturity, affecting bond prices since the time of the principal repayment changes. This means that investors receive the coupon and a portion of the principal paid back over time. These types of bonds reduce credit risk, since it lowers the probability of investors not receiving their principal payment at maturity.

The bond may have a sinking fund option provision allowing the issuer to retire the sinking fund obligation either by purchasing the bonds to be redeemed from the market or by calling the bond via a sinking fund call, whichever is cheaper. If interest rates are high, then the issuer will buy back the requirement amount of bonds from the market since bonds will be cheap, but if interest rates are low (bond prices are high), then most likely the issuer will be buying the bonds at the call price. Unlike a call feature, however, if a bond has a sinking fund option provision, it is an obligation, not an option, for the issuer to buy back the increments of the issue as stated. Because of this, a sinking fund bond trades at a lower price than a non-sinking fund bond.

## Examples

Create a `BKTree` with the following data:

```
ZeroRates = [ 0.035;0.04;0.045];
Compounding = 1;
StartDates = ['jan-1-2007';'jan-1-2008';'jan-1-2009'];
EndDates   = ['jan-1-2008';'jan-1-2009';'jan-1-2010'];
ValuationDate = 'jan-1-2007';
```

Create a `RateSpec`:

```
RateSpec = intenvset('Rates', ZeroRates, 'StartDates', ValuationDate, 'EndDates', ...
EndDates, 'Compounding', Compounding, 'ValuationDate', ValuationDate);
```

Specify a `TimeSpec`:

```
BKTimeSpec = bktimespec(ValuationDate, EndDates, Compounding);
```

Specify a `VolSpec`:

```
VolDates = ['jan-1-2008';'jan-1-2009';'jan-1-2010'];
VolCurve = 0.01;
AlphaDates = 'jan-1-2010';
AlphaCurve = 0.1;
BKVolSpec = bkvolspec(ValuationDate, VolDates, VolCurve, AlphaDates, AlphaCurve);
```

Build a `BKTree`:

```
BKTree = bktree(BKVolSpec, RateSpec, BKTimeSpec);
```

To compute the price of an American puttable bond that pays an annual coupon of 5.25% , matures on January 1, 2010, and is callable on January 1, 2008 and January 1, 2010:

```
BondSettlement = 'jan-1-2007';
BondMaturity   = 'jan-1-2010';
CouponRate = 0.0525;
Period = 1;
OptSpec = 'put';
Strike = [100];
ExerciseDates = {'jan-1-2008' '01-Jan-2010'};
AmericanOpt = 1;

PricePutBondBK = optembndbybk(BKTree, CouponRate, BondSettlement, BondMaturity,...
OptSpec, Strike, ExerciseDates,'Period', 1, 'AmericanOpt', 1)

PricePutBondBK =

102.3820
```

---

Price the following single stepped callable bonds using the following data:

```
% The data for the interest rate term structure is as follows:
Rates = [0.035; 0.042147; 0.047345; 0.052707];
ValuationDate = 'Jan-1-2010';
StartDates = ValuationDate;
EndDates = {'Jan-1-2011'; 'Jan-1-2012'; 'Jan-1-2013'; 'Jan-1-2014'};
Compounding = 1;

% Create RateSpec
RS = intenvset('ValuationDate', ValuationDate, 'StartDates', StartDates,...
'EndDates', EndDates,'Rates', Rates, 'Compounding', Compounding);
```

```
% Instrument
Settle = '01-Jan-2010';
Maturity = {'01-Jan-2013';'01-Jan-2014'};
CouponRate = {{'01-Jan-2012' .0425;'01-Jan-2014' .0750}};
OptSpec='call';
Strike=100;
ExerciseDates='01-Jan-2012';  %Callable in two years

% Build the tree with the following data
VolDates = ['1-Jan-2011'; '1-Jan-2012'; '1-Jan-2013'; '1-Jan-2014'];
VolCurve = 0.01;
AlphaDates = '01-01-2014';
AlphaCurve = 0.1;


BKVolSpec = bkvolspec(RS.ValuationDate, VolDates, VolCurve,...
AlphaDates, AlphaCurve);
BKTimeSpec = bktimespec(RS.ValuationDate, VolDates, Compounding);
BKT = bktree(BKVolSpec, RS, BKTimeSpec);

% The first row corresponds to the price of the callable bond with maturity
% of three years. The second row corresponds to the price of the callable bond
% with maturity of four years.
PBK= optembndbybk(BKT, CouponRate,  Settle, Maturity ,OptSpec, Strike,...
ExerciseDates, 'Period', 1)

PBK =

  100.0945
  100.0945
```

## See Also

bkprice | cfamounts | bktree | instoptembnd

## Purpose

Price bonds with embedded options by Heath-Jarrow-Morton interest-rate tree

## Syntax

```
[Price, PriceTree] = optembndbyhjm(HJMTree, CouponRate,
Settle, Maturity, OptSpec, Strike, ExerciseDates,
'Name1',  Value1, 'Name2', Value2, ...)
```

## Arguments

| | |
|---|---|
| `HJMTree` | Interest-rate tree structure created by `hjmtree`. |
| `CouponRate` | Decimal annual rate. `CouponRate` is a `NINST`-by-`1` vector or `NINST`-by-`1` cell array of decimal annual rates, or decimal annual rate schedules. For the latter case of a variable coupon schedule, each element of the cell array is a `NumDates`-by-`2` cell array, where the first column is dates and the second column is its associated rate. The date indicates the last day that the coupon rate is valid. |
| `Settle` | `NINST`-by-`1` matrix for the settlement date. A vector of serial date numbers or date strings. `Settle` must be earlier than `Maturity`. |
| `Maturity` | `NINST`-by-`1` matrix for the maturity date. A vector of serial date numbers or date strings. |
| `OptSpec` | `NINST`-by-`1` cell array of strings `'call'` or `'put'`. |

| | |
|---|---|
| `Strike` | European option: `NINST`-by-`1` vector of strike price values.<br>Bermuda option: `NINST` by number of strikes (`NSTRIKES`) matrix of strike price values.<br>Each row is the schedule for one option. If an option has fewer than `NSTRIKES` exercise opportunities, the end of the row is padded with `NaN`s.<br>For an American option:<br>`NINST`-by-`1` vector of strike price values for each option. |
| `ExerciseDates` | `NINST`-by-`1` (European option) or `NINST`-by-`NSTRIKES` (Bermuda option) matrix of exercise dates. Each row is the schedule for one option. For a European option, there is only one exercise date, the option expiry date.<br>For an American option:<br>`NINST`-by-`2` vector of exercise date boundaries. For each instrument, the option can be exercised on any coupon date between or including the pair of dates on that row. If only one non-`NaN` date is listed, or if `ExerciseDates` is `NINST`-by-`1`, the option can be exercised between the underlying bond `Settle` and the single listed exercise date. |
| `'Name1', Value1, 'Name2', Value2 ...` | (Optional) The name/value pairs are a variable length list of parameters. All optional inputs are specified as matching parameter name/value pairs. The parameter name is specified as a character string, followed by the corresponding parameter value. Parameter name/value pairs may |

be specified in any order; names are case insensitive and partial string matches are allowed provided no ambiguities exist. Valid parameter names are as follows:

- `AmericanOpt` is a `NINST`-by-`1` matrix for flags options: `0` (European/Bermuda) or `1` (American). Default is `0`.
- `Period` is a `NINST`-by-`1` matrix for coupons per year. Default is `2`.
- `Basis` is a day-count basis of the instrument. `Basis` is a vector of integers with the following supported values:
  - 0 = actual/actual (default)
  - 1 = 30/360 (SIA)
  - 2 = actual/360
  - 3 = actual/365
  - 4 = 30/360 (BMA)
  - 5 = 30/360 (ISDA)
  - 6 = 30/360 (European)
  - 7 = actual/365 (Japanese)
  - 8 = actual/actual (ICMA)
  - 9 = actual/360 (ICMA)
  - 10 = actual/365 (ICMA)
  - 11 = 30/360E (ICMA)
  - 12 = actual/actual (ISDA)
  - 13 = BUS/252

  For more information, see basis.

- `EndMonthRule` is a `NINST`-by-`1` matrix for the end-of-month rule. This rule applies only when `Maturity` is an end-of-month date for a month having 30 or fewer days. When the value is `0`, the end-of-month rule is ignored, meaning that a bond's coupon payment date is always the same numerical day of the month. When the value is `1`, the end-of-month rule is set on (default), meaning that a bond's coupon payment date is always the last actual day of the month.
- `IssueDate` is a `NINST`-by-`1` matrix for the bond issue date.
- `FirstCouponDate` is a `NINST`-by-`1` matrix for a date when a bond makes its first coupon payment; used when bond has an irregular first coupon period. When `FirstCouponDate` and `LastCouponDate` are both specified, `FirstCouponDate` takes precedence in determining the coupon payment structure. If you do not specify a `FirstCouponDate`, the cash flow payment dates are determined from other inputs.
- `LastCouponDate` is a `NINST`-by-`1` matrix for a last coupon date of a bond before the maturity date; used when bond has an irregular last coupon period. In the absence of a specified `FirstCouponDate`, a specified `LastCouponDate` determines the coupon structure of the bond. The coupon structure of a bond is truncated at the `LastCouponDate`, regardless of where it falls, and is followed only by the bond's maturity cash flow date. If you do not

specify a `LastCouponDate`, the cash flow payment dates are determined from other inputs.

- `StartDate` is a `NINST`-by-`1` matrix for date when a bond actually starts (the date from which a bond cash flow is considered). To make an instrument forward-starting, specify this date as a future date. If you do not specify `StartDate`, the effective start date is the `Settle` date.
- Face value. `Face` is a `NINST`-by-`1` vector or `NINST`-by-`1` cell array of face values, or face value schedules. For the latter case, each element of the cell array is a `NumDates`-by-`2` cell array, where the first column is dates and the second column is its associated face value. The date indicates the last day that the face value is valid. Default is 100.
- `Options` is a derivatives pricing options structure created with `derivset`.

---

**Note** The `Settle` date for every bond with embedded option is set to the `ValuationDate` of the HJM tree; the bond's argument for `Settle` date is ignored.

---

## Description

`[Price, PriceTree] = optembndbyhjm(HJMTree, CouponRate, Settle, Maturity, OptSpec, Strike, ExerciseDates, 'Name1', Value1, 'Name2', Value2, ...)` prices bonds with embedded options by an HJM interest-rate tree.

`Price` is a number of instruments (`NINST`)-by-`1` matrix of expected prices at time 0.

`PriceTree` is a structure of trees containing vectors of instrument prices and observation times for each node. Within `PriceTree`:

- `PriceTree.PBush` contains the clean prices.
- `PriceTree.tObs` contains the observation times.

`optembndbyhjm` computes prices of vanilla bonds with embedded options, stepped coupon bonds with embedded options, and bonds with sinking fund option provisions.

## Definitions

### Vanilla Bond with Embedded Option

A vanilla coupon bond is a security representing an obligation to repay a borrowed amount at a designated time and to make periodic interest payments until that time. The issuer of a bond makes the periodic interest payments until the bond matures. At maturity, the issuer pays to the holder of the bond the principal amount owed (face value) and the last interest payment. A vanilla bond with an embedded option is where an option contract has an underlying asset of a vanilla bond.

### Stepped Coupon Bond with Callable and Puttable Features

A step-up and step-down bond is a debt security with a predetermined coupon structure over time. With these instruments, coupons increase (step up) or decrease (step down) at specific times during the life of the bond. Stepped coupon bonds can have options features (call and puts).

### Sinking Fund Bond with Embedded Option

A sinking fund bond is a coupon bond with a sinking fund provision. This provision obligates the issuer to amortize portions of the principal prior to maturity, affecting bond prices since the time of the principal repayment changes. This means that investors receive the coupon and a portion of the principal paid back over time. These types of bonds reduce credit risk, since it lowers the probability of investors not receiving their principal payment at maturity.

The bond may have a sinking fund option provision allowing the issuer to retire the sinking fund obligation either by purchasing the bonds to be redeemed from the market or by calling the bond via a sinking fund call, whichever is cheaper. If interest rates are high, then the issuer will buy back the requirement amount of bonds from the market since bonds will be cheap, but if interest rates are low (bond prices are high), then most likely the issuer will be buying the bonds at the call price. Unlike a call feature, however, if a bond has a sinking fund option provision, it is an obligation, not an option, for the issuer to buy back the increments of the issue as stated. Because of this, a sinking fund bond trades at a lower price than a non-sinking fund bond.

## Examples

Create an `HJMTree` with the following data:

```
Rates = [0.05;0.06;0.07];
Compounding = 1;
StartDates = ['jan-1-2007';'jan-1-2008';'jan-1-2009'];
EndDates   = ['jan-1-2008';'jan-1-2009';'jan-1-2010'];
ValuationDate = 'jan-1-2007';
```

Create a `RateSpec`:

```
RateSpec = intenvset('Rates', Rates, 'StartDates', ValuationDate, 'EndDates', ...
EndDates, 'Compounding', Compounding, 'ValuationDate', ValuationDate);
```

Specify a `VolSpec`:

```
VolSpec = hjmvolspec('Constant', 0.01);
```

Specify a `TimeSpec`:

```
TimeSpec = hjmtimespec(ValuationDate, EndDates, Compounding);
```

Build an `HJMTree`:

```
HJMTree = hjmtree(VolSpec, RateSpec, TimeSpec);
```

To compute the price of an American callable bond that pays a 6% annual coupon and matures and is callable on January 1, 2010:

```
BondSettlement = 'jan-1-2007';
BondMaturity   = 'jan-1-2010';
CouponRate = 0.06;
Period = 1;
OptSpec = 'call';
Strike = [98];
ExerciseDates = '01-Jan-2010';
AmericanOpt = 1;

[PriceCallBond,PT] = optembndbyhjm(HJMTree, CouponRate, BondSettlement, BondMaturity,...
OptSpec, Strike, ExerciseDates, 'Period', 1,'AmericanOp',1)

PriceCallBond =

95.9492

PT =

FinObj: 'HJMPriceTree'
      tObs: [0 1 2 3]
     PBush: {[95.9492]  [1x1x2 double]  [1x2x2 double]  [98 98 98 98]}
```

Price the following single stepped callable bonds using the following data:

```
% The data for the interest rate term structure is as follows:
Rates = [0.035; 0.042147; 0.047345; 0.052707];
ValuationDate = 'Jan-1-2010';
StartDates = ValuationDate;
EndDates = {'Jan-1-2011'; 'Jan-1-2012'; 'Jan-1-2013'; 'Jan-1-2014'};
Compounding = 1;

% Create RateSpec
```

```
RS = intenvset('ValuationDate', ValuationDate, 'StartDates', StartDates,...
'EndDates', EndDates,'Rates', Rates, 'Compounding', Compounding);

% Instrument
Settle = '01-Jan-2010';
Maturity = {'01-Jan-2013';'01-Jan-2014'};
CouponRate = {{'01-Jan-2012' .0425;'01-Jan-2014' .0750}};
OptSpec='call';
Strike=100;
ExerciseDates='01-Jan-2012';  %Callable in two years

% Build the tree with the following data
Volatility = [.2; .19; .18; .17];
CurveTerm = [ 1;  2;   3;   4];
HJMTimeSpec = hjmtimespec(ValuationDate, EndDates);
HJMVolSpec = hjmvolspec('Proportional', Volatility, CurveTerm, 1e6);
HJMT = hjmtree(HJMVolSpec, RS, HJMTimeSpec);

% The first row corresponds to the price of the callable bond with maturity
% of three years. The second row corresponds to the price of the callable
% bond with maturity of four years.
PHJM=  optembndbyhjm(HJMT, CouponRate, Settle, Maturity ,OptSpec, Strike,...
ExerciseDates, 'Period', 1)

PHJM =

  100.0484
   99.8009
```

## See Also

hjmprice | cfamounts | hjmtree | instoptembnd

## Purpose

Price bonds with embedded options by Hull-White interest-rate tree

## Syntax

```
[Price, PriceTree] = optembndbyhw(HWTree, CouponRate,
Settle, Maturity, OptSpec, Strike, ExerciseDates,
'Name1', Value1, 'Name2', Value2, ...)
```

## Arguments

| | |
|---|---|
| `HWTree` | Interest-rate tree structure created by `hwtree`. |
| `CouponRate` | Decimal annual rate. `CouponRate` is a `NINST`-by-`1` vector or `NINST`-by-`1` cell array of decimal annual rates, or decimal annual rate schedules. For the latter case of a variable coupon schedule, each element of the cell array is a `NumDates`-by-`2` cell array, where the first column is dates and the second column is its associated rate. The date indicates the last day that the coupon rate is valid. |
| `Settle` | `NINST`-by-`1` matrix for the settlement date. A vector of serial date numbers or date strings. `Settle` must be earlier than `Maturity`. |
| `Maturity` | `NINST`-by-`1` matrix for the maturity date. A vector of serial date numbers or date strings. |
| `OptSpec` | `NINST`-by-`1` cell array of strings `'call'` or `'put'`. |

| | |
|---|---|
| `Strike` | European option: `NINST`-by-`1` vector of strike price values.<br>Bermuda option: `NINST` by number of strikes (`NSTRIKES`) matrix of strike price values.<br>Each row is the schedule for one option. If an option has fewer than `NSTRIKES` exercise opportunities, the end of the row is padded with `NaN`s.<br>For an American option:<br>`NINST`-by-`1` vector of strike price values for each option. |
| `ExerciseDates` | `NINST`-by-`1` (European option) or `NINST`-by-`NSTRIKES` (Bermuda option) matrix of exercise dates. Each row is the schedule for one option. For a European option, there is only one exercise date, the option expiry date.<br>For an American option:<br>`NINST`-by-`2` vector of exercise date boundaries. For each instrument, the option can be exercised on any coupon date between or including the pair of dates on that row. If only one non-`NaN` date is listed, or if `ExerciseDates` is `NINST`-by-`1`, the option can be exercised between the underlying bond `Settle` and the single listed exercise date. |
| `'Name1', Value1`<br>`'Name2', Value2`<br>`...` | (Optional) The name/value pairs are a variable length list of parameters. All optional inputs are specified as matching parameter name/value pairs. The parameter name is specified as a character string, followed by the corresponding parameter value. Parameter name/value pairs may |

be specified in any order; names are case insensitive and partial string matches are allowed provided no ambiguities exist. Valid parameter names are as follows:

- `AmericanOpt` is a `NINST`-by-`1` matrix for flags options: `0` (European/Bermuda) or `1` (American). Default is `0`.
- `Period` is a `NINST`-by-`1` matrix for coupons per year. Default is `2`.
- `Basis` is a day-count basis of the instrument. `Basis` is a vector of integers with the following supported values:
  - 0 = actual/actual (default)
  - 1 = 30/360 (SIA)
  - 2 = actual/360
  - 3 = actual/365
  - 4 = 30/360 (BMA)
  - 5 = 30/360 (ISDA)
  - 6 = 30/360 (European)
  - 7 = actual/365 (Japanese)
  - 8 = actual/actual (ICMA)
  - 9 = actual/360 (ICMA)
  - 10 = actual/365 (ICMA)
  - 11 = 30/360E (ICMA)
  - 12 = actual/actual (ISDA)
  - 13 = BUS/252

  For more information, see basis.

- `EndMonthRule` is a `NINST`-by-`1` matrix for the end-of-month rule. This rule applies only when `Maturity` is an end-of-month date for a month having 30 or fewer days. When the value is `0`, the end-of-month rule is ignored, meaning that a bond's coupon payment date is always the same numerical day of the month. When the value is `1`, the end-of-month rule is set rule on (default), meaning that a bond's coupon payment date is always the last actual day of the month.
- `IssueDate` is a `NINST`-by-`1` matrix for the bond issue date.
- `FirstCouponDate` is a `NINST`-by-`1` matrix for a date when a bond makes its first coupon payment; used when bond has an irregular first coupon period. When `FirstCouponDate` and `LastCouponDate` are both specified, `FirstCouponDate` takes precedence in determining the coupon payment structure. If you do not specify a `FirstCouponDate`, the cash flow payment dates are determined from other inputs.
- `LastCouponDate` is a `NINST`-by-`1` matrix for a last coupon date of a bond before the maturity date; used when bond has an irregular last coupon period. In the absence of a specified `FirstCouponDate`, a specified `LastCouponDate` determines the coupon structure of the bond. The coupon structure of a bond is truncated at the `LastCouponDate`, regardless of where it falls, and is followed only by the bond's maturity cash flow date. If you do not

specify a `LastCouponDate`, the cash flow payment dates are determined from other inputs.

- `StartDate` is a `NINST`-by-`1` matrix for date when a bond actually starts (the date from which a bond cash flow is considered). To make an instrument forward-starting, specify this date as a future date. If you do not specify `StartDate`, the effective start date is the `Settle` date.
- Face value. `Face` is a `NINST`-by-`1` vector or `NINST`-by-`1` cell array of face values, or face value schedules. For the latter case, each element of the cell array is a `NumDates`-by-`2` cell array, where the first column is dates and the second column is its associated face value. The date indicates the last day that the face value is valid. Default is 100.
- `Options` is a derivatives pricing options structure created with `derivset`.

---

**Note** The `Settle` date for every bond with embedded option is set to the `ValuationDate` of the HW tree; the bond's argument for `Settle` date is ignored.

---

## Description

`[Price, PriceTree] = optembndbyhw(HWTree, CouponRate, Settle, Maturity, OptSpec, Strike, ExerciseDates, 'Name1', Value1, 'Name2', Value2, ...)` prices bonds with embedded options by a HW interest-rate tree.

`Price` is a number of instruments (`NINST`)-by-`1` matrix of expected prices at time 0.

`PriceTree` is a structure of trees containing vectors of instrument prices and observation times for each node. Within `PriceTree`:

- `PriceTree.PTree` contains the clean prices.
- `PriceTree.tObs` contains the observation times.

`optembndbyhw` computes prices of vanilla bonds with embedded options, stepped coupon bonds with embedded options, and bonds with sinking fund option provisions.

## Definitions

### Vanilla Bond with Embedded Option

A vanilla coupon bond is a security representing an obligation to repay a borrowed amount at a designated time and to make periodic interest payments until that time. The issuer of a bond makes the periodic interest payments until the bond matures. At maturity, the issuer pays to the holder of the bond the principal amount owed (face value) and the last interest payment. A vanilla bond with an embedded option is where an option contract has an underlying asset of a vanilla bond.

### Stepped Coupon Bond with Callable and Puttable Features

A step-up and step-down bond is a debt security with a predetermined coupon structure over time. With these instruments, coupons increase (step up) or decrease (step down) at specific times during the life of the bond. Stepped coupon bonds can have options features (call and puts).

### Sinking Fund Bond with Embedded Option

A sinking fund bond is a coupon bond with a sinking fund provision. This provision obligates the issuer to amortize portions of the principal prior to maturity, affecting bond prices since the time of the principal repayment changes. This means that investors receive the coupon and a portion of the principal paid back over time. These types of bonds reduce credit risk, since it lowers the probability of investors not receiving their principal payment at maturity.

The bond may have a sinking fund option provision allowing the issuer to retire the sinking fund obligation either by purchasing the bonds to be redeemed from the market or by calling the bond via a sinking fund call, whichever is cheaper. If interest rates are high, then the issuer will buy back the requirement amount of bonds from the market since bonds will be cheap, but if interest rates are low (bond prices are high), then most likely the issuer will be buying the bonds at the call price. Unlike a call feature, however, if a bond has a sinking fund option provision, it is an obligation, not an option, for the issuer to buy back the increments of the issue as stated. Because of this, a sinking fund bond trades at a lower price than a non-sinking fund bond.

## Examples

Create a `HWTree` with the following data:

```
ZeroRates = [ 0.035;0.04;0.045];
Compounding = 1;
StartDates = ['jan-1-2007';'jan-1-2008';'jan-1-2009'];
EndDates   = ['jan-1-2008';'jan-1-2009';'jan-1-2010'];
ValuationDate = 'jan-1-2007';
```

Create a `RateSpec`:

```
RateSpec = intenvset('Rates', ZeroRates, 'StartDates', ValuationDate, 'EndDates', ...
EndDates, 'Compounding', Compounding, 'ValuationDate', ValuationDate);
```

Specify a `TimeSpec`:

```
HWTimeSpec = hwtimespec(ValuationDate, EndDates, Compounding);
```

Specify a `VolSpec`:

```
VolDates = ['jan-1-2008';'jan-1-2009';'jan-1-2010'];
VolCurve = 0.01;
AlphaDates = 'jan-1-2010';
AlphaCurve = 0.1;
HWVolSpec = hwvolspec(ValuationDate, VolDates, VolCurve, AlphaDates, AlphaCurve);
```

Build a `HWTree`:

```
HWTree = hwtree(HWVolSpec, RateSpec, HWTimeSpec);
```

Compute the price of an American puttable bond that pays an annual coupon of 5.25%, matures on January 1, 2010, and is puttable from January 1, 2008 to January 1, 2010:

```
BondSettlement = 'jan-1-2007';
BondMaturity   = 'jan-1-2010';
CouponRate = 0.0525;
Period = 1;
OptSpec = 'put';
Strike = [100];
ExerciseDates = {'jan-1-2008' '01-Jan-2010'};
AmericanOpt = 1;

PricePutBondHW = optembndbyhw(HWTree, CouponRate, BondSettlement, BondMaturity,...
OptSpec, Strike, ExerciseDates,'Period', 1, 'AmericanOpt', 1)

PricePutBondHW =

102.8801
```

---

Price the following single stepped callable bonds using the following data:

```
% The data for the interest rate term structure is as follows:
Rates = [0.035; 0.042147; 0.047345; 0.052707];
ValuationDate = 'Jan-1-2010';
StartDates = ValuationDate;
EndDates = {'Jan-1-2011'; 'Jan-1-2012'; 'Jan-1-2013'; 'Jan-1-2014'};
Compounding = 1;

% Create RateSpec
RS = intenvset('ValuationDate', ValuationDate, 'StartDates', StartDates,...
'EndDates', EndDates,'Rates', Rates, 'Compounding', Compounding);
```

```
% Instrument
Settle = '01-Jan-2010';
Maturity = {'01-Jan-2013';'01-Jan-2014'};
CouponRate = {{'01-Jan-2012' .0425;'01-Jan-2014' .0750}};
OptSpec='call';
Strike=100;
ExerciseDates='01-Jan-2012';  %Callable in two years

% Build the tree with the following data
VolDates = ['1-Jan-2011'; '1-Jan-2012'; '1-Jan-2013'; '1-Jan-2014'];
VolCurve = 0.01;
AlphaDates = '01-01-2014';
AlphaCurve = 0.1;


HWVolSpec = hwvolspec(RS.ValuationDate, VolDates, VolCurve,...
AlphaDates, AlphaCurve);
HWTimeSpec = hwtimespec(RS.ValuationDate, VolDates, Compounding);
HWT = hwtree(HWVolSpec, RS, HWTimeSpec);

% The first row corresponds to the price of the callable bond with maturity
% of three years. The second row corresponds to the price of the callable
% bond with maturity of four years.

PHW= optembndbyhw(HWT, CouponRate,  Settle, Maturity,OptSpec, Strike,...
ExerciseDates, 'Period', 1)

PHW =

  100.0521
   99.8322
```

---

A corporation issues a two year bond with a sinking fund obligation requiring the company to sink 1/3 of face value after the first year. The company has the option to buy the bonds in the market or call them at

$99. The following data describes the details needed for pricing the sinking fund bond:

```
% The data for the interest rate term structure is as follows:
Rates = [0.1;0.1;0.1;0.1];
ValuationDate = 'Jan-1-2011';
StartDates = ValuationDate;
EndDates = {'Jan-1-2012'; 'Jan-1-2013'; 'Jan-1-2014'; 'Jan-1-2015'};
Compounding = 1;

% Create RateSpec
RateSpec = intenvset('ValuationDate', ValuationDate, 'StartDates',...
StartDates, 'EndDates', EndDates,'Rates', Rates, 'Compounding', Compounding);

% Build the HW tree
% The data to build the tree is as follows:
VolDates = ['1-Jan-2012'; '1-Jan-2013'; '1-Jan-2014';'1-Jan-2015'];
VolCurve = 0.01;
AlphaDates = '01-01-2015';
AlphaCurve = 0.1;

HWVolSpec = hwvolspec(RateSpec.ValuationDate, VolDates, VolCurve,...
AlphaDates, AlphaCurve);
HWTimeSpec = hwtimespec(RateSpec.ValuationDate, VolDates, Compounding);
HWT = hwtree(HWVolSpec, RateSpec, HWTimeSpec);


% Instrument
% The bond has a coupon rate of 9%, a period of one year and matures in
% 1-Jan-2013. Face decreases 1/3 after the first year.
CouponRate = 0.09;
Settle = 'Jan-1-2011';
Maturity =  'Jan-1-2013';
Period = 1;
Face = { ...
            {'Jan-1-2012'  100; ...
             'Jan-1-2013'   66.6666}; ...
```

```
        };

% Option provision
OptSpec = 'call';
Strike = 99;
ExerciseDates = 'Jan-1-2012';

% Price of non-sinking fund bond.
PNSF = bondbyhw(HWT, CouponRate, Settle, Maturity, Period)PNSF =

   98.2645

% Price of the bond with the option sinking provision.
PriceSF = optembndbyhw(HWT, CouponRate, Settle, Maturity,...
OptSpec, Strike, ExerciseDates, 'Period', Period, 'Face', Face)

PriceSF =

   98.1594
```

## See Also

`hwprice` | `cfamounts` | `hwtree` | `instoptembnd`

# optstockbybjs

## Purpose

Price American options using Bjerksund-Stensland 2002 option pricing model

## Syntax

```
Price = optstockbybjs(RateSpec, StockSpec, Settle, Maturity,
OptSpec, Strike)
```

## Arguments

| | |
|---|---|
| `RateSpec` | The annualized continuously compounded rate term structure. For information on the interest rate specification, see `intenvset`. |
| `StockSpec` | Stock specification. See `stockspec`. |
| `Settle` | `NINST`-by-`1` vector of settlement or trade dates. |
| `Maturity` | `NINST`-by-`1` vector of maturity dates. |
| `OptSpec` | `NINST`-by-`1` cell array of strings `'call'` or `'put'`. |
| `Strike` | `NINST`-by-`1` vector of strike price values. |

## Description

`Price = optstockbybjs(RateSpec, StockSpec, Settle, Maturity,`
`OptSpec, Strike)` computes American option prices with continuous dividend yield using the Bjerksund-Stensland 2002 option pricing model.

`Price` is a `NINST`-by-`1` vector of expected option prices.

---

**Note** `optstockbybjs` computes prices of American options with continuous dividend yield using the Bjerksund-Stensland option pricing model.

---

## Examples

Consider two American stock options (a call and a put) with an exercise price of $100. The options expire on April 1, 2008. Assume the underlying stock pays a continuous dividend yield of 4% as of January 1, 2008. The stock has a volatility of 20% per annum and the annualized continuously compounded risk-free rate is 8% per annum. Using this data, calculate the price of the American call and put, assuming the following current prices of the stock: $90 (for the call) and $120 (for the put):

```
Settle = 'Jan-1-2008';
Maturity = 'April-1-2008';
Strike = 100;
AssetPrice = [90;120];
DivYield = 0.04;
Rate = 0.08;
Sigma = 0.20;
```

Define `StockSpec` and `RateSpec`:

```
StockSpec = stockspec(Sigma, AssetPrice, {'continuous'}, DivYield);

RateSpec = intenvset('ValuationDate', Settle, 'StartDates', Settle,...
'EndDates', Maturity, 'Rates', Rate, 'Compounding', -1);
```

Define the option type:

```
OptSpec = {'call'; 'put'};
```

Compute the option prices using the Bjerksund-Stensland 2002 option pricing model:

```
Price = optstockbybjs(RateSpec, StockSpec, Settle, Maturity, OptSpec, Strike)

Price =

    0.8420
    0.1108
```

The first element of the `Price` vector represents the price of the call ($0.84); the second element represents the price of the put option ($0.11).

### References

Bjerksund, P. and G. Stensland, *Closed-Form Approximation of American Options*, Scandinavian Journal of Management, 1993, Vol. 9, Suppl., pp. S88-S99.

Bjerksund, P. and G. Stensland, *Closed Form Valuation of American Options*, Discussion paper 2002 (http://brage.bibsys.no/nhh/bitstream/URN:NBN:no-bibsys_brage_22301/1/bjerksund%

### See Also

`impvbybjs` | `intenvset` | `optstocksensbybjs` | `stockspec`

## Purpose

Price options on futures using Black option pricing model

## Syntax

```
Price = optstockbyblk(RateSpec, StockSpec, Settle, Maturity,
OptSpec, Strike)
```

## Arguments

| | |
|---|---|
| `RateSpec` | The annualized continuously compounded rate term structure. For information on the interest rate specification, see `intenvset`. |
| `StockSpec` | Stock specification. See `stockspec`. |
| `Settle` | `NINST`-by-`1` vector of settlement or trade dates. |
| `Maturity` | `NINST`-by-`1` vector of maturity dates. |
| `OptSpec` | `NINST`-by-`1` cell array of strings `'call'` or `'put'`. |
| `Strike` | `NINST`-by-`1` vector of strike price values. |

## Description

`Price = optstockbyblk(RateSpec, StockSpec, Settle,`
`Maturity,`
`OptSpec, Strike)` computes option prices on futures using the Black option pricing model.

`Price` is a `NINST`-by-`1` vector of expected option prices.

## Examples

Consider two European call options on a futures contract with exercise prices of $20 and $25 that expire on September 1, 2008. Assume that on May 1, 2008 the contract is trading at $20, and has a volatility of 35% per annum. The risk-free rate is 4% per annum. Using this data, calculate the price of the call futures options using the Black model:

```
Strike = [20; 25];
AssetPrice = 20;
Sigma = .35;
Rates = 0.04;
```

```
Settle = 'May-01-08';
Maturity = 'Sep-01-08';
```

Create `RateSpec` and `StockSpec`:

```
RateSpec = intenvset('ValuationDate', Settle, 'StartDates', Settle,...
 'EndDates', Maturity, 'Rates', Rates, 'Compounding', -1);

StockSpec = stockspec(Sigma, AssetPrice);
```

Define the call options:

```
OptSpec = {'call'};
```

Calculate the price using the Black option pricing model:

```
Price = optstockbyblk(RateSpec, StockSpec, Settle, Maturity,...
OptSpec, Strike)
Price =
    1.5903
    0.3037
```

## See Also

`impvbyblk` | `intenvset` | `optstocksensbyblk` | `stockspec`

## Purpose

Price options using Black-Scholes option pricing model

## Syntax

```
Price = optstockbybls(RateSpec, StockSpec, Settle, Maturity,
OptSpec, Strike)
```

## Arguments

| | |
|---|---|
| `RateSpec` | The annualized continuously compounded rate term structure. For information on the interest rate specification, see `intenvset`. |
| `StockSpec` | Stock specification. See `stockspec`. |
| `Settle` | `NINST`-by-`1` vector of settlement or trade dates. |
| `Maturity` | `NINST`-by-`1` vector of maturity dates. |
| `OptSpec` | `NINST`-by-`1` cell array of strings `'call'` or `'put'`. |
| `Strike` | `NINST`-by-`1` vector of strike price values. |

## Description

`Price = optstockbybls(RateSpec, StockSpec, Settle,`
`Maturity,`
`OptSpec, Strike)` computes option prices using the Black-Scholes option pricing model.

`Price` is a `NINST`-by-`1` vector of expected option prices.

**Note** When using `StockSpec` with `optstockbybls`, you can modify `StockSpec` to handle other types of underliers when pricing instruments that use the Black-Scholes model.

When pricing Futures (Black model), enter the following in `StockSpec`:

```
DivType = 'Continuous';
DivAmount = RateSpec.Rates;
```

When pricing Foreign Currencies (Garman-Kohlhagen model), enter the following in `StockSpec`:

```
DivType = 'Continuous';
DivAmount = ForeignRate;
```

where `ForeignRate` is the continuously compounded, annualized risk free interest rate in the foreign country.

## Examples

Consider two European options, a call and a put, with an exercise price of $29 on January 1, 2008. The options expire on May 1, 2008. Assume that the underlying stock for the call option provides a cash dividend of $0.50 on February 15, 2008. The underlying stock for the put option provides a continuous dividend yield of 4.5% per annum. The stocks are trading at $30 and have a volatility of 25% per annum. The annualized continuously compounded risk-free rate is 5% per annum. Using this data, compute the price of the options using the Black-Scholes model:

```
Strike = 29;
AssetPrice = 30;
Sigma = .25;
Rates = 0.05;
Settle = 'Jan-01-2008';
Maturity = 'May-01-2008';
```

Define `RateSpec`:

```
RateSpec = intenvset('ValuationDate', Settle, 'StartDates', Settle, 'EndDates',...
Maturity, 'Rates', Rates, 'Compounding', -1);
```

Define `StockSpec`:

```
DividendType = {'cash';'continuous'};
DividendAmounts = [0.50; 0.045];
ExDividendDates = {'Feb-15-2008';NaN};

StockSpec = stockspec(Sigma, AssetPrice, DividendType, DividendAmounts,...
ExDividendDates);
```

Price the call and the put options using the Black-Scholes model:

```
OptSpec = {'call'; 'put'};

Price = optstockbybls(RateSpec, StockSpec, Settle, Maturity, OptSpec, Strike)

Price =

    2.2030
    1.2025
```

## See Also

`impvbybls` | `intenvset` | `optstocksensbybls` | `stockspec`

# optstockbycrr

**Purpose** Price stock option from Cox-Ross-Rubinstein tree

**Syntax**

```
[Price, PriceTree] = optstockbycrr(CRRTree, OptSpec, Strike,
Settle, ExerciseDates, AmericanOpt)
```

**Arguments**

| | |
|---|---|
| `CRRTree` | Stock tree structure created by `crrtree`. |
| `OptSpec` | Number of instruments (`NINST`)-by-1 cell array of strings `'call'` or `'put'`. |

---

**Note** The interpretation of the `Strike` and `ExerciseDates` arguments depends upon the setting of the `AmericanOpt` argument. If `AmericanOpt = 0`, `NaN`, or is unspecified, the option is a European or Bermuda option. If `AmericanOpt = 1`, the option is an American option.

---

| | |
|---|---|
| `Strike` | European option: `NINST`-by-1 vector of strike price values.<br>Bermuda option: `NINST` by number of strikes (`NSTRIKES`) matrix of strike price values.<br>Each row is the schedule for one option. If an option has fewer than `NSTRIKES` exercise opportunities, the end of the row is padded with `NaN`s.<br>American option: `NINST`-by-1 vector of strike price values for each option. |

| | |
|---|---|
| `Settle` | `NINST`-by-`1` vector of settlement or trade dates. |
| `ExerciseDates` | `NINST`-by-`1` (European option) or `NINST`-by-`NSTRIKES` (Bermuda option) matrix of exercise dates. Each row is the schedule for one option. For a European option, there is only one exercise date, the option expiry date.<br>For an American option:<br>`NINST`-by-`2` vector of exercise date boundaries. For each instrument, the option can be exercised on any coupon date between or including the pair of dates on that row. If only one non-`NaN` date is listed, or if `ExerciseDates` is `NINST`-by-`1`, the option can be exercised between the underlying bond `Settle` and the single listed exercise date. |

Data arguments are `NINST`-by-`1` vectors, scalar, or empty. Fill unspecified entries in vectors with `NaN`. Only one data argument is required to create the instrument. The others may be omitted or passed as empty matrices `[]`.

## Description

`[Price, PriceTree] = optstockbycrr(CRRTree, OptSpec, Strike,`
`Settle, ExerciseDates, AmericanOpt)` computes the price of a European, Bermuda, or American stock option.

`Price` is a `NINST`-by-`1` vector of expected option prices at time 0.

`PriceTree` is a tree structure with a vector of instrument prices at each node.

## Examples

Price a stock option using a CRR binomial tree.

Load the file `deriv.mat,` which provides `CRRTree`. The `CRRTree` structure contains the stock specification and time information needed to price the option.

```
load deriv.mat;
```

Set the required values. Other arguments will use defaults.

```
OptSpec = 'Call';
Strike = 105;
Settle = '01-Jan-2003';
ExerciseDates = '01-Jan-2005';

Price = optstockbycrr(CRRTree, OptSpec, Strike, Settle, ...
ExerciseDates)

Price =

 8.2863
```

## See Also

`crrtree` | `instoptstock`

## Purpose

Price stock option from Equal Probabilities binomial tree

## Syntax

```
[Price, PriceTree] = optstockbyeqp(EQPTree, OptSpec, Strike,
Settle, ExerciseDates, AmericanOpt)
```

## Arguments

| | |
|---|---|
| `EQPTree` | Stock tree structure created by `eqptree`. |
| `OptSpec` | Number of instruments (`NINST`)-by-1 cell array of strings `'call'` or `'put'`. |

---

**Note** The interpretation of the `Strike` and `ExerciseDates` arguments depends upon the setting of the `AmericanOpt` argument. If `AmericanOpt = 0`, `NaN`, or is unspecified, the option is a European or Bermuda option. If `AmericanOpt = 1`, the option is an American option.

---

| | |
|---|---|
| `Strike` | European option: `NINST`-by-1 vector of strike price values.<br>Bermuda option: `NINST` by number of strikes (`NSTRIKES`) matrix of strike price values.<br>Each row is the schedule for one option. If an option has fewer than `NSTRIKES` exercise opportunities, the end of the row is padded with `NaN`s.<br>American option: `NINST`-by-1 vector of strike price values for each option. |

| | |
|---|---|
| `Settle` | `NINST`-by-`1` vector of settlement or trade dates. |
| `ExerciseDates` | `NINST`-by-`1` (European option) or `NINST`-by-`NSTRIKES` (Bermuda option) matrix of exercise dates. Each row is the schedule for one option. For a European option, there is only one exercise date, the option expiry date.<br>For an American option:<br>`NINST`-by-`2` vector of exercise date boundaries. For each instrument, the option can be exercised on any coupon date between or including the pair of dates on that row. If only one non-`NaN` date is listed, or if `ExerciseDates` is `NINST`-by-`1`, the option can be exercised between the underlying bond `Settle` and the single listed exercise date. |

Data arguments are `NINST`-by-`1` vectors, scalar, or empty. Fill unspecified entries in vectors with `NaN`. Only one data argument is required to create the instrument. The others may be omitted or passed as empty matrices `[]`.

## Description

`[Price, PriceTree] = optstockbyeqp(EQPTree, OptSpec, Strike,`
`Settle, ExerciseDates, AmericanOpt)` computes the price of a European/Bermuda or American stock option.

`Price` is a `NINST`-by-`1` vector of expected option prices at time 0.

`PriceTree` is a tree structure with a vector of instrument prices at each node.

## Examples

Price a stock option using an EQP equity tree.

Load the file `deriv.mat`, which provides `EQPTree`. The `EQPTree` structure contains the stock specification and time information needed to price the option.

```
load deriv.mat
```

Set the required values. Other arguments will use defaults.

```
OptSpec = 'Call';
Strike = 105;
Settle = '01-Jan-2003';
ExerciseDates = '01-Jan-2005';

Price = optstockbyeqp(EQPTree, OptSpec, Strike, Settle, ...
ExerciseDates)

Price =

 8.4791
```

## See Also

`eqptree` | `instoptstock`

# optstockbyitt

## Purpose

Price options on stocks using implied trinomial tree (ITT)

## Syntax

```
[Price, PriceTree] = optstockbyitt(ITTTree, OptSpec, Strike,
Settle, ExerciseDates, AmericanOpt)
```

## Arguments

| | |
|---|---|
| `ITTTree` | Stock tree structure created by `itttree`. |
| `OptSpec` | Number of instruments (`NINST`)-by-`1` cell array of strings `'call'` or `'put'`. |

---

**Note** The interpretation of the `Strike` and `ExerciseDates` arguments depends on the setting of the `AmericanOpt` argument. If `AmericanOpt = 0`, `NaN`, or is unspecified, the option is a European or Bermuda option. If `AmericanOpt = 1`, the option is an American option.

---

| | |
|---|---|
| `Strike` | European option: `NINST`-by-`1` vector of strike price values.<br>Bermuda option: `NINST` by number of strikes (`NSTRIKES`) matrix of strike price values.<br>Each row is the schedule for one option. If an option has fewer than `NSTRIKES` exercise opportunities, the end of the row is padded with `NaN`s.<br>American option: `NINST`-by-`1` vector of strike price values for each option. |
| `Settle` | `NINST`-by-`1` vector of settlement or trade dates. |

| | |
|---|---|
| `ExerciseDates` | For a European or Bermuda option:<br>`NINST`-by-`1` (European option) or `NINST`-by-`NSTRIKES` (Bermuda option) matrix of exercise dates. Each row is the schedule for one option. For a European option, there is only one exercise date, the option expiry date.<br>For an American option:<br>`NINST`-by-`2` vector of exercise date boundaries. For each instrument, the option can be exercised on any coupon date between or including the pair of dates on that row. If only one non-`NaN` date is listed, or if `ExerciseDates` is `NINST`-by-`1`, the option can be exercised between the underlying bond `Settle` and the single listed exercise date. |
| `AmericanOpt` | (Optional) If `AmericanOpt = 0`, `NaN`, or is unspecified, the option is a European or Bermuda option. If `AmericanOpt = 1`, the option is an American option. |

---

**Note** Data arguments for `optstockbyitt` are `NINST`-by-`1` vectors, scalar, or empty. Fill unspecified entries in vectors with `NaN`. Only one data argument is required to create the instrument; the others may be omitted or passed as empty matrices `[]`.

---

## Description

`[Price, PriceTree] = optstockbyitt(ITTTree, OptSpec, Strike,`
`Settle, ExerciseDates, AmericanOpt)` computes the price of a European/Bermuda or American stock option.

`Price` is a `NINST`-by-`1` vector of expected option prices at time 0.

`PriceTree` is a tree structure with a vector of instrument prices at each node.

---

**Note** The `Settle` date for every option is set to the `ValuationDate` of the stock tree. The option argument, `Settle`, is ignored.

---

## Examples

Price a stock option using an ITT equity tree.

Load the file `deriv.mat` which provides the `ITTTree`. The `ITTTree` structure contains the stock specification and time information needed to price the option.

```
load deriv.mat
```

Set the required values. Other arguments will use defaults.

```
OptSpec = 'Put';
Strike = 80;
Settle = '01-Jan-2006';
ExerciseDates = ' 01-Jan-2010 ';


Price = optstockbyitt(ITTTree, OptSpec, Strike, Settle,ExerciseDates)

Price =

 10.68
```

## References

Chriss, Neil A., E. Derman, and I. Kani, “Implied trinomial trees of the volatility smile,” *Journal of Derivatives*, 1996.

## See Also

`instoptstock` | `itttree` | `stockoptspec`

## Purpose

Price options on stocks using Leisen-Reimer binomial tree model

## Syntax

```
[Price, PriceTree] = optstockbylr(LRTree, OptSpec, Strike,
Settle, ExerciseDates)
[Price, PriceTree] = optstockbylr(LRTree, OptSpec, Strike,
Settle, ExerciseDates, Name,Value)
```

## Description

`[Price, PriceTree] = optstockbylr(LRTree, OptSpec, Strike, Settle, ExerciseDates)` computes option prices on stocks using the Leisen-Reimer binomial tree model.

`[Price, PriceTree] = optstockbylr(LRTree, OptSpec, Strike, Settle, ExerciseDates, Name,Value)` computes option prices on stocks using the Leisen-Reimer binomial tree model with additional options specified by one or more `Name,Value` pair arguments.

## Input Arguments

### LRTree

Stock tree structure created by `lrtree`.

### OptSpec

`NINST`-by-`1` cell array of strings `'call'` or `'put'`.

### Strike

`NINST`-by-`1` (European/American) or `NINST`-by-`NSTRIKES` (Bermuda) matrix of strike price values. Each row is the schedule for one option. If an option has fewer than `NSTRIKES` exercise opportunities, the end of the row is padded with `NaN`s.

### Settle

`NINST`-by-`1` matrix of settlement or trade dates.

**Note** The settle date for every option is set to the `ValuationDate` of the stock tree. The option argument, `Settle`, is ignored.

**ExerciseDates**

`NINST`-by-`1` (European/American) or `NINST`-by-`NSTRIKEDATES` (Bermuda) matrix of exercise dates. Each row is the schedule for one option. For a European option, there is only one `ExerciseDate` on the option expiry date. For the American type, the option can be exercised on any tree data between the `ValuationDate` and tree maturity. The last element of each row must be the same as the maturity of the tree.

### Name-Value Pair Arguments

Specify optional comma-separated pairs of `Name,Value` arguments, where `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside single quotes (`' '`). You can specify several name and value pair arguments in any order as `Name1,Value1,...,NameN,ValueN`.

**AmericanOpt**

`NINST`-by-`1` flags with a value of `0` (European/Bermuda) or `1` (American).

**Default:** `0`

## Output Arguments

**Price**

`NINST`-by-`1` expected prices at time 0.

**PriceTree**

Tree structure with a vector of instrument prices at each node.

## Examples

Consider European call and put options with an exercise price of $95 that expire on July 1, 2010. The underlying stock is trading at $100 on January 1, 2010, provides a continuous dividend yield of 3% per annum and has a volatility of 20% per annum. The annualized continuously compounded risk-free rate is 8% per annum. Using this data, compute the price of the options using the Leisen-Reimer model with a tree of 15 and 55 time steps.

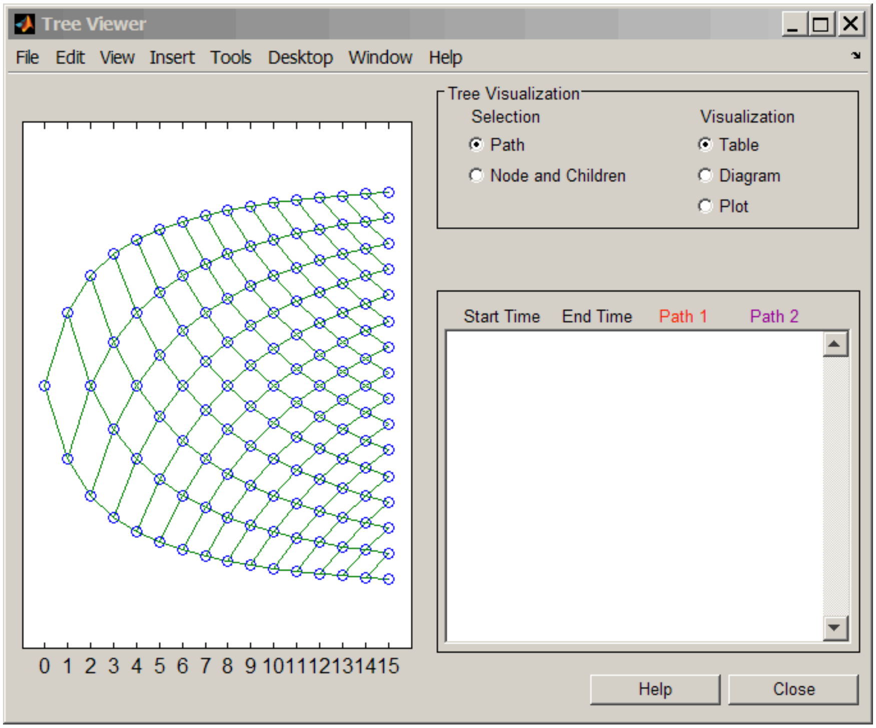
```
AssetPrice  = 100;
```

```
Strike = 95;

ValuationDate = 'Jan-1-2010';
Maturity = 'July-1-2010';

% Define StockSpec
Sigma = 0.2;
DividendType = 'continuous';
DividendAmounts = 0.03;

StockSpec = stockspec(Sigma, AssetPrice, DividendType, DividendAmounts);


% Define RateSpec
Rates = 0.08;
Settle = ValuationDate;
Basis = 1;
Compounding = -1;

RateSpec = intenvset('ValuationDate', ValuationDate, 'StartDates', Settle, ...
'EndDates', Maturity, 'Rates', Rates, 'Compounding', Compounding, 'Basis', Basis);

% Build the Leisen-Reimer (LR) tree with 15 and 55 time steps
LRTimeSpec15  = lrtimespec(ValuationDate, Maturity, 15);
LRTimeSpec55  = lrtimespec(ValuationDate, Maturity, 55);

% Use the PP2 method
LRMethod  = 'PP2';

LRTree15 = lrtree(StockSpec, RateSpec, LRTimeSpec15, Strike, 'method', LRMethod);
LRTree55 = lrtree(StockSpec, RateSpec, LRTimeSpec55, Strike, 'method', LRMethod);


% Price the call and the put options using the LR model:
OptSpec = {'call'; 'put'};

PriceLR15 = optstockbylr(LRTree15, OptSpec, Strike, Settle, Maturity);
```

```
PriceLR55 = optstockbylr(LRTree55, OptSpec, Strike, Settle, Maturity);

% Calculate price using the Black-Scholes model (BLS) to compare values with
% the LR model:
PriceBLS = optstockbybls(RateSpec, StockSpec, Settle, Maturity, OptSpec, Strike);

% Compare values of BLS and LR
[PriceBLS PriceLR15 PriceLR55]

ans =

    9.7258    9.7252    9.7257
    2.4896    2.4890    2.4895
```

Use `treeviewer` to display the `LRTree` of 15 time steps:

```
treeviewer(LRTree15)
```



Tree Visualization

Selection
● Path
○ Node and Children

Visualization
● Table
○ Diagram
○ Plot

## References

Leisen D.P., M. Reimer, “Binomial Models for Option Valuation – Examining and Improving Convergence,” *Applied Mathematical Finance*, Number 3, 1996, pp. 319-346.

## See Also

| `instoptstock` | `lrtree` | `optstocksensbylr`

# optstockbyrgw

## Purpose

Determine American call option prices using Roll-Geske-Whaley option pricing model

## Syntax

```
Price = optstockbyrgw(RateSpec, StockSpec, Settle,
Maturity, Strike)
```

## Arguments

| | |
|---|---|
| `RateSpec` | The annualized continuously compounded rate term structure. For information on the interest rate specification, see `intenvset`. |
| `StockSpec` | Stock specification. See `stockspec`. |
| `Settle` | `NINST`-by-`1` vector of settlement or trade dates. |
| `Maturity` | `NINST`-by-`1` vector of maturity dates. |
| `Strike` | `NINST`-by-`1` vector of strike price values. |

## Description

`Price = optstockbyrgw(RateSpec, StockSpec, Settle, Maturity, Strike)` computes the American call option prices using the Roll-Geske-Whaley option pricing model.

`Price` is a `NINST`-by-1 vector of expected call option prices.

---

**Note** `optstockbyrgw` computes prices of American calls with a single cash dividend using the Roll-Geske-Whaley option pricing model.

---

## Examples

Consider an American call option with an exercise price of $22 that expires on February 1, 2009. The underlying stock is trading at $20 on June 1, 2008 and has a volatility of 20% per annum. The annualized continuously compounded risk-free rate is 6.77% per annum. The stock pays a single dividend of $2 on September 1, 2008. Using this data, compute price of the American call option using the Roll-Geske-Whaley option pricing model:

```
Settle = 'Jun-01-2008';
Maturity = 'Feb-01-2009';
AssetPrice = 20;
Strike = 22;
Sigma  = 0.2;
Rate = 0.0677;
DivAmount = 2;
DivDate = 'Sep-01-2008';
```

Define `StockSpec` and `RateSpec`:

```
RateSpec = intenvset('ValuationDate', Settle, 'StartDates', Settle, 'EndDates',...
Maturity, 'Rates', Rate, 'Compounding', -1, 'Basis', 0);

StockSpec = stockspec(Sigma, AssetPrice, {'cash'}, DivAmount, DivDate);
```

Compute the price of the American call :

```
Price  = optstockbyrgw(RateSpec, StockSpec, Settle, Maturity,Strike)

Price =

    0.3359
```

## See Also

`impvbyrgw` | `intenvset` | `optstocksensbyrgw` | `stockspec`

**Purpose** Determine American option prices and sensitivities using Bjerksund-Stensland 2002 option pricing model

**Syntax**

```
PriceSens = optstocksensbybjs(RateSpec, StockSpec, Settle,
Maturity, OptSpec, Strike, 'Name1', Value1...)
```

## Arguments

| | |
|---|---|
| `RateSpec` | The annualized continuously compounded rate term structure. For information on the interest rate specification, see `intenvset`. |
| `StockSpec` | Stock specification. See `stockspec`. |
| `Settle` | `NINST`-by-`1` vector of settlement or trade dates. |
| `Maturity` | `NINST`-by-`1` vector of maturity dates. |
| `OptSpec` | `NINST`-by-`1` cell array of strings `'call'` or `'put'`. |
| `Strike` | `NINST`-by-`1` vector of strike price values. |
| `OutSpec` | (Optional) All optional inputs are specified as matching parameter name/value pairs. The parameter name is specified as a character string, followed by the corresponding parameter value. Parameter name/value pairs may be specified in any order; names are case-insensitive and partial string matches are allowed provided no ambiguities exist. Valid parameter names are:<br>• `NOUT`-by-`1` or `1`-by-`NOUT` cell array of strings indicating the nature and order of the outputs for the function. Possible values are: `'Price'`, `'Delta'`, `'Gamma'`, `'Vega'`, `'Lambda'`, `'Rho'`, `'Theta'`, or `'All'`.<br>For example, `OutSpec = {'Price'; 'Lambda'; 'Rho'}` specifies that the output should be `Price`, `Lambda`, and `Rho`, in that order. |

To invoke from a function: `[Price, Lambda, Rho] = optstocksensbybjs(..., 'OutSpec', {'Price', 'Lambda', 'Rho'})`

`OutSpec = {'All'}` specifies that the output should be `Delta`, `Gamma`, `Vega`, `Lambda`, `Rho`, `Theta`, and `Price`, in that order. This is the same as specifying `OutSpec` as `OutSpec = {'Delta', 'Gamma', 'Vega', 'Lambda', 'Rho', 'Theta', 'Price'};`.

- Default is `OutSpec` = `{'Price'}`.

## Description

`PriceSens = optstocksensbybjs(RateSpec, StockSpec, Settle, Maturity, OptSpec, Strike, 'Name1', Value1...)` computes American option prices and sensitivities using the Bjerksund-Stensland 2002 option pricing model.

`optstocksensbybjs` can be used to compute six sensitivities for the Bjerksund-Stensland 2002 model: `delta`, `gamma`, `vega`, `lambda`, `rho`, and `theta`. This function is also capable of returning the price of the option. The selection of output parameters and their order is determined by the optional input parameter `OutSpec`. This parameter is a cell array of strings, each one specifying a desired output parameter. The order in which these output parameters are returned by the function is the same as the order of the strings contained in `OutSpec`.

`PriceSens` is a `NINST`-by-`1` vector of expected prices or sensitivities values.

---

**Note** `optstocksensbybjs` computes prices of American options with continuous dividend yield using the Bjerksund-Stensland option pricing model. All sensitivities are evaluated by computing a discrete approximation of the partial derivative. This means that the option is revalued with a fractional change for each relevant parameter, and the change in the option value divided by the increment, is the approximated sensitivity value.

---

## Examples

Consider four American put options with an exercise price of $100. The options expire on October 1, 2008. Assume the underlying stock pays a continuous dividend yield of 4% and has a volatility of 40% per annum. The annualized continuously compounded risk-free rate is 8% per annum. Using this data, calculate the `delta`, `gamma`, and `price` of the American put options, assuming the following current prices of the stock on July 1, 2008: $90, $100, $110 and $120:

```
Settle = 'July-1-2008';
Maturity = 'October-1-2008';
Strike = 100;
AssetPrice = [90;100;110;120];
Rate = 0.08;
Sigma = 0.40;
DivYield = 0.04;
```

Define `StockSpec` and `RateSpec`:

```
StockSpec = stockspec(Sigma, AssetPrice, {'continuous'}, DivYield);

RateSpec = intenvset('ValuationDate', Settle, 'StartDates', Settle,...
'EndDates', Maturity, 'Rates', Rate, 'Compounding', -1);
```

Define the option type:

```
OptSpec = {'put'};
```

Compute `delta`, `gamma`, and `price` of the put options using the Bjerksund-Stensland 2002 option pricing model:

```
OutSpec = {'Delta', 'Gamma', 'Price'};

[Delta, Gamma, Price] = optstocksensbybjs(RateSpec, StockSpec, Settle, Maturity,...
OptSpec, Strike, 'OutSpec', OutSpec)

Delta =

   -0.6572
   -0.4434
   -0.2660
   -0.1442

Gamma =

    0.0217
    0.0202
    0.0150
    0.0095

Price =

   12.9467
    7.4571
    3.9539
    1.9495
```

## References

Bjerksund, P. and G. Stensland, *Closed-Form Approximation of American Options*, Scandinavian Journal of Management, 1993, Vol. 9, Suppl., pp. S88-S99.

Bjerksund, P. and G. Stensland, *Closed Form Valuation of American Options*, Discussion paper 2002 (http://brage.bibsys.no/nhh/bitstream/URN:NBN:no-bibsys_brage_22301/1/bjerksun

**See Also** `impvbybjs` | `intenvset` | `optstockbybjs` | `stockspec`

## Purpose

Determine option prices and sensitivities on futures using Black pricing model

## Syntax

```
PriceSens = optstocksensbyblk(RateSpec, StockSpec, Settle,
Maturity, OptSpec, Strike, 'Name1', Value1...)
```

## Arguments

| | |
|---|---|
| `RateSpec` | The annualized continuously compounded rate term structure. For information on the interest rate specification, see `intenvset`. |
| `StockSpec` | Stock specification. See `stockspec`. |
| `Settle` | `NINST`-by-`1` vector of settlement or trade dates. |
| `Maturity` | `NINST`-by-`1` vector of maturity dates. |
| `OptSpec` | `NINST`-by-`1` cell array of strings `'call'` or `'put'`. |
| `Strike` | `NINST`-by-`1` vector of strike price values. |
| `OutSpec` | (Optional) All optional inputs are specified as matching parameter name/value pairs. The parameter name is specified as a character string, followed by the corresponding parameter value. Parameter name/value pairs may be specified in any order; names are case-insensitive and partial string matches are allowed provided no ambiguities exist. Valid parameter names are:<br>• `NOUT`-by-`1` or `1`-by-`NOUT` cell array of strings indicating the nature and order of the outputs for the function. Possible values are: `'Price'`, `'Delta'`, `'Gamma'`, `'Vega'`, `'Lambda'`, `'Rho'`, `'Theta'`, or `'All'`.<br>For example, `OutSpec = {'Price'; 'Lambda'; 'Rho'}` specifies that the output should be `Price`, `Lambda`, and `Rho`, in that order. |

To invoke from a function: `[Price, Lambda, Rho] = optstocksensbyblk(..., 'OutSpec', {'Price', 'Lambda', 'Rho'})`

`OutSpec = {'All'}` specifies that the output should be `Delta`, `Gamma`, `Vega`, `Lambda`, `Rho`, `Theta`, and `Price`, in that order. This is the same as specifying `OutSpec` as `OutSpec = {'Delta', 'Gamma', 'Vega', 'Lambda', 'Rho', 'Theta', 'Price'};`.

- Default is `OutSpec = {'Price'}`.

## Description

`PriceSens = optstocksensbyblk(RateSpec, StockSpec, Settle, Maturity, OptSpec, Strike, 'Name1', Value1...)` computes option prices and sensitivities on futures using the Black pricing model.

`PriceSens` is a `NINST-by-1` vector of expected future prices or sensitivities values.

## Examples

Consider a European put option on a futures contract with an exercise price of $60 that expires on June 30, 2008. On April 1, 2008 the underlying stock is trading at $58 and has a volatility of 9.5% per annum. The annualized continuously compounded risk-free rate is 5% per annum. Using this data, compute `delta`, `gamma`, and the `price` of the put option.

```
AssetPrice = 58;
Strike = 60;
Sigma = .095;
Rates = 0.05;
Settle = 'April-01-08';
Maturity = 'June-30-08';
```

Create `RateSpec` and `StockSpec`:

```
RateSpec = intenvset('ValuationDate', Settle, 'StartDates', Settle, 'EndDates',...
Maturity, 'Rates', Rates, 'Compounding', -1, 'Basis', 1);

StockSpec = stockspec(Sigma, AssetPrice);
```

Define the options:

```
OptSpec = {'put'};
```

Compute `Delta`, `Gamma` and `Price` for the European put option:

```
OutSpec = {'Delta','Gamma','Price'};
[Delta, Gamma, Price] = optstocksensbyblk(RateSpec, StockSpec, Settle,...
Maturity, OptSpec, Strike,'OutSpec', OutSpec)

Delta =

   -0.7469


Gamma =

    0.1130


Price =

    2.3569
```

## See Also

`impvbyblk` | `intenvset` | `optstockbyblk` | `stockspec`

# optstocksensbybls

**Purpose** Determine option prices and sensitivities using Black-Scholes option pricing model

**Syntax**

```
PriceSens = optstocksensbybls(RateSpec, StockSpec, Settle,
Maturity, OptSpec, Strike, 'Name1', Value1...)
```

**Arguments**

| | |
|---|---|
| `RateSpec` | The annualized continuously compounded rate term structure. For information on the interest rate specification, see `intenvset`. |
| `StockSpec` | Stock specification. See `stockspec`. |
| `Settle` | `NINST`-by-`1` vector of settlement or trade dates. |
| `Maturity` | `NINST`-by-`1` vector of maturity dates. |
| `OptSpec` | `NINST`-by-`1` cell array of strings `'call'` or `'put'`. |
| `Strike` | `NINST`-by-`1` vector of strike price values. |
| `OutSpec` | (Optional) All optional inputs are specified as matching parameter name/value pairs. The parameter name is specified as a character string, followed by the corresponding parameter value. Parameter name/value pairs may be specified in any order; names are case-insensitive and partial string matches are allowed provided no ambiguities exist. Valid parameter names are:<br>• `NOUT`-by-`1` or `1`-by-`NOUT` cell array of strings indicating the nature and order of the outputs for the function. Possible values are: `'Price'`, `'Delta'`, `'Gamma'`, `'Vega'`, `'Lambda'`, `'Rho'`, `'Theta'`, or `'All'`.<br>For example, `OutSpec = {'Price'; 'Lambda'; 'Rho'}` specifies that the output should be `Price`, `Lambda`, and `Rho`, in that order. |

To invoke from a function: `[Price, Lambda, Rho] = optstocksensbybls(..., 'OutSpec', {'Price', 'Lambda', 'Rho'})`

`OutSpec = {'All'}` specifies that the output should be `Delta`, `Gamma`, `Vega`, `Lambda`, `Rho`, `Theta`, and `Price`, in that order. This is the same as specifying `OutSpec` as `OutSpec = {'Delta', 'Gamma', 'Vega', 'Lambda', 'Rho', 'Theta', 'Price'};`.

- Default is `OutSpec` = `{'Price'}`.

## Description

`PriceSens = optstocksensbybls(RateSpec, StockSpec, Settle, Maturity, OptSpec, Strike, 'Name1', Value1...)` computes option prices and sensitivities using the Black-Scholes option pricing model.

`PriceSens` is a `NINST`-by-`1` vector of expected prices or sensitivities values.

---

**Note** When using `StockSpec` with `optstocksensbybls`, you can modify `StockSpec` to handle other types of underliers when pricing instruments that use the Black-Scholes model.

When pricing Futures (Black model), enter the following in `StockSpec`:

```
DivType = 'Continuous';
DivAmount = RateSpec.Rates;
```

When pricing Foreign Currencies (Garman-Kohlhagen model), enter the following in `StockSpec`:

```
DivType = 'Continuous';
DivAmount = ForeignRate;
```

where `ForeignRate` is the continuously compounded, annualized risk free interest rate in the foreign country.

---

## Examples

Consider a European call and put options with an exercise price of $30 that expires on June 1, 2008. The underlying stock is trading at $30 on January 1, 2008 and has a volatility of 30% per annum. The annualized continuously compounded risk-free rate is 5% per annum. Using this data, compute the `delta`, `gamma`, and `price` of the options using the Black-Scholes model.

```
AssetPrice = 30;
Strike = 30;
Sigma = .30;
Rates = 0.05;
Settle = 'January-01-2008';
Maturity = 'June -01-2008';
```

Define `RateSpec` and `StockSpec` :

```
RateSpec = intenvset('ValuationDate', Settle, 'StartDates', Settle, 'EndDates',...
```

```
Maturity, 'Rates', Rates, 'Compounding',-1, 'Basis', 1);

StockSpec = stockspec(Sigma, AssetPrice);
```

Define the options:

```
OptSpec = {'call', 'put'};
```

Compute `delta`, `gamma`, and `price` for the European options:

```
OutSpec = {'Delta','Gamma','Price'};
[Delta, Gamma, Price] = optstocksensbybls(RateSpec, StockSpec, Settle,...
Maturity, OptSpec, Strike,'OutSpec', OutSpec)

Delta =

    0.5810
   -0.4190


Gamma =

    0.0673
    0.0673


Price =

    2.6126
    1.9941
```

## See Also

`impvbybls` | `intenvset` | `optstockbybls` | `stockspec`

# optstocksensbylr

## Purpose

Determine option prices and sensitivities using Leisen-Reimer binomial tree model

## Syntax

```
PriceSens = optstocksensbylr(LRTree, OptSpec, Strike, Settle,
ExerciseDates)
PriceSens = optstocksensbylr(LRTree, OptSpec, Strike, Settle,
ExerciseDates, Name,Value)
```

## Description

`PriceSens = optstocksensbylr(LRTree, OptSpec, Strike, Settle, ExerciseDates)` calculates option prices and sensitivities using a Leisen-Reimer binomial tree model.

`PriceSens = optstocksensbylr(LRTree, OptSpec, Strike, Settle, ExerciseDates, Name,Value)` constructs a Leisen-Reimer stock tree with additional options specified by one or more `Name,Value` pair arguments.

## Input Arguments

### LRTree

Stock tree structure created by `lrtree`.

### OptSpec

`NINST`-by-1 cell array of strings `'call'` or `'put'`.

### Strike

`NINST`-by-1 (European/American) or `NINST`-by-`NSTRIKES` (Bermuda) matrix of strike price values. Each row is the schedule for one option. If an option has fewer than `NSTRIKES` exercise opportunities, the end of the row is padded with `NaN`s.

### Settle

`NINST`-by-1 matrix of settlement or trade dates.

### ExerciseDates

`NINST`-by-`1`(European/American) or `NINST`-by-`NSTRIKEDATES` (Bermuda) matrix of exercise dates. Each row is the schedule for one option. For a European option, there is only one `ExerciseDate` on the option expiry date. For the American type, the option can be exercised on any tree data between the `ValuationDate` and tree maturity. The last element of each row must be the same as the maturity of the tree.

### Name-Value Pair Arguments

Specify optional comma-separated pairs of `Name,Value` arguments, where `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside single quotes (`' '`). You can specify several name and value pair arguments in any order as `Name1,Value1,...,NameN,ValueN`.

#### AmericanOpt

`NINST`-by-`1` flags with values of `0` (European/Bermuda) or `1` (American).

**Default:** `0`

#### OutSpec

`NOUT`-by-`1` or `1`-by-`NOUT` cell array of strings indicating the nature and order of the outputs for the function. Possible values are `Price`, `Delta`, `Gamma`, `Vega`, `Lambda`, `Rho`, and `All`.

**Default:** `Price`

## Output Arguments

#### PriceSens

`NINST`-by-`1` expected prices or sensitivities values.

## Examples

Consider European call and put options with an exercise price of $100 that expire on December 1, 2010. The underlying stock is trading at $100 on June 1, 2010 and has a volatility of 30% per annum. The annualized continuously compounded risk-free rate is 7% per annum. Using this data, compute the price, delta and gamma of the options using the Leisen-Reimer model with a tree of 25 time steps and the `PP2` method.

```
AssetPrice  = 100;
Strike = 100;

ValuationDate = 'June-1-2010';
Maturity = 'December-1-2010';

%Define StockSpec
Sigma = 0.3;

StockSpec = stockspec(Sigma, AssetPrice);

% Define RateSpec
Rates = 0.07;
Settle = ValuationDate;
Basis = 1;
Compounding = -1;

RateSpec = intenvset('ValuationDate', ValuationDate, 'StartDates', Settle, ...
'EndDates', Maturity, 'Rates', Rates, 'Compounding', Compounding, 'Basis', Basis);

%Build the Leisen-Reimer (LR) tree with 25 time steps
LRTimeSpec  = lrtimespec(ValuationDate, Maturity, 25);

% Use the PP2 method
LRMethod  = 'PP2';

TreeLR = lrtree(StockSpec, RateSpec, LRTimeSpec, Strike, 'method', LRMethod);

%Compute prices and sensitivities using the LR model:
OptSpec = {'call'; 'put'};
OutSpec = {'Price', 'Delta', 'Gamma'};

[Price, Delta, Gamma] = optstocksensbylr(TreeLR, OptSpec, Strike, Settle, ...
Maturity, 'OutSpec', OutSpec)
```

```
Price =

   10.1332
    6.6937

Delta =

    0.6056
   -0.3944

Gamma =

    0.0185
    0.0185
```

## References

Leisen D.P., M. Reimer, “Binomial Models for Option Valuation – Examining and Improving Convergence,” *Applied Mathematical Finance*, Number 3, 1996, pp. 319-346.

## See Also

| `optstockbylr` | `lrtree`

# optstocksensbyrgw

## Purpose

Determine American call option prices and sensitivities using Roll-Geske-Whaley option pricing model

## Syntax

```
PriceSens = optstocksensbyrgw(RateSpec, StockSpec, Settle,
Maturity, OptSpec, Strike, 'Name1', Value1...)
```

## Arguments

| | |
|---|---|
| `RateSpec` | The annualized continuously compounded rate term structure. For information on the interest rate specification, see `intenvset`. |
| `StockSpec` | Stock specification. See `stockspec`. |
| `Settle` | `NINST`-by-`1` vector of settlement or trade dates. |
| `Maturity` | `NINST`-by-`1` vector of maturity dates. |
| `OptSpec` | `NINST`-by-`1` cell array of strings `'call'` or `'put'`. |
| `Strike` | `NINST`-by-`1` vector of strike price values. |
| `OutSpec` | (Optional) All optional inputs are specified as matching parameter name/value pairs. The parameter name is specified as a character string, followed by the corresponding parameter value. Parameter name/value pairs may be specified in any order; names are case-insensitive and partial string matches are allowed provided no ambiguities exist. Valid parameter names are:<br>• `NOUT`-by-`1` or `1`-by-`NOUT` cell array of strings indicating the nature and order of the outputs for the function. Possible values are: `'Price'`, `'Delta'`, `'Gamma'`, `'Vega'`, `'Lambda'`, `'Rho'`, `'Theta'`, or `'All'`.<br>For example, `OutSpec = {'Price'; 'Lambda'; 'Rho'}` specifies that the output should be `Price`, `Lambda`, and `Rho`, in that order. |

To invoke from a function: `[Price, Lambda, Rho] = optstocksensbyrgw(..., 'OutSpec', {'Price', 'Lambda', 'Rho'})`

`OutSpec = {'All'}` specifies that the output should be `Delta`, `Gamma`, `Vega`, `Lambda`, `Rho`, `Theta`, and `Price`, in that order. This is the same as specifying `OutSpec` as `OutSpec = {'Delta', 'Gamma', 'Vega', 'Lambda', 'Rho', 'Theta', 'Price'};`.

- Default is `OutSpec` = `{'Price'}`.

## Description

`PriceSens = optstocksensbyrgw(RateSpec, StockSpec, Settle, Maturity, OptSpec, Strike, 'Name1', Value1...)` computes American call option prices and sensitivities using the Roll-Geske-Whaley option pricing model.

`PriceSens` is a `NINST`-by-`1` vector of expected prices or sensitivities values.

---

**Note** `optstocksensbyrgw` computes prices of American calls with a single cash dividend using the Roll-Geske-Whaley option pricing model. All sensitivities are evaluated by computing a discrete approximation of the partial derivative. This means that the option is revalued with a fractional change for each relevant parameter, and the change in the option value divided by the increment, is the approximated sensitivity value.

---

## Examples

Consider an American stock option with an exercise price of $82 on January 1, 2008 that expires on May 1, 2008. Assume the underlying stock pays dividends of $4 on April 1, 2008. The stock is trading at $80 and has a volatility of 30% per annum. The risk-free rate is 6% per annum. Using this data, calculate the price and the value of `delta` and `gamma` of the American call using the Roll-Geske-Whaley option pricing model:

```
AssetPrice = 80;
Settle = 'Jan-01-2008';
Maturity = 'May-01-2008';
Strike = 82;
Rate = 0.06;
Sigma  = 0.3;
DivAmount = 4;
DivDate = 'Apr-01-2008';
```

Define `StockSpec` and `RateSpec`:

```
StockSpec = stockspec(Sigma, AssetPrice, {'cash'}, DivAmount, DivDate);

RateSpec = intenvset('ValuationDate', Settle, 'StartDates', Settle,...
'EndDates', Maturity, 'Rates', Rate, 'Compounding', -1, 'Basis', 1);
```

Define `OutSpec`:

```
OutSpec = {'Price', 'Delta', 'Gamma'};
```

Calculate the call `Price`, `Delta`, and `Gamma`:

```
[Price, Delta, Gamma]  = optstocksensbyrgw(RateSpec, StockSpec, Settle,...
Maturity, Strike,'OutSpec', OutSpec)

Price =

    4.3860


Delta =

    0.5022
```

```
Gamma =

    0.0336
```

## See Also

`impvbyrgw` | `intenvset` | `optstockbyrgw` | `stockspec`

# rangefloatbybdt

## Purpose

Price range floating note using Black-Derman-Toy tree

## Syntax

```
Price = rangefloatbybdt(BDTTree,Spread,Settle,Maturity,
RateSched)
[Price,PriceTree] = rangefloatbybdt(BDTTree,Spread,Settle,
Maturity, RateSched, Name,Value)
```

## Description

`Price = rangefloatbybdt(BDTTree,Spread,Settle,Maturity, RateSched)` calculates the price of the range note instrument at the valuation date using a BDT model.

`[Price,PriceTree] = rangefloatbybdt(BDTTree,Spread,Settle, Maturity, RateSched, Name,Value)` calculates the price of the range note instrument at the valuation date and the price evolution for one or more range instruments using a BDT model with additional options specified by one or more `Name,Value` pair arguments.

## Input Arguments

### BDTTree

Interest-rate tree structure created by `bdttree`.

### Spread

`NINST`-by-`1` vector of the number of basis points over the reference rate.

### Settle

`NINST`-by-`1` vector of dates representing the settle date of the range floating note.

---

**Note** The `Settle` date for every range floating instrument is set to the `ValuationDate` of the BDT tree. The range floating note argument `Settle` is ignored.

---

### Maturity

`NINST`-by-`1` vector of dates representing the maturity date of the floating-rate note.

### RateSched

`NINST`-by-`1` vector of structures representing the range of rates within which cash flows are nonzero. Each element of the structure array contains two fields:

- `RateSched.Dates` — `NDates`-by-`1` cell array of dates corresponding to the range schedule.
- `RateSched.Rates` — `NDates`-by-`2` array with the first column containing the lower bound of the range and the second column containing the upper bound of the range. Cash flow for date `RateSched.Dates`(*n*) is nonzero for rates in the range `RateSched.Rates`(*n*,1) < `Rate` < `RateSched.Rate` (*n*,2).

## Name-Value Pair Arguments

Specify optional comma-separated pairs of `Name,Value` arguments, where `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside single quotes (`' '`). You can specify several name and value pair arguments in any order as `Name1,Value1,...,NameN,ValueN`.

### Basis

`NINST`-by-`1` vector representing the day-count basis used when annualizing the input forward rate tree.

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (BMA)
- 5 = 30/360 (ISDA)

- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/actual (ISDA)
- 13 = BUS/252

For more information, see basis.

**Default:** `0` (actual/actual)

### EndMonthRule

`NINST`-by-`1` vector for end-of-month rule. Values are `1` (in effect) and `0` (not in effect).

**Default:** `1` (in effect)

### Options

Structure created with `derivset` containing derivatives pricing options.

**Default:** None

### Principal

`NINST`-by-`1` vector of the notional principal amount.

**Default:** `100`

### Reset

`NINST`-by-`1` vector representing the frequency of payments per year.

**Default:** `1`

## Output Arguments

### Price

`NINST`-by-`1` vector for expected prices at time 0.

### PriceTree

Structure containing trees of vectors of instrument prices and accrued interest, and a vector of observation times for each node. Values are:

- `PriceTree.PTree` contains the clean prices.
- `PriceTree.AITree` contains the accrued interest.
- `PriceTree.tObs` contains the observation times.

## Definitions

### Range Note Instrument

A range note is a structured (market-linked) security whose coupon rate is equal to the reference rate as long as the reference rate is within a certain range. If the reference rate is outside of the range, the coupon rate is 0 for that period. This type of instrument entitles the holder to cash flows that depend on the level of some reference interest rate and are floored to be positive. The note holder gets directs exposure to the reference rate. In return for the drawback that no interest will be paid for the time the range is left, they offer higher coupon rates than comparable standard products, like vanilla floating notes.

## Examples

Compute the price of a range note:

```
% The data for the interest rate term structure is as follows:
Rates = [0.035; 0.042147; 0.047345; 0.052707];
ValuationDate = 'Jan-1-2011';
StartDates = ValuationDate;
EndDates = {'Jan-1-2012'; 'Jan-1-2013'; 'Jan-1-2014'; 'Jan-1-2015'};
Compounding = 1;

%  Create RateSpec
RS = intenvset('ValuationDate', ValuationDate, 'StartDates', StartDates,...
```

```
'EndDates', EndDates, 'Rates', Rates, 'Compounding', Compounding);

% Instrument
% The range note matures in Jan-1-2014 and has the following RateSchedule:
Spread = 100;
Settle = 'Jan-1-2011';
Maturity = 'Jan-1-2014';
RateSched(1).Dates = {'Jan-1-2012'; 'Jan-1-2013'  ; 'Jan-1-2014'};
RateSched(1).Rates = [0.045 0.055 ; 0.0525  0.0675; 0.06 0.08];

% The data to build the tree is as follows:
% Assume the volatility to be 10%.
Sigma = 0.1;
BDTTS = bdttimespec(ValuationDate, EndDates, Compounding);
BDTVS = bdtvolspec(ValuationDate, EndDates, Sigma*ones(1, length(EndDates))');
BDTT = bdttree(BDTVS, RS, BDTTS);

%Price the instrument
Price = rangefloatbybdt(BDTT, Spread, Settle, Maturity, RateSched)

Price =

   97.5267
```

## References

Jarrow, Robert, *Modelling Fixed Income Securities and Interest Rate Options*, Stanford Economics and Finance, 2nd edition, 2002.

## See Also

| `bdttree` | `cfbybdt` | `floatbybdt` | `swapbybdt` | `floorbybdt` | `fixedbybdt` | `bondbybdt` | `rangefloatbyhjm` | `rangefloatbybk` | `instrangefloat` | `rangefloatbyhw` |

## Purpose

Price range floating note using Black-Karasinski tree

## Syntax

```
Price = rangefloatbybk(BKTree,Spread,Settle,Maturity,
RateSched)
[Price,PriceTree] = rangefloatbybk(BKTree,Spread,Settle,
Maturity, RateSched, Name,Value)
```

## Description

`Price = rangefloatbybk(BKTree,Spread,Settle,Maturity, RateSched)` calculates the price of the range note instrument at the valuation date using a BK model.

`[Price,PriceTree] = rangefloatbybk(BKTree,Spread,Settle, Maturity, RateSched, Name,Value)` calculates the price of the range note instrument at the valuation date and the price evolution for one or more range instruments using a BK model with additional options specified by one or more `Name,Value` pair arguments.

## Input Arguments

### BKTree

Interest-rate tree structure created by `bktree`.

### Spread

`NINST`-by-`1` vector of the number of basis points over the reference rate.

### Settle

`NINST`-by-`1` vector of dates representing the settle date of the range floating note.

---

**Note** The `Settle` date for every range floating instrument is set to the `ValuationDate` of the BK tree. The range floating note argument `Settle` is ignored.

---

### Maturity

`NINST`-by-`1` vector of dates representing the maturity date of the floating-rate note.

### RateSched

`NINST`-by-`1` vector of structures representing the range of rates within which cash flows are nonzero. Each element of the structure array contains two fields:

- `RateSched.Dates` — `NDates`-by-`1` cell array of dates corresponding to the range schedule.
- `RateSched.Rates` — `NDates`-by-`2` array with the first column containing the lower bound of the range and the second column containing the upper bound of the range. Cash flow for date `RateSched.Dates`(*n*) is nonzero for rates in the range `RateSched.Rates`(*n*,1) < `Rate` < `RateSched.Rate` (*n*,2).

## Name-Value Pair Arguments

Specify optional comma-separated pairs of `Name,Value` arguments, where `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside single quotes (`' '`). You can specify several name and value pair arguments in any order as `Name1,Value1,...,NameN,ValueN`.

### Basis

`NINST`-by-`1` vector representing the day-count basis used when annualizing the input forward rate tree.

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (BMA)
- 5 = 30/360 (ISDA)

- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/actual (ISDA)
- 13 = BUS/252

For more information, see basis.

**Default:** `0` (actual/actual)

### EndMonthRule

`NINST`-by-`1` vector for end-of-month rule. Values are `1` (in effect) and `0` (not in effect).

**Default:** `1` (in effect)

### Options

Structure created with `derivset` containing derivatives pricing options.

**Default:** None

### Principal

`NINST`-by-`1` vector of the notional principal amount.

**Default:** `100`

### Reset

`NINST`-by-`1` vector representing the frequency of payments per year.

**Default:** `1`

## Output Arguments

### Price

`NINST`-by-`1` vector for expected prices at time 0.

### PriceTree

Structure containing trees of vectors of instrument prices and accrued interest, and a vector of observation times for each node. Values are:

- `PriceTree.PTree` contains the clean prices.
- `PriceTree.AITree` contains the accrued interest.
- `PriceTree.tObs` contains the observation times.

## Definitions

### Range Note Instrument

A range note is a structured (market-linked) security whose coupon rate is equal to the reference rate as long as the reference rate is within a certain range. If the reference rate is outside of the range, the coupon rate is 0 for that period. This type of instrument entitles the holder to cash flows that depend on the level of some reference interest rate and are floored to be positive. The note holder gets directs exposure to the reference rate. In return for the drawback that no interest will be paid for the time the range is left, they offer higher coupon rates than comparable standard products, like vanilla floating notes.

## Examples

Compute the price of a range note:

```
% The data for the interest rate term structure is as follows:
Rates = [0.035; 0.042147; 0.047345; 0.052707];
ValuationDate = 'Jan-1-2011';
StartDates = ValuationDate;
EndDates = {'Jan-1-2012'; 'Jan-1-2013'; 'Jan-1-2014'; 'Jan-1-2015'};
Compounding = 1;

%  Create RateSpec
RS = intenvset('ValuationDate', ValuationDate, 'StartDates', StartDates,...
```

```
'EndDates', EndDates, 'Rates', Rates, 'Compounding', Compounding);

% Instrument
% The range note matures in Jan-1-2014 and has the following RateSchedule:
Spread = 100;
Settle = 'Jan-1-2011';
Maturity = 'Jan-1-2014';
RateSched(1).Dates = {'Jan-1-2012'; 'Jan-1-2013'  ; 'Jan-1-2014'};
RateSched(1).Rates = [0.045 0.055 ; 0.0525  0.0675; 0.06 0.08];

% The data to build the tree is as follows:
VolDates = ['1-Jan-2012'; '1-Jan-2013'; '1-Jan-2014';'1-Jan-2015'];
VolCurve = 0.01;
AlphaDates = '01-01-2015';
AlphaCurve = 0.1;

BKVS = bkvolspec(RS.ValuationDate, VolDates, VolCurve,...
AlphaDates, AlphaCurve);
BKTS = bktimespec(RS.ValuationDate, VolDates, Compounding);
BKT = bktree(BKVS, RS, BKTS);

%Price the instrument
Price = rangefloatbybk(BKT, Spread, Settle, Maturity, RateSched)

Price =

  102.7574
```

## References

Jarrow, Robert, *Modelling Fixed Income Securities and Interest Rate Options*, Stanford Economics and Finance, 2nd edition, 2002.

## See Also

| `bktree` | `cfbybk` | `capbybk` | `swapbybk` | `floorbybk` | `fixedbybk` | `bondbybk` | `rangefloatbyhjm` | `rangefloatbybdt` | `rangefloatbyhw` | `instrangefloat` |

# rangefloatbyhjm

## Purpose

Price range floating note using Heath-Jarrow-Morton tree

## Syntax

`Price = rangefloatbyhjm(HJMTree,Spread,Settle,Maturity, RateSched)`
`[Price,PriceTree] = rangefloatbyhjm(HJMTree,Spread,Settle, Maturity, RateSched, Name,Value)`

## Description

`Price = rangefloatbyhjm(HJMTree,Spread,Settle,Maturity, RateSched)` calculates the price of the range note instrument at the valuation date using an HJM model.

`[Price,PriceTree] = rangefloatbyhjm(HJMTree,Spread,Settle, Maturity, RateSched, Name,Value)` calculates the price of the range note instrument at the valuation date and the price evolution for one or more range instruments using an HJM model with additional options specified by one or more `Name,Value` pair arguments.

## Input Arguments

### HJMTree

Interest-rate tree structure created by `hjmtree`.

### Spread

`NINST`-by-`1` vector of the number of basis points over the reference rate.

### Settle

`NINST`-by-`1` vector of dates representing the settle date of the range floating note.

---

**Note** The `Settle` date for every range floating instrument is set to the `ValuationDate` of the HJM tree. The range floating note argument `Settle` is ignored.

---

### Maturity

`NINST`-by-`1` vector of dates representing the maturity date of the floating-rate note.

### RateSched

`NINST`-by-`1` vector of structures representing the range of rates within which cash flows are nonzero. Each element of the structure array contains two fields:

- `RateSched.Dates` — `NDates`-by-`1` cell array of dates corresponding to the range schedule.
- `RateSched.Rates` — `NDates`-by-`2` array with the first column containing the lower bound of the range and the second column containing the upper bound of the range. Cash flow for date `RateSched.Dates`(*n*) is nonzero for rates in the range `RateSched.Rates`(*n*,1) < `Rate` < `RateSched.Rate` (*n*,2).

## Name-Value Pair Arguments

Specify optional comma-separated pairs of `Name,Value` arguments, where `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside single quotes (`' '`). You can specify several name and value pair arguments in any order as `Name1,Value1,...,NameN,ValueN`.

### Basis

`NINST`-by-`1` vector representing the day-count basis used when annualizing the input forward rate tree.

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (BMA)
- 5 = 30/360 (ISDA)

- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/actual (ISDA)
- 13 = BUS/252

For more information, see basis.

**Default:** `0` (actual/actual)

### EndMonthRule

`NINST`-by-`1` vector for end-of-month rule. Values are `1` (in effect) and `0` (not in effect).

**Default:** `1` (in effect)

### Options

Structure created with `derivset` containing derivatives pricing options.

**Default:** None

### Principal

`NINST`-by-`1` vector of the notional principal amount.

**Default:** `100`

### Reset

`NINST`-by-`1` vector representing the frequency of payments per year.

**Default:** `1`

## Output Arguments

### Price

`NINST`-by-`1` vector for expected prices at time 0.

### PriceTree

Structure containing trees of vectors of instrument prices and accrued interest, and a vector of observation times for each node. Values are:

- `PriceTree.PTree` contains the clean prices.
- `PriceTree.AITree` contains the accrued interest.
- `PriceTree.tObs` contains the observation times.

## Definitions

### Range Note Instrument

A range note is a structured (market-linked) security whose coupon rate is equal to the reference rate as long as the reference rate is within a certain range. If the reference rate is outside of the range, the coupon rate is 0 for that period. This type of instrument entitles the holder to cash flows that depend on the level of some reference interest rate and are floored to be positive. The note holder gets directs exposure to the reference rate. In return for the drawback that no interest will be paid for the time the range is left, they offer higher coupon rates than comparable standard products, like vanilla floating notes.

## Examples

Compute the price of a range note:

```
% The data for the interest rate term structure is as follows:
Rates = [0.035; 0.042147; 0.047345; 0.052707];
ValuationDate = 'Jan-1-2011';
StartDates = ValuationDate;
EndDates = {'Jan-1-2012'; 'Jan-1-2013'; 'Jan-1-2014'; 'Jan-1-2015'};
Compounding = 1;

%  Create RateSpec
RS = intenvset('ValuationDate', ValuationDate, 'StartDates', StartDates,...
```

```
'EndDates', EndDates, 'Rates', Rates, 'Compounding', Compounding);

% Instrument
% The range note matures in Jan-1-2014 and has the following RateSchedule:
Spread = 100;
Settle = 'Jan-1-2011';
Maturity = 'Jan-1-2014';
RateSched(1).Dates = {'Jan-1-2012'; 'Jan-1-2013'  ; 'Jan-1-2014'};
RateSched(1).Rates = [0.045 0.055 ; 0.0525  0.0675; 0.06 0.08];

% The data to build the tree is as follows:
Volatility = [.2; .19; .18; .17];
CurveTerm = [ 1;  2;   3;   4];
MaTree = {'Jan-1-2012'; 'Jan-1-2013'; 'Jan-1-2014'; 'Jan-1-2015'};
HJMTS = hjmtimespec(ValuationDate, MaTree);
HJMVS = hjmvolspec('Proportional', Volatility, CurveTerm, 1e6);
HJMT = hjmtree(HJMVS, RS, HJMTS);

%Price the instrument
Price = rangefloatbyhjm(HJMT, Spread, Settle, Maturity, RateSched)

Price =

   90.2348
```

## References

Jarrow, Robert, *Modelling Fixed Income Securities and Interest Rate Options*, Stanford Economics and Finance, 2nd edition, 2002.

## See Also

| `hjmtree` | `cfbyhjm` | `floatbyhjm` | `swapbyhjm` | `floorbyhjm` | `fixedbyhjm` | `bondbyhjm` | `rangefloatbybk` | `rangefloatbybdt` | `rangefloatbyhw` | `instrangefloat` |

## Purpose

Price range floating note using Hull-White tree

## Syntax

```
Price = rangefloatbyhw(HWTree,Spread,Settle,Maturity,
RateSched)
[Price,PriceTree] = rangefloatbyhw(HWTree,Spread,Settle,
Maturity, RateSched, Name,Value)
```

## Description

`Price = rangefloatbyhw(HWTree,Spread,Settle,Maturity, RateSched)` calculates the price of the range note instrument at the valuation date using an HW model.

`[Price,PriceTree] = rangefloatbyhw(HWTree,Spread,Settle, Maturity, RateSched, Name,Value)` calculates the price of the range note instrument at the valuation date and the price evolution for one or more range instruments using an HW model with additional options specified by one or more `Name,Value` pair arguments.

## Input Arguments

### HWTree

Interest-rate tree structure created by `hwtree`.

### Spread

`NINST`-by-`1` vector of the number of basis points over the reference rate.

### Settle

`NINST`-by-`1` vector of dates representing the settle date of the range floating note.

---

**Note** The `Settle` date for every range floating instrument is set to the `ValuationDate` of the HW tree. The range floating note argument `Settle` is ignored.

---

### Maturity

`NINST`-by-`1` vector of dates representing the maturity date of the floating-rate note.

#### RateSched

`NINST`-by-`1` vector of structures representing the range of rates within which cash flows are nonzero. Each element of the structure array contains two fields:

- `RateSched.Dates` — `NDates`-by-`1` cell array of dates corresponding to the range schedule.
- `RateSched.Rates` — `NDates`-by-`2` array with the first column containing the lower bound of the range and the second column containing the upper bound of the range. Cash flow for date `RateSched.Dates`(*n*) is nonzero for rates in the range `RateSched.Rates`(*n*,1) < `Rate` < `RateSched.Rate` (*n*,2).

### Name-Value Pair Arguments

Specify optional comma-separated pairs of `Name,Value` arguments, where `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside single quotes (`' '`). You can specify several name and value pair arguments in any order as `Name1,Value1,...,NameN,ValueN`.

#### Basis

`NINST`-by-`1` vector representing the day-count basis used when annualizing the input forward rate tree.

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (BMA)
- 5 = 30/360 (ISDA)

- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/actual (ISDA)
- 13 = BUS/252

For more information, see basis.

**Default:** `0` (actual/actual)

### EndMonthRule

`NINST`-by-`1` vector for end-of-month rule. Values are `1` (in effect) and `0` (not in effect).

**Default:** `1` (in effect)

### Options

Structure created with `derivset` containing derivatives pricing options.

**Default:** None

### Principal

`NINST`-by-`1` vector of the notional principal amount.

**Default:** `100`

### Reset

`NINST`-by-`1` vector representing the frequency of payments per year.

**Default:** `1`

## Output Arguments

### Price

`NINST`-by-`1` vector for expected prices at time 0.

### PriceTree

Structure containing trees of vectors of instrument prices and accrued interest, and a vector of observation times for each node. Values are:

- `PriceTree.PTree` contains the clean prices.
- `PriceTree.AITree` contains the accrued interest.
- `PriceTree.tObs` contains the observation times.

## Definitions

### Range Note Instrument

A range note is a structured (market-linked) security whose coupon rate is equal to the reference rate as long as the reference rate is within a certain range. If the reference rate is outside of the range, the coupon rate is 0 for that period. This type of instrument entitles the holder to cash flows that depend on the level of some reference interest rate and are floored to be positive. The note holder gets directs exposure to the reference rate. In return for the drawback that no interest will be paid for the time the range is left, they offer higher coupon rates than comparable standard products, like vanilla floating notes.

## Examples

Compute the price of a range note:

```
% The data for the interest rate term structure is as follows:
Rates = [0.035; 0.042147; 0.047345; 0.052707];
ValuationDate = 'Jan-1-2011';
StartDates = ValuationDate;
EndDates = {'Jan-1-2012'; 'Jan-1-2013'; 'Jan-1-2014'; 'Jan-1-2015'};
Compounding = 1;

%  Create RateSpec
RS = intenvset('ValuationDate', ValuationDate, 'StartDates', StartDates,...
```

```
'EndDates', EndDates, 'Rates', Rates, 'Compounding', Compounding);

% Instrument
% The range note matures in Jan-1-2014 and has the following RateSchedule:
Spread = 100;
Settle = 'Jan-1-2011';
Maturity = 'Jan-1-2014';
RateSched(1).Dates = {'Jan-1-2012'; 'Jan-1-2013'  ; 'Jan-1-2014'};
RateSched(1).Rates = [0.045 0.055 ; 0.0525  0.0675; 0.06 0.08];

% The data to build the tree is as follows:
VolDates = ['1-Jan-2012'; '1-Jan-2013'; '1-Jan-2014';'1-Jan-2015'];
VolCurve = 0.01;
AlphaDates = '01-01-2015';
AlphaCurve = 0.1;

HWVS = hwvolspec(RS.ValuationDate, VolDates, VolCurve,...
AlphaDates, AlphaCurve);
HWTS = hwtimespec(RS.ValuationDate, VolDates, Compounding);
HWT = hwtree(HWVS, RS, HWTS);

%Price the instrument
Price = rangefloatbyhw(HWT, Spread, Settle, Maturity, RateSched)

Price =

   96.6501
```

## References

Jarrow, Robert, *Modelling Fixed Income Securities and Interest Rate Options*, Stanford Economics and Finance, 2nd edition, 2002.

## See Also

| `hwtree` | `cfbyhw` | `capbyhw` | `swapbyhw` | `floorbyhw` | `fixedbyhw` | `bondbyhw` | `rangefloatbybk` | `rangefloatbybdt` | `rangefloatbyhjm` | `instrangefloat` |

## Purpose

Discount factors from interest rates

## Syntax

**Usage 1**: Interval points are input as times in periodic units.

```
Disc = rate2disc(Compounding, Rates, EndTimes)
Disc = rate2disc(Compounding, Rates, EndTimes, StartTimes)
```

**Usage 2**: `ValuationDate` is passed and interval points are input as dates.

```
[Disc, EndTimes, StartTimes] = rate2disc(Compounding, Rates,
EndDates, StartDates, ValuationDate)
[Disc, EndTimes, StartTimes] = rate2disc(Compounding, Rates,
EndDates, StartDates, ValuationDate, Basis, EndMonthRule)
```

## Arguments

| | |
|---|---|
| `Compounding` | Scalar value representing the rate at which the input zero rates were compounded when annualized. This argument determines the formula for the discount factors:<br>`Compounding` = 1, 2, 3, 4, 6, 12<br>`Disc = (1 + Z/F)^(-T)`, where `F` is the compounding frequency, `Z` is the zero rate, and `T` is the time in periodic units; for example, `T = F` is 1 year.<br>`Compounding` = 365<br>`Disc = (1 + Z/F)^(-T)`, where `F` is the number of days in the basis year and `T` is a number of days elapsed computed by basis.<br>`Compounding` = -1<br>`Disc = exp(-T*Z)`, where `T` is time in years. |
| `Rates` | Number of points (`NPOINTS`) by number of curves (`NCURVES`) matrix of rates in decimal form. For example, 5% is 0.05 in `Rates`. `Rates` are the yields over investment intervals from |

| | |
|---|---|
| | `StartTimes`, when the cash flow is valued, to `EndTimes`, when the cash flow is received. |
| `EndTimes` | `NPOINTS`-by-`1` vector or scalar of times in periodic units ending the interval to discount over. |
| `StartTimes` | (Optional) `NPOINTS`-by-`1` vector or scalar of times in periodic units starting the interval to discount over. Default = `0`. |

**Note** When `ValuationDate` is not passed, the third and fourth arguments (`EndTimes` and `StartTimes`) are interpreted as times.

| | |
|---|---|
| `EndDates` | `NPOINTS`-by-`1` vector or scalar of serial maturity dates ending the interval to discount over. |

**Note** : When `ValuationDate` is passed, the third and fourth arguments (`EndDates` and `StartDates`) are interpreted as dates. The date `ValuationDate` is used as the zero point for computing the times.

| | |
|---|---|
| `StartDates` | (Optional) `NPOINTS`-by-`1` vector or scalar of serial dates starting the interval to discount over. `StartDates` must be earlier than `EndDates`.<br>Default = `ValuationDate`. |
| `ValuationDate` | Scalar value in serial date number form representing the observation date of the investment horizons entered in `StartDates` and `EndDates`. Required in **Usage 2**. Omitted or passed as an empty matrix to invoke **Usage 1**. |

| | |
|---|---|
| `Basis` | (Optional) Day-count basis of the instrument when using **Usage 2**. A vector of integers.<br>• 0 = actual/actual (default)<br>• 1 = 30/360 (SIA)<br>• 2 = actual/360<br>• 3 = actual/365<br>• 4 = 30/360 (BMA)<br>• 5 = 30/360 (ISDA)<br>• 6 = 30/360 (European)<br>• 7 = actual/365 (Japanese)<br>• 8 = actual/actual (ICMA)<br>• 9 = actual/360 (ICMA)<br>• 10 = actual/365 (ICMA)<br>• 11 = 30/360E (ICMA)<br>• 12 = actual/actual (ISDA)<br>• 13 = BUS/252<br>For more information, see basis. |
| `EndMonthRule` | (Optional) End-of-month rule when using **Usage 2**. A vector. This rule applies only when `Maturity` is an end-of-month date for a month having 30 or fewer days. `0` = ignore rule, meaning that a bond's coupon payment date is always the same numerical day of the month. `1` = set rule on (default), meaning that a bond's coupon payment date is always the last actual day of the month. |

## Description

**Usage 1**: `Disc = rate2disc(Compounding, Rates, EndTimes)` or `Disc = rate2disc(Compounding, Rates, EndTimes, StartTimes)` where interval points are input as times in periodic units.

**Usage 2**: `[Disc, EndTimes, StartTimes] = rate2disc(Compounding, Rates, EndDates, StartDates, ValuationDate)` or `[Disc, EndTimes, StartTimes] = rate2disc(Compounding, Rates, EndDates, StartDates, ValuationDate, Basis, EndMonthRule)` where `ValuationDate` is passed and interval points are input as dates.

`rate2disc` computes the discounts over a series of `NPOINTS` time intervals given the annualized yield over those intervals. `NCURVES` different rate curves can be translated at once if they have the same time structure. The time intervals can represent a zero curve or a forward curve.

The output `Disc` is an `NPOINTS`-by-`NCURVES` column vector of discount factors in decimal form representing the value at time `StartTime` of a unit cash flow received at time `EndTime`.

You can specify the investment intervals either with input times (**Usage 1**) or with input dates (**Usage 2**). Entering `ValuationDate` invokes the date interpretation; omitting `ValuationDate` invokes the default time interpretations.

For **Usage 1**:

- `StartTimes` is an `NPOINTS`-by-`1` column vector of times starting the interval to discount over, measured in periodic units.
- `EndTimes` is an `NPOINTS`-by-`1` column vector of times ending the interval to discount over, measured in periodic units.

For **Usage 2**:

- `StartDates` is an `NPOINTS`-by-`1` column vector of serial dates starting the interval to discount over, measured in days.
- `EndDates` is an `NPOINTS`-by-`1` column vector of serial dates ending the interval to discount over, measured in days.

If `Compounding = 365` (daily), `StartDates` and `EndDates` are measured in days as in **Usage 2**. Otherwise, in **Usage 1**, the arguments contain values, `T`, computed from SIA semiannual time factors, `Tsemi`, by the formula `T = Tsemi/2*F`, where `F` is the compounding frequency.

## Examples

**Example 1** demonstrates **Usage 1**. Compute discounts from a zero curve at 6 months, 12 months, and 24 months. The times to the cash flows are 1, 2, and 4. You are computing the present value (at time 0) of the cash flows.

```
Compounding = 2;
Rates = [0.05; 0.06; 0.065];
EndTimes   = [1; 2; 4];
Disc = rate2disc(Compounding, Rates, EndTimes)

Disc =
    0.9756
    0.9426
    0.8799
```

**Example 2** demonstrates **Usage 2**. Compute discounts from a zero curve at 6 months, 12 months, and 24 months. Use dates to specify the ending time horizon.

```
Compounding = 2;
Rates = [0.05; 0.06; 0.065];
EndDates = ['10/15/97'; '04/15/98'; '04/15/99'];
ValuationDate = '4/15/97';
Disc = rate2disc(Compounding, Rates, EndDates, [], ValuationDate)

Disc =
    0.9756
    0.9426
    0.8799
```

**Example 3** demonstrates **Usage 1**. Compute discounts from the 1-year forward rates beginning now, in 6 months, and in 12 months. Use monthly compounding. The times to the cash flows are 12, 18, 24, and the forward times are 0, 6, 12.

```
Compounding = 12;
Rates = [0.05; 0.04; 0.06];
EndTimes = [12; 18; 24];
StartTimes = [0; 6; 12];
Disc = rate2disc(Compounding, Rates, EndTimes, StartTimes)
Disc =
    0.9513
    0.9609
    0.9419
```

## See Also

`disc2rate` | `ratetimes`

## Purpose

Change time intervals defining interest-rate environment

## Syntax

```
[Rates, EndTimes, StartTimes] = ratetimes(Compounding,
RefRates, RefEndTimes, RefStartTimes, EndTimes, StartTimes)
[Rates, EndTimes, StartTimes] = ratetimes(Compounding,
RefRates, RefEndDates, RefStartDates, EndDates, StartDates,
ValuationDate)
```

Usage 1: `ValuationDate` not passed; third through sixth arguments are interpreted as times.

```
[Rates, EndTimes, StartTimes] = ratetimes(Compounding,
RefRates, RefEndTimes, RefStartTimes, EndTimes, StartTimes)
```

Usage 2: `ValuationDate` passed and interval points input as dates.

```
[Rates, EndTimes, StartTimes] = ratetimes(Compounding,
RefRates, RefEndDates, RefStartDates, EndDates, StartDates,
ValuationDate)
```

## Arguments

| | |
|---|---|
| `Compounding` | Scalar value representing the rate at which the input zero rates were compounded when annualized. This argument determines the formula for the discount factors:<br>`Compounding = 1, 2, 3, 4, 6, 12`<br>`Disc = (1 + Z/F)^(-T)`, where `F` is the compounding frequency, `Z` is the zero rate, and `T` is the time in periodic units; for example, `T = F` is 1 year.<br>`Compounding = 365`<br>`Disc = (1 + Z/F)^(-T)`, where `F` is the number of days in the basis year and `T` is a number of days elapsed computed by basis.<br>`Compounding = -1` |

| | |
|---|---|
| | `Disc = exp(-T*Z)`, where `T` is time in years. |
| `RefRates` | `NREFPTS`-by-`NCURVES` matrix of reference rates in decimal form. `RefRates` are the yields over investment intervals from `RefStartTimes`, when the cash flow is valued, to `RefEndTimes`, when the cash flow is received. |
| `RefEndTimes` | `NREFPTS`-by-`1` vector or scalar of times in periodic units ending the intervals corresponding to `RefRates`. |
| `RefStartTimes` | (Optional) `NREFPTS`-by-`1` vector or scalar of times in periodic units starting the intervals corresponding to `RefRates`. Default = `0`. |
| `EndTimes` | `NPOINTS`-by-`1` vector or scalar of times in periodic units ending the interval to discount over. |
| `StartTimes` | (Optional) `NPOINTS`-by-`1` vector or scalar of times in periodic units starting the interval to discount over. Default = `0`. |
| `RefEndDates` | `NREFPTS`-by-`1` vector or scalar of serial dates ending the intervals corresponding to `RefRates`. |
| `RefStartDates` | (Optional) `NREFPTS`-by-`1` vector or scalar of serial dates starting the intervals corresponding to `RefRates`. Default = `ValuationDate`. |
| `EndDates` | `NPOINTS`-by-`1` vector or scalar of serial maturity dates ending the interval to discount over. |

| | |
|---|---|
| `StartDates` | (Optional) `NPOINTS`-by-`1` vector or scalar of serial dates starting the interval to discount over. `StartDates` must be earlier than `EndDates`.<br>Default = `ValuationDate`. |
| `ValuationDate` | Scalar value in serial date number form representing the observation date of the investment horizons entered in `StartDates` and `EndDates`. Required in Usage 2. Omitted or passed as an empty matrix to invoke Usage 1. |

## Description

`[Rates, EndTimes, StartTimes] = ratetimes(Compounding, RefRates, RefEndTimes, RefStartTimes, EndTimes, StartTimes)` and `[Rates, EndTimes, StartTimes] = ratetimes(Compounding, RefRates, RefEndDates, RefStartDates, EndDates, StartDates, ValuationDate)` change time intervals defining an interest-rate environment.

`ratetimes` takes an interest-rate environment defined by yields over one collection of time intervals and computes the yields over another set of time intervals. The zero rate is assumed to be piece-wise linear in time.

`Rates` is an `NPOINTS`-by-`NCURVES` matrix of rates implied by the reference interest-rate structure and sampled at new intervals.

`StartTimes` is an `NPOINTS`-by-`1` column vector of times starting the new intervals where rates are desired, measured in periodic units.

`EndTimes` is an `NPOINTS`-by-`1` column vector of times ending the new intervals, measured in periodic units.

If `Compounding = 365` (daily), `StartTimes` and `EndTimes` are measured in days. The arguments otherwise contain values, `T`, computed from SIA semiannual time factors, `Tsemi`, by the formula `T = Tsemi/2 * F`, where `F` is the compounding frequency.

You can specify the investment intervals either with input times (Usage 1) or with input dates (Usage 2). Entering the argument `ValuationDate` invokes the date interpretation; omitting `ValuationDate` invokes the default time interpretations.

## Examples

**Example 1**. The reference environment is a collection of zero rates at 6, 12, and 24 months. Create a collection of 1-year forward rates beginning at 0, 6, and 12 months.

```
RefRates = [0.05; 0.06; 0.065];
RefEndTimes = [1; 2; 4];
StartTimes = [0; 1; 2];
EndTimes   = [2; 3; 4];
Rates = ratetimes(2, RefRates, RefEndTimes, 0, EndTimes,...
StartTimes)

Rates =
 0.0600
 0.0688
 0.0700
```

**Example 2**. Interpolate a zero yield curve to different dates. Zero curves start at the default date of `ValuationDate`.

```
RefRates = [0.04; 0.05; 0.052];
RefDates = [729756; 729907; 730121];
Dates    = [730241; 730486];
ValuationDate   = 729391;
Rates = ratetimes(2, RefRates, RefDates, [], Dates, [],...
ValuationDate)
Rates =
 0.0520
 0.0520
```

## See Also

`disc2rate` | `rate2disc`

# stockoptspec

## Purpose

Specify European stock option structure

## Syntax

`[StockOptSpec] = stockoptspec(OptPrice, Strike, Settle, Maturity, OptSpec, InterpMethod)`

## Arguments

| | |
|---|---|
| `OptPrice` | `NINST`-by-`1` vector of European option prices. |
| `Strike` | `NINST`-by-`1` vector of strike prices. |
| `Settle` | Scalar date marking the settlement date. |
| `Maturity` | `NINST`-by-`1` vector of maturity dates. |
| `OptSpec` | `NINST`-by-`1` cell array of strings `'call'` or `'put'`. |
| `InterpMethod` | (Optional) Method of interpolation to use for option prices. `InterpMethod` is `[{'price'} \| 'vol']`. The default is `'price'`. By specifying `'vol'`, implied volatilities will be used for interpolation purposes. The interpolated values will then be used to calculate the implicit interpolated prices. |

## Description

`[StockOptSpec] = stockoptspec(OptPrice, Strike, Settle, Maturity, OptSpec, InterpMethod)` creates a structure encapsulating the properties of a stock option structure.

## Examples

Consider the following data quoted from liquid options in the market with varying strikes and maturity. You specify these parameters in MATLAB as follows:

```
Settle =   '01/01/06';

Maturity =    ['07/01/06';
    '07/01/06';
    '07/01/06';
    '07/01/06';
```

```
    '01/01/07';
    '01/01/07';
    '01/01/07';
    '01/01/07';
    '07/01/07';
    '07/01/07';
    '07/01/07';
    '07/01/07';
    '01/01/08';
    '01/01/08';
    '01/01/08';
    '01/01/08'];

Strike = [113;
   101;
   100;
    88;
   128;
   112;
   100;
    78;
   144;
   112;
   100;
    69;
   162;
   112;
   100;
    61];

OptPrice =[                 0;
   4.807905472659144;
   1.306321897011867;
   0.048039195057173;
                   0;
   2.310953054191461;
   1.421950392866235;
```

```
   0.020414826276740;
                   0;
   5.091986935627730;
   1.346534812295291;
   0.005101325584140;
                   0;
   8.047628153217246;
   1.219653432150932;
   0.001041436654748];


OptSpec = { 'call';
    'call';
    'put';
    'put';
    'call';
    'call';
    'put';
    'put';
    'call';
    'call';
    'put';
    'put';
    'call';
    'call';
    'put';
    'put'};

StockOptSpec = stockoptspec(OptPrice, Strike, Settle, Maturity, OptSpec)

StockOptSpec =

          FinObj: 'StockOptSpec'
        OptPrice: [16x1 double]
          Strike: [16x1 double]
          Settle: 732678
        Maturity: [16x1 double]
```

```
     OptSpec: {16x1 cell}
InterpMethod: 'price'
```

## See Also

`ittprice` | `itttree` | `stockspec`

# stockspec

**Purpose** Create stock structure

**Syntax**

```
StockSpec = stockspec(Sigma, AssetPrice, DividendType,
DividendAmounts, ExDividendDates)
```

**Arguments**

| | |
|---|---|
| `Sigma` | `NINST`-by-`1` decimal annual price volatility of underlying security. |
| `AssetPrice` | `NINST`-by-`1` vector of underlying asset price values at time 0. |
| `DividendType` | (Optional) `NINST`-by-`1` cell array of strings specifying each stock's dividend type. Dividend type must be either `cash` for actual dollar dividends, `constant` for constant dividend yield, or `continuous` for continuous dividend yield. This function does not handle stock option dividends.<br>**Note** Dividends are assumed to be paid in cash. Noncash dividends (stock) are not allowed. When combining two or more type of dividends, shorter rows should be padded with the value `NaN`. |

| | |
|---|---|
| `DividendAmounts` | (Optional) `NINST`-by-`NDIV` matrix of cash dividends or `NINST`-by-`1` vector representing a constant or continuous annualized dividend yield. |
| `ExDividendDates` | (Optional) `NINST`-by-`NDIV` matrix of ex-dividend dates for `cash` type or `NINST`-by-`1` vector of ex-dividend dates for `constant` dividend type. For `continuous` dividend type, this argument should be ignored. |

## Description

`StockSpec = stockspec(Sigma, AssetPrice, DividendType, DividendAmounts, ExDividendDates)` creates a MATLAB structure containing the properties of a stock.

## Examples

**Example 1**. Consider a stock that provides four cash dividends of $0.50 on January 3, 2008, April 1, 2008, July 5, 2008 and October 1, 2008. The stock is trading at $50, and has a volatility of 20% per annum. Using this data, create the structure `StockSpec`:

```
AssetPrice = 50;
Sigma = 0.20;

DividendType = {'cash'};
DividendAmounts = [0.50, 0.50, 0.50, 0.50];
ExDividendDates = {'03-Jan-2008', '01-Apr-2008', '05-July-2008', '01-Oct-2008'};

StockSpec = stockspec(Sigma, AssetPrice, DividendType, DividendAmounts, ExDividendDates)

StockSpec =

             FinObj: 'StockSpec'
              Sigma: 0.2000
         AssetPrice: 50
       DividendType: {'cash'}
    DividendAmounts: [0.5000 0.5000 0.5000 0.5000]
```

```
    ExDividendDates: [733410 733499 733594 733682]
```

Examine the `StockSpec` structure:

```
datedisp(StockSpec.ExDividendDates)
03-Jan-2008   01-Apr-2008   05-Jul-2008   01-Oct-2008

StockSpec.DividendType

ans =

    'cash'
```

The `StockSpec` structure encapsulates the information of the stock and its four cash dividends.

**Example 2**. Consider two stocks that are trading at $40 and $35. The first one provides two cash dividends of $0.25 on March 1, 2008 and June 1, 2008. The second stock provides a continuous dividend yield of 3%. The stocks have a volatility of 30% per annum. Using this data, create the structure `StockSpec`:

```
AssetPrice = [40; 35];
Sigma = .30;

DividendType = {'cash'; 'continuous'};
DividendAmount = [0.25, 0.25 ; 0.03 NaN];

DividendDate1 = 'March-01-2008';
DividendDate2 = 'Jun-01-2008';

StockSpec = stockspec(Sigma, AssetPrice, DividendType, DividendAmount,...
{ DividendDate1, DividendDate2 ; NaN NaN})
StockSpec =

             FinObj: 'StockSpec'
              Sigma: [2x1 double]
         AssetPrice: [2x1 double]
```

```
       DividendType: {2x1 cell}
    DividendAmounts: [2x2 double]
    ExDividendDates: [2x2 double]
```

Examine the `StockSpec` structure:

```
datedisp(StockSpec.ExDividendDates)
01-Mar-2008   01-Jun-2008
   NaN           NaN

StockSpec.DividendType

ans =

    'cash'
    'continuous'
```

The `StockSpec` structure encapsulates the information of the two stocks and their dividends.

## See Also

`crrprice` | `crrtree` | `intenvset` | `optstockbybjs` | `optstockbyblk` | `optstockbybls` | `optstockbyrgw`

# supersharebybls

## Purpose

Calculate price of supershare digital options using Black-Scholes model

## Syntax

```
Price = supersharebybls(RateSpec, StockSpec, Settle, Maturity,
OptSpec, StrikeLow, StrikeHigh)
```

## Arguments

| | |
|---|---|
| `RateSpec` | The annualized, continuously compounded rate term structure. For information on the interest rate specification, see `intenvset`. |
| `StockSpec` | Stock specification. See `stockspec`. |
| `Settle` | `NINST`-by-`1` vector of settlement or trade dates. |
| `Maturity` | `NINST`-by-`1` vector of maturity dates. |
| `StrikeLow` | `NINST`-by-`1` vector of low strike price values. |
| `StrikeHigh` | `NINST`-by-`1` vector of high strike price values. |

## Description

`Price = supersharebybls(RateSpec, StockSpec, Settle, Maturity, OptSpec, StrikeLow, StrikeHigh)` computes supershare digital option prices using the Black-Scholes model.

`Price` is a `NINST`-by-`1` vector of expected option prices.

## Examples

Consider a supershare based on a portfolio of nondividend paying stocks with a lower strike of 350 and an upper strike of 450. The value of the portfolio on November 1, 2008 is 400. The risk-free rate is 4.5% and the volatility is 18%. Using this data, calculate the price of the supershare option on February 1, 2009.

Create the `RateSpec`:

```
Settle = 'Nov-1-2008';
```

```
Maturity = 'Feb-1-2009';
Rates = 0.045;
Basis = 1;
Compounding = -1;
RateSpec = intenvset('ValuationDate', Settle, 'StartDates', Settle,...
'EndDates', Maturity, 'Rates', Rates, 'Compounding', Compounding, 'Basis', Basis);
```

Define the `StockSpec`:

```
AssetPrice = 400;
Sigma = .18;
StockSpec = stockspec(Sigma, AssetPrice);
```

Define the high and low strike points:

```
StrikeLow = 350;
StrikeHigh = 450;
```

Calculate the price:

```
Pssh = supersharebybls(RateSpec, StockSpec, Settle, Maturity,...
StrikeLow, StrikeHigh)

Pssh =

    0.9411
```

## See Also

`assetbybls` | `cashbybls` | `gapbybls` | `supersharesensbybls`

# supersharesensbybls

## Purpose

Calculate price and sensitivities of supershare digital options using Black-Scholes model

## Syntax

```
PriceSens = supersharesensbybls(RateSpec, StockSpec, Settle,
Maturity, StrikeLow, StrikeHigh)
PriceSens = supersharesensbybls(RateSpec, StockSpec, Settle,
Maturity, StrikeLow, StrikeHigh, OutSpec)
```

## Arguments

| | |
|---|---|
| `RateSpec` | The annualized, continuously compounded rate term structure. For information on the interest rate specification, see `intenvset`. |
| `StockSpec` | Stock specification. See `stockspec`. |
| `Settle` | `NINST`-by-`1` vector of settlement or trade dates. |
| `Maturity` | `NINST`-by-`1` vector of maturity dates. |
| `StrikeLow` | `NINST`-by-`1` vector of low strike price values. |
| `StrikeHigh` | `NINST`-by-`1` vector of high strike price values. |
| `OutSpec` | (Optional) All optional inputs are specified as matching parameter name/value pairs. The parameter name is specified as a character string, followed by the corresponding parameter value. You can specify parameter name/value pairs in any order. Names are case-insensitive and partial string matches are allowed provided no ambiguities exist. Valid parameter names are:<br>• `NOUT`-by-`1` or `1`-by-`NOUT` cell array of strings indicating the nature and order of the outputs for the function. Possible values are `'Price'`, `'Delta'`, `'Gamma'`, `'Vega'`, `'Lambda'`, `'Rho'`, `'Theta'`, or `'All'`.<br>For example, `OutSpec = {'Price'; 'Lambda'; 'Rho'}` specifies that the output |

should be `Price`, `Lambda`, and `Rho`, in that order.

To invoke from a function: `[Price, Lambda, Rho] = supersharesensbybls(..., 'OutSpec', {'Price', 'Lambda', 'Rho'})`

`OutSpec = {'All'}` specifies that the output should be `Delta`, `Gamma`, `Vega`, `Lambda`, `Rho`, `Theta`, and `Price`, in that order. This is the same as specifying `OutSpec` as `OutSpec = {'Delta', 'Gamma', 'Vega', 'Lambda', 'Rho', 'Theta', 'Price'};`.

- Default is `OutSpec = {'Price'}`.

## Description

`PriceSens = supersharesensbybls(RateSpec, StockSpec, Settle, Maturity, StrikeLow, StrikeHigh)` computes supershare option prices using the Black-Scholes option pricing model.

`PriceSens = supersharesensbybls(RateSpec, StockSpec, Settle, Maturity, StrikeLow, StrikeHigh, OutSpec)` includes an `OutSpec` argument defined as parameter/value pairs, and computes supershare option prices and sensitivities using the Black-Scholes option pricing model.

`PriceSens` is a `NINST`-by-`1` vector of expected option prices and sensitivities.

## Examples

Consider a supershare based on a portfolio of nondividend paying stocks with a lower strike of 350 and an upper strike of 450. The value of the portfolio on November 1, 2008 is 400. The risk-free rate is 4.5% and the volatility is 18%. Using this data, calculate the price and sensitivity of the supershare option on February 1, 2009.

Create the `RateSpec`:

```
Settle = 'Nov-1-2008';
Maturity = 'Feb-1-2009';
Rates = 0.045;
Basis = 1;
Compounding = -1;
RateSpec = intenvset('ValuationDate', Settle, 'StartDates', Settle,...
'EndDates', Maturity, 'Rates', Rates, 'Compounding', Compounding, 'Basis', Basis);
```

Define the `StockSpec`:

```
AssetPrice = 400;
Sigma = .18;
StockSpec = stockspec(Sigma, AssetPrice);
```

Define the high and low strike points:

```
StrikeLow = 350;
StrikeHigh = 450;
```

Calculate the price:

```
Pssh = supersharebybls(RateSpec, StockSpec, Settle, Maturity,...
StrikeLow, StrikeHigh)

Pssh =

    0.9411
```

Compute the delta and theta of the supershare option:

```
OutSpec = { 'delta';'theta'};
[Delta, Theta]= supersharesensbybls(RateSpec, StockSpec, Settle,...
Maturity, StrikeLow, StrikeHigh, 'OutSpec', OutSpec)

Delta =

   -0.0010
```

```
Theta =

   -1.0102
```

## See Also

`supersharebybls`

## Purpose

Price swap instrument from Black-Derman-Toy interest-rate tree

## Syntax

```
[Price, PriceTree, CFTree, SwapRate] = swapbybdt(BDTTree,
LegRate, Settle, Maturity)
[Price, PriceTree, CFTree, SwapRate] = swapbybdt(BDTTree,
LegRate, Settle, Maturity, LegReset, Basis, Principal,
LegType, EndMonthRule)
[Price, PriceTree, CFTree, SwapRate] = swapbybdt(BDTTree,
LegRate, Settle, Maturity, Name,Value)
```

## Input Arguments

| | |
|---|---|
| `BDTTree` | Interest-rate tree structure created by `bdttree`. |
| `LegRate` | Number of instruments (`NINST`)-by-2 matrix, with each row defined as:<br>`[CouponRate Spread]` or `[Spread CouponRate]`<br>`CouponRate` is the decimal annual rate. `Spread` is the number of basis points over the reference rate. The first column represents the receiving leg, while the second column represents the paying leg. |
| `Settle` | Settlement date. `NINST`-by-`1` vector of serial date numbers or date strings representing the settlement date for each swap. `Settle` must be earlier than `Maturity`. |
| `Maturity` | Maturity date. `NINST`-by-`1` vector of dates representing the maturity date for each swap. |

The `Settle` date for every swap is set to the `ValuationDate` of the BDT tree. The swap argument `Settle` is ignored.

This function also calculates the `SwapRate` (fixed rate) so that the value of the swap is initially 0. To do this, enter `CouponRate` as `NaN`.

### Ordered Input or Name-Value Pair Arguments

Enter the following optional inputs using an ordered syntax or as name-value pair arguments. You cannot mix ordered syntax with name-value pair arguments.

#### LegReset

`NINST`-by-`2` matrix representing the reset frequency per year for each swap. `NINST`-by-`1` vector representing the frequency of payments per year.

**Default:** `[1 1]`

#### Basis

Day-count basis of the instrument. A vector of integers.

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (PSA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ISMA)
- 9 = actual/360 (ISMA)
- 10 = actual/365 (ISMA)
- 11 = 30/360E (ISMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see basis.

**Default:** `0` (actual/actual)

### Principal

`NINST`-by-`1` vector or `NINST`-by-`1` cell array of the notional principal amounts or principal value schedules. For the latter case, each element of the cell array is a `NumDates`-by-`2` call array where the first column is dates and the second column is its associated notional principal value. The date indicates the last day that the principal value is valid.

**Default:** `100`

### LegType

`NINST`-by-`2` matrix. Each row represents an instrument. Each column indicates if the corresponding leg is fixed (`1`) or floating (`0`). This matrix defines the interpretation of the values entered in `LegRate`.

**Default:** `[1 0]` for each instrument

### Options

Derivatives pricing options structure created with `derivset`.

### EndMonthRule

End-of-month rule. `NINST`-by-`1` vector. This rule applies only when `Maturity` is an end-of-month date for a month having 30 or fewer days.

- `0` = Ignore rule, meaning that a bond coupon payment date is always the same numerical day of the month.
- `1` = Set rule on, meaning that a bond coupon payment date is always the last actual day of the month.

**Default:** `1`

### Name-Value Pair Arguments

Specify optional comma-separated pairs of `Name,Value` arguments, where `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside single quotes (`' '`). You can specify several name and value pair arguments in any order as `Name1,Value1,...,NameN,ValueN`.

#### AdjustCashFlowsBasis

Adjust the cash flows based on the actual period day count. `NINST`-by-`1` of logicals.

**Default:** `false`

#### BusinessDayConvention

Require payment dates to be business dates. `NINST`-by-`1` cell array with possible choices of business day convention:

- `actual`
- `follow`
- `modifiedfollow`
- `previous`
- `modifiedprevious`

**Default:** `actual`

#### Holidays

Holidays used for business day convention. `NHOLIDAYS`-by-`1` of MATLAB date numbers.

**Default:** If no dates are specified, `holidays.m` is used.

#### StartDate

`NINST`-by-`1` vector of dates when the swap actually starts. Use this argument to price forward swaps, i.e., swaps that start in a future date

**Default:** `Settle` date

## Description

`[Price, PriceTree, CFTree, SwapRate] = swapbybdt(BDTTree, LegRate, Settle, Maturity)` computes the price of a swap instrument from a BDT interest-rate tree.

`[Price, PriceTree, CFTree, SwapRate] = swapbybdt(BDTTree, LegRate, Settle, Maturity, LegReset, Basis, Principal, LegType, EndMonthRule)` computes the price of a swap instrument from a BDT interest-rate tree using optional input arguments.

`[Price, PriceTree, CFTree, SwapRate] = swapbybdt(BDTTree, LegRate, Settle, Maturity, Name,Value)` computes the price of a swap instrument from a BDT interest-rate tree with additional options specified by one or more `Name,Value` pair arguments.

`Price` is number of instruments (`NINST`)-by-`1` expected prices of the swap at time 0.

`PriceTree` is a tree structure with a vector of the swap values at each node.

`CFTree` is a tree structure with a vector of the swap cash flows at each node. This structure contains only `NaN`s because with binomial recombining trees, cash flows cannot be computed accurately at each node of a tree.

`SwapRate` is a `NINST`-by-`1` vector of rates applicable to the fixed leg such that the swaps' values are zero at time 0. This rate is used in calculating the swaps' prices when the rate specified for the fixed leg in `LegRate` is `NaN`. `SwapRate` is padded with `NaN` for those instruments in which `CouponRate` is not set to `NaN`.

## Definitions

### Amortizing Swap

In an amortizing swap, the notional principal decreases periodically because it is tied to an underlying financial instrument with a declining (amortizing) principal balance, such as a mortgage.

### Forward Swap

Agreement to enter into an interest-rate swap arrangement on a fixed date in future.

## Examples

### Price Interest-Rate Swap

Price an interest-rate swap with a fixed receiving leg and a floating paying leg. Payments are made once a year, and the notional principal amount is $100. The values for the remaining arguments are:

- Coupon rate for fixed leg: 0.15 (15%)
- Spread for floating leg: 10 basis points
- Swap settlement date: Jan. 01, 2000
- Swap maturity date: Jan. 01, 2003

Based on the information above, set the required arguments and build the `LegRate`, `LegType`, and `LegReset` matrices:

```
Settle = '01-Jan-2000';
Maturity = '01-Jan-2003';
Basis = 0;
Principal = 100;
LegRate = [0.15 10]; % [CouponRate Spread]
LegType = [1 0]; % [Fixed Float]
LegReset = [1 1]; % Payments once per year
```

Price the swap using the `BDTTree` included in the MAT-file `deriv.mat`. `BDTTree` contains the time and forward-rate information needed to price the instrument.

```
load deriv.mat;
```

Use `swapbybdt` to compute the price of the swap.

```
Price  = swapbybdt(BDTTree, LegRate, Settle, Maturity,...
LegReset, Basis, Principal, LegType)

Price =

  7.4222
```

Using the previous data, calculate the swap rate, the coupon rate for the fixed leg, such that the swap price at time = 0 is zero.

```
LegRate = [NaN 20];

[Price, PriceTree, CFTree, SwapRate] = swapbybdt(BDTTree,...
LegRate, Settle, Maturity, LegReset, Basis, Principal, LegType)

Price =

 -1.4211e-014

PriceTree =

    FinObj: 'BDTPriceTree'
      tObs: [0 1 2 3 4]
     PTree: {1x5 cell}
CFTree =

    FinObj: 'BDTCFTree'
      tObs: [0 1 2 3 4]
    CFTree: {[NaN] [NaN NaN] [NaN NaN NaN] [NaN NaN NaN NaN] ...}

SwapRate =

    0.1205
```

### Price an Amortizing Swap

Price an amortizing swap using the `Principal` input argument to define the amortization schedule.

Create the `RateSpec`.

```
Rates = 0.035;
ValuationDate = '1-Jan-2011';
StartDates = ValuationDate;
EndDates = '1-Jan-2017';
Compounding = 1;

RateSpec = intenvset('ValuationDate', ValuationDate,'StartDates', StartDates,...
'EndDates', EndDates,'Rates', Rates, 'Compounding', Compounding)
```

```
RateSpec = 

           FinObj: 'RateSpec'
      Compounding: 1
             Disc: 0.8135
            Rates: 0.0350
         EndTimes: 6
       StartTimes: 0
         EndDates: 736696
       StartDates: 734504
    ValuationDate: 734504
            Basis: 0
     EndMonthRule: 1
```

Create the swap instrument using the following data:

```
Settle ='1-Jan-2011';
Maturity = '1-Jan-2017';
Period = 1;
LegRate = [0.04 10];
```

Define the swap amortizing schedule.

```
Principal ={{'1-Jan-2013' 100;'1-Jan-2014' 80;'1-Jan-2015' 60;'1-Jan-2016' 40; '1-Jan-2017' 20}};
```

Build the BDT tree and assume volatility is 10%.

```
MatDates = {'1-Jan-2012'; '1-Jan-2013';'1-Jan-2014';'1-Jan-2015';'1-Jan-2016';'1-Jan-2017'};
BDTTimeSpec = bdttimespec(ValuationDate, MatDates);
Volatility = 0.10;
BDTVolSpec = bdtvolspec(ValuationDate, MatDates, Volatility*ones(1,length(MatDates))');
BDTT = bdttree(BDTVolSpec, RateSpec, BDTTimeSpec);
```

Compute the price of the amortizing swap.

```
Price = swapbybdt(BDTT, LegRate, Settle, Maturity, 'Principal' , Principal)
```

```
Price =

    1.4574
```

### Price a Forward Swap

Price a forward swap using the `StartDate` input argument to define the future starting date of the swap.

Create the `RateSpec`.

```
Rates = 0.0325;
ValuationDate = '1-Jan-2012';
StartDates = ValuationDate;
EndDates = '1-Jan-2018';
Compounding = 1;

RateSpec = intenvset('ValuationDate', ValuationDate,'StartDates', StartDates,...
'EndDates', EndDates,'Rates', Rates, 'Compounding', Compounding)
```

```
RateSpec =

           FinObj: 'RateSpec'
      Compounding: 1
             Disc: 0.8254
```

```
           Rates: 0.0325
        EndTimes: 6
      StartTimes: 0
        EndDates: 737061
      StartDates: 734869
   ValuationDate: 734869
           Basis: 0
    EndMonthRule: 1
```

Build the tree with a volatility of 10%.

```
MatDates = {'1-Jan-2013'; '1-Jan-2014';'1-Jan-2015';'1-Jan-2016';'1-Jan-2017';'1-Jan-2018'};
BDTTimeSpec = bdttimespec(ValuationDate, MatDates);
Volatility = 0.10;
BDTVolSpec = bdtvolspec(ValuationDate, MatDates, Volatility*ones(1,length(MatDates))');
BDTT = bdttree(BDTVolSpec, RateSpec, BDTTimeSpec);
```

Compute the price of a forward swap that starts in two years (Jan 1, 2014) and matures in three years with a forward swap rate of 3.85%.

```
Settle ='1-Jan-2012';
Maturity = '1-Jan-2017';
StartDate = '1-Jan-2014';
LegRate = [0.0385 10];

Price = swapbybdt(BDTT, LegRate, Settle, Maturity, 'StartDate', StartDate)
```

```
Price =

    1.3203
```

Using the previous data, compute the forward swap rate, the coupon rate for the fixed leg, such that the forward swap price at time = 0 is zero.

```
LegRate = [NaN 10];
[Price, ~,~, SwapRate] = swapbybdt(BDTT, LegRate, Settle, Maturity, 'StartDate', StartDate)
```

```
Price =

    -7.1054e-14


SwapRate =

    0.0335
```

**See Also** `bdttree` | `capbybdt` | `cfbybdt` | `floorbybdt`

## Purpose

Price swap instrument from Black-Karasinski interest-rate tree

## Syntax

```
[Price, PriceTree, SwapRate] = swapbybk(BKTree,
LegRate, Settle, Maturity)
[Price, PriceTree, SwapRate] = swapbybk(BKTree,
LegRate, Settle, Maturity, LegReset, Basis, Principal,
LegType, EndMonthRule)
[Price, PriceTree, SwapRate] = swapbybk(BKTree,
LegRate, Settle, Maturity, Name,Value)
```

## Input Arguments

| | |
|---|---|
| `BKTree` | Interest-rate tree structure created by `bktree`. |
| `LegRate` | Number of instruments (`NINST`)-by-`2` matrix, with each row defined as:<br>`[CouponRate Spread]` or `[Spread CouponRate]`<br>`CouponRate` is the decimal annual rate. `Spread` is the number of basis points over the reference rate. The first column represents the receiving leg, while the second column represents the paying leg. |
| `Settle` | Settlement date. `NINST`-by-`1` vector of serial date numbers or date strings representing the settlement date for each swap. `Settle` must be earlier than `Maturity`. |
| `Maturity` | Maturity date. `NINST`-by-`1` vector of dates representing the maturity date for each swap. |

The `Settle` date for every swap is set to the `ValuationDate` of the BK tree. The swap argument `Settle` is ignored.

This function also calculates the `SwapRate` (fixed rate) so that the value of the swap is initially zero. To do this, enter `CouponRate` as `NaN`.

## Ordered Input or Name-Value Pair Arguments

Enter the following optional inputs using an ordered syntax or as name-value pair arguments. You cannot mix ordered syntax with name-value pair arguments.

### LegReset

`NINST`-by-`2` matrix representing the reset frequency per year for each swap. `NINST`-by-`1` vector representing the frequency of payments per year.

**Default:** `[1 1]`

### Basis

Day-count basis of the instrument. A vector of integers.

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (PSA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ISMA)
- 9 = actual/360 (ISMA)
- 10 = actual/365 (ISMA)
- 11 = 30/360E (ISMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see basis.

**Default:** `0` (actual/actual)

### Principal

`NINST`-by-`1` vector or `NINST`-by-`1` cell array of the notional principal amounts or principal value schedules. For the latter case, each element of the cell array is a `NumDates`-by-`2` call array where the first column is dates and the second column is its associated notional principal value. The date indicates the last day that the principal value is valid.

**Default:** `100`

### LegType

`NINST`-by-`2` matrix. Each row represents an instrument. Each column indicates if the corresponding leg is fixed (`1`) or floating (`0`). This matrix defines the interpretation of the values entered in `LegRate`.

**Default:** `[1 0]` for each instrument

### Options

Derivatives pricing options structure created with `derivset`.

### EndMonthRule

End-of-month rule. A `NINST`-by-`1` vector. This rule applies only when `Maturity` is an end-of-month date for a month having 30 or fewer days.

- `0` = Ignore rule, meaning that a bond coupon payment date is always the same numerical day of the month.
- `1` = Set rule on, meaning that a bond coupon payment date is always the last actual day of the month.

**Default:** `1`

### StartDate

`NINST`-by-1 vector of dates when the swap actually starts. Use this argument to price forward swaps, i.e., swaps that start in a future date

**Default:** `Settle` date

## Name-Value Pair Arguments

Specify optional comma-separated pairs of `Name,Value` arguments, where `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside single quotes (`' '`). You can specify several name and value pair arguments in any order as `Name1,Value1,...,NameN,ValueN`.

### AdjustCashFlowsBasis

Adjust the cash flows based on the actual period day count. `NINST`-by-1 of logicals.

**Default:** `false`

### BusinessDayConvention

Require payment dates to be business dates. `NINST`-by-1 cell array with possible choices of business day convention:

- `actual`
- `follow`
- `modifiedfollow`
- `previous`
- `modifiedprevious`

**Default:** `actual`

### Holidays

Holidays used for business day convention. `NHOLIDAYS`-by-1 of MATLAB date numbers.

**Default:** If no dates are specified, `holidays.m` is used.

## Description

`[Price, PriceTree, SwapRate] = swapbybk(BKTree, LegRate, Settle, Maturity)` computes the price of a swap instrument from a Black-Karasinski interest-rate tree.

`[Price, PriceTree, SwapRate] = swapbybk(BKTree, LegRate, Settle, Maturity, LegReset, Basis, Principal, LegType, EndMonthRule)` computes the price of a swap instrument from a Black-Karasinski interest-rate tree using optional input arguments.

`[Price, PriceTree, SwapRate] = swapbybk(BKTree, LegRate, Settle, Maturity, Name,Value)` computes the price of a swap instrument from a Black-Karasinski interest-rate tree with additional options specified by one or more `Name,Value` pair arguments.

`Price` is the number of instruments (`NINST`)-by-`1` expected prices of the swap at time 0.

`PriceTree` is the tree structure with a vector of the swap values at each node.

`SwapRate` is a `NINST`-by-`1` vector of rates applicable to the fixed leg such that the swaps' values are zero at time 0. This rate is used in calculating the swaps' prices when the rate specified for the fixed leg in `LegRate` is `NaN`. The `SwapRate` output is padded with `NaN` for those instruments in which `CouponRate` is not set to `NaN`.

## Definitions

### Amortizing Swap

In an amortizing swap, the notional principal decreases periodically because it is tied to an underlying financial instrument with a declining (amortizing) principal balance, such as a mortgage.

### Forward Swap

Agreement to enter into an interest-rate swap arrangement on a fixed date in future.

## Examples

### Price Interest-Rate Swap

Price an interest-rate swap with a fixed receiving leg and a floating paying leg. Payments are made once a year, and the notional principal amount is $100. The values for the remaining arguments are:

- Coupon rate for fixed leg: 0.15 (15%)
- Spread for floating leg: 10 basis points
- Swap settlement date: Jan. 01, 2005
- Swap maturity date: Jan. 01, 2008

Based on the information above, set the required arguments and build the `LegRate`, `LegType`, and `LegReset` matrices:

```
Settle = '01-Jan-2005';
Maturity = '01-Jan-2008';
Basis = 0;
Principal = 100;
LegRate = [0.15 10]; % [CouponRate Spread]
LegType = [1 0]; % [Fixed Float]
LegReset = [1 1]; % Payments once per year
```

Price the swap using the `BKTree` included in the MAT-file `deriv.mat`. The `BKTree` structure contains the time and forward-rate information needed to price the instrument.

```
load deriv.mat;
```

Use `swapbybk` to compute the price of the swap.

```
Price  = swapbybk(BKTree, LegRate, Settle, Maturity, LegReset,...
Basis, Principal, LegType)
```

```
Price =

  39.1827
```

Using the previous data, calculate the swap rate, which is the coupon rate for the fixed leg, such that the swap price at time = 0 is zero.

```
LegRate = [NaN 20];

[Price, PriceTree, SwapRate] = swapbybk(BKTree, LegRate, ...
Settle, Maturity, LegReset, Basis, Principal, LegType)
```

```
Price =

     0

PriceTree =

     FinObj: 'BKPriceTree'
       PTree: {1x5 cell}
        tObs: [0 1 2 3 4]
     Connect: {[2]  [2 3 4]  [2 2 3 4 4]}
      Probs: {[3x1 double]  [3x3 double]  [3x5 double]}
SwapRate =

    0.0438
```

### Price an Amortizing Swap

Price an amortizing swap using the `Principal` input argument to define the amortization schedule.

Create the `RateSpec`.

```
Rates = 0.035;
ValuationDate = '1-Jan-2011';
StartDates = ValuationDate;
EndDates = '1-Jan-2017';
Compounding = 1;

RateSpec = intenvset('ValuationDate', ValuationDate,'StartDates', StartDates,...
'EndDates', EndDates,'Rates', Rates, 'Compounding', Compounding)
```

```
RateSpec =

           FinObj: 'RateSpec'
      Compounding: 1
             Disc: 0.8135
            Rates: 0.0350
         EndTimes: 6
       StartTimes: 0
         EndDates: 736696
       StartDates: 734504
    ValuationDate: 734504
            Basis: 0
     EndMonthRule: 1
```

Create the swap instrument using the following data:

```
Settle ='1-Jan-2011';
Maturity = '1-Jan-2017';
Period = 1;
LegRate = [0.04 10];
```

Define the swap amortizing schedule.

```
Principal ={{'1-Jan-2013' 100;'1-Jan-2014' 80;'1-Jan-2015' 60;'1-Jan-2016' 40; '1-Jan-2017' 20}};
```

Build the BK tree and assume volatility is 10%.

```
MatDates = {'1-Jan-2012'; '1-Jan-2013';'1-Jan-2014';'1-Jan-2015';'1-Jan-2016';'1-Jan-2017'};
BKTimeSpec = bktimespec(ValuationDate, MatDates);
Volatility = 0.10;
AlphaDates = '01-01-2017';
AlphaCurve = 0.1;
BKVolSpec = bkvolspec(ValuationDate, MatDates, Volatility*ones(1,length(MatDates))',...
AlphaDates, AlphaCurve);
BKT = bktree(BKVolSpec, RateSpec, BKTimeSpec);
```

Compute the price of the amortizing swap.

```
Price = swapbybk(BKT, LegRate, Settle, Maturity, 'Principal' , Principal)
```

```
Price =

    1.4574
```

### Price a Forward Swap

Price a forward swap using the `StartDate` input argument to define the future starting date of the swap.

Create the `RateSpec`.

```
Rates = 0.0374;
ValuationDate = '1-Jan-2012';
StartDates = ValuationDate;
EndDates = '1-Jan-2018';
Compounding = 1;

RateSpec = intenvset('ValuationDate', ValuationDate,'StartDates', StartDates,...
'EndDates', EndDates,'Rates', Rates, 'Compounding', Compounding)
```

```
RateSpec =

           FinObj: 'RateSpec'
      Compounding: 1
             Disc: 0.8023
            Rates: 0.0374
         EndTimes: 6
       StartTimes: 0
         EndDates: 737061
       StartDates: 734869
    ValuationDate: 734869
            Basis: 0
     EndMonthRule: 1
```

Build a BK tree.

```
VolDates = {'1-Jan-2013'; '1-Jan-2014';'1-Jan-2015';'1-Jan-2016';'1-Jan-2017';'1-Jan-2018'};
VolCurve = 0.1;
AlphaDates = '01-01-2018';
```

```
AlphaCurve = 0.1;

BKVolSpec = bkvolspec(RateSpec.ValuationDate, VolDates, VolCurve,...
AlphaDates, AlphaCurve);
BKTimeSpec = bktimespec(RateSpec.ValuationDate, VolDates, Compounding);
BKT = bktree(BKVolSpec, RateSpec, BKTimeSpec);
```

Compute the price of a forward swap that starts in a year (Jan 1, 2013) and matures in four years with a forward swap rate of 4.25%.

```
Settle ='1-Jan-2012';
Maturity = '1-Jan-2017';
StartDate = '1-Jan-2013';
LegRate = [0.0425 10];

Price = swapbybk(BKT, LegRate, Settle, Maturity, 'StartDate', StartDate)
```

```
Price =

     1.4434
```

Using the previous data, compute the forward swap rate, the coupon rate for the fixed leg, such that the forward swap price at time = 0 is zero.

```
LegRate = [NaN 10];
[Price, ~,SwapRate] = swapbybk(BKT, LegRate, Settle, Maturity, 'StartDate', StartDate)
```

```
Price =

     1.4211e-14


SwapRate =

    0.0384
```

## See Also

bktree | bondbybk | capbybk | fixedbybk | floorbybk

## Purpose

Price swap instrument from Heath-Jarrow-Morton interest-rate tree

## Syntax

```
[Price, PriceTree, CFTree, SwapRate] = swapbyhjm(HJMTree,
LegRate, Settle, Maturity)
[Price, PriceTree, CFTree, SwapRate] = swapbyhjm(HJMTree,
LegRate, Settle, Maturity, LegReset, Basis, Principal,
LegType, EndMonthRule)
[Price, PriceTree, CFTree, SwapRate] = swapbyhjm(HJMTree,
LegRate, Settle, Maturity, Name,Value)
```

## Input Arguments

| | |
|---|---|
| `HJMTree` | Forward-rate tree structure created by `hjmtree`. |
| `LegRate` | Number of instruments (`NINST`)-by-`2` matrix, with each row defined as:<br>`[CouponRate Spread]` or `[Spread CouponRate]`<br>`CouponRate` is the decimal annual rate. `Spread` is the number of basis points over the reference rate. The first column represents the receiving leg, while the second column represents the paying leg. |
| `Settle` | Settlement date. `NINST`-by-`1` vector of serial date numbers or date strings representing the settlement date for each swap. `Settle` must be earlier than `Maturity`. |
| `Maturity` | Maturity date. `NINST`-by-`1` vector of dates representing the maturity date for each swap. |

The `Settle` date for every swap is set to the `ValuationDate` of the HJM tree. The swap argument `Settle` is ignored.

This function also calculates the `SwapRate` (fixed rate) so that the value of the swap is initially zero. To do this, enter `CouponRate` as `NaN`.

### Ordered Input or Name-Value Pair Arguments

Enter the following optional inputs using an ordered syntax or as name-value pair arguments. You cannot mix ordered syntax with name-value pair arguments.

#### LegReset

`NINST`-by-`2` matrix representing the reset frequency per year for each swap. `NINST`-by-`1` vector representing the frequency of payments per year.

**Default:** `[1 1]`

#### Basis

Day-count basis of the instrument. A vector of integers.

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (PSA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ISMA)
- 9 = actual/360 (ISMA)
- 10 = actual/365 (ISMA)
- 11 = 30/360E (ISMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see basis.

**Default:** `0` (actual/actual)

### Principal

`NINST`-by-`1` vector or `NINST`-by-`1` cell array of the notional principal amounts or principal value schedules. For the latter case, each element of the cell array is a `NumDates`-by-`2` call array where the first column is dates and the second column is its associated notional principal value. The date indicates the last day that the principal value is valid.

**Default:** `100`

### LegType

`NINST`-by-`2` matrix. Each row represents an instrument. Each column indicates if the corresponding leg is fixed (`1`) or floating (`0`). This matrix defines the interpretation of the values entered in `LegRate`.

**Default:** `[1 0]` for each instrument

### Options

Derivatives pricing options structure created with `derivset`.

### EndMonthRule

End-of-month rule. A `NINST`-by-`1` vector. This rule applies only when `Maturity` is an end-of-month date for a month having 30 or fewer days.

- `0` = Ignore rule, meaning that a bond coupon payment date is always the same numerical day of the month.
- `1` = Set rule on, meaning that a bond coupon payment date is always the last actual day of the month.

**Default:** `1`

### Name-Value Pair Arguments

Specify optional comma-separated pairs of `Name,Value` arguments, where `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside single quotes (`' '`). You can specify several name and value pair arguments in any order as `Name1,Value1,...,NameN,ValueN`.

#### AdjustCashFlowsBasis

Adjust the cash flows based on the actual period day count. `NINST`-by-`1` of logicals.

**Default:** `false`

#### BusinessDayConvention

Require payment dates to be business dates. `NINST`-by-`1` cell array with possible choices of business day convention:

- `actual`
- `follow`
- `modifiedfollow`
- `previous`
- `modifiedprevious`

**Default:** `actual`

#### Holidays

Holidays used for business day convention. `NHOLIDAYS`-by-`1` of MATLAB date numbers.

**Default:** If no dates are specified, `holidays.m` is used.

#### StartDate

`NINST`-by-`1` vector of dates when the swap actually starts. Use this argument to price forward swaps, i.e., swaps that start in a future date

**Default:** `Settle` date

## Description

`[Price, PriceTree, CFTree, SwapRate] = swapbyhjm(HJMTree, LegRate, Settle, Maturity)` computes the price of a swap instrument from an HJM interest-rate tree.

`[Price, PriceTree, CFTree, SwapRate] = swapbyhjm(HJMTree, LegRate, Settle, Maturity, LegReset, Basis, Principal, LegType, EndMonthRule)` computes the price of a swap instrument from an HJM interest-rate tree with optional input arguments.

`[Price, PriceTree, CFTree, SwapRate] = swapbyhjm(HJMTree, LegRate, Settle, Maturity, Name,Value)` computes the price of a swap instrument from an HJM interest-rate tree with additional options specified by one or more `Name,Value` pair arguments.

`Price` is the number of instruments (`NINST`-by-`1`) expected prices of the swap at time 0.

`PriceTree` is the tree structure with a vector of the swap values at each node.

`CFTree` is the tree structure with a vector of the swap cash flows at each node.

`SwapRate` is a `NINST`-by-`1` vector of rates applicable to the fixed leg such that the swaps' values are zero at time 0. This rate is used in calculating the swaps' prices when the rate specified for the fixed leg in `LegRate` is `NaN`. The `SwapRate` output is padded with `NaN` for those instruments in which `CouponRate` is not set to `NaN`.

## Definitions

### Amortizing Swap

In an amortizing swap, the notional principal decreases periodically because it is tied to an underlying financial instrument with a declining (amortizing) principal balance, such as a mortgage.

### Forward Swap

Agreement to enter into an interest-rate swap arrangement on a fixed date in future.

## Examples

### Price an Interest-Rate Swap

Price an interest-rate swap with a fixed receiving leg and a floating paying leg. Payments are made once a year, and the notional principal amount is $100. The values for the remaining arguments are:

- Coupon rate for fixed leg: 0.06 (6%)
- Spread for floating leg: 20 basis points
- Swap settlement date: Jan. 01, 2000
- Swap maturity date: Jan. 01, 2003

Based on the information above, set the required arguments and build the `LegRate`, `LegType`, and `LegReset` matrices:

```
Settle = '01-Jan-2000';
Maturity = '01-Jan-2003';
Basis = 0;
Principal = 100;
LegRate = [0.06 20]; % [CouponRate Spread]
LegType = [1 0]; % [Fixed Float]
LegReset = [1 1]; % Payments once per year
```

Price the swap using the `HJMTree` included in the MAT-file `deriv.mat`. The `HJMTree` structure contains the time and forward-rate information needed to price the instrument.

```
load deriv.mat;
```

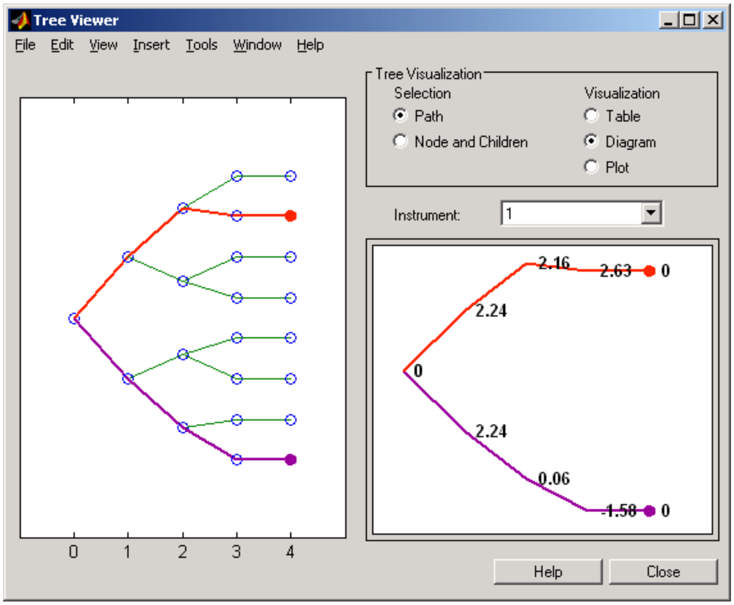
Use `swapbyhjm` to compute the price of the swap.

```
[Price, PriceTree, CFTree] = swapbyhjm(HJMTree, LegRate,...
Settle, Maturity, LegReset, Basis, Principal, LegType)
```

```
Price =

   3.6923

PriceTree =

    FinObj: 'HJMPriceTree'
      tObs: [0 1 2 3 4]
     PBush: {1x5 cell}

CFTree =

    FinObj: 'HJMCFTree'
      tObs: [0 1 2 3 4]
    CFBush: {[0] [1x1x2 double] [1x2x2 double] ... [1x8 double]}
```

Use `treeviewer` to examine `CFTree` graphically and see the cash flows from the swap along both the up and the down branches. A positive cash flow indicates an inflow (income - payments > 0), while a negative cash flow indicates an outflow (income - payments < 0).

```
treeviewer(CFTree)
```



Tree Visualization

Selection

● Path

○ Node and Children

Visualization

○ Table

● Diagram

○ Plot

Instrument: 1

---

**Note** `treeviewer` price tree diagrams follow the convention that increasing prices appear on the upper branch of a tree and, consequently, decreasing prices appear on the lower branch. Conversely, for interest-rate displays, *decreasing* interest rates appear on the upper branch (prices are rising) and *increasing* interest rates on the lower branch (prices are falling).

---

In this example, you have sold a swap (receive fixed rate and pay floating rate). At time `t = 3`, if interest rates go down, your cash flow is positive ($2.63), meaning that you will receive this amount. But if interest rates go up, your cash flow is negative (-$1.58), meaning that you owe this amount.

Using the previous data, calculate the swap rate, which is the coupon rate for the fixed leg, such that the swap price at time = 0 is zero.

```
LegRate = [NaN 20];

[Price, PriceTree, CFTree, SwapRate] = swapbyhjm(HJMTree,...
LegRate, Settle, Maturity, LegReset, Basis, Principal, LegType)

Price =

   0

PriceTree =

FinObj: 'HJMPriceTree'
  tObs: [0 1 2 3 4]
 PBush:{[0] [1x1x2 double] [1x2x2 double] ... [1x8 double]}

CFTree =

FinObj: 'HJMCFTree'
  tObs: [0 1 2 3 4]
CFBush:{[0] [1x1x2 double] [1x2x2 double] ... [1x8 double]}

SwapRate =

   0.0466
```

### Price an Amortizing Swap

Price an amortizing swap using the `Principal` input argument to define the amortization schedule.

Create the `RateSpec`.

```
Rates = 0.035;
ValuationDate = '1-Jan-2011';
StartDates = ValuationDate;
EndDates = '1-Jan-2017';
Compounding = 1;
```

```
RateSpec = intenvset('ValuationDate', ValuationDate,'StartDates', StartDates,...
'EndDates', EndDates,'Rates', Rates, 'Compounding', Compounding)
```

```
RateSpec =

           FinObj: 'RateSpec'
      Compounding: 1
             Disc: 0.8135
            Rates: 0.0350
         EndTimes: 6
       StartTimes: 0
         EndDates: 736696
       StartDates: 734504
    ValuationDate: 734504
            Basis: 0
     EndMonthRule: 1
```

Create the swap instrument using the following data:

```
Settle ='1-Jan-2011';
Maturity = '1-Jan-2017';
Period = 1;
LegRate = [0.04 10];
```

Define the swap amortizing schedule.

```
Principal ={{'1-Jan-2013' 100;'1-Jan-2014' 80;'1-Jan-2015' 60;'1-Jan-2016' 40; '1-Jan-2017' 20}};
```

Build the HJM tree using the following data:

```
MatDates = {'1-Jan-2012'; '1-Jan-2013';'1-Jan-2014';'1-Jan-2015';'1-Jan-2016';'1-Jan-2017'};
HJMTimeSpec = hjmtimespec(RateSpec.ValuationDate, MatDates);
Volatility = [.10; .08; .06; .04];
CurveTerm = [ 1; 2; 3; 4];
HJMVolSpec = hjmvolspec('Proportional', Volatility, CurveTerm, 1e6);
HJMT = hjmtree(HJMVolSpec,RateSpec,HJMTimeSpec);
```

Compute the price of the amortizing swap.

```
Price = swapbyhjm(HJMT, LegRate, Settle, Maturity, 'Principal', Principal)
```

```
Price =

    1.4574
```

## Price a Forward Swap

Price a forward swap using the `StartDate` input argument to define the future starting date of the swap.

Create the `RateSpec`.

```
Rates = 0.0374;
ValuationDate = '1-Jan-2012';
StartDates = ValuationDate;
EndDates = '1-Jan-2018';
Compounding = 1;

RateSpec = intenvset('ValuationDate', ValuationDate,'StartDates', StartDates,...
'EndDates', EndDates,'Rates', Rates, 'Compounding', Compounding)
```

```
RateSpec =

           FinObj: 'RateSpec'
      Compounding: 1
             Disc: 0.8023
            Rates: 0.0374
         EndTimes: 6
       StartTimes: 0
         EndDates: 737061
       StartDates: 734869
    ValuationDate: 734869
            Basis: 0
     EndMonthRule: 1
```

Build an HJM tree.

```
MatDates = {'1-Jan-2013'; '1-Jan-2014';'1-Jan-2015';'1-Jan-2016';'1-Jan-2017';'1-Jan-2018'};
HJMTimeSpec = hjmtimespec(RateSpec.ValuationDate, MatDates);
Volatility = [.10; .08; .06; .04];
CurveTerm = [ 1; 2; 3; 4];
HJMVolSpec = hjmvolspec('Proportional', Volatility, CurveTerm, 1e6);
HJMT = hjmtree(HJMVolSpec,RateSpec,HJMTimeSpec);
```

Compute the price of a forward swap that starts in a year (Jan 1, 2013) and matures in four years with a forward swap rate of 4.25%.

```
Settle ='1-Jan-2012';
Maturity = '1-Jan-2017';
StartDate = '1-Jan-2013';
LegRate = [0.0425 10];

Price = swapbyhjm(HJMT, LegRate, Settle, Maturity, 'StartDate', StartDate
```

```
Price =

     1.4434
```

Using the previous data, compute the forward swap rate, the coupon rate for the fixed leg, such that the forward swap price at time = 0 is zero.

```
LegRate = [NaN 10];
[Price, ~,~, SwapRate] = swapbyhjm(HJMT, LegRate, Settle, Maturity, 'StartDate', StartDate)
```

```
Price =

     0


SwapRate =

    0.0384
```

## See Also

`capbyhjm` | `cfbyhjm` | `floorbyhjm` | `hjmtree`

## Purpose

Price swap instrument from Hull-White interest-rate tree

## Syntax

```
[Price, PriceTree, SwapRate] = swapbyhw(HWTree,
LegRate, Settle, Maturity)
[Price, PriceTree, SwapRate] = swapbyhw(HWTree,
LegRate, Settle, Maturity, LegReset, Basis,
Principal, LegType, Options, EndMonthRule)
[Price, PriceTree, SwapRate] = swapbyhw(HWTree,
LegRate, Settle, Maturity, Name,Value)
```

## Input Arguments

| | |
|---|---|
| `HWTree` | Forward-rate tree structure created by `hwtree`. |
| `LegRate` | Number of instruments (`NINST`)-by-`2` matrix, with each row defined as:<br>`[CouponRate Spread]` or `[Spread CouponRate]`<br>`CouponRate` is the decimal annual rate. `Spread` is the number of basis points over the reference rate. The first column represents the receiving leg, while the second column represents the paying leg. |
| `Settle` | Settlement date. `NINST`-by-`1` vector of serial date numbers or date strings representing the settlement date for each swap. `Settle` must be earlier than `Maturity`. |
| `Maturity` | Maturity date. `NINST`-by-`1` vector of dates representing the maturity date for each swap. |

The `Settle` date for every swap is set to the `ValuationDate` of the HW tree. The swap argument `Settle` is ignored.

This function also calculates the `SwapRate` (fixed rate) so that the value of the swap is initially zero. To do this, enter `CouponRate` as `NaN`.

### Ordered Input or Name-Value Pair Arguments

Enter the following optional inputs using an ordered syntax or as name-value pair arguments. You cannot mix ordered syntax with name-value pair arguments.

#### LegReset

`NINST`-by-`2` matrix representing the reset frequency per year for each swap. `NINST`-by-`1` vector representing the frequency of payments per year.

**Default:** `[1 1]`

#### Basis

Day-count basis of the instrument. A vector of integers.

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (PSA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ISMA)
- 9 = actual/360 (ISMA)
- 10 = actual/365 (ISMA)
- 11 = 30/360E (ISMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see basis.

**Default:** `0` (actual/actual)

### Principal

`NINST`-by-`1` vector or `NINST`-by-`1` cell array of the notional principal amounts or principal value schedules. For the latter case, each element of the cell array is a `NumDates`-by-`2` call array where the first column is dates and the second column is its associated notional principal value. The date indicates the last day that the principal value is valid.

**Default:** `100`

### LegType

`NINST`-by-`2` matrix. Each row represents an instrument. Each column indicates if the corresponding leg is fixed (`1`) or floating (`0`). This matrix defines the interpretation of the values entered in `LegRate`.

**Default:** `[1 0]` for each instrument

### Options

Derivatives pricing options structure created with `derivset`.

### EndMonthRule

End-of-month rule. `NINST`-by-`1` vector. This rule applies only when `Maturity` is an end-of-month date for a month having 30 or fewer days.

- `0` = Ignore rule, meaning that a bond coupon payment date is always the same numerical day of the month.
- `1` = Set rule on, meaning that a bond coupon payment date is always the last actual day of the month.

**Default:** `1`

### Name-Value Pair Arguments

Specify optional comma-separated pairs of `Name,Value` arguments, where `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside single quotes (`' '`). You can specify several name and value pair arguments in any order as `Name1,Value1,...,NameN,ValueN`.

#### AdjustCashFlowsBasis

Adjust the cash flows based on the actual period day count. `NINST`-by-`1` of logicals.

**Default:** `false`

#### BusinessDayConvention

Require payment dates to be business dates. `NINST`-by-`1` cell array with possible choices of business day convention:

- `actual`
- `follow`
- `modifiedfollow`
- `previous`
- `modifiedprevious`

**Default:** `actual`

#### Holidays

Holidays used for business day convention. `NHOLIDAYS`-by-`1` of MATLAB date numbers.

**Default:** If no dates are specified, `holidays.m` is used.

#### StartDate

`NINST`-by-`1` vector of dates when the swap actually starts. Use this argument to price forward swaps, i.e., swaps that start in a future date

**Default:** `Settle` date

## Description

`[Price, PriceTree, SwapRate] = swapbyhw(HWTree, LegRate, Settle, Maturity)` computes the price of a swap instrument from a Hull-White interest-rate tree.

`[Price, PriceTree, SwapRate] = swapbyhw(HWTree, LegRate, Settle, Maturity, LegReset, Basis, Principal, LegType, Options, EndMonthRule)` computes the price of a swap instrument from a Hull-White interest-rate tree with optional input arguments.

`[Price, PriceTree, SwapRate] = swapbyhw(HWTree, LegRate, Settle, Maturity, Name,Value)` computes the price of a swap instrument from a Hull-White interest-rate tree with additional options specified by one or more `Name,Value` pair arguments.

`Price` is number of instruments (`NINST`)-by-`1` expected prices of the swap at time 0.

`PriceTree` is the tree structure with a vector of the swap values at each node.

`SwapRate` is a `NINST`-by-`1` vector of rates applicable to the fixed leg such that the swaps' values are zero at time 0. This rate is used in calculating the swaps' prices when the rate specified for the fixed leg in `LegRate` is `NaN`. The `SwapRate` output is padded with `NaN`s for those instruments in which `CouponRate` is not set to `NaN`.

## Definitions

### Amortizing Swap

In an amortizing swap, the notional principal decreases periodically because it is tied to an underlying financial instrument with a declining (amortizing) principal balance, such as a mortgage.

### Forward Swap

Agreement to enter into an interest-rate swap arrangement on a fixed date in future.

## Examples

### Price an Interest-Rate Swap

Price an interest-rate swap with a fixed receiving leg and a floating paying leg. Payments are made once a year, and the notional principal amount is $100. The values for the remaining arguments are:

- Coupon rate for fixed leg: 0.06 (6%)
- Spread for floating leg: 20 basis points
- Swap settlement date: Jan. 01, 2005
- Swap maturity date: Jan. 01, 2008

Based on the information above, set the required arguments and build the `LegRate`, `LegType`, and `LegReset` matrices:

```
Settle = '01-Jan-2005';
Maturity = '01-Jan-2008';
Basis = 0;
Principal = 100;
LegRate = [0.06 20]; % [CouponRate Spread]
LegType = [1 0]; % [Fixed Float]
LegReset = [1 1]; % Payments once per year
```

Price the swap using the `HWTree` included in the MAT-file `deriv.mat`. The `HWTree` structure contains the time and forward-rate information needed to price the instrument.

```
load deriv.mat;
```

Use `swapbyhw` to compute the price of the swap.

```
[Price, PriceTree, SwapRate] = swapbyhw(HWTree, LegRate, ...
Settle, Maturity, LegReset, Basis, Principal, LegType)
```

```
Price =

    5.9109

PriceTree =

     FinObj: 'HWPriceTree'
      PTree: {1x5 cell}
       tObs: [0 1 2 3 4]
    Connect: {[2]  [2 3 4]  [2 2 3 4 4]}
      Probs: {[3x1 double]  [3x3 double]  [3x5 double]}

SwapRate =

   NaN
```

Using the previous data, calculate the swap rate, which is the coupon rate for the fixed leg, such that the swap price at time = 0 is zero.

```
LegRate = [NaN 20];

[Price, PriceTree, SwapRate] = swapbyhw(HWTree, LegRate, ...
Settle, Maturity, LegReset, Basis, Principal, LegType)

Price =

   1.4211e-014

PriceTree =

FinObj: 'HWPriceTree'
       PTree: {1x5 cell}
        tObs: [0 1 2 3 4]
     Connect: {[2]  [2 3 4]  [2 2 3 4 4]}
      Probs: {[3x1 double]  [3x3 double]  [3x5 double]}

SwapRate =
```

```
   0.0438
```

### Price an Amortizing Swap

Price an amortizing swap using the `Principal` input argument to define the amortization schedule.

Create the `RateSpec`.

```
Rates = 0.035;
ValuationDate = '1-Jan-2011';
StartDates = ValuationDate;
EndDates = '1-Jan-2017';
Compounding = 1;

RateSpec = intenvset('ValuationDate', ValuationDate,'StartDates', StartDates,...
'EndDates', EndDates,'Rates', Rates, 'Compounding', Compounding)
```

```
RateSpec = 

           FinObj: 'RateSpec'
      Compounding: 1
             Disc: 0.8135
            Rates: 0.0350
         EndTimes: 6
       StartTimes: 0
         EndDates: 736696
       StartDates: 734504
    ValuationDate: 734504
            Basis: 0
     EndMonthRule: 1
```

Create the swap instrument using the following data:

```
Settle ='1-Jan-2011';
Maturity = '1-Jan-2017';
Period = 1;
```

```
LegRate = [0.04 10];
```

Define the swap amortizing schedule.

```
Principal ={{'1-Jan-2013' 100;'1-Jan-2014' 80;'1-Jan-2015' 60;'1-Jan-2016' 40; '1-Jan-2017' 20}};
```

Build the HW tree using the following data:

```
VolDates = ['1-Jan-2012'; '1-Jan-2013';'1-Jan-2014';'1-Jan-2015';'1-Jan-2016';'1-Jan-2017'];
VolCurve = 0.1;
AlphaDates = '01-01-2017';
AlphaCurve = 0.1;

HWVolSpec = hwvolspec(RateSpec.ValuationDate, VolDates, VolCurve,...
AlphaDates, AlphaCurve);
HWTimeSpec = hwtimespec(RateSpec.ValuationDate, VolDates, Compounding);
HWT = hwtree(HWVolSpec, RateSpec, HWTimeSpec);
```

Compute the price of the amortizing swap.

```
Price = swapbyhw(HWT, LegRate, Settle, Maturity, 'Principal', Principal)
```

```
Price =

    1.4574
```

### Price a Forward Swap

Price a forward swap using the `StartDate` input argument to define the future starting date of the swap.

Create the `RateSpec`.

```
Rates = 0.0374;
ValuationDate = '1-Jan-2012';
StartDates = ValuationDate;
EndDates = '1-Jan-2018';
Compounding = 1;
```

```
RateSpec = intenvset('ValuationDate', ValuationDate,'StartDates', StartDates,...
'EndDates', EndDates,'Rates', Rates, 'Compounding', Compounding)
```

```
RateSpec =

           FinObj: 'RateSpec'
      Compounding: 1
             Disc: 0.8023
            Rates: 0.0374
         EndTimes: 6
       StartTimes: 0
         EndDates: 737061
       StartDates: 734869
    ValuationDate: 734869
            Basis: 0
     EndMonthRule: 1
```

Build an HW tree.

```
VolDates = {'1-Jan-2013'; '1-Jan-2014';'1-Jan-2015';'1-Jan-2016';'1-Jan-2017';'1-Jan-2018'};
VolCurve = 0.1;
AlphaDates = '01-01-2018';
AlphaCurve = 0.1;

HWVolSpec = hwvolspec(RateSpec.ValuationDate, VolDates, VolCurve,...
AlphaDates, AlphaCurve);
HWTimeSpec = hwtimespec(RateSpec.ValuationDate, VolDates, Compounding);
HWT = hwtree(HWVolSpec, RateSpec, HWTimeSpec);
```

Compute the price of a forward swap that starts in a year (Jan 1, 2013) and matures in four years with a forward swap rate of 4.25%.

```
Settle ='1-Jan-2012';
Maturity = '1-Jan-2017';
StartDate = '1-Jan-2013';
LegRate = [0.0425 10];

Price = swapbyhw(HWT, LegRate, Settle, Maturity, 'StartDate', StartDate)
```

```
Price =

     1.4434
```

Using the previous data, compute the forward swap rate, the coupon rate for the fixed leg, such that the forward swap price at time = 0 is zero.

```
LegRate = [NaN 10];
[Price, ~,SwapRate] = swapbyhw(HWT, LegRate, Settle, Maturity, 'StartDate', StartDate)
```

```
Price =

     1.4211e-14


SwapRate =

    0.0384
```

## See Also

`bondbyhw` | `capbyhw` | `cfbyhw` | `floorbyhw` | `fixedbyhw` | `hwtree`

# swapbyzero

## Purpose

Price swap instrument from set of zero curves

## Syntax

```
[Price, SwapRate AI, RecCF, RecCFDates, PayCF, PayCFDates] =
swapbyzero(RateSpec, LegRate, Settle, Maturity)
[Price, SwapRate AI, RecCF, RecCFDates, PayCF, PayCFDates] =
swapbyzero(RateSpec, LegRate, Settle, Maturity,
LegReset, Basis,  Principal, LegType, EndMonthRule)
[Price, SwapRate, AI, RecCF, RecCFDates, PayCF, PayCFDates] =
swapbyzero(RateSpec, LegRate, Settle, Maturity,
Name, Value)
```

## Description

`[Price, SwapRate AI, RecCF, RecCFDates, PayCF, PayCFDates] = swapbyzero(RateSpec, LegRate, Settle, Maturity)` prices a swap instrument from a set of zero coupon bond rates. All inputs are either scalars or `NINST`-by-1 vectors unless otherwise specified. Any date can be a serial date number or date string. An optional argument can be passed as an empty matrix `[]`.

`[Price, SwapRate AI, RecCF, RecCFDates, PayCF, PayCFDates] = swapbyzero(RateSpec, LegRate, Settle, Maturity, LegReset, Basis, Principal, LegType, EndMonthRule)` prices a swap instrument from a set of zero coupon bond rates with optional input arguments. All inputs are either scalars or `NINST`-by-1 vectors unless otherwise specified. Any date can be a serial date number or date string. An optional argument can be passed as an empty matrix `[]`.

`[Price, SwapRate, AI, RecCF, RecCFDates, PayCF, PayCFDates] = swapbyzero(RateSpec, LegRate, Settle, Maturity, Name, Value)` prices a swap instrument from a set of zero coupon bond rates with additional options specified by one or more `Name, Value` pair arguments.

## Input Arguments

**RateSpec**

Structure containing the properties of an interest-rate structure. See `intenvset` for information on creating `RateSpec`.

`RateSpec` can also be a `1`-by-`2` input variable of `RateSpecs`, with the second `RateSpec` structure containing the discount curve(s) for the paying leg. If only one `RateSpec` structure is specified, then this `RateSpec` is used to discount both legs.

**LegRate**

Number of instruments (`NINST`)-by-`2` matrix, with each row defined as:

`[CouponRate Spread]` or `[Spread CouponRate]`

`CouponRate` is the decimal annual rate. `Spread` is the number of basis points over the reference rate. The first column represents the receiving leg, while the second column represents the paying leg.

**Settle**

Settlement date. `Settle` must be either a scalar or `NINST`-by-`1` vector of serial date numbers or date strings of the same value which represent the settlement date for each swap. `Settle` must be earlier than `Maturity`.

**Maturity**

Maturity date. `NINST`-by-`1` vector of dates representing the maturity date for each swap.

### Ordered Input or Name-Value Pair Arguments

Enter the following optional inputs using an ordered syntax or as name-value pair arguments. You cannot mix ordered syntax with name-value pair arguments.

**LegReset**

`NINST`-by-`2` matrix representing the reset frequency per year for each swap. `NINST`-by-`1` vector representing the frequency of payments per year.

**Default:** `[1 1]`

#### Basis

Day-count basis of the instrument. A vector of integers.

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (PSA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ISMA)
- 9 = actual/360 (ISMA)
- 10 = actual/365 (ISMA)
- 11 = 30/360E (ISMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see basis.

**Default:** `0` (actual/actual)

#### Principal

`NINST`-by-`1` vector or `NINST`-by-`1` cell array of the notional principal amounts or principal value schedules. For the latter case, each element of the cell array is a `NumDates`-by-`2` call array where the first column is dates and the second column is its associated notional principal value. The date indicates the last day that the principal value is valid.

**Default:** `100`

#### LegType

`NINST`-by-`2` matrix. Each row represents an instrument. Each column indicates if the corresponding leg is fixed (`1`) or floating (`0`). This matrix defines the interpretation of the values entered in `LegRate`.

**Default:** `[1 0]` for each instrument

#### Options

Derivatives pricing options structure created with `derivset`.

#### EndMonthRule

End-of-month rule. `NINST`-by-`1` vector. This rule applies only when `Maturity` is an end-of-month date for a month having 30 or fewer days.

- `0` = Ignore rule, meaning that a bond coupon payment date is always the same numerical day of the month.
- `1` = Set rule on, meaning that a bond coupon payment date is always the last actual day of the month.

**Default:** `1`

### Name-Value Pair Arguments

Specify optional comma-separated pairs of `Name,Value` arguments, where `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside single quotes (`' '`). You can specify several name and value pair arguments in any order as `Name1,Value1,...,NameN,ValueN`.

#### AdjustCashFlowsBasis

Adjust the cash flows based on the actual period day count. `NINST`-by-`1` of logicals.

> **Default:** `false`

### BusinessDayConvention

Require payment dates to be business dates. `NINST`-by-`1` cell array with possible choices of business day convention:

- `actual`
- `follow`
- `modifiedfollow`
- `previous`
- `modifiedprevious`

> **Default:** `actual`

### ForwardRateSpec

Forward rate spec to use in generating cash flows.

> **Default:** If not specified, then the `RateSpec` is used both for discounting cash flows and generating floating cash flows.

### Holidays

Holidays used for business day convention. `NHOLIDAYS`-by-`1` of MATLAB date numbers.

> **Default:** If none specified, `holidays.m` is used.

### LatestFloatingRate

Rate for the next floating payment, set at the last reset date. `NINST`-by-`1` of scalars.

**Default:** If not specified, then the `RateSpec` must contain this information.

### StartDate

`NINST`-by-`1` vector of dates when the swap actually starts. Use this argument to price forward swaps, i.e., swaps that start in a future date

**Default:** `Settle` date

## Output Arguments

### Price

Number of instruments (`NINST`) by number of curves (`NUMCURVES`) matrix of swap prices. Each column arises from one of the zero curves.

### SwapRate

`NINST`-by-`NUMCURVES` matrix of rates applicable to the fixed leg such that the swap's values are zero at time 0. This rate is used in calculating the swaps' prices when the rate specified for the fixed leg in `LegRate` is `NaN`. The `SwapRate` output is padded with `NaN` for those instruments in which `CouponRate` is not set to `NaN`.

Output cash flows, cash flow dates, and accrued interest.

### AI

`NINST`-by-`NUMCURVES` matrix of accrued interest.

### RecCF

`NINST`-by-`NUMCURVES` matrix of cash flows for the receiving leg.

---

**Note** If there is more than one curve specified in the `RateSpec` input, then the first `NCURVES` row corresponds to the first swap, the second `NCURVES` row correspond to the second swap, and so on.

---

**RecCFDates**

`NINST`-by-`NUMCURVES` matrix of payment dates for the receiving leg.

**PayCF**

`NINST`-by-`NUMCURVES` matrix of cash flows for the paying leg.

**PayCFDates**

`NINST`-by-`NUMCURVES` matrix of payment dates for the paying leg.

## Definitions

### Amortizing Swap

In an amortizing swap, the notional principal decreases periodically because it is tied to an underlying financial instrument with a declining (amortizing) principal balance, such as a mortgage.

### Forward Swap

Agreement to enter into an interest-rate swap arrangement on a fixed date in future.

## Examples

### Price an Interest-Rate Swap

Price an interest-rate swap with a fixed receiving leg and a floating paying leg. Payments are made once a year, and the notional principal amount is $100. The values for the remaining arguments are:

- Coupon rate for fixed leg: 0.06 (6%)
- Spread for floating leg: 20 basis points
- Swap settlement date: Jan. 01, 2000
- Swap maturity date: Jan. 01, 2003

Based on the information above, set the required arguments and build the `LegRate`, `LegType`, and `LegReset` matrices:

```
Settle = '01-Jan-2000';
Maturity = '01-Jan-2003';
Basis = 0;
```

```
Principal = 100;
LegRate = [0.06 20]; % [CouponRate Spread]
LegType = [1 0]; % [Fixed Float]
LegReset = [1 1]; % Payments once per year
```

Load the file `deriv.mat`, which provides `ZeroRateSpec`, the interest-rate term structure needed to price the bond.

```
load deriv.mat;
```

Use `swapbyzero` to compute the price of the swap.

```
Price = swapbyzero(ZeroRateSpec, LegRate, Settle, Maturity,...
LegReset, Basis, Principal, LegType)

Price =
   3.6923
```

Using the previous data, calculate the swap rate, which is the coupon rate for the fixed leg, such that the swap price at time = 0 is zero.

```
LegRate = [NaN 20];

[Price, SwapRate] = swapbyzero(ZeroRateSpec, LegRate, Settle,...
Maturity, LegReset, Basis, Principal, LegType)

Price =

 -1.4211e-014

SwapRate =
   0.0466
```

Use `swapbyzero` with name-value pair arguments for `LegRate`, `LegType`, `LatestFloatingRate`, `AdjustCashFlowsBasis`, and

`BusinessDayConvention` to calculate output for `Price`, `SwapRate`, `AI`, `RecCF`, `RecCFDates`, `PayCF`, and `PayCFDates`:

```
Settle = datenum('08-Jun-2010');
RateSpec = intenvset('Rates', [.005 .0075 .01 .014 .02 .025 .03]',...
'StartDates',Settle, 'EndDates',{'08-Dec-2010','08-Jun-2011',...
'08-Jun-2012','08-Jun-2013','08-Jun-2015','08-Jun-2017','08-Jun-2020'}');
Maturity = datenum('15-Sep-2020');
LegRate = [.025 50];
LegType = [1 0]; % fixed/floating
LatestFloatingRate = .005;

[Price, SwapRate, AI, RecCF, RecCFDates, PayCF,PayCFDates] = ...
swapbyzero(RateSpec, LegRate, Settle, Maturity,'LegType',LegType,...
'LatestFloatingRate',LatestFloatingRate,'AdjustCashFlowsBasis',true,...
'BusinessDayConvention','modifiedfollow')
```

```
Price =

   -3.3937

SwapRate =

   NaN

AI =

    1.4575


RecCF =

Columns 1 through 10

-1.8219    1.2603    1.2603    1.2740    1.2671   1.2466    1.2534    1.2603   1.2603  1.2740

Columns 11 through 12
```

```
1.2671  101.2534


RecCFDates =

Columns 1 through 8

734297      734396      734761     735129     735493      735857    736222   736588

Columns 9 through 12

736953      737320      737684      738049


PayCF =

Columns 1 through 10

-0.3644    0.2521    0.7082    1.0116    1.4423    1.6380   1.9161    2.1038  2.2768  2.2766

Columns 11 through 12

2.4370  102.3432


PayCFDates =

Columns 1 through 8

734297      734396      734761      735129      735493      735857    736222     736588

Columns 9 through 12

736953      737320      737684      738049
```

### Price Swaps By Specifying Multiple Term Structures Using RateSpec

Price three swaps using two interest-rate curves.

Define data for the interest-rate term structure.

```
StartDates = '01-May-2012';
EndDates = {'01-May-2013'; '01-May-2014';'01-May-2015';'01-May-2016'};
Rates = [[0.0356;0.041185;0.04489;0.047741],[0.0366;0.04218;0.04589;0.04974]];
```

Create the `RateSpec` using `intenvset`.

```
RateSpec = intenvset('Rates', Rates, 'StartDates',StartDates,...
'EndDates', EndDates, 'Compounding', 1)
```

```
RateSpec =

           FinObj: 'RateSpec'
      Compounding: 1
             Disc: [4x2 double]
            Rates: [4x2 double]
         EndTimes: [4x1 double]
       StartTimes: [4x1 double]
         EndDates: [4x1 double]
       StartDates: 734990
    ValuationDate: 734990
            Basis: 0
     EndMonthRule: 1
```

Look at the `Rates` for the two interest-rate curves.

```
RateSpec.Rates
```

```
ans =

    0.0356    0.0366
    0.0412    0.0422
    0.0449    0.0459
```

```
    0.0477    0.0497
```

Define the swap instruments.

```
Settle = '01-May-2012';
Maturity = '01-May-2015';
LegRate = [0.06 10];
Principal = [100;50;100];  % Three notional amounts
```

Price three swaps using two curves.

```
Price = swapbyzero(RateSpec, LegRate, Settle, Maturity, 'Principal', Principal)
```

```
Price =

    3.9688    3.6869
    1.9844    1.8434
    3.9688    3.6869
```

### Price an Amortizing Swap

Price an amortizing swap using the `Principal` input argument to define the amortization schedule.

Create the `RateSpec`.

```
Rates = 0.035;
ValuationDate = '1-Jan-2011';
StartDates = ValuationDate;
EndDates = '1-Jan-2017';
Compounding = 1;

RateSpec = intenvset('ValuationDate', ValuationDate,'StartDates', StartDates,...
'EndDates', EndDates,'Rates', Rates, 'Compounding', Compounding);
```

```
RateSpec =

          FinObj: 'RateSpec'
     Compounding: 1
```

```
                Disc: 0.8135
               Rates: 0.0350
            EndTimes: 6
          StartTimes: 0
            EndDates: 736696
          StartDates: 734504
       ValuationDate: 734504
               Basis: 0
        EndMonthRule: 1
```

Create the swap instrument using the following data:

```
Settle ='1-Jan-2011';
Maturity = '1-Jan-2017';
Period = 1;
LegRate = [0.04 10];
```

Define the swap amortizing schedule.

```
Principal ={{'1-Jan-2013' 100;'1-Jan-2014' 80;'1-Jan-2015' 60;'1-Jan-2016' 40; '1-Jan-2017' 20}};
```

Compute the price of the amortizing swap.

```
Price = swapbyzero(RateSpec, LegRate, Settle, Maturity, 'Principal' , Principal)
```

```
Price =

    1.4574
```

### Price a Forward Swap

Price a forward swap using the `StartDate` input argument to define the future starting date of the swap.

Create the `RateSpec`.

```
Rates = 0.0325;
ValuationDate = '1-Jan-2012';
StartDates = ValuationDate;
EndDates = '1-Jan-2018';
```

```
Compounding = 1;

RateSpec = intenvset('ValuationDate', ValuationDate,'StartDates', StartDates,...
'EndDates', EndDates,'Rates', Rates, 'Compounding', Compounding)
```

```
RateSpec =

           FinObj: 'RateSpec'
      Compounding: 1
             Disc: 0.8254
            Rates: 0.0325
         EndTimes: 6
       StartTimes: 0
         EndDates: 737061
       StartDates: 734869
    ValuationDate: 734869
            Basis: 0
     EndMonthRule: 1
```

Compute the price of a forward swap that starts in a year (Jan 1, 2013) and matures in three years with a forward swap rate of 4.27%.

```
Settle ='1-Jan-2012';
StartDate = '1-Jan-2013';
Maturity = '1-Jan-2016';
LegRate = [0.0427 10];

Price = swapbyzero(RateSpec, LegRate, Settle, Maturity, 'StartDate' , StartDate)
```

```
Price =

     2.5083
```

Using the previous data, compute the forward swap rate, the coupon rate for the fixed leg, such that the forward swap price at time = 0 is zero.

```
LegRate = [NaN 10];
```

```
[Price, SwapRate] = swapbyzero(RateSpec, LegRate, Settle, Maturity,...
'StartDate' , StartDate)
```

```
Price =

     0


SwapRate =

    0.0335
```

## See Also

| `bondbyzero` | `cfbyzero` | `fixedbyzero` | `floatbyzero`

## Purpose

Price swaption from Black-Derman-Toy interest-rate tree

## Syntax

```
[Price, PriceTree] = swaptionbybdt(BDTTree, OptSpec, Strike,
ExerciseDates, Spread, Settle, Maturity,
'Name1', Value1, 'Name2', Value2)
```

## Arguments

| | |
|---|---|
| `BDTTree` | Interest-rate tree structure created by `bdttree`. |
| `OptSpec` | `NINST`-by-`1` cell array of strings `'call'` or `'put'`. A call swaption entitles the buyer to pay the fixed rate. A put swaption entitles the buyer to receive the fixed rate. |
| `Strike` | `NINST`-by-`1` vector for strike swap rate values. |
| `ExerciseDates` | For a European option: `NINST`-by-`1` vector of exercise dates. Each row is the schedule for one option. For a European option, there is only one `ExerciseDate` on the option expiry date.<br>For an American option: `NINST`-by-`2` vector of exercise date boundaries. For each instrument, the option can be exercised on any coupon date between or including the pair of dates on that row. If only one non-`NaN` date is listed, or if `ExerciseDates` is `NINST`-by-`1`, the option can be exercised between the underlying swap `Settle` and the single listed `ExerciseDate`. |
| `Spread` | `NINST`-by-`1` vector representing the number of basis points over the reference rate. |
| `Settle` | `NINST`-by-`1` vector of dates representing the settle date for each swap. |

| | |
|---|---|
| `Maturity` | `NINST`-by-`1` vector of dates representing the maturity date for each swap. |

**Note** All optional inputs that follow are specified as matching parameter name/value pairs. The parameter name is specified as a character string, followed by the corresponding parameter value. Parameter name/value pairs may be specified in any order; names are case-insensitive and partial string matches are allowed provided no ambiguities exist.

| | |
|---|---|
| `AmericanOpt` | (Optional) `NINST`-by-`1` flags options:<br>• 0 for European options<br>• 1 for American options |
| `SwapReset` | (Optional) `NINST`-by-`1` vector representing the reset frequency per year for the underlying swap. Default is 1. |
| `Basis` | (Optional) Day-count basis of the instrument. A vector of integers.<br>• 0 = actual/actual (default)<br>• 1 = 30/360 (SIA)<br>• 2 = actual/360<br>• 3 = actual/365<br>• 4 = 30/360 (BMA)<br>• 5 = 30/360 (ISDA)<br>• 6 = 30/360 (European)<br>• 7 = actual/365 (Japanese)<br>• 8 = actual/actual (ICMA)<br>• 9 = actual/360 (ICMA) |

| | |
|---|---|
| | - 10 = actual/365 (ICMA)<br>- 11 = 30/360E (ICMA)<br>- 12 = actual/actual (ISDA)<br>- 13 = BUS/252<br>For more information, see basis. |
| `Principal` | (Optional) `NINST`-by-`1` vector of the notional principal amounts. Default is 100. |
| `Options` | (Optional) Derivatives pricing options structure created with `derivset`. |

## Description

`[Price, PriceTree] = swaptionbybdt(BDTTree, OptSpec, Strike,`
`ExerciseDates, Spread, Settle, Maturity,`
`'Name1', Value1, 'Name2', Value2)` computes the price of a swaption from a BDT interest-rate tree. The swaption may be a call swaption or a put swaption.

---

**Note** The `Settle` date for every swaption is set to the `ValuationDate` of the BDT tree. The swap argument `Settle` is ignored.

---

A call swaption or payer swaption allows the option buyer to enter into an interest rate swap in which the buyer of the option pays the fixed rate and receives the floating rate.

A put swaption or receiver swaption allows the option buyer to enter into an interest rate swap in which the buyer of the option receives the fixed rate and pays the floating rate.

`Price` is a `NINST`-by-`1` vector of expected swaption prices at time 0.

`PriceTree` is a MATLAB structure of trees containing vectors of swaption instrument prices and a vector of observation times for each node. Within `PriceTree`:

- `PriceTree.PTree` contains the clean prices.
- `PriceTree.tObs` contains the observation times.

## Examples

Price a 5-year call swaption using a BDT interest-rate tree.

Assume that interest rate and volatility are fixed at 6% and 20% annually between the valuation date of the tree until its maturity. Build a tree with the following data:

```
Rates = 0.06 * ones (10,1);
StartDates = ['jan-1-2007';'jan-1-2008';'jan-1-2009';'jan-1-2010';'jan-1-2011';...
'jan-1-2012';'jan-1-2013';'jan-1-2014';'jan-1-2015';'jan-1-2016'];

EndDates =['jan-1-2008';'jan-1-2009';'jan-1-2010';'jan-1-2011';'jan-1-2012';...
'jan-1-2013';'jan-1-2014';'jan-1-2015';'jan-1-2016';'jan-1-2017'];
ValuationDate = 'jan-1-2007';
Compounding = 1;
```

Determine the `RateSpec`:

```
RateSpec = intenvset('Rates', Rates, 'StartDates', StartDates, 'EndDates', EndDates, ...
'Compounding', Compounding);
```

Use `VolSpec` to compute the interest rate volatility:

```
Volatility = 0.20 * ones (10,1);  VolSpec = bdtvolspec(ValuationDate,...
EndDates, Volatility);
```

Use `TimeSpec` to specify the structure of the time layout for an equal probabilities tree:

```
TimeSpec = bdttimespec(ValuationDate, EndDates, Compounding);
```

Build the BDT tree:

```
BDTTree = bdttree(VolSpec, RateSpec, TimeSpec);
```

Use the following swaption arguments:

```
SwapSettlement = 'jan-1-2007';
SwapMaturity   = 'jan-1-2015';
Spread = 0;
SwapReset = 1;
Principal = 100;
OptSpec = 'call';
Strike=.062;
ExerciseDates = 'jan-1-2012';
Basis=1;
```

Price the swaption

```
[Price, PriceTree] = swaptionbybdt(BDTTree, OptSpec, Strike, ExerciseDates, ...
Spread, SwapSettlement, SwapMaturity, 'SwapReset', SwapReset, ...
'Basis', Basis, 'Principal', Principal)
```

to return

```
Price =        2.0592

PriceTree =
FinObj: 'BDTPriceTree'
tObs: [0 1 2 3 4 5 6 7 8 9 10]
PTree: {1x11 cell}
```

## See Also

`bdttree` | `instswaption` | `swapbybdt`

# swaptionbybk

**Purpose** Price swaption from Black-Karasinski interest-rate tree

**Syntax**

```
[Price, PriceTree] = swaptionbybk(BKTTree, OptSpec, Strike,
ExerciseDates, Spread, Settle, Maturity,
'Name1', Value1, 'Name2', Value2)
```

**Arguments**

| | |
|---|---|
| `BKTree` | Interest-rate tree structure created by `bktree`. |
| `OptSpec` | `NINST`-by-`1` cell array of strings `'call'` or `'put'`. A call swaption entitles the buyer to pay the fixed rate. A put swaption entitles the buyer to receive the fixed rate. |
| `Strike` | `NINST`-by-`1` vector for strike swap rate values. |
| `ExerciseDates` | For a European option: `NINST`-by-`1` vector of exercise dates. Each row is the schedule for one option. For a European option, there is only one `ExerciseDate` on the option expiry date.<br>For an American option: `NINST`-by-`2` vector of exercise date boundaries. For each instrument, the option can be exercised on any coupon date between or including the pair of dates on that row. If only one non-`NaN` date is listed, or if `ExerciseDates` is `NINST`-by-`1`, the option can be exercised between the underlying swap `Settle` and the single listed `ExerciseDate`. |
| `Spread` | `NINST`-by-`1` vector representing the number of basis points over the reference rate. |
| `Settle` | `NINST`-by-`1` vector of dates representing the settle date for each swap. |

| | |
|---|---|
| `Maturity` | `NINST`-by-`1` vector of dates representing the maturity date for each swap. |

---

**Note** All optional inputs that follow are specified as matching parameter name/value pairs. The parameter name is specified as a character string, followed by the corresponding parameter value. Parameter name/value pairs may be specified in any order; names are case-insensitive and partial string matches are allowed provided no ambiguities exist.

---

| | |
|---|---|
| `AmericanOpt` | (Optional) `NINST`-by-`1` flags options:<br>• 0 for European options<br>• 1 for American options |
| `SwapReset` | (Optional) `NINST`-by-`1` vector representing the reset frequency per year for the underlying swap. Default is 1. |
| `Basis` | (Optional) Day-count basis of the instrument. A vector of integers.<br>• 0 = actual/actual (default)<br>• 1 = 30/360 (SIA)<br>• 2 = actual/360<br>• 3 = actual/365<br>• 4 = 30/360 (BMA)<br>• 5 = 30/360 (ISDA)<br>• 6 = 30/360 (European)<br>• 7 = actual/365 (Japanese)<br>• 8 = actual/actual (ICMA)<br>• 9 = actual/360 (ICMA) |

- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/actual (ISDA)
- 13 = BUS/252

For more information, see basis.

| | |
|---|---|
| `Principal` | (Optional) `NINST`-by-`1` vector of the notional principal amounts. Default is 100. |
| `Options` | (Optional) Derivatives pricing options structure created with `derivset`. |

## Description

`[Price, PriceTree] = swaptionbybk(BKTTree, OptSpec, Strike, ExerciseDates, Spread, Settle, Maturity, 'Name1', Value1, 'Name2', Value2)` computes the price of a swaption from a BK interest-rate tree.

---

**Note** The `Settle` date for every swaption is set to the `ValuationDate` of the BK tree. The swap argument `Settle` is ignored.

---

The swaption may be a call swaption or a put swaption.

A call swaption or payer swaption allows the option buyer to enter into an interest rate swap in which the buyer of the option pays the fixed rate and receives the floating rate.

A put swaption or receiver swaption allows the option buyer to enter into an interest rate swap in which the buyer of the option receives the fixed rate and pays the floating rate.

`Price` is a `NINST`-by-`1` vector of expected swaption prices at time 0.

`PriceTree` is a MATLAB structure of trees containing vectors of swaption instrument prices and a vector of observation times for each node. Within `PriceTree`:

- `PriceTree.PTree` contains the clean prices.
- `PriceTree.tObs` contains the observation times.

## Examples

Price a 4-year call and put swaption using a BK interest-rate tree with the following data.

Specify the `RateSpec`, assuming the interest rate is fixed at 7% annually:

```
 Rates =0.07 * ones (10,1);
Compounding = 2;
StartDates = ['jan-1-2007';'jul-1-2007';'jan-1-2008';'jul-1-2008';'jan-1-2009'; ...
'jul-1-2009'; 'jan-1-2010'; 'jul-1-2010';'jan-1-2011';'jul-1-2011'];
EndDates =['jul-1-2007';'jan-1-2008';'jul-1-2008';'jan-1-2009';'jul-1-2009'; ...
'jan-1-2010'; 'jul-1-2010';'jan-1-2011';'jul-1-2011';'jan-1-2012'];
ValuationDate = 'jan-1-2007';
RateSpec = intenvset('Rates', Rates, 'StartDates', StartDates, 'EndDates', EndDates,...
'Compounding', Compounding);
```

Use `BKVolSpec` to compute the interest rate volatility:

```
Volatility = 0.10*ones(10,1);
AlphaCurve = 0.05*ones(10,1);
AlphaDates = EndDates;
BKVolSpec = bkvolspec(ValuationDate, EndDates, Volatility, AlphaDates, AlphaCurve);
```

Use `BKTimeSpec` to specify the structure of the time layout for the BK interest-rate tree.

```
BKTimeSpec = bktimespec(ValuationDate, EndDates, Compounding);
```

Build the BK tree:

```
BKTree = bktree(BKVolSpec, RateSpec, BKTimeSpec);
```

Use the following arguments for a 5-year swap and 4-year swaption:

```
SwapSettlement = 'jan-1-2007';
SwapMaturity   = 'jan-1-2012';
Spread = 0;
SwapReset = 2 ;
Principal = 100;
OptSpec = {'call' ;'put'};
Strike= [ 0.07 ; 0.0725];
ExerciseDates = 'jan-1-2011';
Basis=1;
```

Price the swaption

```
PriceSwaption = swaptionbybk(BKTree, OptSpec, Strike, ExerciseDates, ...
Spread, SwapSettlement, SwapMaturity, 'SwapReset', SwapReset, 'Basis', Basis, ...
'Principal', Principal)
```

to return

```
PriceSwaption =
0.3593
0.4756
```

## See Also

`bktree` | `instswaption` | `swapbybk`

## Purpose

Price European swaption instrument using Black model

## Syntax

```
Price = swaptionbyblk(RateSpec, OptSpec, Strike,
Settle, ExerciseDates, Maturity, Volatility)
Price = swaptionbyblk(RateSpec, OptSpec, Strike, Settle,
ExerciseDates, Maturity, Volatility, Name, Value)
```

## Description

`Price = swaptionbyblk(RateSpec, OptSpec, Strike, Settle, ExerciseDates, Maturity, Volatility)` prices swaptions using the Black option pricing model.

`Price = swaptionbyblk(RateSpec, OptSpec, Strike, Settle, ExerciseDates, Maturity, Volatility, Name, Value)` prices swaptions using the Black option pricing model with additional options specified by one or more `Name,Value` pair arguments.

---

**Note** When pricing swaptions using the Black model, the underlying is not a regular swap, but a forward on a swap.

---

## Input Arguments

### RateSpec

Structure containing the properties of an interest-rate structure. See `intenvset` for information on creating `RateSpec`.

`RateSpec` can be a `NINST`-by-`2` input variable of `RateSpec`s, with the second input being the discount curve for the paying leg if different than the receiving leg. If only one curve is specified, than it is used to discount both legs.

### OptSpec

`NINST`-by-`1` cell array of strings `'call'` or `'put'`. A call swaption entitles the buyer to pay the fixed rate. A put swaption entitles the buyer to receive the fixed rate.

### Strike

`NINST`-by-`1` vector of strike swap rate values.

**Settle**

Settlement date. `NINST`-by-`1` vector of serial date numbers or date strings representing the settlement date for each swap. `Settle` must be earlier than `Maturity`.

**ExerciseDate**

For a European option: `NINST`-by-`1` vector of exercise dates. Each row is the schedule for one option. When using an European option, there is only one `ExerciseDate` on the option expiry date.

**Maturity**

Maturity date. `NINST`-by-`1` vector of dates representing the maturity date for each swap.

**Volatility**

Volatilities values. `NINST`-by-`1` vector of volatilities.

## Name-Value Pair Arguments

Specify optional comma-separated pairs of `Name,Value` arguments, where `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside single quotes (`' '`). You can specify several name and value pair arguments in any order as `Name1,Value1,...,NameN,ValueN`.

**Basis**

Day-count basis of the instrument. A vector of integers.

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365

- 4 = 30/360 (PSA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ISMA)
- 9 = actual/360 (ISMA)
- 10 = actual/365 (ISMA)
- 11 = 30/360E (ISMA)
- 12 = actual/365 (ISDA)
- 13 = BUS/252

For more information, see basis.

**Default:** `0` (actual/actual)

### Principal

`NINST`-by-`1` vector of the notional principal amount.

**Default:** `100`

### SwapReset

`NINST`-by-`1` vector representing the reset frequency per year for the underlying forward swap.

**Default:** `1`

## Output Arguments

### Price

`NINST`-by-`1` vector of prices for the swaptions at time 0.

## Definitions

### Forward Swap

A forward swap is a swap that starts at a future date.

## Examples

### Price a European Swaption Using the Black Model Where the Yield Curve is Flat at 6%

Price a European swaption that gives the holder the right to enter in five years into a three-year paying swap where a fixed-rate of 6.2% is paid and floating is received. Assume that the yield curve is flat at 6% per annum with continuous compounding, the volatility of the swap rate is 20%, the principal is \$100, and payments are exchanged semiannually.

Create the `RateSpec`.

```
Rate = 0.06;
Compounding  = -1;
ValuationDate = 'Jan-1-2010';
EndDates =   'Jan-1-2020';
Basis = 1;

RateSpec = intenvset('ValuationDate', ValuationDate,'StartDates', ValuationDate, ...
'EndDates', EndDates, 'Rates', Rate, 'Compounding', Compounding, 'Basis', Basis);
```

```
RateSpec =

           FinObj: 'RateSpec'
      Compounding: -1
             Disc: 0.5488
            Rates: 0.0600
         EndTimes: 10
       StartTimes: 0
         EndDates: 737791
       StartDates: 734139
    ValuationDate: 734139
            Basis: 1
     EndMonthRule: 1
```

Price the swaption using the Black model.

```
Settle = 'Jan-1-2011';
ExerciseDates = 'Jan-1-2016';
```

```
Maturity = 'Jan-1-2019';
Reset = 2;
Principal = 100;
Strike = 0.062;
Volatility = 0.2;
OptSpec = 'call';

Price= swaptionbyblk(RateSpec, OptSpec, Strike, Settle, ExerciseDates, Maturity, ...
Volatility, 'Reset', Reset, 'Principal', Principal, 'Basis', Basis)
```

```
Price =

    2.0710
```

### Price a European Swaption Using the Black Model Where the Yield Curve Is Incrementally Increasing

Price a European swaption that gives the holder the right to enter into a 5-year receiving swap in a year, where a fixed rate of 3% is received and floating is paid. Assume that the 1-year, 2-year, 3-year, 4-year and 5- year zero rates are 3%, 3.4%, 3.7%, 3.9% and 4% with continuous compounding. The swap rate volatility is 21%, the principal is $1000, and payments are exchanged semiannually.

Create the `RateSpec`.

```
ValuationDate = 'Jan-1-2010';
EndDates = {'Jan-1-2011';'Jan-1-2012';'Jan-1-2013';'Jan-1-2014';'Jan-1-2015'};
Rates = [0.03; 0.034 ; 0.037; 0.039; 0.04;];
Compounding = -1;
Basis = 1;

RateSpec = intenvset('ValuationDate', ValuationDate, 'StartDates', ValuationDate, ...
'EndDates', EndDates, 'Rates', Rates, 'Compounding', Compounding,'Basis', Basis)
```

```
RateSpec =

           FinObj: 'RateSpec'
      Compounding: -1
```

```
             Disc: [5x1 double]
            Rates: [5x1 double]
         EndTimes: [5x1 double]
       StartTimes: [5x1 double]
         EndDates: [5x1 double]
       StartDates: 734139
    ValuationDate: 734139
            Basis: 1
     EndMonthRule: 1
```

Price the swaption using the Black model.

```
Settle = 'Jan-1-2011';
ExerciseDates = 'Jan-1-2012';
Maturity = 'Jan-1-2017';
Strike = 0.03;
Volatility = 0.21;
Principal =1000;
Reset = 2;
OptSpec = 'put';

Price = swaptionbyblk(RateSpec, OptSpec, Strike, Settle,  ExerciseDates, ...
Maturity, Volatility,'Basis', Basis, 'Reset', Reset,'Principal', Principal)
```

```
Price =

    0.5903
```

## See Also

| `bondbyzero` | `cfbyzero` | `fixedbyzero` | `floatbyzero`

# swaptionbyhjm

**Purpose** Price swaption from Heath-Jarrow-Morton interest-rate tree

**Syntax**

```
[Price, PriceTree] = swaptionbyhjm(HJMTree, OptSpec, Strike,
ExerciseDates, Spread, Settle, Maturity,
'Name1', Value1, 'Name2', Value2)
```

## Arguments

| | |
|---|---|
| `HJMTree` | Interest-rate tree structure created by `hjmtree`. |
| `OptSpec` | `NINST`-by-`1` cell array of strings `'call'` or `'put'`. A call swaption entitles the buyer to pay the fixed rate. A put swaption entitles the buyer to receive the fixed rate. |
| `Strike` | `NINST`-by-`1` vector of strike swap rate values. |
| `ExerciseDates` | For a European option: `NINST`-by-`1` vector of exercise dates. Each row is the schedule for one option. For a European option, there is only one `ExerciseDate` on the option expiry date.<br>For an American option: `NINST`-by-`2` vector of exercise date boundaries. For each instrument, the option can be exercised on any coupon date between or including the pair of dates on that row. If only one non-`NaN` date is listed, or if `ExerciseDates` is `NINST`-by-`1`, the option can be exercised between the underlying swap `Settle` and the single listed `ExerciseDate`. |
| `Spread` | `NINST`-by-`1` vector representing the number of basis points over the reference rate. |
| `Settle` | `NINST`-by-`1` vector of dates representing the settle date for each swap. |

| | |
|---|---|
| `Maturity` | `NINST`-by-`1` vector of dates representing the maturity date for each swap. |

---

**Note** All optional inputs that follow are specified as matching parameter name/value pairs. The parameter name is specified as a character string, followed by the corresponding parameter value. Parameter name/value pairs may be specified in any order; names are case-insensitive and partial string matches are allowed provided no ambiguities exist.

---

| | |
|---|---|
| `AmericanOpt` | (Optional) `NINST`-by-`1` flags options:<br>• 0 for European options<br>• 1 for American options |
| `SwapReset` | (Optional) `NINST`-by-`1` vector representing the reset frequency per year for the underlying swap. Default is 1. |
| `Basis` | (Optional) Day-count basis of the instrument. A vector of integers.<br>• 0 = actual/actual (default)<br>• 1 = 30/360 (SIA)<br>• 2 = actual/360<br>• 3 = actual/365<br>• 4 = 30/360 (BMA)<br>• 5 = 30/360 (ISDA)<br>• 6 = 30/360 (European)<br>• 7 = actual/365 (Japanese)<br>• 8 = actual/actual (ICMA)<br>• 9 = actual/360 (ICMA) |

| | |
|---|---|
| | • 10 = actual/365 (ICMA)<br>• 11 = 30/360E (ICMA)<br>• 12 = actual/actual (ISDA)<br>• 13 = BUS/252<br>For more information, see basis. |
| `Principal` | (Optional) `NINST-by-1` vector of the notional principal amounts. Default is 100. |
| `Options` | (Optional) Derivatives pricing options structure created with `derivset`. |

## Description

`[Price, PriceTree] = swaptionbyhjm(HJMTree, OptSpec, Strike, ExerciseDates, Spread, Settle, Maturity, 'Name1', Value1, 'Name2', Value2)` computes the price of a swaption from a HJM interest-rate tree.

---

**Note** The `Settle` date for every swaption is set to the `ValuationDate` of the HJM tree. The swap argument `Settle` is ignored.

---

The swaption may be a call swaption or a put swaption.

A call swaption or payer swaption allows the option buyer to enter into an interest rate swap in which the buyer of the option pays the fixed rate and receives the floating rate.

A put swaption or receiver swaption allows the option buyer to enter into an interest rate swap in which the buyer of the option receives the fixed rate and pays the floating rate.

`Price` is a (`NINST-by-1` vector of expected swaption prices at time 0.

`PriceTree` is a MATLAB structure of trees containing vectors of swaption instrument prices and a vector of observation times for each node. Within `PriceTree`:

- `PriceTree.PTree` contains the clean prices.
- `PriceTree.tObs` contains the observation times.

## Examples

Price a 1-year call swaption using an HJM interest-rate tree.

Assume that interest rate is fixed at 5% annually between the valuation date of the tree until its maturity. Build a tree with the following data.

Specify the `RateSpec`:

```
Rates = [ 0.05;0.05;0.05;0.05];
StartDates = 'jan-1-2007';
EndDates =['jan-1-2008';'jan-1-2009';'jan-1-2010';'jan-1-2011'];
ValuationDate = StartDates;
Compounding = 1;
RateSpec = intenvset('Rates', Rates, 'StartDates', StartDates, 'EndDates',...
EndDates, 'Compounding', Compounding);
```

Use `VolSpec` to compute the interest rate volatility:

```
VolSpec=hjmvolspec('Constant',0.01);
```

Use `TimeSpec` to specify the structure of the time layout for the HJM interest-rate tree:

```
 TimeSpec = hjmtimespec(ValuationDate, EndDates, Compounding);
```

Build the HJM tree:

```
HJMTree = hjmtree(VolSpec, RateSpec, TimeSpec);
```

Use the following swaption arguments:

```
SwapSettlement = 'jan-1-2007';
SwapMaturity   = 'jan-1-2010';
Spread = [0];
```

```
SwapReset = 1;
Basis  = 1;
Principal = 100;
OptSpec = 'call';
Strike=0.05;
ExerciseDates = '01-Jan-2008';
```

Price the swaption

```
[Price, PriceTree] = swaptionbyhjm(HJMTree, OptSpec, Strike, ExerciseDates, ...
Spread, SwapSettlement, SwapMaturity,'SwapReset', SwapReset, ...
'Basis', Basis, 'Principal', Principal)
```

to return

```
Price =

    0.9296

PriceTree =

    FinObj: 'HJMPriceTree'
    tObs: [5x1 double]
    PBush: {[0.9296]  [1x1x2 double]  [1x2x2 double]  [1x4x2 double]  [0 0 0 0 0 0 0 0]}
```

## See Also

`hjmtree` | `instswaption` | `swapbyhjm`

# swaptionbyhw

## Purpose

Price swaption from Hull-White interest-rate tree

## Syntax

```
[Price, PriceTree] = swaptionbyhw(HWTree, OptSpec, Strike,
ExerciseDates, Spread, Settle, Maturity,
'Name1', Value1, 'Name2', Value2)
```

## Arguments

| | |
|---|---|
| `HWTree` | Interest-rate tree structure created by `hwtree`. |
| `OptSpec` | `NINST`-by-`1` cell array of strings `'call'` or `'put'`. A call swaption entitles the buyer to pay the fixed rate. A put swaption entitles the buyer to receive the fixed rate. |
| `Strike` | `NINST`-by-`1` vector for strike swap rate values. |
| `ExerciseDates` | For a European option: `NINST`-by-`1` vector of exercise dates. Each row is the schedule for one option. For a European option, there is only one `ExerciseDate` on the option expiry date.<br>For an American option: `NINST`-by-`2` vector of exercise date boundaries. For each instrument, the option can be exercised on any coupon date between or including the pair of dates on that row. If only one non-`NaN` date is listed, or if `ExerciseDates` is `NINST`-by-`1`, the option can be exercised between the underlying swap `Settle` and the single listed `ExerciseDate`. |
| `Spread` | `NINST`-by-`1` vector representing the number of basis points over the reference rate. |
| `Settle` | `NINST`-by-`1` vector of dates representing the settle date for each swap. |

| | |
|---|---|
| `Maturity` | `NINST`-by-`1` vector of dates representing the maturity date for each swap. |

---

**Note** All optional inputs that follow are specified as matching parameter name/value pairs. The parameter name is specified as a character string, followed by the corresponding parameter value. Parameter name/value pairs may be specified in any order; names are case-insensitive and partial string matches are allowed provided no ambiguities exist.

---

| | |
|---|---|
| `AmericanOpt` | (Optional) `NINST`-by-`1` flags options:<br>• 0 for European options<br>• 1 for American options |
| `SwapReset` | (Optional) `NINST`-by-`1` vector representing the reset frequency per year for the underlying swap. Default is 1. |
| `Basis` | (Optional) Day-count basis of the instrument. A vector of integers.<br>• 0 = actual/actual (default)<br>• 1 = 30/360 (SIA)<br>• 2 = actual/360<br>• 3 = actual/365<br>• 4 = 30/360 (BMA)<br>• 5 = 30/360 (ISDA)<br>• 6 = 30/360 (European)<br>• 7 = actual/365 (Japanese)<br>• 8 = actual/actual (ICMA)<br>• 9 = actual/360 (ICMA) |

| | |
|---|---|
| | - 10 = actual/365 (ICMA)<br>- 11 = 30/360E (ICMA)<br>- 12 = actual/actual (ISDA)<br>- 13 = BUS/252<br>For more information, see basis. |
| `Principal` | (Optional) `NINST`-by-`1` vector of the notional principal amounts. Default is 100. |
| `Options` | (Optional) Derivatives pricing options structure created with `derivset`. |

## Description

`[Price, PriceTree] = swaptionbyhw(HWTree, OptSpec, Strike, ExerciseDates, Spread, Settle, Maturity, 'Name1', Value1, 'Name2', Value2)` computes the price of a swaption from a HW interest-rate tree.

---

**Note** The `Settle` date for every swaption is set to the `ValuationDate` of the HW tree. The swap argument `Settle` is ignored.

---

The swaption may be a call swaption or a put swaption.

A call swaption or payer swaption allows the option buyer to enter into an interest rate swap in which the buyer of the option pays the fixed rate and receives the floating rate.

A put swaption or receiver swaption allows the option buyer to enter into an interest rate swap in which the buyer of the option receives the fixed rate and pays the floating rate.

`Price` is a `NINST`-by-`1` vector of expected swaption prices at time 0.

`PriceTree` is a MATLAB structure of trees containing vectors of swaption instrument prices and a vector of observation times for each node. Within `PriceTree`:

- `PriceTree.PTree` contains the clean prices.
- `PriceTree.tObs` contains the observation times.

## Examples

Price a 3-year put swaption using an HW interest-rate tree with the following data.

Specify the `RateSpec`:

```
Rates =0.075 * ones (10,1);
Compounding = 2;
StartDates = ['jan-1-2007';'jul-1-2007';'jan-1-2008';'jul-1-2008';'jan-1-2009';...
'jul-1-2009';'jan-1-2010'; 'jul-1-2010';'jan-1-2011';'jul-1-2011'];
EndDates =['jul-1-2007';'jan-1-2008';'jul-1-2008';'jan-1-2009';'jul-1-2009';...
'jan-1-2010';'jul-1-2010';'jan-1-2011';'jul-1-2011';'jan-1-2012'];
ValuationDate = 'jan-1-2007';
RateSpec = intenvset('Rates', Rates, 'StartDates', StartDates, 'EndDates',...
EndDates, 'Compounding', Compounding);
```

Use `HWVolSpec` to compute the interest rate volatility:

```
Volatility = 0.05*ones(10,1);
AlphaCurve = 0.01*ones(10,1);
AlphaDates = EndDates;
HWVolSpec = hwvolspec(ValuationDate, EndDates, Volatility, AlphaDates, AlphaCurve);
```

Use `HWTimeSpec` to specify the structure of the time layout for an HW interest-rate tree:

```
HWTimeSpec = hwtimespec(ValuationDate, EndDates, Compounding);
```

Build the HW tree:

```
HWTree = hwtree(HWVolSpec, RateSpec, HWTimeSpec);
```

Use the following arguments for a 5-year swap and 3-year swaption:

```
SwapSettlement = 'jan-1-2007';
SwapMaturity   = 'jan-1-2012';
Spread = 0;
SwapReset = 2 ;
Principal = 100;
OptSpec = 'put';
Strike= 0.04;
ExerciseDates = 'jan-1-2010';
Basis=1;
```

Price the swaption

```
PriceSwaption = swaptionbyhw(HWTree, OptSpec, Strike, ExerciseDates, ...
Spread, SwapSettlement, SwapMaturity,'SwapReset', SwapReset, ...
'Basis', Basis,'Principal', Principal)
```

to return

```
PriceSwaption =

    2.9081
```

## See Also

`hwtree` | `instswaption` | `swapbyhw`

## Purpose

Dates from time and frequency

## Syntax

```
Dates = time2date(Settle, Times, Compounding, Basis,
EndMonthRule)
```

## Arguments

| | |
|---|---|
| `Settle` | Settlement date. A vector of serial date numbers or date strings. |
| `Times` | Vector of times corresponding to the compounding value. `Times` must be equal to or greater than 0. |
| `Compounding` | (Optional) Scalar value representing the rate at which the input zero rates were compounded when annualized. Default = `2`. This argument determines the formula for the discount factors:<br>`Compounding` = `1`, `2`, `3`, `4`, `6`, `12`<br>`Disc = (1 + Z/F)^(-T)`, where `F` is the compounding frequency, `Z` is the zero rate, and `T` is the time in periodic units; for example, `T = F` is 1 year.<br>`Compounding` = `365`<br>`Disc = (1 + Z/F)^(-T)`, where `F` is the number of days in the basis year and `T` is a number of days elapsed computed by basis.<br>`Compounding` = `-1`<br>`Disc = exp(-T*Z)`, where `T` is time in years. |

| | |
|---|---|
| `Basis` | (Optional) Day-count basis of the instrument. A vector of integers.<br>• 0 = actual/actual (default)<br>• 1 = 30/360 (SIA)<br>• 2 = actual/360<br>• 3 = actual/365<br>• 4 = 30/360 (BMA)<br>• 5 = 30/360 (ISDA)<br>• 6 = 30/360 (European)<br>• 7 = actual/365 (Japanese)<br>• 8 = actual/actual (ICMA)<br>• 9 = actual/360 (ICMA)<br>• 10 = actual/365 (ICMA)<br>• 11 = 30/360E (ICMA)<br>• 12 = actual/actual (ISDA)<br>• 13 = BUS/252<br>For more information, see basis. |
| `EndMonthRule` | (Optional) End-of-month rule. A vector. This rule applies only when `Maturity` is an end-of-month date for a month having 30 or fewer days. `0` = ignore rule, meaning that a bond's coupon payment date is always the same numerical day of the month. `1` = set rule on (default), meaning that a bond's coupon payment date is always the last actual day of the month. |

## Description

`Dates = time2date(Settle, Times, Compounding, Basis, EndMonthRule)` computes dates corresponding to the times occurring beyond the settlement date.

---

**Note** To obtain accurate results from this function, the `Basis` and `Dates` arguments must be consistent. If the `Dates` argument contains months that have 31 days, `Basis` must be one of the values that allow months to contain more than 30 days; for example, `Basis` = `0`, `3`, or `7`.

---

The `time2date` function is the inverse of `date2time`.

## Examples

Show that `date2time` and `time2date` are the inverse of each other. First compute the time factors using `date2time`.

```
Settle = '1-Sep-2002';
Dates = datenum(['31-Aug-2005'; '28-Feb-2006'; '15-Jun-2006';
                 '31-Dec-2006']);
Compounding = 2;
Basis = 0;
EndMonthRule = 1;
Times = date2time(Settle, Dates, Compounding, Basis,...
                  EndMonthRule)

Times =

    5.9945
    6.9945
    7.5738
    8.6576
```

Now use the calculated `Times` in `time2date` and compare the calculated dates with the original set.

```
Dates_calc = time2date(Settle, Times, Compounding, Basis,...
                  EndMonthRule)
```

```
Dates_calc =

      732555
      732736
      732843
      733042

datestr(Dates_calc)

ans =

31-Aug-2005
28-Feb-2006
15-Jun-2006
31-Dec-2006
```

## See Also

`cftimes` | `date2time` | `disc2rate` | `rate2disc`

## Purpose

Entries from node of recombining binomial tree

## Syntax

```
Values = treepath(Tree, BranchList)
```

## Arguments

| | |
|---|---|
| `Tree` | Recombining binomial tree. |
| `BranchList` | Number of paths (`NUMPATHS`) by path length (`PATHLENGTH`) matrix containing the sequence of branchings. |

## Description

`Values = treepath(Tree, BranchList)` extracts entries of a node of a recombining binomial tree. The node path is described by the sequence of branchings taken, starting at the root. The top branch is number one, the second-to-top is two, and so on. Set the branch sequence to zero to obtain the entries at the root node.

`Values` is a number of values (`NUMVALS`)-by-`NUMPATHS` matrix containing the retrieved entries of a recombining tree.

## Examples

Create a BDT tree by loading the example file.

```
load deriv.mat;
```

Then

```
FwdRates = treepath(BDTTree.FwdTree, [1 2 1])
```

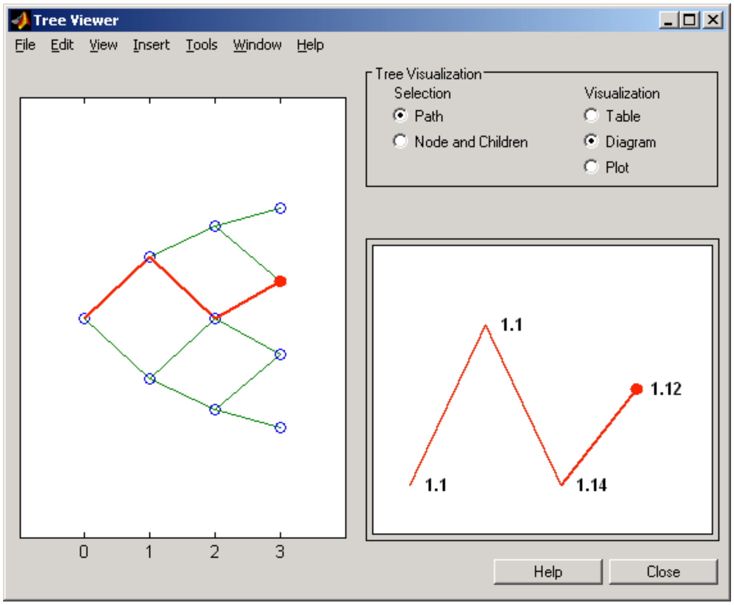
returns the rates at the tree nodes located by taking the up branch, then the down branch, and finally the up branch again.

```
FwdRates =

    1.1000
    1.0979
    1.1377
```

```
   1.1183
```

You can visualize this with the `treeviewer` function.

```
treeviewer(BDTTree)
```



Tree Visualization

Selection
● Path
○ Node and Children

Visualization
○ Table
● Diagram
○ Plot

## See Also

`mktree` | `treeshape`

## Purpose

Shape of recombining binomial tree

## Syntax

```
[NumLevels, NumPos, IsPriceTree] = treeshape(Tree)
```

## Arguments

| | |
|---|---|
| `Tree` | Recombining binomial tree. |

## Description

`[NumLevels, NumPos, IsPriceTree] = treeshape(Tree)` returns information on a recombining binomial tree's shape.

`NumLevels` is the number of time levels of the tree.

`NumPos` is a `1`-by-`NUMLEVELS` vector containing the length of the state vectors in each level.

`IsPriceTree` is a Boolean determining if a final horizontal branch is present in the tree.

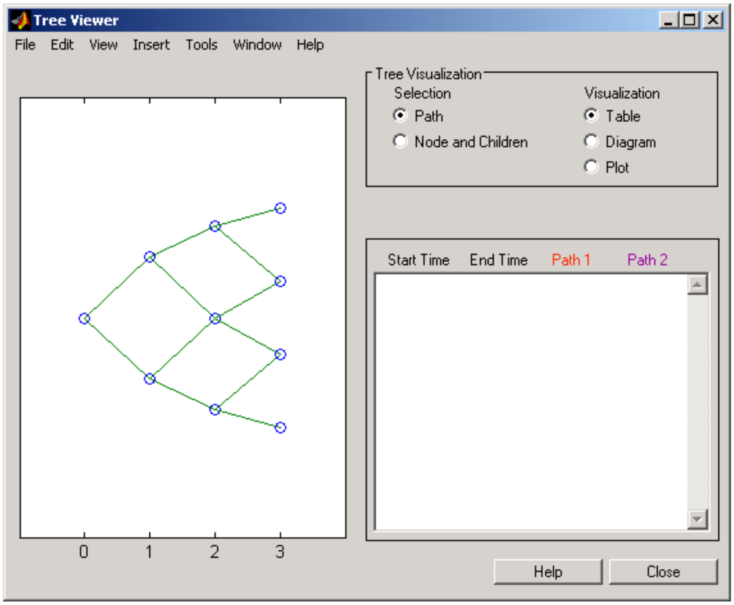
## Examples

Create a BDT tree by loading the example file.

```
load deriv.mat;
```

With `treeviewer` you can see the general shape of the BDT interest-rate tree.

```
treeviewer(BDTTree)
```



Tree Visualization

Selection
● Path
○ Node and Children

Visualization
● Table
○ Diagram
○ Plot

With this tree

```
[NumLevels, NumPos, IsPriceTree] = treeshape(BDTTree.FwdTree)
```

returns

```
NumLevels   =
      4

NumPos      =
      1       1       1       1

IsPriceTree =
      0
```

## See Also

`mktree` | `treepath`

## Purpose

Tree information

## Syntax

```
treeviewer(Tree)
treeviewer(PriceTree, InstSet)
treeviewer(CFTree, InstSet)
```

## Arguments

| | |
|---|---|
| `Tree` | `Tree` can be any of the following types of trees.<br>*Interest-rate trees:*<br>• Black-Derman-Toy (`BDTTree`)<br>• Black-Karasinski (`BKTree`)<br>• Heath-Jarrow-Morton (`HJMTree`)<br>• Hull-White (`HWTree`)<br>For information on creating interest-rate trees, see:<br>• `bktree` for information on creating `BKTree`.<br>• `bdttree` for information on creating `BDTTree`.<br>• `hjmtree` for information on creating `HJMTree`.<br>• `hwtree` for information on creating `HWTree`.<br>*Money market trees:*<br>• Money market tree (`MMktTree`)<br>For information on creating money-market trees, see:<br>• `mmktbybdt` for information on creating a money-market tree from a BDT interest-rate tree.<br>• `mmktbyhjm` for information on creating a money-market tree from an HJM interest-rate tree. |

---

**Note** Money market trees cannot be created from BK or HW interest-rate trees.

---

*Stock price trees:*

- Cox-Ross-Rubinstein (`CRRTree`)
- Implied Trinomial tree (`ITTTree`)
- Leisen-Reimer stock tree (`LRTree`)
- Equal probabilities (`EQPTree`)

  For information on creating stock price trees, see:

- `crrtree` for information on creating `CRRTree`.
- `eqptree` for information on creating `EQPTree`.
- `itttree` for information on creating `ITTTree`.
- `lrtree` for information on creating `LRTree`.

*Cash flow trees:*

- Black-Derman-Toy (`BDTCFTree`)
- Heath-Jarrow-Morton (`HJMCFTree`)

  Cash flow trees are created as outputs from the swap functions `swapbyhjm` and `swapbybdt`.

---

**Note** For the function `swapbybdt`, which uses a recombining binomial tree, this structure contains only `NaN`s because cash flows cannot be accurately calculated at every tree node for floating-rate notes.

---

| | |
|---|---|
| `PriceTree` | `PriceTree` is a Black-Derman-Toy (`BDTPriceTree`), Black-Karasinski (`BKPriceTree`), Heath-Jarrow-Morton (`HJMPriceTree`), Hull-White (`HWPriceTree`), Cox-Ross-Rubinstein (`crrprice`), Equal probabilities (`eqpprice`), or Implied Trinomial tree (`ittprice`) tree of instrument prices. |
| `CFTree` | `CFTree` is a tree of swap cash flows. You create cash flow trees when executing the Black-Derman-Toy and Heath-Jarrow-Morton swap functions. (Black-Derman-Toy cash flow trees contain only `NaN`s.) |
| `InstSet` | (Optional) Variable containing a collection of instruments whose prices or cash flows are contained in a tree. The collection can be created with the function `instadd` or as a cell array containing the names of the instruments. To display the names of the instruments, the field `Name` should exist in `InstSet`. If `InstSet` is not passed, `treeviewer` uses default instruments names (numbers) when displaying prices or cash flows. |

## Description

`treeviewer(Tree)` displays an interest rate, stock price, or money-market tree.

`treeviewer(PriceTree, InstSet)` displays a tree of instrument prices. If you provide the name of an instrument set (`InstSet`) and you have named the instruments using the field `Name`, the `treeviewer` display identifies the instrument being displayed with its name. (See Example 3 for a description.) If you do not provide the optional `InstSet` argument, the instruments are identified by their sequence number in the instrument set. (See Example 6 for a description.)

`treeviewer(CFTree, InstSet)` displays a cash flow tree that has been created with `swapbybdt` or `swapbyhjm`. If you provide the name of an instrument set (`InstSet`) containing cash flow names, the `treeviewer` display identifies the instrument being displayed with its name. (See Example 3 for a description.) If the optional `InstSet` argument is not present, the instruments are identified by their sequence number in the instrument set. See Example 6 for a description.)

`treeviewer` price tree diagrams follow the convention that increasing prices appear on the upper branch of a tree and, consequently, decreasing prices appear on the lower branch. Conversely, for interest rate displays, *decreasing* interest rates appear on the upper branch (prices are rising) and *increasing* interest rates on the lower branch (prices are falling).

`treeviewer` provides an interactive display of prices or interest rates. The display is activated by clicking the nodes along the price or interest rate path shown in the left pane when the function is called. For HJM trees you select the endpoints of the path, and `treeviewer` displays all data from beginning to end. With recombining trees, such as BDT, BK and HW, you must click *each* node in succession from the beginning (`t = 1`) to the last node (`t = n`). Do not include the *root node*, the node at `t = 0`. If you do not click the nodes in the proper order, you are reminded with the message

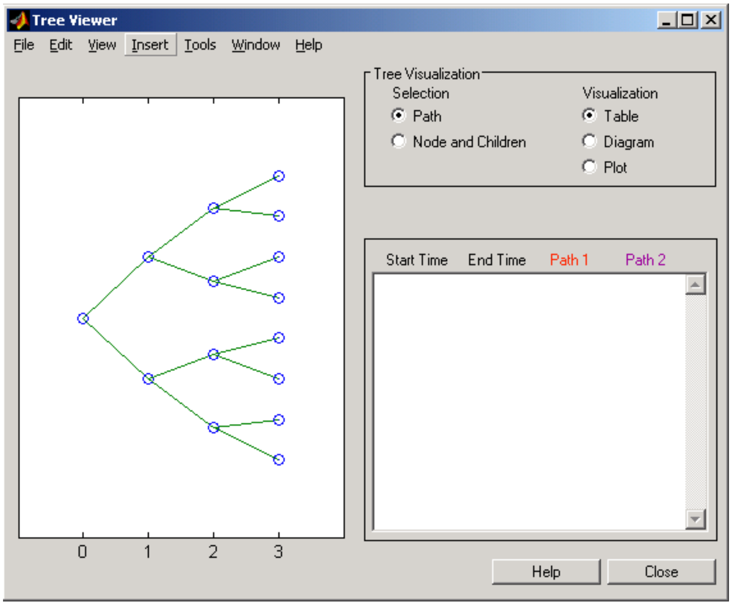
```
Parent of selected node must be selected.
```

## Examples

### Example 1. Display an HJM Interest-Rate Tree.

```
load deriv.mat
treeviewer(HJMTree)
```

The `treeviewer` function displays the structure of an HJM tree in the left pane. The tree visualization in the right pane is blank.

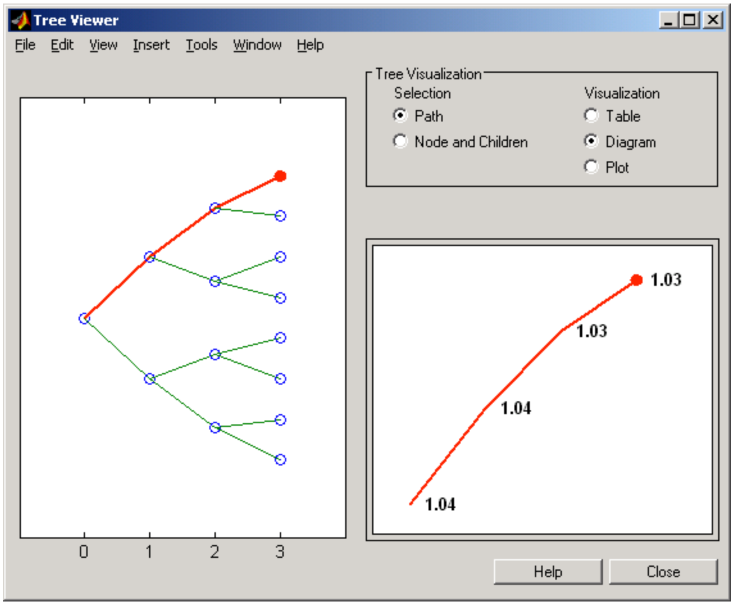


Tree Visualization

Selection
● Path
○ Node and Children

Visualization
● Table
○ Diagram
○ Plot

To visualize the actual interest-rate tree, go to the **Tree Visualization** pane and click on**Path**(the default) and **Diagram**. Now, select the first path by clicking on the last node (`t = 3`) of the upper branch.



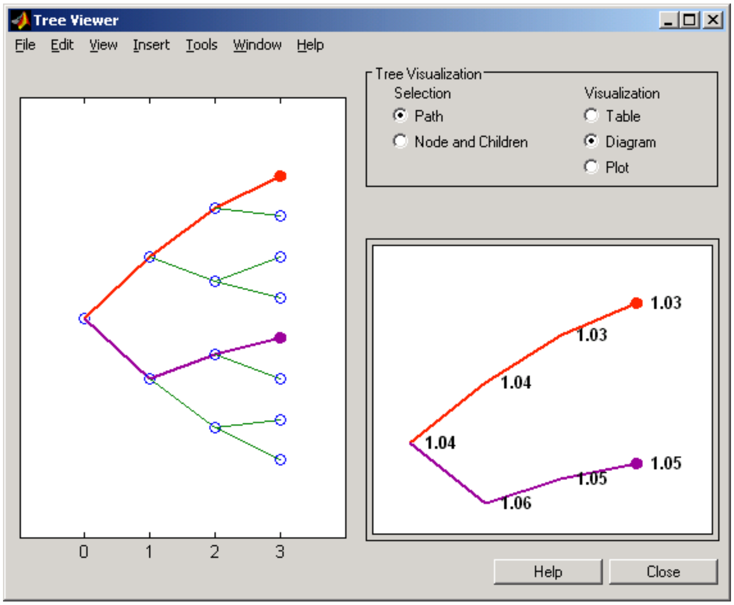
Tree Visualization

Selection
● Path
○ Node and Children

Visualization
○ Table
● Diagram
○ Plot

Note that the entire upper path is highlighted in red.

To complete the process, select a second path by clicking on the last node (`t = 3`) of another branch. The second path is highlighted in purple. The final display looks like this.



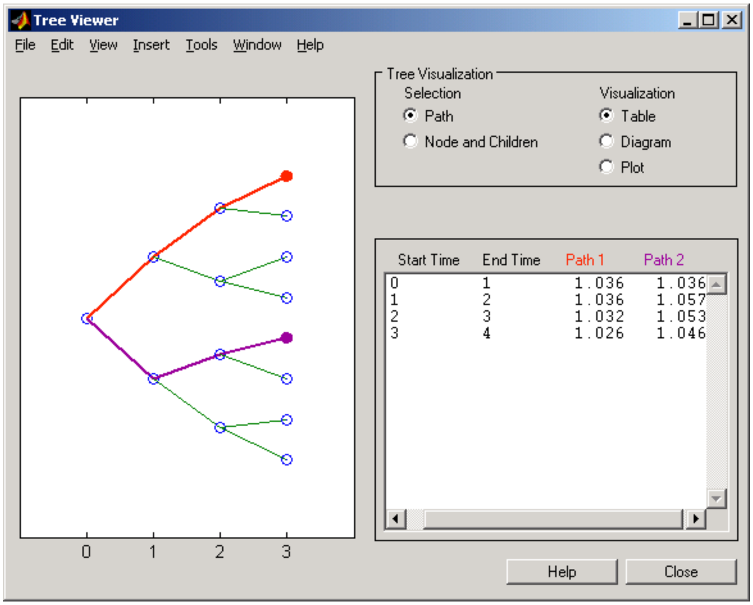
Tree Visualization

Selection
● Path
○ Node and Children

Visualization
○ Table
● Diagram
○ Plot

## Alternative Forms of Display

The **Tree Visualization** pane allows you to select alternative ways to display tree data. For example, if you select **Path** and **Table** as your visualization choices, the final display above instead appears in tabular form.



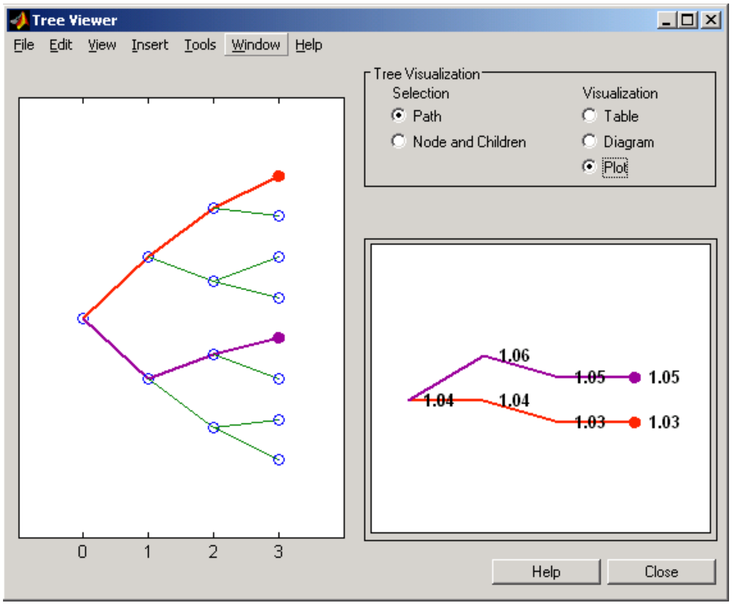
Tree Visualization

Selection
● Path
○ Node and Children

Visualization
● Table
○ Diagram
○ Plot

| Start Time | End Time | Path 1 | Path 2 |
|---|---|---|---|
| 0 | 1 | 1.036 | 1.036 |
| 1 | 2 | 1.036 | 1.057 |
| 2 | 3 | 1.032 | 1.053 |
| 3 | 4 | 1.026 | 1.046 |

To see a plot of interest rates along the chosen branches, click **Path** and **Plot** in the **Tree Visualization** pane.



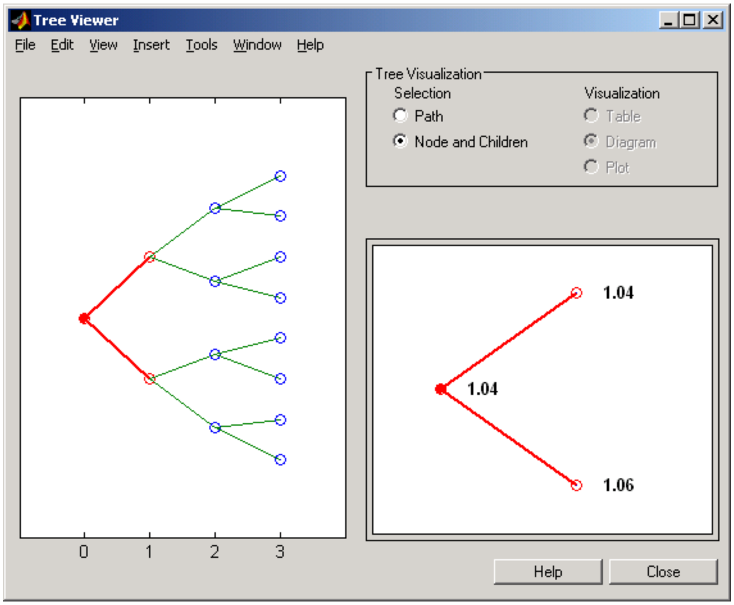
Tree Visualization

Selection
● Path
○ Node and Children

Visualization
○ Table
○ Diagram
● Plot

Note that with **Plot** selected, rising interest rates are shown on the upper branch and declining interest rates on the lower.

Finally, if you clicked **Node and Children** under **Tree Visualization**, you restrict the data displayed to just the selected parent node and its children.

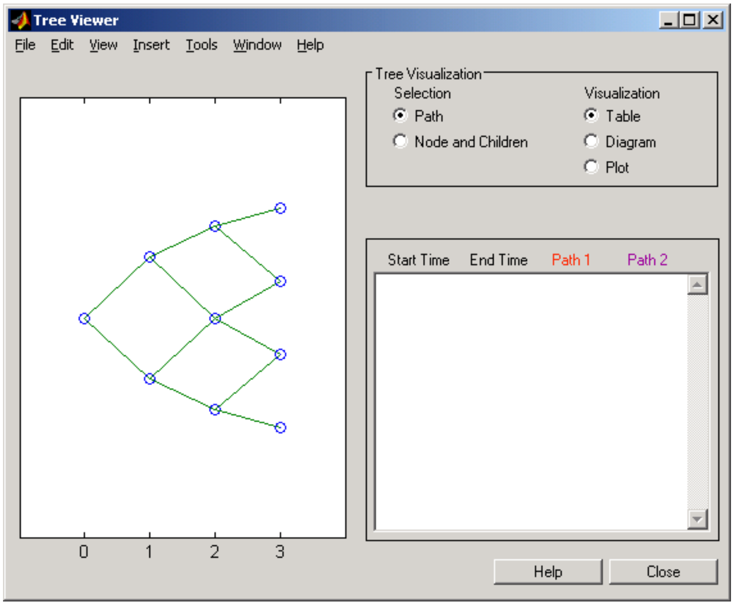


With **Node and Children** selected, the choices under **Visualization** are unavailable.

### Example 2. Display a BDT Interest-Rate Tree.

```
load deriv.mat
treeviewer(BDTTree)
```

The `treeviewer` function displays the structure of a BDT tree in the left pane. The tree visualization in the right pane is blank.



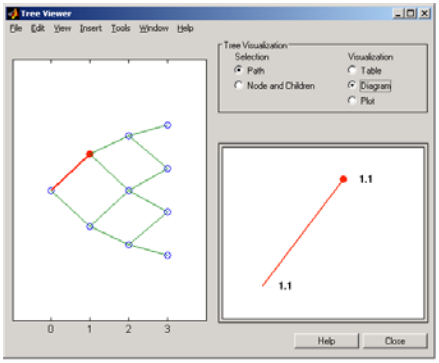
Tree Visualization

Selection
- ● Path
- ○ Node and Children

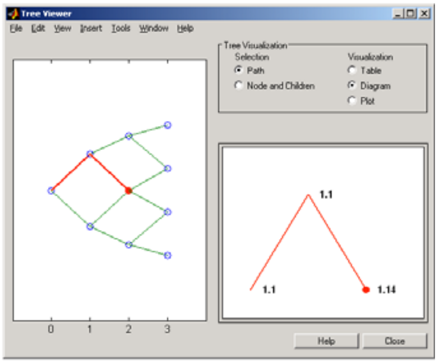
Visualization
- ● Table
- ○ Diagram
- ○ Plot

| Start Time | End Time | Path 1 | Path 2 |
|---|---|---|---|

To visualize the actual interest-rate tree, go to the **Tree Visualization** pane and click **Path** (the default) and **Diagram**. Now, select the first path by clicking on the first node of the up branch (`t = 1`). Continue by clicking the down branch at the next node (`t = 2`). The two figures below show the treeviewer path diagrams for these selections.



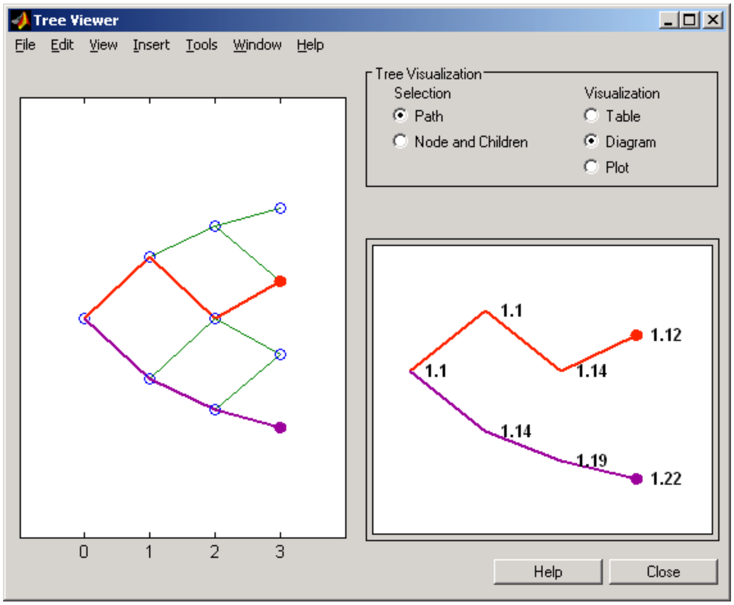
t = 1



t = 2

Continue clicking all nodes in succession until you reach the end of the branch. Note that the entire path you have selected is highlighted in red.

Select a second path by clicking the first node of the lower branch (`t = 1`). Continue clicking lower nodes as you did on the first branch. Note that the second branch is highlighted in purple. The final display looks like this.



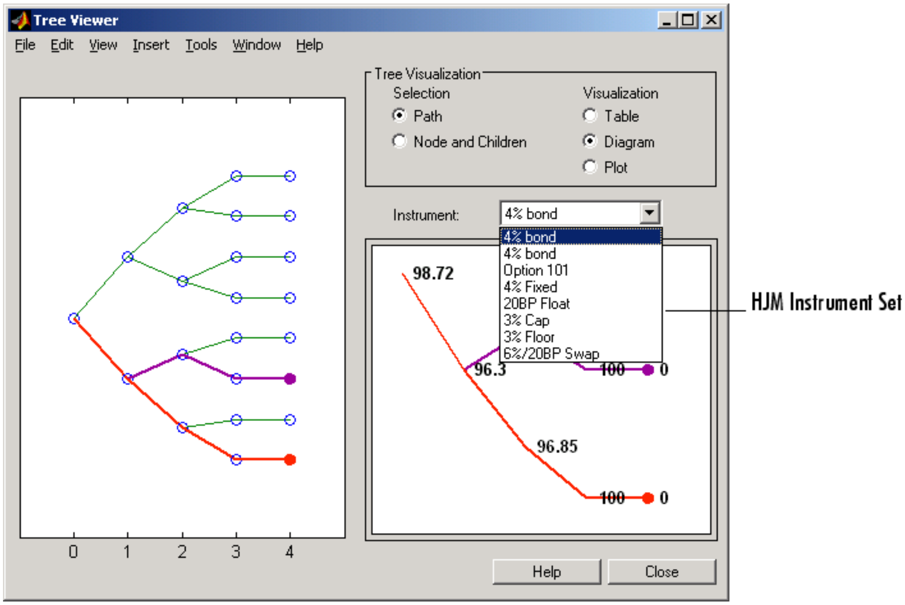
Tree Visualization

Selection
● Path
○ Node and Children

Visualization
○ Table
● Diagram
○ Plot

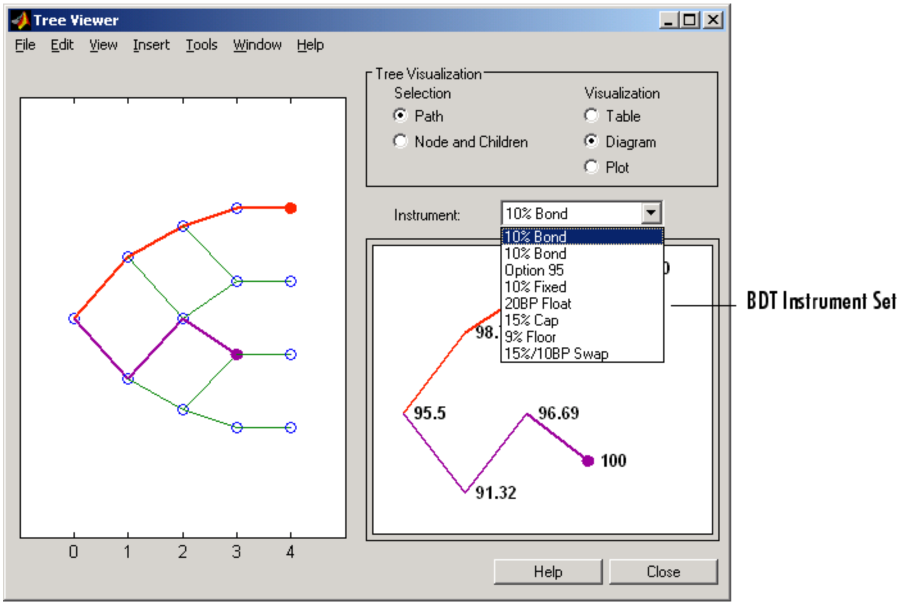
### Example 3. Display an HJM Price Tree for Named Instruments.

```
load deriv.mat
[Price, PriceTree] = hjmprice(HJMTree, HJMInstSet);
treeviewer(PriceTree, HJMInstSet)
```



## Example 4. Display a BDT Price Tree for Named Instruments.

```
load deriv.mat
[Price, PriceTree] = bdtprice(BDTTree, BDTInstSet);
treeviewer(PriceTree, BDTInstSet)
```



Tree Visualization

Selection

● Path

○ Node and Children

Visualization

○ Table

● Diagram

○ Plot

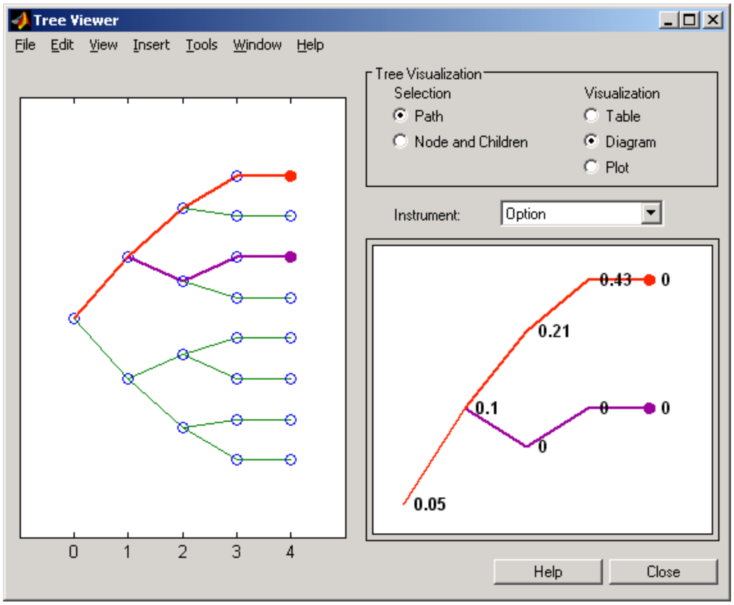
Instrument: 10% Bond

10% Bond
10% Bond
Option 95
10% Fixed
20BP Float
15% Cap
9% Floor
15%/10BP Swap

BDT Instrument Set

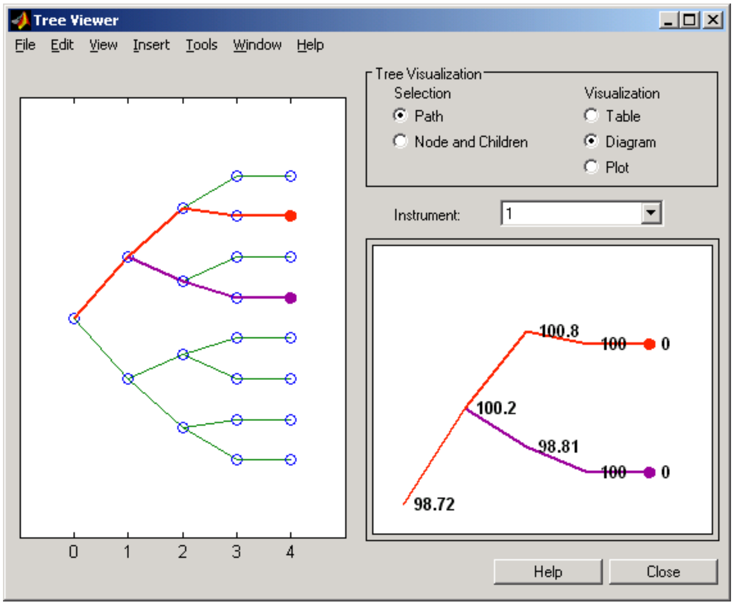
### Example 5. Display an HJM Price Tree with Renamed Instruments.

```
load deriv.mat
[Price, PriceTree] = hjmprice(HJMTree, HJMInstSet);
Names = {'Bond1', 'Bond2', 'Option', 'Fixed','Float', 'Cap',...
'Floor', 'Swap'};
treeviewer(PriceTree, Names)
```



### Example 6. Display an HJM Price Tree Using Default Instrument Names (Numbers).

```
load deriv.mat
[Price, PriceTree] = hjmprice(HJMTree, HJMInstSet);
treeviewer(PriceTree)
```



Tree Visualization

Selection
● Path
○ Node and Children

Visualization
○ Table
● Diagram
○ Plot

Instrument: 1

## See Also

`bdttree` | `bktree` | `crrtree` | `eqptree` | `hjmtree` | `hwtree` | `instadd` | `itttree` | `lrtree` | `mmktbybdt` | `mmktbyhjm` | `swapbybdt` | `swapbyhjm`

# trintreepath

## Purpose

Entries from node of recombining trinomial tree

## Syntax

`Values = trintreepath(TrinTree, BranchList)`

## Arguments

| | |
|---|---|
| `TrinTree` | Recombining price or interest-rate trinomial tree. |
| `BranchList` | Number of paths (`NUMPATHS`) by path length (`PATHLENGTH`) matrix containing the sequence of branchings. |

## Description

`Values = trintreepath(TrinTree, BranchList)` extracts entries of a node of a recombining trinomial tree. The node path is described by the sequence of branchings taken, starting at the root. The top branch is number 1, the middle branch is 2, and the bottom branch is 3. Set the branch sequence to 0 to obtain the entries at the root node.

`Values` is a number of values (`NUMVALS`)-by-`NUMPATHS` matrix containing the retrieved entries of a recombining tree.

## Examples

Create a Hull-White tree by loading the example file.

```
load deriv.mat;
```

Then, for example

```
FwdRates = trintreepath(HWTree, [1 2 3])
```

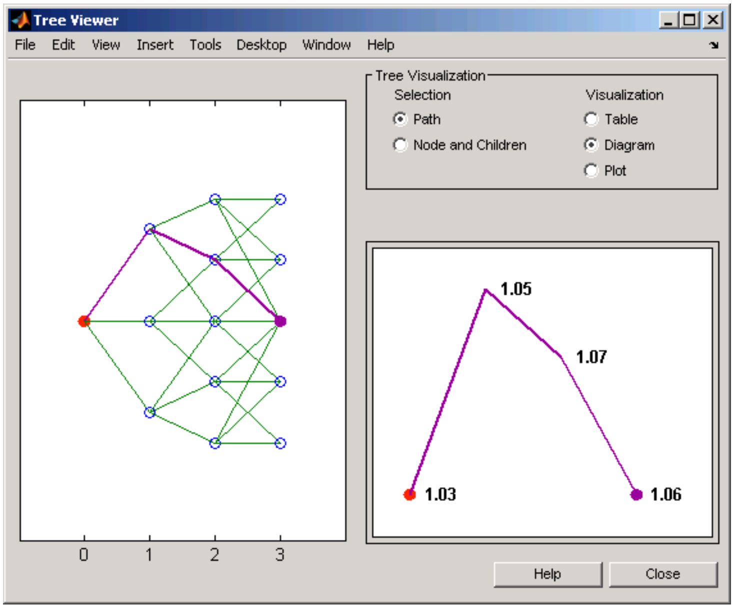
returns the rates at the tree nodes located by starting at 0, taking the up branch at the first node, the middle branch at the second node, and finally the bottom branch at the third node.

```
FwdRates =

    1.0279
```

```
    1.0528
    1.0652
    1.0591
```

You can visualize this with the `treeviewer` function.

```
treeviewer(HWTree)
```



## See Also

`mktrintree` | `trintreeshape`

# trintreeshape

## Purpose

Shape of recombining trinomial tree

## Syntax

```
[NumLevels, NumPos, NumStates] = trintreeshape(TrinTree)
```

## Arguments

| | |
|---|---|
| `TrinTree` | Recombining price or interest-rate trinomial tree. |

## Description

`[NumLevels, NumPos, NumStates] = trintreeshape(TrinTree)` returns information on a recombining trinomial tree's shape.

`NumLevels` is the number of time levels of the tree.

`NumPos` is a `1`-by-`NUMLEVELS` vector containing the length of the state vectors in each level.

`NumStates` is a `1`-by-`NUMLEVELS` vector containing the number of state vectors in each level.

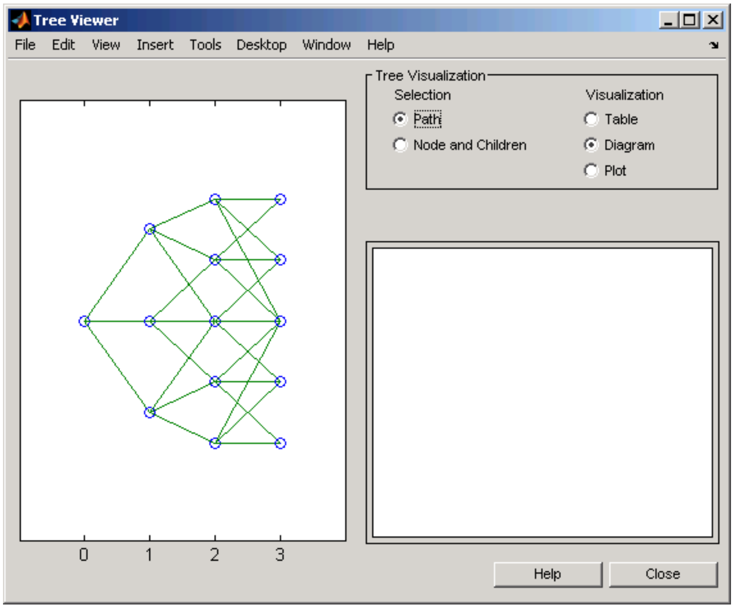
## Examples

Create a Hull-White tree by loading the example file.

```
load deriv.mat;
```

With `treeviewer` you can see the general shape of the HW interest-rate tree.

```
treeviewer(HWTree)
```



With this tree

```
[NumLevels, NumPos, NumStates] = trintreeshape(HWTree)
```

returns

```
NumLevels  =
     4

NumPos     =
     1     1     1     1

NumStates =
     1     3     5     5
```

## See Also

`mktrintree` | `trintreepath`

## Purpose

Determine option-adjusted spread of callable bond using Agency OAS model

## Syntax

```
OAS = agencyoas(ZeroData, Price, CouponRate, Settle, Maturity,
Vol, CallDate)
OAS = agencyoas(ZeroData, Price, CouponRate, Settle, Maturity,
Vol, CallDate, Name,Value)
```

## Description

`OAS = agencyoas(ZeroData, Price, CouponRate, Settle, Maturity, Vol, CallDate)` computes OAS of a callable bond given price using the Agency OAS model.

`OAS = agencyoas(ZeroData, Price, CouponRate, Settle, Maturity, Vol, CallDate, Name,Value)` computes OAS of a callable bond given price using the Agency OAS model with additional options specified by one or more `Name,Value` pair arguments.

## Input Arguments

### ZeroData

Zero curve represented as a `numRates`-by-`2` matrix where the first column is zero dates and the second column is the accompanying zero rates.

### Price

`numBonds`-by-`1` vector of prices.

### CouponRate

`numBonds`-by-`1` vector of coupon rates in decimal form.

### Settle

Scalar MATLAB date number for the settlement date for all bonds and the zero data.

---

**Note** The `Settle` date must be an identical settlement date for all the bonds and the zero curve.

---

### Maturity

`numBonds`-by-`1` vector of maturity dates.

### Vol

`numBonds`-by-`1` vector of volatilities in decimal form. This is the volatility of interest rates corresponding to the time of the `CallDate`.

### CallDate

`numBonds`-by-`1` vector of call dates.

## Name-Value Pair Arguments

Specify optional comma-separated pairs of `Name,Value` arguments, where `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside single quotes (`' '`). You can specify several name and value pair arguments in any order as `Name1,Value1,...,NameN,ValueN`.

### Basis

`N`-by-`1` vector of day-count basis:

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (BMA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)

- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/actual (ISDA)
- 13 = BUS/252

For more information, see basis.

**Default:** `0` (actual/actual)

### CurveBasis

Basis of the zero curve, where the choices are identical to `Basis`.

**Default:** `0` (actual/actual)

### CurveCompounding

Compounding frequency of the zero curve. Possible values include: –1, 0, 1, 2 , 3, 4, 6, 12.

**Default:** `2` (Semi-annual)

### EndMonthRule

End-of-month rule; `1`, indicating in effect, and `0`, indicating rule not in effect for the bond(s). When `1`, the rule is in effect for the bond(s), this means that a security that pays coupon interest on the last day of the month will always make payment on the last day of the month.

**Default:** `1` — Indicates in effect

### Face

Face value of the bond.

**Default:** `100`

### FirstCouponDate

Date when a bond makes its first coupon payment; used when bond has an irregular first coupon period. When `FirstCouponDate` and `LastCouponDate` are both specified, `FirstCouponDate` takes precedence in determining the coupon payment structure.

**Default:** If you do not specify a `FirstCouponDate`, the cash flow payment dates are determined from other inputs.

### InterpMethod

Interpolation method used to obtain points from the zero curve. Values are:

- `linear` — linear interpolation
- `cubic` — piecewise cubic spline interpolation
- `pchip` — piecewise cubic Hermite interpolation

**Default:** `linear`

### IssueDate

Bond issue date.

**Default:** If you do not specify an `IssueDate`, the cash flow payment dates are determined from other inputs.

### LastCouponDate

Last coupon date of a bond before the maturity date; used when bond has an irregular last coupon period. In the absence of a specified `FirstCouponDate`, a specified `LastCouponDate` determines the coupon structure of the bond. The coupon structure of a bond is truncated at

the `LastCouponDate`, regardless of where it falls, and is followed only by the bond's maturity cash flow date.

> **Default:** If you do not specify a `LastCouponDate`, the cash flow payment dates are determined from other inputs.

### Period

Number of coupon payments per year. Possible values include: 0, 1, 2 , 3, 4, 6, 12.

> **Default:** `2`

### StartDate

Forward starting date of payments.

> **Default:** If you do not specify a `StartDate`, the effective start date is the `Settle` date.

## Output Arguments

### OAS

`numBonds`-by-`1` matrix of option-adjusted spreads.

## Definitions

### Agency OAS Model

The BMA European Callable Securities Formula provides a standard methodology for computing price and option-adjusted spread for European Callable Securities (ECS).

## Examples

Compute the agency `OAS` value:

```
Settle = datenum('20-Jan-2010');
ZeroRates = [.07 .164 .253 1.002 1.732 2.226 2.605 3.316 ...
3.474 4.188 4.902]'/100;
ZeroDates = daysadd(Settle,360*[.25 .5 1 2 3 4 5 7 10 20 30],1);
ZeroData = [ZeroDates ZeroRates];

Maturity = datenum('30-Dec-2013');
```

```
CouponRate = .022;
Price = 99.155;
Vol = .5117;
CallDate = datenum('30-Dec-2010');
OAS = agencyoas(ZeroData, Price, CouponRate, Settle, Maturity, Vol, CallDate)
OAS =

8.6279
```

## References

SIFMA, The BMA European Callable Securities Formula, `http://www.sifma.org`.

## See Also

`| agencyprice |`

## Tutorials

- "Agency Option-Adjusted Spreads" on page 6-2

# agencyprice

## Purpose

Price callable bond using Agency OAS model

## Syntax

```
Price = agencyprice(ZeroData, OAS, CouponRate, Settle, Maturity,
Vol, CallDate)
Price = agencyprice(ZeroData, OAS, CouponRate, Settle, Maturity,
Vol, CallDate, Name,Value)
```

## Description

`Price = agencyprice(ZeroData, OAS, CouponRate, Settle, Maturity, Vol, CallDate)` computes the price for a callable bond, given OAS, using the Agency OAS model.

`Price = agencyprice(ZeroData, OAS, CouponRate, Settle, Maturity, Vol, CallDate, Name,Value)` computes the price for a callable bond, given OAS, using the Agency OAS model with additional options specified by one or more `Name,Value` pair arguments.

## Input Arguments

### ZeroData

Zero curve represented as a `numRates`-by-`2` matrix where the first column is zero dates and the second column is the accompanying zero rates.

### OAS

`numBonds`-by-`1` vector of option-adjusted spreads, expressed as a decimal (i.e., 50 basis points is entered as `.005`).

### CouponRate

`numBonds`-by-`1` vector of coupon rates in decimal form.

### Settle

Scalar MATLAB date number for the settlement date for all the bonds and the zero data.

---

**Note** The `Settle` date must be an identical settlement date for all bonds and the zero curve.

---

#### Maturity

`numBonds`-by-`1` vector of maturity dates.

#### Vol

`numBonds`-by-`1` vector of volatilities in decimal form. This is the volatility of interest rates corresponding to the time of the `CallDate`.

#### CallDate

`numBonds`-by-`1` vector of call dates.

### Name-Value Pair Arguments

Specify optional comma-separated pairs of `Name,Value` arguments, where `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside single quotes (`' '`). You can specify several name and value pair arguments in any order as `Name1,Value1,...,NameN,ValueN`.

#### Basis

`N`-by-`1` vector of day-count basis:

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (BMA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)

- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/actual (ISDA)
- 13 = BUS/252

For more information, see basis.

**Default:** `0` (actual/actual)

### CurveBasis

Basis of the zero curve, where the choices are identical to `Basis`.

**Default:** `0` (actual/actual)

### CurveCompounding

Compounding frequency of the curve. Possible values include: –1, 0, 1, 2 , 3, 4, 6, 12.

**Default:** `2` (Semi-annual)

### EndMonthRule

End-of-month rule; `1`, indicating in effect, and `0`, indicating rule not in effect for the bond(s). When `1`, the rule is in effect for the bond(s). This means that a security that pays coupon interest on the last day of the month will always make payment on the last day of the month.

**Default:** `1` — Indicates in effect

### Face

Face value of the bond.

**Default:** `100`

### FirstCouponDate

Date when a bond makes its first coupon payment; used when bond has an irregular first coupon period. When `FirstCouponDate` and `LastCouponDate` are both specified, `FirstCouponDate` takes precedence in determining the coupon payment structure.

**Default:** If you do not specify a `FirstCouponDate`, the cash flow payment dates are determined from other inputs.

### InterpMethod

Interpolation method used to obtain points from the zero curve. Values are:

- `linear` — linear interpolation
- `cubic` — piecewise cubic spline interpolation
- `pchip` — piecewise cubic Hermite interpolation

**Default:** `linear`

### IssueDate

Bond issue date.

**Default:** If you do not specify an `IssueDate`, the cash flow payment dates are determined from other inputs.

### LastCouponDate

Last coupon date of a bond before the maturity date; used when bond has an irregular last coupon period. In the absence of a specified `FirstCouponDate`, a specified `LastCouponDate` determines the coupon structure of the bond. The coupon structure of a bond is truncated at

the `LastCouponDate`, regardless of where it falls, and is followed only by the bond's maturity cash flow date.

> **Default:** If you do not specify a `LastCouponDate`, the cash flow payment dates are determined from other inputs.

**Period**

Number of coupon payments per year. Possible values include: 0, 1, 2 , 3, 4, 6, 12.

> **Default:** `2`

**StartDate**

Forward starting date of payments.

> **Default:** If you do not specify a `StartDate`, the effective start date is the `Settle` date.

## Output Arguments

**Price**

`numBonds`-by-`1` matrix of the price.

## Definitions

### Agency OAS Model

The BMA European Callable Securities Formula provides a standard methodology for computing price and option-adjusted spread for European Callable Securities (ECS).

## Examples

Compute the agency `Price`:

```
Settle = datenum('20-Jan-2010');
ZeroRates = [.07 .164 .253 1.002 1.732 2.226 2.605 3.316 ...
3.474 4.188 4.902]'/100;
ZeroDates = daysadd(Settle,360*[.25 .5 1 2 3 4 5 7 10 20 30],1);
ZeroData = [ZeroDates ZeroRates];

Maturity = datenum('30-Dec-2013');
```

```
CouponRate = .022;
OAS = 6.53/10000;
Vol = .5117;
CallDate = datenum('30-Dec-2010');
Price = agencyprice(ZeroData, OAS, CouponRate, Settle, Maturity, Vol, CallDate)
Price =

99.4226
```

## References

SIFMA, The BMA European Callable Securities Formula, `http://www.sifma.org`.

## See Also

| `agencyoas` |

## Tutorials

- "Agency Option-Adjusted Spreads" on page 6-2

## Purpose

Price European call option on bonds using Black model

## Syntax

```
CallPrice = bkcall(Strike, ZeroData, Sigma, BondData, Settle,
Expiry, Period, Basis, EndMonthRule, InterpMethod,
StrikeConvention)
```

## Arguments

| | |
|---|---|
| `Strike` | Scalar or number of options (`NOPT`)-by-`1` vector of strike prices. |
| `ZeroData` | Two-column (optionally three-column) matrix containing zero (spot) rate information used to discount future cash flows.<br>• Column 1: Serial maturity date associated with the zero rate in the second column.<br>• Column 2: Annualized zero rates, in decimal form, appropriate for discounting cash flows occurring on the date specified in the first column. All dates must occur after `Settle` (dates must correspond to future investment horizons) and must be in ascending order.<br>• Column 3 (optional): Annual compounding frequency. Values are 1 (annual), 2 (semiannual, default), 3 (three times per year), 4 (quarterly), 6 (bimonthly), 12 (monthly), and -1 (continuous). |
| `Sigma` | Scalar or `NOPT`-by-`1` vector of annualized price volatilities required by Black's model. |

| | |
|---|---|
| `BondData` | Row vector with three (optionally four) columns or `NOPT`-by-`3` (optionally `NOPT`-by-`4`) matrix specifying characteristics of underlying bonds in the form:<br>`[CleanPrice CouponRate Maturity Face]`<br>`CleanPrice` is the price excluding accrued interest.<br>`CouponRate` is the decimal coupon rate.<br>`Maturity` is the bond maturity date in serial date number format.<br>`Face` is the face value of the bond. If unspecified, the face value is assumed to be 100. |
| `Settle` | Settlement date of the options. May be a serial date number or date string. `Settle` also represents the starting reference date for the input zero curve. |
| `Expiry` | Scalar or `NOPT`-by-`1` vector of option maturity dates. May be a serial date number or date string. |
| `Period` | (Optional) Number of coupons per year for the underlying bond. Default = 2 (semiannual). Supported values are 0, 1, 2, 3, 4, 6, and 12. |

| | |
|---|---|
| `Basis` | (Optional) Day-count basis of the bond. A vector of integers.<br>• 0 = actual/actual (default)<br>• 1 = 30/360 (SIA)<br>• 2 = actual/360<br>• 3 = actual/365<br>• 4 = 30/360 (BMA)<br>• 5 = 30/360 (ISDA)<br>• 6 = 30/360 (European)<br>• 7 = actual/365 (Japanese)<br>• 8 = actual/actual (ICMA)<br>• 9 = actual/360 (ICMA)<br>• 10 = actual/365 (ICMA)<br>• 11 = 30/360E (ICMA)<br>• 12 = actual/actual (ISDA)<br>• 13 = BUS/252<br>For more information, see basis. |
| `EndMonthRule` | (Optional) End-of-month rule. This rule applies only when `Maturity` is an end-of-month date for a month having 30 or fewer days. 0 = ignore rule, meaning that a bond's coupon payment date is always the same numerical day of the month. 1 = set rule on (default), meaning that a bond's coupon payment date is always the last actual day of the month. |

| | |
|---|---|
| `InterpMethod` | (Optional) Scalar integer zero curve interpolation method. For cash flows that do not fall on a date found in the `ZeroData` spot curve, indicates the method used to interpolate the appropriate zero discount rate. Available methods are (0) nearest, (1) linear, and (2) cubic. Default = 1. See `interp1` for more information. |
| `StrikeConvention` | (Optional) Scalar or `NOPT`-by-`1` vector of option contract strike price conventions.<br>`StrikeConvention = 0` (default) defines the strike price as the cash (dirty) price paid for the underlying bond.<br>`StrikeConvention = 1` defines the strike price as the quoted (clean) price paid for the underlying bond. When evaluating Black's model, the accrued interest of the bond at option expiration is added to the input strike price. |

## Description

`CallPrice = bkcall(Strike, ZeroData, Sigma, BondData, Settle, Expiry, Period, Basis, EndMonthRule, InterpMethod, StrikeConvention)` using Black's model, derives an `NOPT`-by-`1` vector of prices of European call options on bonds.

If cash flows occur beyond the dates spanned by `ZeroData`, the input zero curve, the appropriate zero rate for discounting such cash flows is obtained by extrapolating the nearest rate on the curve (that is, if a cash flow occurs before the first or after the last date on the input zero curve, a flat curve is assumed).

In addition, you can use the Financial Instruments Toolbox method `getZeroRates` for an `IRDataCurve` object with a `Dates` property to create a vector of dates and data acceptable for `bkcall`. For more information, see "Converting an IRDataCurve or IRFunctionCurve Object" on page 9-37.

## Examples

This example is based on Example 22.1, page 512, of Hull. (See References below.)

Consider a European call option on a bond maturing in 9.75 years. The underlying bond has a clean price of $935, a face value of $1000, and pays 10% semiannual coupons. Since the bond matures in 9.75 years, a $50 coupon will be paid in 3 months and again in 9 months. Also, assume that the annualized volatility of the forward bond price is 9%. Furthermore, suppose the option expires in 10 months and has a strike price of $1000, and that the annualized continuously compounded risk-free discount rates for maturities of 3, 9, and 10 months are 9%, 9.5%, and 10%, respectively.

```
% Specify the option information.
Settle       =  '15-Mar-2004';
Expiry       =  '15-Jan-2005'; % 10 months from settlement
Strike       =  1000;
Sigma        =  0.09;
Convention   =  [0 1]';

% Specify the interest-rate environment.
ZeroData     = [datenum('15-Jun-2004')  0.09   -1;  % 3 months
                datenum('15-Dec-2004')  0.095  -1;  % 9 months
                datenum(Expiry)         0.10   -1]; % 10 months

% Specify the bond information.
CleanPrice   =  935;
CouponRate   =  0.1;
Maturity     = '15-Dec-2013';  % 9.75 years from settlement
Face         =  1000;
BondData     = [CleanPrice CouponRate datenum(Maturity) Face];
Period       =  2;
Basis        =  1;

% Call Black's model.
CallPrices = bkcall(Strike, ZeroData, Sigma, BondData, Settle,...
Expiry, Period, Basis, [], [], Convention)
```

```
CallPrices =

  9.4873
  7.9686
```

When the strike price is the dirty price (`Convention = 0`), the call option value is $9.49. When the strike price is the clean price (`Convention = 1`), the call option value is $7.97.

## References

[1] Hull, John C., *Options, Futures, and Other Derivatives*, Prentice Hall, 5th edition, 2003, pp. 287-288, 508-515.

## See Also

`bkput`

# bkcaplet

**Purpose** Price interest-rate caplet using Black model

**Syntax**

```
CapPrices = bkcaplet(CapData, FwdRates, ZeroPrice, Settle,
StartDate, EndDate, Sigma)
```

**Arguments**

| | |
|---|---|
| `CapData` | Number of caps (`NCAP`)-by-`2` matrix containing cap rates and bases: `[CapRates Basis]`.<br>Values for bases may be:<br>• 0 = actual/actual (default)<br>• 1 = 30/360 (SIA)<br>• 2 = actual/360<br>• 3 = actual/365<br>• 4 = 30/360 (BMA)<br>• 5 = 30/360 (ISDA)<br>• 6 = 30/360 (European)<br>• 7 = actual/365 (Japanese)<br>• 8 = actual/actual (ICMA)<br>• 9 = actual/360 (ICMA)<br>• 10 = actual/365 (ICMA)<br>• 11 = 30/360E (ICMA)<br>• 12 = actual/actual (ISDA)<br>• 13 = BUS/252<br>For more information, see basis. |
| `FwdRates` | Scalar or `NCAP`-by-`1` vector containing forward rates in decimal. `FwdRates` accrue on the same basis as `CapRates`. |

| | |
|---|---|
| `ZeroPrice` | Scalar or `NCAP`-by-`1` vector containing zero coupon prices with maturities corresponding to those of each cap in `CapData`, per $100 nominal value. |
| `Settle` | Scalar or `NCAP`-by-`1` vector of identical elements containing settlement date of caplets. |
| `StartDate` | Scalar or `NCAP`-by-`1` vector containing start dates of the caplets. |
| `EndDate` | Scalar or `NCAP`-by-`1` vector containing maturity dates of caplets. |
| `Sigma` | Scalar or `NCAP`-by-`1` vector containing volatility of forward rates in decimal, corresponding to each caplet. |

## Description

`CapPrices = bkcaplet(CapData, FwdRates, ZeroPrice, Settle, StartDate, EndDate, Sigma)` computes the prices of interest-rate caplets for every $100 face value of principal.

## Examples

Given a notional amount of $1,000,000, compute the value of a caplet on October 15, 2002 that starts on October 15, 2003 and ends on January 15, 2004.

```
CapData = [0.08, 1];
FwdRates = 0.07;
ZeroPrice = 100*exp(-0.065*1.25);
Settle = datenum('15-Oct-2002');
BeginDates = datenum('15-Oct-2003');
EndDates = datenum('15-Jan-2004');
Sigma = 0.20;
```

Because the caplet is $100 notional, divide $1,000,000 by $100.

```
Notional   = 1000000/100;

CapPrice = Notional*bkcaplet(CapData, FwdRates, ZeroPrice, ...
Settle, BeginDates, EndDates, Sigma)
```

```
CapPrice =

  519.0046
```

**See Also** `bkfloorlet`

## Purpose

Price interest-rate floorlet using Black model

## Syntax

```
FloorPrices = bkfloorlet(FloorData, FwdRates, ZeroPrice, Settle,
StartDate, EndDate, Sigma)
```

## Arguments

| | |
|---|---|
| `FloorData` | Number of floors (`NFLR`)-by-`2` matrix containing floor rates and bases: `[FloorRate Basis]`.<br>Values for bases may be:<br>• 0 = actual/actual (default)<br>• 1 = 30/360 (SIA)<br>• 2 = actual/360<br>• 3 = actual/365<br>• 4 = 30/360 (BMA)<br>• 5 = 30/360 (ISDA)<br>• 6 = 30/360 (European)<br>• 7 = actual/365 (Japanese)<br>• 8 = actual/actual (ICMA)<br>• 9 = actual/360 (ICMA)<br>• 10 = actual/365 (ICMA)<br>• 11 = 30/360E (ICMA)<br>• 12 = actual/actual (ISDA)<br>• 13 = BUS/252<br>For more information, see basis. |
| `FwdRates` | Scalar or `NFLR`-by-`1` vector containing forward rates in decimal. `FwdRates` accrue on the same basis as `FloorRates`. |

| | |
|---|---|
| `ZeroPrice` | Scalar or `NFLR`-by-`1` vector containing zero coupon prices with maturities corresponding to those of each floor in `FloorData`, per $100 nominal value. |
| `Settle` | Scalar or `NFLR`-by-`1` vector of identical elements containing settlement date of floorlets. |
| `StartDate` | Scalar or `NFLR`-by-`1` vector containing start dates of the floorlets. |
| `EndDate` | Scalar or `NFLR`-by-`1` vector containing maturity dates of floorlets. |
| `Sigma` | Scalar or `NFLR`-by-`1` vector containing volatility of forward rates in decimal, corresponding to each floorlet. |

## Description

`FloorPrices = bkfloorlet(FloorData, FwdRates, ZeroPrice, Settle, StartDate, EndDate, Sigma)` computes the prices of interest-rate floorlets for every $100 of notional value.

## Examples

Given a notional amount of $1,000,000, compute the value of a floorlet on October 15, 2002 that starts on October 15, 2003 and ends on January 15, 2004.

```
FloorData = [0.08, 1];
FwdRates = 0.07;
ZeroPrice = 100*exp(-0.065*1.25);
Settle = datenum('15-Oct-2002');
BeginDates = datenum('15-Oct-2003');
EndDates = datenum('15-Jan-2004');
Sigma = 0.20;

% Because floorlet is $100 notional, divide $1,000,000 by $100.
Notional = 1000000/100;

FloorPrice = Notional*bkfloorlet(FloorData, FwdRates, ...
ZeroPrice, Settle, BeginDates, EndDates, Sigma)
```

```
FloorPrice =

      2823.91
```

## See Also

`bkcaplet`

## Purpose

Price European put option on bonds using Black model

## Syntax

```
PutPrice = bkput(Strike, ZeroData, Sigma, BondData, Settle, Expiry,
Period, Basis, EndMonthRule, InterpMethod,
StrikeConvention)
```

## Arguments

| | |
|---|---|
| `Strike` | Scalar or number of options (`NOPT`)-by-`1` vector of strike prices. |
| `ZeroData` | Two-column (optionally three-column) matrix containing zero (spot) rate information used to discount future cash flows.<br>• Column 1: Serial maturity date associated with the zero rate in the second column.<br>• Column 2: Annualized zero rates, in decimal form, appropriate for discounting cash flows occurring on the date specified in the first column. All dates must occur after `Settle` (dates must correspond to future investment horizons) and must be in ascending order.<br>• Column 3 (optional): Annual compounding frequency. Values are 1 (annual), 2 (semiannual, default), 3 (three times per year), 4 (quarterly), 6 (bimonthly), 12 (monthly), and -1 (continuous). |
| `Sigma` | Scalar or `NOPT`-by-`1` vector of annualized price volatilities required by Black's model. |

| | |
|---|---|
| `BondData` | Row vector with three (optionally four) columns or `NOPT`-by-`3` (optionally `NOPT`-by-`4`) matrix specifying characteristics of underlying bonds in the form `[CleanPrice CouponRate Maturity Face] where`:<br>• `CleanPrice` is the price excluding accrued interest.<br>• `CouponRate` is the decimal coupon rate.<br>• `Maturity` is the bond maturity date in serial date number format.<br>• `Face` is the face value of the bond. If unspecified, the face value is assumed to be 100. |
| `Settle` | Settlement date of the options. May be a serial date number or date string. `Settle` also represents the starting reference date for the input zero curve. |
| `Expiry` | Scalar or `NOPT`-by-`1` vector of option maturity dates. May be a serial date number or date string. |
| `Period` | (Optional) Number of coupons per year for the underlying bond. Default = 2 (semiannual). Supported values are 0, 1, 2, 3, 4, 6, and 12. |

| | |
|---|---|
| `Basis` | (Optional) Day-count basis of the bond. A vector of integers.<br>• 0 = actual/actual (default)<br>• 1 = 30/360 (SIA)<br>• 2 = actual/360<br>• 3 = actual/365<br>• 4 = 30/360 (BMA)<br>• 5 = 30/360 (ISDA)<br>• 6 = 30/360 (European)<br>• 7 = actual/365 (Japanese)<br>• 8 = actual/actual (ICMA)<br>• 9 = actual/360 (ICMA)<br>• 10 = actual/365 (ICMA)<br>• 11 = 30/360E (ICMA)<br>• 12 = actual/actual (ISDA)<br>• 13 = BUS/252<br>For more information, see basis. |
| `EndMonthRule` | (Optional) End-of-month rule. This rule applies only when `Maturity` is an end-of-month date for a month having 30 or fewer days. 0 = ignore rule, meaning that a bond's coupon payment date is always the same numerical day of the month. 1 = set rule on (default), meaning that a bond's coupon payment date is always the last actual day of the month. |

| | |
|---|---|
| `InterpMethod` | (Optional) Scalar integer zero curve interpolation method. For cash flows that do not fall on a date found in the `ZeroData` spot curve, indicates the method used to interpolate the appropriate zero discount rate. Available methods are (0) nearest, (1) linear, and (2) cubic. Default = 1. See `interp1` for more information. |
| `StrikeConvention` | (Optional) Scalar or `NOPT`-by-`1` vector of option contract strike price conventions.<br>`StrikeConvention = 0` (default) defines the strike price as the cash (dirty) price paid for the underlying bond.<br>`StrikeConvention = 1` defines the strike price as the quoted (clean) price paid for the underlying bond. The accrued interest of the bond at option expiration is added to the input strike price when evaluating Black's model. |

## Description

`PutPrice = bkput(Strike, ZeroData, Sigma, BondData, Settle, Expiry, Period, Basis, EndMonthRule, InterpMethod, StrikeConvention)` using Black's model, derives an `NOPT`-by-`1` vector of prices of European put options on bonds.

If cash flows occur beyond the dates spanned by `ZeroData`, the input zero curve, the appropriate zero rate for discounting such cash flows is obtained by extrapolating the nearest rate on the curve (that is, if a cash flow occurs before the first or after the last date on the input zero curve, a flat curve is assumed).

In addition, you can use the Financial Instruments Toolbox method `getZeroRates` for an `IRDataCurve` object with a `Dates` property to create a vector of dates and data acceptable for `bkput`. For more information, see "Converting an IRDataCurve or IRFunctionCurve Object" on page 9-37.

## Examples

This example is based on example 22.2, page 514, of Hull. (See References below.)

Consider a European put option on a bond maturing in 10 years. The underlying bond has a clean price of $122.82, a face value of $100, and pays 8% semiannual coupons. Also, assume that the annualized volatility of the forward bond yield is 20%. Furthermore, suppose the option expires in 2.25 years and has a strike price of $115, and that the annualized continuously compounded risk free zero (spot) curve is flat at 5%. For a hypothetical settlement date of March 15, 2004, the following code illustrates the use of Black's model to duplicate the put prices in Example 22.2 of the Hull reference. In particular, it illustrates how to convert a broker's yield volatility to a price volatility suitable for Black's model.

```
% Specify the option information.
Settle       =  '15-Mar-2004';
Expiry       =  '15-Jun-2006';  % 2.25 years from settlement
Strike       =  115;
YieldSigma   =  0.2;
Convention   =  [0; 1];

% Specify the interest-rate environment. Since the
% zero curve is flat, interpolation into the curve always returns
% 0.05. Thus, the following curve is not unique to the solution.
ZeroData     = [datenum('15-Jun-2004') 0.05   -1;
                datenum('15-Dec-2004') 0.05   -1;
                datenum(Expiry)        0.05   -1];

% Specify the bond information.
CleanPrice   =  122.82;
CouponRate   =  0.08;
Maturity     = '15-Mar-2014';  % 10 years from settlement
Face         =  100;
BondData     = [CleanPrice CouponRate datenum(Maturity) Face];
Period       =  2;  % semiannual coupons
Basis        =  1;  % 30/360 day-count basis
```

```
% Convert a broker's yield volatility quote to a price volatility
% required by Black's model. To duplicate Example 22.2 in Hull,
% first compute the periodic (semiannual) yield to maturity from
% the clean bond price.
Yield  = bndyield(CleanPrice, CouponRate, Settle, Maturity,...
Period, Basis);

% Compute the duration of the bond at option expiration. Most
% fixed-income sensitivity analyses use the modified duration
% statistic to examine the impact of small changes in periodic
% yields on bond prices. However, Hull's example operates in
% continuous time (annualized instantaneous volatilities and
% continuously compounded zero yields for discounting coupons).
% To duplicate Hull's results, use the second output of BNDDURY,
% the Macaulay duration.
[Modified, Macaulay] = bnddury(Yield, CouponRate, Expiry,...
Maturity, Period, Basis);

% Convert the yield-to-maturity from a periodic to a
% continuous yield.
Yield  = Period .* log(1 + Yield./Period);

% Finally, convert the yield volatility to a price volatility via
% Hull's Equation 22.6 (page 514).
PriceSigma = Macaulay .* Yield .* YieldSigma;

% Finally, call Black's model.
PutPrices  = bkput(Strike, ZeroData, PriceSigma, BondData,...
Settle, Expiry, Period, Basis, [], [], Convention)
PutPrices =

    1.7838
    2.4071
```

When the strike price is the dirty price (`Convention = 0`), the call option value is $1.78. When the strike price is the clean price (`Convention = 1`), the call option value is $2.41.

**References**

[1] Hull, John C., *Options, Futures, and Other Derivatives*, Prentice Hall, 5th edition, 2003, pp. 287-288, 508-515.

**See Also**

`bkcall`

## Purpose

Implied repo rates for bond future given price

## Syntax

```
ImpRepo = bndfutimprepo(Price, FutPrice, FutSettle,
Delivery, ConvFactor, CouponRate, Maturity)
ImpRepo = bndfutimprepo(Price, FutPrice, FutSettle,
Delivery, ConvFactor, CouponRate, Maturity,
'ParameterName', 'ParameterValue ...)
```

## Description

`ImpRepo = bndfutimprepo(Price, FutPrice, FutSettle, Delivery, ConvFactor, CouponRate, Maturity)` computes the implied repo rate for a bond future given the price of a bond, the bond properties, the price of the bond future, and the bond conversion factor.

`ImpRepo = bndfutimprepo(Price, FutPrice, FutSettle, Delivery, ConvFactor, CouponRate, Maturity, 'ParameterName', 'ParameterValue ...)` accepts optional inputs as one or more comma-separated parameter/value pairs. `'ParameterName'` is the name of the parameter inside single quotes. `ParameterValue` is the value corresponding to `'ParameterName'`. Specify parameter-value pairs in any order. Names are case-insensitive.

## Input Arguments

### Price

`numBonds`-by-`1` vector of bond prices.

### FutPrice

`numBonds`-by-`1` vector of future prices

### FutSettle

`numBonds`-by-`1` vector of future settle dates.

### Delivery

`numBonds`-by-`1` vector of future delivery dates.

**ConvFactor**

`numBonds`-by-`1` vector of bond conversion factors. For more information, see `convfactor`.

**CouponRate**

`numBonds`-by-`1` vector of coupon rates in decimal form.

**Maturity**

`numBonds`-by-`1` vector of coupon rates in decimal form.

## Parameter–Value Pairs

**Basis**

Day-count basis. Possible values include

- 0 = actual/actual (default)
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (BMA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/actual (ISDA)
- 13 = BUS/252

For more information, see basis.

**Default:** 0

### EndMonthRule

End-of-month rule. Values are:

- `0` — Rule is not in effect for the bond.
- 1 — Rule is in effect for the bond. This means that a security that pays coupon interest on the last day of the month always makes payment on the last day of the month.

**Default:** 1

### Face

Face value of the bond. `Face` has no impact on key rate duration. This calling sequence is preserved for consistency.

**Default:** 100

### FirstCouponDate

Date when a bond makes its first coupon payment; used when bond has an irregular first coupon period. When `FirstCouponDate` and `LastCouponDate` are both specified, `FirstCouponDate` takes precedence in determining the coupon payment structure.

**Default:** If you do not specify a `FirstCouponDate`, the cash flow payment dates are determined from other inputs.

### IssueDate

Issue date for a bond.

### LastCouponDate

Last coupon date of a bond before the maturity date; used when bond has an irregular last coupon period. In the absence of a specified `FirstCouponDate`, a specified `LastCouponDate` determines the coupon structure of the bond. The coupon structure of a bond is truncated at the `LastCouponDate`, regardless of where it falls, and is followed only by the bond's maturity cash flow date.

> **Default:** If you do not specify a `LastCouponDate`, the cash flow payment dates are determined from other inputs.

### Period

Number of coupons payments per year. Possible values include:

- 0
- 1
- 2
- 3
- 4
- 6
- 12

> **Default:** 2

### ReinvestBasis

Day count basis for resinvestment rate.

> **Default:** Identical to `RepoBasis`.

### ReinvestRate

Rate for reinvesting intermediate coupons from the bond.

> **Default:** Identical to `ImpRepo`.

#### RepoBasis

Day count basis for `ImpRepo`.

**Default:** 2

#### StartDate

Date when a bond actually starts (the date from which a bond cash flow is considered). To make an instrument forward-starting, specify this date as a future date. If you do not specify `StartDate`, the effective start date is the `Settle` date.

## Output Arguments

#### ImpRepo

Implied repo rate, or the repo rate that would produce the price input.

## Definitions

`bndfutimprepo` computes the implied repo rate for a bond future given:

- Price of a bond
- Bond properties
- Price of the bond future
- Bond conversion factor

The default behavior is that the coupon reinvestment rate matches the repo rate. However, you can specify a separate reinvestment rate using optional inputs.

## Examples

Compute the repro rate for a bond future:

```
bndfutimprepo(129,98,'9/21/2000','12/29/2000',1.3136,.0875,'8/15/2020')
```

This returns:

```
ans =
    0.0584
```

## References

Burghardt, G., T. Belton, M. Lane, and J. Papa, *The Treasury Bond Basis*, McGraw-Hill, 2005.

Krgin, Dragomir, *Handbook of Global Fixed Income Calculations*, John Wiley & Sons, 2002.

## See Also

`bndfutprice` | `convfactor`

## How To

- "Bond Futures" on page 7-12

## Purpose

Price bond future given repo rates

## Syntax

```
[FutPrice,AccrInt] = bndfutprice(RepoRate, Price, FutSettle,
Delivery, ConvFactor, CouponRate, Maturity)
FutPrice,AccrInt] = bndfutprice(RepoRate, FutPrice, FutSettle,
Delivery, ConvFactor, CouponRate, Maturity,
'ParameterName','ParameterValue ...)
```

## Description

`[FutPrice,AccrInt] = bndfutprice(RepoRate, Price, FutSettle, Delivery, ConvFactor, CouponRate, Maturity)` computes the price of a bond futures contract for one or more bonds given a repo rate, and bond properties, including the bond conversion factor.

`FutPrice,AccrInt] = bndfutprice(RepoRate, FutPrice, FutSettle, Delivery, ConvFactor, CouponRate, Maturity, 'ParameterName','ParameterValue ...)` accepts optional inputs as one or more comma-separated parameter/value pairs. `'ParameterName'` is the name of the parameter inside single quotes. `ParameterValue` is the value corresponding to `'ParameterName'`. Specify parameter-value pairs in any order. Names are case-insensitive.

## Input Arguments

### RepoRate

`numBonds`-by-`1` vector of repo rates.

### Price

`numBonds`-by-`1` vector of bond prices

### FutSettle

`numBonds`-by-`1` vector of future settle dates.

### Delivery

`numBonds`-by-`1` vector of future delivery dates.

### ConvFactor

`numBonds`-by-`1` vector of bond conversion factors. For more information, see `convfactor`.

### CouponRate

`numBonds`-by-`1` vector of coupon rates in decimal form.

### Maturity

`numBonds`-by-`1` vector of coupon rates in decimal form.

## Parameter–Value Pairs

### Basis

Day-count basis. Possible values include

- 0 = actual/actual (default)
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (BMA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/actual (ISDA)

- 13 = BUS/252

For more information, see basis.

**Default:** 0

### EndMonthRule

End-of-month rule. Values are:

- `0` — Rule is not in effect for the bond.
- 1 — Rule is in effect for the bond. This means that a security that pays coupon interest on the last day of the month always makes payment on the last day of the month.

**Default:** 1

### IssueDate

Issue date for a bond.

### Face

Face value of the bond. `Face` has no impact on key rate duration. This calling sequence is preserved for consistency.

**Default:** 100

### FirstCouponDate

Date when a bond makes its first coupon payment; used when bond has an irregular first coupon period. When `FirstCouponDate` and `LastCouponDate` are both specified, `FirstCouponDate` takes precedence in determining the coupon payment structure.

**Default:** If you do not specify a `FirstCouponDate`, the cash flow payment dates are determined from other inputs.

### LastCouponDate

Last coupon date of a bond before the maturity date; used when bond has an irregular last coupon period. In the absence of a specified `FirstCouponDate`, a specified `LastCouponDate` determines the coupon structure of the bond. The coupon structure of a bond is truncated at the `LastCouponDate`, regardless of where it falls, and is followed only by the bond's maturity cash flow date.

> **Default:** If you do not specify a `LastCouponDate`, the cash flow payment dates are determined from other inputs.

#### Period

Number of coupons payments per year. Possible values include:

- 0
- 1
- 2
- 3
- 4
- 6
- 12

> **Default:** 2

#### ReinvestBasis

Day count basis for reinvestment rate.

> **Default:** Identical to `RepoBasis`.

#### ReinvestRate

Compounding convention for reinvestment rate.

> **Default:** Identical to `RepoRate`.

**RepoBasis**

Day count basis for `RepoRate`.

**Default:** 2

**StartDate**

Date when a bond actually starts (the date from which a bond cash flow is considered). To make an instrument forward-starting, specify this date as a future date. If you do not specify `StartDate`, the effective start date is the `Settle` date.

## Output Arguments

**FutPrice**

Quoted futures price, per $100 notional.

**AccrInt**

Accrued interest due at delivery date, per $100 notional.

## Definitions

`bndfutprice` computes the price of a bond futures contract for one or more bonds, given a repo rate, and bond properties, including the bond conversion factor. The default behavior is that the coupon reinvestment rate matches the repo rate. However, you can specify a separate reinvestment rate using optional inputs.

## Examples

Compute the price for a bond future:

```
bndfutprice(.064, 129, '9/21/2000','12/29/2000', 1.3136, .0875, '8/15/2020')
```

The returns:

```
ans =
   98.1516
```

## References

Burghardt, G., T. Belton, M. Lane, and J. Papa, *The Treasury Bond Basis*, McGraw-Hill, 2005.

Krgin, Dragomir, *Handbook of Global Fixed Income Calculations*, John Wiley & Sons, 2002.

## See Also

`bndfutimprepo` | `convfactor`

## How To

- "Bond Futures" on page 7-12

## Purpose

Bootstrap interest-rate curve from market data

## Class

`@IRDataCurve`

## Syntax

```
Dcurve = IRDataCurve.bootstrap(Type, Settle, InstrumentTypes,
Instruments)
```

```
Dcurve = IRDataCurve.bootstrap(Type, Settle, InstrumentTypes,
Instruments, 'Parameter1', Value1, 'Parameter2',
Value2, ...)
```

## Arguments

| | |
|---|---|
| `Type` | Type of interest-rate curve. Acceptable values are: `discount`, `forward`, or `zero`. |
| `Settle` | Scalar or column vector of settlement dates. |
| `InstrumentTypes` | `N`-by-`1` cell array (where `N` is the number of instruments) indicating what kind of instrument is in the `Instruments` matrix. Acceptable values are `deposit`, `futures`, `swap`, and `bond`. |
| `Instruments` | `N`-by-`3` data matrix for `Instruments` where the first column is `Settle` date, the second column is `Maturity`, and the third column is the market quote (dates must be MATLAB date numbers). |

---

**Note** The market quote represents the following for each instrument:

- `deposit`: rate
- `futures`: price (e.g., 9628.54)
- `swap`: rate
- `bond`: clean price

---

| | |
|---|---|
| `Compounding` | (Optional) Scalar that sets the compounding frequency per year for an `IRDataCurve` object:<br>• -1 = Continuous compounding<br>• 1 = Annual compounding<br>• 2 = Semiannual compounding (default)<br>• 3 = Compounding three times per year<br>• 4 = Quarterly compounding<br>• 6 = Bimonthly compounding<br>• 12 = Monthly compounding |

| | |
|---|---|
| `Basis` | (Optional) Day-count basis of the interest-rate curve. A scalar of integers.<br>• 0 = actual/actual (default)<br>• 1 = 30/360 (SIA)<br>• 2 = actual/360<br>• 3 = actual/365<br>• 4 = 30/360 (BMA)<br>• 5 = 30/360 (ISDA)<br>• 6 = 30/360 (European)<br>• 7 = actual/365 (Japanese)<br>• 8 = actual/actual (ICMA)<br>• 9 = actual/360 (ICMA)<br>• 10 = actual/365 (ICMA)<br>• 11 = 30/360E (ICMA)<br>• 12 = actual/actual (ISDA)<br>• 13 = BUS/252<br>For more information, see basis. |
| `InterpMethod` | (Optional) Values are:<br>• `'linear'` — Linear interpolation (default).<br>• `'constant'` — Piecewise constant interpolation.<br>• `'pchip'` — Piecewise cubic Hermite interpolation. |

| | |
|---|---|
| | - `'spline'` — Cubic spline interpolation. |
| `IRBootstrapOptionsObj` | (Optional) An `IRBootstrapOptions` object. |

## Instrument Parameters

For each bond `Instrument`, you can specify the following additional instrument parameters as parameter/value pairs. For example, `InstrumentBasis` distinguishes a bond instrument's `Basis` value from the curve's `Basis` value.

| | |
|---|---|
| `InstrumentCouponRate` | (Optional) Decimal number indicating the annual percentage rate used to determine the coupons payable on an instrument. |
| `InstrumentPeriod` | (Optional) Coupons per year of the instrument. A vector of integers. Allowed values are 0, 1, 2 (default), 3, 4, 6, and 12. |
| `InstrumentBasis` | (Optional) Day-count basis of the instrument. A vector of integers.<br>• 0 = actual/actual (default)<br>• 1 = 30/360 (SIA)<br>• 2 = actual/360<br>• 3 = actual/365<br>• 4 = 30/360 (BMA)<br>• 5 = 30/360 (ISDA)<br>• 6 = 30/360 (European)<br>• 7 = actual/365 (Japanese)<br>• 8 = actual/actual (ICMA)<br>• 9 = actual/360 (ICMA)<br>• 10 = actual/365 (ICMA) |

| | |
|---|---|
| | • 11 = 30/360E (ICMA)<br>• 12 = actual/actual (ISDA)<br>• 13 = BUS/252<br>For more information, see basis. |
| `InstrumentEndMonthRule` | (Optional) End-of-month rule. A vector. This rule applies only when `Maturity` is an end-of-month date for a month having 30 or fewer days. 0 = ignore rule, meaning that an instrument's coupon payment date is always the same numerical day of the month. 1 = set rule on (default), meaning that an instrument's coupon payment date is always the last actual day of the month. |
| `InstrumentIssueDate` | (Optional) Date when an instrument was issued. |
| `InstrumentFirstCouponDate` | (Optional) Date when a bond makes its first coupon payment; used when bond has an irregular first coupon period. When `FirstCouponDate` and `LastCouponDate` are both specified, `FirstCouponDate` takes precedence in determining the coupon payment structure. If you do not specify a `FirstCouponDate`, the cash flow payment dates are determined from other inputs. |

| | |
|---|---|
| `InstrumentLastCouponDate` | (Optional) Last coupon date of a bond before the maturity date; used when bond has an irregular last coupon period. In the absence of a specified `FirstCouponDate`, a specified `LastCouponDate` determines the coupon structure of the bond. The coupon structure of a bond is truncated at the `LastCouponDate`, regardless of where it falls, and is followed only by the bond's maturity cash flow date. If you do not specify a `LastCouponDate`, the cash flow payment dates are determined from other inputs. |
| `InstrumentFace` | (Optional) Face or par value. Default = 100. |

---

**Note** When using `Instrument` parameter/value pairs, you can specify simple interest for an `Instrument` by specifying the `InstrumentPeriod` value as `0`. If `InstrumentBasis` and `InstrumentPeriod` are not specified for an `Instrument`, the following default values are used:

- `deposit` instrument uses `Basis` as `2` (act/360) and `Period` is `0` (simple interest).
- `futures` instrument uses `Basis` as `2` (act/360) and `Period` is `4` (quarterly).
- `swap` instrument uses `Basis` as `2` (act/360) and `Period` is `2`.
- `bond` instrument uses `Basis` as `0` (act/act) and `Period` is `2`.

---

## Description

`Dcurve = IRDataCurve.bootstrap(Type, Settle, InstrumentTypes, Instruments, 'Parameter1', Value1, 'Parameter2', Value2, ...)` bootstraps an interest-rate curve from market data. The dates of the bootstrapped curve correspond to the maturity dates of the input instruments. You must enter the optional arguments for `Basis`, `Compounding`, `Interpmethod`, and `IRBootstrapOptionsObj` as parameter/value pairs.

## Examples

In this bootstrapping example, `InstrumentTypes`, `Instruments`, and a `Settle` date are defined:

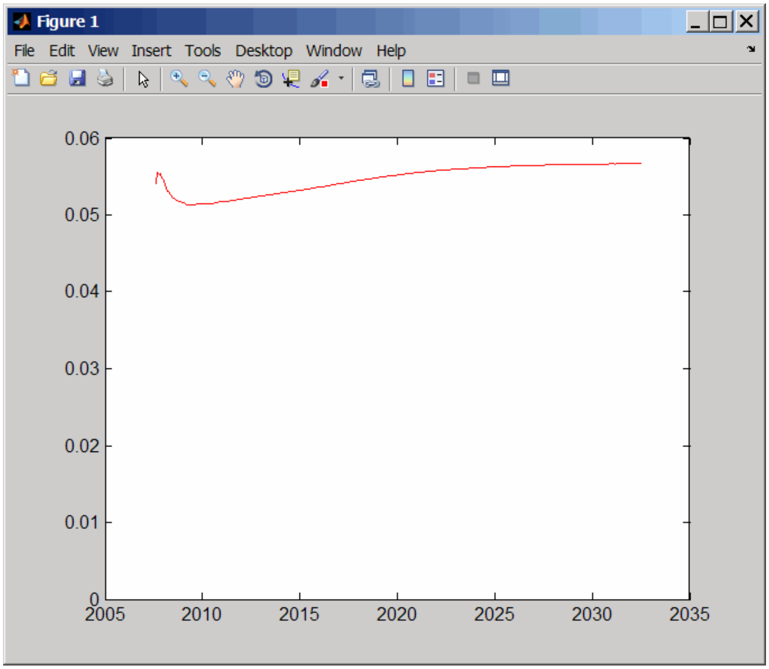
```
InstrumentTypes = {'Deposit';'Deposit';...
'Futures';'Futures';'Futures';'Futures';'Futures';'Futures';...
'Swap';'Swap';'Swap';'Swap';};

Instruments = [datenum('08/10/2007'),datenum('09/17/2007'),.0532000; ...
datenum('08/10/2007'),datenum('11/17/2007'),.0535866; ...
datenum('08/08/2007'),datenum('19-Dec-2007'),9485; ...
datenum('08/08/2007'),datenum('19-Mar-2008'),9502; ...
datenum('08/08/2007'),datenum('18-Jun-2008'),9509.5; ...
datenum('08/08/2007'),datenum('17-Sep-2008'),9509; ...
datenum('08/08/2007'),datenum('17-Dec-2008'),9505.5; ...
datenum('08/08/2007'),datenum('18-Mar-2009'),9501; ...
datenum('08/08/2007'),datenum('08/08/2014'),.0530; ...
datenum('08/08/2007'),datenum('08/08/2019'),.0551; ...
datenum('08/08/2007'),datenum('08/08/2027'),.0565; ...
datenum('08/08/2007'),datenum('08/08/2037'),.0566];

CurveSettle = datenum('08/10/2007');
```

Use the `bootstrap` method to create an `IRDataCurve` object.

```
bootModel = IRDataCurve.bootstrap('Forward', CurveSettle, ...
InstrumentTypes, Instruments,'InterpMethod','pchip')

bootModel =

 IRDataCurve

    Type: Forward
     Settle: 733264 (10-Aug-2007)
   Compounding: 2
   Basis: 0 (actual/actual)
  InterpMethod: pchip
   Dates: [12x1 double]
    Data: [12x1 double]
```

To create the plot for the bootstrapped market data:

```
PlottingDates = (datenum('08/11/2007'):30:CurveSettle+365*25)';
plot(PlottingDates,bootModel.getParYields(PlottingDates),'r')
set(gca,'ylim',[0 .06])
datetick
```



For an example of bootstrapping using instrument parameters support for prepending the word `Instrument` to the parameter field, see “IRDataCurve Bootstrapping Based on Market Instruments” on page 9-7.

## How To

- “@IRDataCurve” on page A-7
- “@IRBootstrapOptions” on page A-2

## Purpose

Price convertible bond

## Syntax

```
[CbMatrix, UndMatrix, DebtMatrix, EqtyMatrix] =
cbprice(RiskFreeRate, StaticSpread, Sigma, Price, ConvRatio,
NumSteps, IssueDate, Settle, Maturity, CouponRate)
[CbMatrix, UndMatrix, DebtMatrix, EqtyMatrix] =
cbprice(RiskFreeRate, StaticSpread, Sigma, Price, ConvRatio,
NumSteps, IssueDate, Settle, Maturity, CouponRate,
Name,Value)
```

## Description

`[CbMatrix, UndMatrix, DebtMatrix, EqtyMatrix] = cbprice(RiskFreeRate, StaticSpread, Sigma, Price, ConvRatio, NumSteps, IssueDate, Settle, Maturity, CouponRate)` price a convertible bond with a one-factor lattice method.

`[CbMatrix, UndMatrix, DebtMatrix, EqtyMatrix] = cbprice(RiskFreeRate, StaticSpread, Sigma, Price, ConvRatio, NumSteps, IssueDate, Settle, Maturity, CouponRate, Name,Value)` price a convertible bond with a one-factor lattice method with additional options specified by one or more `Name,Value` pair arguments.

## Input Arguments

### RiskFreeRate

Annual yield of the risk-free bond with the same maturity as the convertible, compounded continuously. Scalar value of risk-free rates is in decimal. (Recommended value is the yield of a risk-free bond with the same maturity as the convertible.)

### StaticSpread

Scalar value of the constant spread to risk-free rate. Adding `StaticSpread` to the `RiskFreeRate` produces the issuer's yield, which reflects the credit risk.

### Sigma

Scalar value of the annual volatility environment in decimal.

**Price**

Scalar value of the price of the asset at the settlement or valuation date.

**ConvRatio**

Scalar value of the number of assets convertible to one bond.

**NumSteps**

Scalar value of the number of steps within the binomial tree.

**IssueDate**

Scalar value of the issue date of the convertible bond.

**Settle**

Scalar value of the settlement date of the convertible bond.

**Maturity**

Scalar value of the maturity date of the convertible bond.

**CouponRate**

Scalar value of the coupon rate in decimal form or a `C`-by-`2` vector of dates and associated coupon rates.

## Name-Value Pair Arguments

Specify optional comma-separated pairs of `Name,Value` arguments, where `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside single quotes (`' '`). You can specify several name and value pair arguments in any order as `Name1,Value1,...,NameN,ValueN`.

**Basis**

Day-count basis of the bond. A vector of integers.

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (BMA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/actual (ISDA)
- 13 = BUS/252

For more information, see basis.

**Default:** `0` (actual/actual)

### CallType

Scalar value for the call type. Values are `0` for a call on cash price, or `1` for a call on clean price.

**Default:** `0` (cash price)

### CallInfo

Two-column matrix containing the call information. The first column is the call dates and the second column is the call prices for every $100 face of the bond. The call, in the amount of call prices, is activated *after* the corresponding call date.

**Default:** No call feature

### ConvInfo

Two-column matrix containing convertible information. The first column is the convertible dates and the second column is whether the issue is convertible or not.

**Default:** Bond is always convertible

### DividendInfo

Two-column matrix of dividend information. The first column is the ex-dividend date and the second column is the corresponding amount. Enter any amount known at any time; only the amounts that are within the lifespan of the option are used. If the `DividendType` is `2`, `DividendInfo` is a `1`-by-`2` matrix where the first entry is the `Settle` date and the second entry is the continuous dividend yield.

**Default:** No dividend

### DividendType

Scalar value for dividend type. Values are:

- `0` — Dollar dividend
- `1` — Dividend yield
- `2` — Continuous dividend yield

**Default:** `0` (Dollar dividend)

### EndMonthRule

`NINST`-by-1 vector for end-of-month rule. Values are `1` (on, in effect) and `0` (off, not in effect).

**Default:** `1` (on, in effect)

### Period

Scalar value for number of coupon payments. Values are:

- `1` — One coupon per year
- `2` — Semiannual
- `3` — Three times a year
- `4` — Quarterly
- `6` — Bimonthly compounding
- `12` — Monthly

> **Default:** `2` (Semiannual)

### IssueDate

`NINST`-by-`1` vector of bond issue date.

> **Default:** If you do not specify an `IssueDate`, the cash flow payment date is determined from other inputs.

### FirstCouponDate

Date when a bond makes its first coupon payment; used when bond has an irregular first coupon period. When `FirstCouponDate` and `LastCouponDate` are both specified, `FirstCouponDate` takes precedence in determining the coupon payment structure.

> **Default:** If you do not specify a `FirstCouponDate`, the cash flow payment dates are determined from other inputs.

### LastCouponDate

Last coupon date of a bond before the maturity date; used when bond has an irregular last coupon period. In the absence of a specified `FirstCouponDate`, a specified `LastCouponDate` determines the coupon structure of the bond. The coupon structure of a bond is truncated at

the `LastCouponDate`, regardless of where it falls, and is followed only by the bond's maturity cash flow date.

> **Default:** If you do not specify a `LastCouponDate`, the cash flow payment dates are determined from other inputs.

### Period

`NINST`-by-`1` vector for coupons per year.

> **Default:** `2` per year

### PutInfo

Two-column matrix containing put information. The first column is the put dates and the second column is the put prices for every $100 face of the bond. The put, in the amount of put prices, is activated *after* the corresponding put date.

> **Default:** No put feature

### PutType

Scalar value for put type. Value are `0` for a put on cash price or `1` for a put on clean price.

> **Default:** `0` (put on cash price)

### TreeType

Scalar value for tree type. Values are `0` for binomial lattice or `1` for trinomial lattice.

> **Default:** `0` (binomial lattice)

## Output Arguments

### CbMatrix

Matrix of CB prices in binomial format. Price of convertible is `CbMatrix(1,1)`.

**UndMatrix**

Matrix of stock prices in binomial format.

**DebtMatrix**

Matrix of CB debt component in binomial format.

**EqtyMatrix**

Matrix of CB equity component in binomial format.

## Definitions

### Convertible Bond

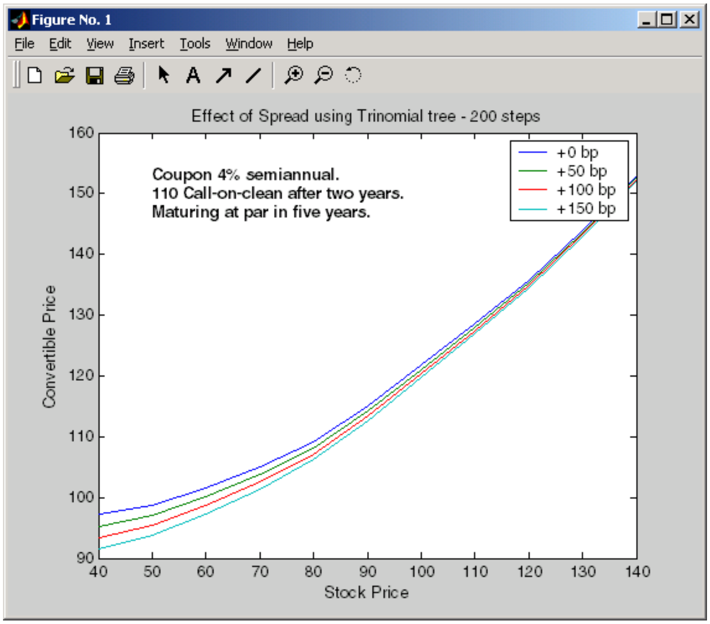
A convertible bond (CB) is a debt instrument that you can convert into a predetermined amount of the issuing company's equity at certain times before the bond's maturity. In addition to standard bond features (for example, maturity date, face value, coupon), a convertible bond often has callable and puttable features.

## Examples

Perform a spread effect analysis of a 4% coupon convertible bond callable at 110 at the end of the second year, maturing at par in 5 years, with yield to maturity of 5%, and spread (of yield to maturity versus 5-year treasury) of 0, 50, 100, and 150 basis points. The underlying stock pays no dividend.

```
RiskFreeRate = 0.05;
Sigma        = 0.3;
ConvRatio    = 1;
NumSteps     = 200;
IssueDate    = '2-Jan-2002';
Settle       = '2-Jan-2002';
Maturity     = '2-Jan-2007';
CouponRate   = 0.04;
Period       = 2;
Basis        = 1;
EndMonthRule = 1;
DividendType = 0;
DividendInfo = [];
```

```
CallInfo     = [datenum('2-Jan-2004') , 110];
CallType     = 1;
TreeType     = 1;
Spreads = 0:0.005:0.015;
Prices = 40:10:140;
stock = repmat(Prices',1,length(Spreads));

convprice = zeros(length(Prices),length(Spreads));

  for spreadidx = 1:length(Spreads)
      for priceidx = 1:length(Prices)
        [CbMatrix, UndMatrix, DebtMatrix, EqtyMatrix] = ...
           cbprice(RiskFreeRate, Spreads(spreadidx), Sigma, Prices(priceidx), ...
           ConvRatio, NumSteps, IssueDate, Settle, ...
           Maturity, CouponRate, Period, Basis, EndMonthRule, ...
           DividendType, DividendInfo, CallType, CallInfo, TreeType);

        convprice(priceidx,spreadidx) = CbMatrix(1,1);
      end
  end

 plot(stock,convprice);
 legend({'+0 bp'; '+50 bp'; '+100 bp'; '+150 bp'});
 title ('Effect of Spread using Trinomial tree - 200 steps')
 xlabel('Stock Price');
 ylabel('Convertible Price');
 text(50, 150, ['Coupon 4 semiannual,', sprintf('\n'), ...
 '110 Call-on-clean after 2 years,' sprintf('\n'), ...
 'maturing par in 5 years'],'fontweight','Bold')
```



## References

Andersen, L. and D. Buffum, “Calibration and implementation of convertible bonds models,” Working paper, Banc of America Securities, 2003.

Ayache, E., P.A. Forsyth, and K.R. Vetzal, “Valuation of Convertible Bonds with Credit Risk,” *Journal of Derivatives*, 11 (Fall 2003), 9-29.

Tsiveriotis, K. and C. Fernandes, “Valuing Convertible Bonds with Credit Risk,” *Journal of Fixed Income* 8, 95-102, 1998

Zabolotnyuk, Yuriy, Jones, Robert A. and Veld, Chris H., “An Empirical Comparison of Convertible Bond Valuation Models,” (October 15, 2009). Available at SSRN: `http://ssrn.com/abstract=994805`.

## Tutorials

- “Convertible Bond Valuation” on page 7-10

**Purpose** Accrued interest on certificate of deposit

**Syntax**

```
AccrInt = cdai(CouponRate, Settle, Maturity, IssueDate, Basis)
```

**Arguments**

| | |
|---|---|
| `CouponRate` | Annual interest rate in decimal. |
| `Settle` | Settlement date. `Settle` must be earlier than `Maturity`. |
| `Maturity` | Maturity date. |
| `IssueDate` | Issue date. |
| `Basis` | (Optional) Day-count basis of the instrument.<br>• 0 = actual/actual (default)<br>• 1 = 30/360 (SIA)<br>• 2 = actual/360<br>• 3 = actual/365<br>• 4 = 30/360 (BMA)<br>• 5 = 30/360 (ISDA)<br>• 6 = 30/360 (European)<br>• 7 = actual/365 (Japanese)<br>• 8 = actual/actual (ICMA)<br>• 9 = actual/360 (ICMA)<br>• 10 = actual/365 (ICMA)<br>• 11 = 30/360E (ICMA)<br>• 12 = actual/actual (ISDA)<br>• 13 = BUS/252<br>For more information, see basis. |

Each required input must be some certificates of deposit (`NCDS`)-by-`1` or `1`-by-`NCDS` conforming vector or scalar. The optional `Basis` argument may be either a `NCDS`-by-`1` or a `1`-by-`NCDS` vector, a scalar, or the empty matrix (`[]`).

## Description

`AccrInt = cdai(CouponRate, Settle, Maturity, IssueDate, Basis)` computes the accrued interest on a certificate of deposit.

`AccrInt` represents the accrued interest per $100 of face value.

This function assumes that the certificates of deposit pay interest at maturity. Because of the simple interest treatment of these securities, the function is best used for short-term maturities (less than 1 year). The default simple interest calculation is the actual/360 convention (SIA).

## Examples

Given a certificate of deposit with these characteristics, compute the accrued interest due.

```
CouponRate      =  0.05;
Settle          =  '02-Jan-02';
Maturity        =  '31-Mar-02';
IssueDate       =  '1-Oct-01';

AccrInt = cdai(CouponRate, Settle, Maturity, IssueDate)

AccrInt =

    1.2917
```

## See Also

`accrfrac` | `bndyield` | `stepcpnyield` | `tbillyield` | `zeroyield`

# cdprice

## Purpose

Price of certificate of deposit

## Syntax

```
[Price, AccrInt] = cdprice(Yield, CouponRate, Settle, Maturity,
IssueDate, Basis)
```

## Arguments

| | |
|---|---|
| `Yield` | Simple yield to maturity over the basis denominator. |
| `CouponRate` | Coupon interest rate in decimal. |
| `Settle` | Settlement date. `Settle` must be earlier than `Maturity`. |
| `Maturity` | Maturity date. |
| `IssueDate` | Issue date. |
| `Basis` | (Optional) Day-count basis of the instrument.<br>• 0 = actual/actual (default)<br>• 1 = 30/360 (SIA)<br>• 2 = actual/360<br>• 3 = actual/365<br>• 4 = 30/360 (BMA)<br>• 5 = 30/360 (ISDA)<br>• 6 = 30/360 (European)<br>• 7 = actual/365 (Japanese)<br>• 8 = actual/actual (ICMA)<br>• 9 = actual/360 (ICMA)<br>• 10 = actual/365 (ICMA)<br>• 11 = 30/360E (ICMA)<br>• 12 = actual/actual (ISDA) |

- 13 = BUS/252

For more information, see basis.

Each required input must be some certificates of deposit (`NCDS`)-by-`1` or `1`-by-`NCDS` conforming vector or scalar. The optional `Basis` argument may be either a `NCDS`-by-`1` or a `1`-by-`NCDS` vector, a scalar, or the empty matrix (`[]`).

## Description

`[Price, AccrInt] = cdprice(Yield, CouponRate, Settle, Maturity, IssueDate, Basis)` computes the price of a certificate of deposit given its yield.

`Price` is the clean price of the certificate of deposit per $100 of face value.

`AccruedInt` is the accrued interest payable at settlement per unit of face value.

This function assumes that the certificates of deposit pay interest at maturity. Because of the simple interest treatment of these securities, the function is best used for short-term maturities (less than 1 year). The default simple interest calculation is the actual/360 convention.

## Examples

Given a certificate of deposit with these characteristics, compute the price and the accrued interest due on the settlement date.

```
Yield          =  0.0525;
CouponRate     =  0.05;
Settle         =  '02-Jan-02';
Maturity       =  '31-Mar-02';
IssueDate      =  '1-Oct-01';

[Price, AccruedInt] = cdprice(Yield, CouponRate, Settle, ...
Maturity, IssueDate)

Price =

    99.9233
```

```
AccruedInt =

    1.2917
```

**See Also** `bndprice` | `cdai` | `cdyield` | `stepcpnprice` | `tbillprice`

## Purpose

Bootstrap default probability curve from credit default swap market quotes

## Syntax

`[ProbData, HazData] = cdsbootstrap(ZeroData, MarketData, Settle)`
`[ProbData, HazData] = cdsbootstrap(ZeroData, MarketData, Settle, Name,Value)`

## Description

`[ProbData, HazData] = cdsbootstrap(ZeroData, MarketData, Settle)` bootstraps the default probability curve using credit default swap (CDS) market quotes. The market quotes can be expresed as a list of maturity dates and corresponding CDS market spreads, or as a list of maturities and corresponding upfronts and standard spreads for standard CDS contracts. The estimation uses the standard model of the survival probability.

`[ProbData, HazData] = cdsbootstrap(ZeroData, MarketData, Settle, Name,Value)` bootstraps the default probability curve using CDS market quotes with additional options specified by one or more `Name,Value` pair arguments. The market quotes can be expresed as a list of maturity dates and corresponding CDS market spreads, or as a list of maturities and corresponding upfronts and standard spreads for standard CDS contracts. The estimation uses the standard model of the survival probability.

## Input Arguments

### ZeroData

`M`-by-`2` vector of dates and zero rates or `IRCurve` of zero rates.

### MarketData

`N`-by-`2` matrix of dates and corresponding market spreads or `N`-by-`2` matrix of dates, upfronts, and standard spreads of CDS contracts.

### Settle

Settlement date is a serial date number or date string. This must be earlier than or equal to the dates in `MarketData`.

### Name-Value Pair Arguments

Specify optional comma-separated pairs of `Name,Value` arguments, where `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside single quotes (`' '`). You can specify several name and value pair arguments in any order as `Name1,Value1,...,NameN,ValueN`.

---

**Note** Any optional input of size `N`-by-`1` is also acceptable as an array of size `1`-by-`N`, or as a single value applicable to all contracts. Single values are internally expanded to an array of size `N`-by-`1`.

---

#### Basis

`N`-by-`1` vector of day-count basis of the CDS:

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (BMA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/actual (ISDA)
- 13 = BUS/252

For more information, see basis.

**Default:** `2` (actual/360)

### BusDayConvention

String or `N`-by-`1` cell array of strings of business day conventions. Values are:

- `actual`
- `follow`
- `modifiedfollow`
- `previous`
- `modifiedprevious`

**Default:** `actual`

### PayAccruedPremium

`N`-by-`1` vector of Boolean flags, `True` (default), if accrued premiums are paid upon default, `False` otherwise.

**Default:** `True`

### Period

`N`-by-`1` vector of the number of premiums per year of the CDS. Allowed values are 1, 2, 3, 4, 6, and 12.

**Default:** `4`

### ProbDates

`P`-by-`1` vector of dates for `ProbData`.

**Default:** Column of dates in `MarketData`

**RecoveryRate**

`N`-by-`1` vector of recovery rates, expressed as a decimal from `0` to `1`.

**Default:** `0.4`

**TimeStep**

Positive integer indicating the number of days to take as time step for the numerical integration.

**Default:** `10` (days)

**ZeroBasis**

Basis of the zero curve. Choices are identical to `Basis`.

**Default:** `0` (actual/actual)

**ZeroCompounding**

Compounding frequency of the zero curve. Allowed values are:

- 1 — Annual compounding
- 2 — Semiannual compounding
- 3 — Compounding three times per year
- 4 — Quarterly compounding
- 6 — Bimonthly compounding
- 12 — Monthly compounding
- -1 — Continuous compounding

---

**Note** When `ZeroData` is an `IRCurve` object, the arguments `ZeroCompounding` and `ZeroBasis` are implicit in `ZeroData` and are redundant inside this function. In that case, specify these optional arguments when constructing the `IRCurve` object before calling this function.

---

**Default:** `2` (Semiannual compounding)

## Output Arguments

### ProbData

`P`-by-`2` matrix with dates and corresponding cumulative default probability values. The dates match those in `MarketData`, unless the optional input parameter `ProbDates` is provided.

### HazData

`N`-by-`2` matrix with dates and corresponding hazard rate values for the standard survival probability model. The dates match those in `MarketData`.

---

**Note** A warning is displayed when non-monotone default probabilities (i.e., negative hazard rates) are found.

---

## Examples

Use `cdsbootstrap` with market quotes for CDS contracts to generate `ProbData` and `HazData` values:

```
Settle = '17-Jul-2009';
Spread_Time = [1 2 3 5 7]';
Spread = [140 175 210 265 310]';
Market_Dates = daysadd(datenum(Settle),360*Spread_Time,1);
MarketData = [Market_Dates Spread];
Zero_Time = [.5 1 2 3 4 5]';
Zero_Rate = [1.35 1.43 1.9 2.47 2.936 3.311]'/100;
Zero_Dates = daysadd(datenum(Settle),360*Zero_Time,1);
```

```
ZeroData = [Zero_Dates Zero_Rate];

[ProbData,HazData] = cdsbootstrap(ZeroData,MarketData,Settle)
ProbData =

  1.0e+005 *

    7.3434    0.0000
    7.3470    0.0000
    7.3507    0.0000
    7.3580    0.0000
    7.3653    0.0000

HazData =

  1.0e+005 *

    7.3434    0.0000
    7.3470    0.0000
    7.3507    0.0000
    7.3580    0.0000
    7.3653    0.0000
```

## Algorithms

If the time to default is denoted by $\tau$, the default probability curve, or function, $PD(t)$, and its complement, the survival function $Q(t)$, are given by:

$$PD(t) = P[\tau \leq t] = 1 - P[\tau > t] = 1 - Q(t)$$

In the standard model, the survival probability is defined in terms of a piecewise constant hazard rate $h(t)$. For example, if $h(t)$ =

$\lambda_1$, for $0 \leq t \leq t_1$

$\lambda_2$, for $t_1 < t \leq t_2$

$\lambda_3$, for $t_2 < t$

then the survival function is given by $Q(t)$ =

$e^{-\lambda_1 t}$, for $0 \le t \le t_1$

$e^{-\lambda_1 t - \lambda_2 (t - t_1)}$, for $t_1 < t \le t_2$

$e^{-\lambda_1 t_1 - \lambda_2 (t_2 - t_1) - \lambda_3 (t - t_2)}$, for $t_2 < t$

Given $n$ market dates $t_1,\ldots,t_n$ and corresponding market CDS spreads $S_1,\ldots,S_n$, `cdsbootstrap` calibrates the parameters $\lambda_1,\ldots,\lambda_n$ and evaluates *PD(t)* on the market dates, or an optional user-defined set of dates.

## References

Beumee, J., D. Brigo, D. Schiemert, and G. Stoyle. “Charting a Course Through the CDS Big Bang,” *Fitch Solutions, Quantitative Research*, Global Special Report. April 7, 2009.

Hull, J., and A. White, “Valuing Credit Default Swaps I: No Counterparty Default Risk,” *Journal of Derivatives* 8, 29-40.

O’Kane, D. and S. Turnbull, “Valuation of Credit Default Swaps.” *Lehman Brothers, Fixed Income Quantitative Credit Research*, April, 2003.

## See Also

| `cdsspread` | `cdsprice`

## Tutorials

- “Credit Default Swap (CDS)” on page 8-2

# cdsoptprice

## Purpose

Price payer and receiver credit default swap options

## Syntax

```
[Payer, Receiver] = cdsoptprice(ZeroData, ProbData, Settle,
OptionMaturity, CDSMaturity, Strike, SpreadVol)
[Payer, Receiver] = cdsoptprice(ZeroData, ProbData, Settle,
OptionMaturity, CDSMaturity, Strike, SpreadVol,
Name,Value)
```

## Description

`[Payer, Receiver] = cdsoptprice(ZeroData, ProbData, Settle, OptionMaturity, CDSMaturity, Strike, SpreadVol)` computes the price of payer and receiver credit default swap options.

`[Payer, Receiver] = cdsoptprice(ZeroData, ProbData, Settle, OptionMaturity, CDSMaturity, Strike, SpreadVol, Name,Value)` computes the price of payer and receiver credit default swap options with additional options specified by one or more `Name,Value` pair arguments.

## Input Arguments

### ZeroData

`M`-by-`2` vector of dates and zero rates or `IRCurve` of zero rates.

### ProbData

`P`-by-`2` array of dates and default probabilities.

### Settle

Settlement date is a serial date number or date string. `Settle` must be earlier than the maturity date.

### OptionMaturity

`N`-by-`1` vector of serial date numbers or date strings containing the option maturity dates.

### CDSMaturity

`N`-by-`1` vector of serial date numbers or date strings containing the CDS maturity dates.

**Strike**

`N`-by-`1` vector of option strikes expressed in basis points.

**SpreadVol**

`N`-by-`1` vector of annualized credit spread volatilities expressed as a positive decimal number.

## Name-Value Pair Arguments

Specify optional comma-separated pairs of `Name,Value` arguments, where `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside single quotes (`' '`). You can specify several name and value pair arguments in any order as `Name1,Value1,...,NameN,ValueN`.

---

**Note** Any optional input of size `N`-by-`1` is also acceptable as an array of size `1`-by-`N`, or as a single value applicable to all contracts. Single values are internally expanded to an array of size `N`-by-`1`.

---

**Basis**

`N`-by-`1` vector of contract day-count basis:

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (BMA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)

- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/actual (ISDA)
- 13 = BUS/252

For more information, see basis.

**Default:** `2` (actual/360)

### BusDayConvention

String or `N`-by-`1` cell array of strings of business day conventions. Values are:

- `actual`
- `follow`
- `modifiedfollow`
- `previous`
- `modifiedprevious`

**Default:** `actual`

### Knockout

`N`-by-`1` vector of Boolean flags. If the credit default swaptions is a knockout, the flag is `True`, otherwise it is `False`.

**Default:** `True`

### PayAccruedPremium

`N`-by-`1` vector of Boolean flags. If accrued premiums are paid upon default, the flag is `True`, otherwise it is `False`.

**Default:** `True`

### Period

`N`-by-`1` vector of the number of premiums per year of the CDS. Allowed values are 1, 2, 3, 4, 6, and 12.

**Default:** `4`

### RecoveryRate

`N`-by-`1` vector of recovery rates, expressed as a decimal from `0` to `1`.

**Default:** `0.4`

### ZeroBasis

Basis of the zero curve. Choices are identical to `Basis`.

**Default:** `0` (actual/actual)

### ZeroCompounding

Compounding frequency of the zero curve. Allowed values are:

- 1 — Annual compounding
- 2 — Semiannual compounding
- 3 — Compounding three times per year
- 4 — Quarterly compounding
- 6 — Bimonthly compounding
- 12 — Monthly compounding
- −1 — Continuous compounding

---

**Note** When `ZeroData` is an `IRCurve` object, the arguments `ZeroCompounding` and `ZeroBasis` are implicit in `ZeroData` and are redundant inside this function. In that case, specify these optional arguments when constructing the `IRCurve` object before calling this function.

---

**Default:** `2` (Semiannual compounding)

## Output Arguments

### Payer

`N`-by-`1` vector of prices for payer swap options in `Basis` points.

### Receiver

`N`-by-`1` vector of prices for receiver swap options in `Basis` points.

## Definitions

### Credit Default Swap Option

A credit default swap (CDS) option, or credit default swaption, is a contract that provides the option holder with the right, but not the obligation, to enter into a credit default swap in the future. CDS options can either be payer swaptions or receiver swaptions. In a payer swaption, the option holder has the right to enter into a CDS in which they are paying premiums and in a receiver swaptions, the option holder is receiving premiums.

## Examples

### Obtain Payer and Receiver Values for a Credit Default Swap Option

Use `cdsoptprice` to generate `Payer` and `Receiver` values for a credit default swap option.

```
Settle = datenum('12-Jun-2012');
OptionMaturity = datenum('20-Sep-2012');
CDSMaturity = datenum('20-Sep-2017');
OptionStrike = 200;
```

```
SpreadVolatility = .4;

Zero_Time = [.5 1 2 3 4 5]';
Zero_Rate = [.5 .75 1.5 1.7 1.9 2.2]'/100;
Zero_Dates = daysadd(Settle,360*Zero_Time,1);
ZeroData = [Zero_Dates Zero_Rate];

Market_Time = [1 2 3 5 7 10]';
Market_Rate = [100 120 145 220 245 270]';
Market_Dates = daysadd(Settle,360*Market_Time,1);
MarketData = [Market_Dates Market_Rate];

ProbData = cdsbootstrap(ZeroData, MarketData, Settle);

[Payer,Receiver] = cdsoptprice(ZeroData, ProbData, Settle,...
OptionMaturity, CDSMaturity, OptionStrike, SpreadVolatility)
```

```
Payer =

  199.9797


Receiver =

   28.2353
```

## References

O'Kane, D., *Modelling Single-name and Multi-name Credit Derivatives*, Wiley, 2008.

## See Also

| `cdsbootstrap` | `cdsspread` | `cdsprice`

## Tutorials

- "Credit Default Swap Option" on page 8-35

# cdsprice

## Purpose

Determine price for credit default swap

## Syntax

```
[Price, AccPrem, PaymentDates, PaymentTimes,
PaymentCF] = cdsprice(ZeroData, ProbData, Settle,
Maturity, ContractSpread)
[Price, AccPrem, PaymentDates, PaymentTimes,
PaymentCF] = cdsprice(ZeroData, ProbData,
Settle, Maturity, ContractSpread,
Name,Value)
```

## Description

`[Price, AccPrem, PaymentDates, PaymentTimes, PaymentCF] = cdsprice(ZeroData, ProbData, Settle, Maturity, ContractSpread)` computes the price, or the mark-to-market value for CDS instruments.

`[Price, AccPrem, PaymentDates, PaymentTimes, PaymentCF] = cdsprice(ZeroData, ProbData, Settle, Maturity, ContractSpread, Name,Value)` computes the price, or the mark-to-market value for CDS instruments with additional options specified by one or more `Name,Value` pair arguments.

## Input Arguments

### ZeroData

`M`-by-`2` vector of dates and zero rates or `IRCurve` of zero rates.

### ProbData

`P`-by-`2` array of dates and default probabilities.

### Settle

Settlement date is a serial date number or date string. This must be earlier than or equal to the dates in `MarketData`.

### Maturity

`N`-by-`1` vector of serial date numbers or date strings containing the maturity dates.

**ContractSpread**

`N`-by-`1` vector of contract spreads, expressed in basis points.

## Name-Value Pair Arguments

Specify optional comma-separated pairs of `Name,Value` arguments, where `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside single quotes (`' '`). You can specify several name and value pair arguments in any order as `Name1,Value1,...,NameN,ValueN`.

---

**Note** Any optional input of size `N`-by-`1` is also acceptable as an array of size `1`-by-`N`, or as a single value applicable to all contracts. Single values are internally expanded to an array of size `N`-by-`1`.

---

**Basis**

`N`-by-`1` vector of day-count basis of the CDS:

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (BMA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)

- 12 = actual/actual (ISDA)
- 13 = BUS/252

For more information, see basis.

**Default:** `2` (actual/360)

#### BusDayConvention

String or `N`-by-`1` cell array of strings of business day conventions. Values are:

- `actual`
- `follow`
- `modifiedfollow`
- `previous`
- `modifiedprevious`

**Default:** `actual`

#### Notional

`N`-by-`1` vector of contract notional values. Use positive values for long positions and negative values for short positions.

**Default:** `10MM`

#### PayAccruedPremium

`N`-by-`1` vector of Boolean flags. `True`, if accrued premiums are paid upon default, `False` otherwise.

**Default:** `True`

#### Period

`N`-by-`1` vector of number of premiums per year of the CDS. Allowed values are 1, 2, 3, 4, 6, and 12.

**Default:** `4`

### RecoveryRate

`N`-by-`1` vector of recovery rates, expressed as a decimal from `0` to `1`.

**Default:** `0.4`

### TimeStep

Positive integer indicating the number of days to take as time step for the numerical integration.

**Default:** `10` (days)

### ZeroBasis

Basis of the zero curve, where the choices are identical to `Basis`.

**Default:** `0` (actual/actual)

### ZeroCompounding

Compounding frequency of the zero curve. Allowed values are:

- 1 — Annual compounding
- 2 — Semiannual compounding
- 3 — Compounding three times per year
- 4 — Quarterly compounding
- 6 — Bimonthly compounding
- 12 — Monthly compounding
- -1 — Continuous compounding

---

**Note** When `ZeroData` is an `IRCurve` object, the arguments `ZeroCompounding` and `ZeroBasis` are implicit in `ZeroData` and are redundant inside this function. In that case, specify these optional arguments when constructing the `IRCurve` object before calling this function.

---

**Default:** `2` (Semiannual compounding)

## Output Arguments

### Price

`N`-by-`1` vector of CDS prices.

### AccPrem

`N`-by-`1` vector of accrued premiums.

### PaymentDates

`N`-by-`numCF` matrix of payment dates.

### PaymentTimes

`N`-by-`numCF` matrix of accrual fractions.

### PaymentCF

`N`-by-`numCF` matrix of payments.

## Definitions

### CDS Price

The price or mark-to-market (MtM) value of an existing CDS contract is computed using the following formula:

```
CDS price = Notional * (Current Spread - Contract Spread)
* RPV01
```

`Current Spread` is the current breakeven spread for a similar contract, according to current market conditions. `RPV01` is the ’risky present value of a basis point,' the present value of the premium payments, taking into consideration the default probability. This formula assumes a long position, and the right side is multiplied by -1 for short positions.

## Examples

Use `cdsprice` to compute the clean price for a CDS contract:

```
Settle = '17-Jul-2009';
Zero_Time = [.5 1 2 3 4 5]';
Zero_Rate = [1.35 1.43 1.9 2.47 2.936 3.311]'/100;
Zero_Dates = daysadd(Settle,360*Zero_Time,1);
ZeroData = [Zero_Dates Zero_Rate];

ProbData = [daysadd(datenum(Settle),360,1), 0.0247];
Maturity = '20-Sep-2010';
ContractSpread = 135;

[Price,AccPrem] = cdsprice(ZeroData,ProbData,Settle,Maturity,ContractSpread);
CleanPrice = Price - AccPrem
CleanPrice =

  4.9381e+003
```

## Algorithms

The premium leg is computed as the product of a spread $S$ and the risky present value of a basis point (`RPV01`). The `RPV01` is given by:

$$RPV01 = \sum_{j=1}^{N} Z(t_j)\Delta(t_{j-1}, t_j, B)Q(t_j)$$

when no accrued premiums are paid upon default, and it can be approximated by

$$RPV01 \approx \frac{1}{2}\sum_{j=1}^{N} Z(t_j)\Delta(t_{j-1}, t_j, B)(Q(t_{j-1}) + Q(t_j))$$

when accrued premiums are paid upon default. Here, $t_0 = 0$ is the valuation date, and $t_1,...,t_n = T$ are the premium payment dates over the life of the contract, $T$ is the maturity of the contract, $Z(t)$ is the discount factor for a payment received at time $t$, and $\Delta(t_{j-1}, t_j, B)$ is a day count between dates $t_{j-1}$ and $t_j$ corresponding to a basis $B$.

The protection leg of a CDS contract is given by the following formula:

$$ProtectionLeg = \int_0^T Z(\tau)(1-R)dPD(\tau)$$

$$\approx (1-R)\sum_{i=1}^{M} Z(\tau_i)(PD(\tau_i) - PD(\tau_{i-1}))$$

$$= (1-R)\sum_{i=1}^{M} Z(\tau_i)(Q(\tau_{i-1}) - Q(\tau_i))$$

where the integral is approximated with a finite sum over the discretization $\tau_0 = 0, \tau_1, \ldots, \tau_M = T$.

If the spread of an existing CDS contract is $S_C$, and the current breakeven spread for a comparable contract is $S_0$, the current price, or mark-to-market value of the contract is given by:

`MtM` = `Notional` $(S_0 - S_C)$`RPV01`

This assumes a long position from the protection standpoint (protection was bought). For short positions, the sign is reversed.

## References

Beumee, J., D. Brigo, D. Schiemert, and G. Stoyle. “Charting a Course Through the CDS Big Bang,” *Fitch Solutions, Quantitative Research*, Global Special Report. April 7, 2009.

Hull, J., and A. White, “Valuing Credit Default Swaps I: No Counterparty Default Risk,” *Journal of Derivatives* 8, 29-40.

O’Kane, D. and S. Turnbull, “Valuation of Credit Default Swaps.” *Lehman Brothers, Fixed Income Quantitative Credit Research*, April, 2003.

## See Also

| `cdsspread` | `cdsbootstrap`

## Tutorials

- "Credit Default Swap (CDS)" on page 8-2

# cdsspread

## Purpose

Determine spread of credit default swap

## Syntax

```
[Spread, PaymentDates, PaymentTimes] = cdsspread(ZeroData,
ProbData, Settle, Maturity)
[Spread, PaymentDates, PaymentTimes] = cdsspread(ZeroData,
 ProbData, Settle, Maturity,
Name,Value)
```

## Description

`[Spread, PaymentDates, PaymentTimes] = cdsspread(ZeroData, ProbData, Settle, Maturity)` computes the spread of the CDS.

`[Spread, PaymentDates, PaymentTimes] = cdsspread(ZeroData, ProbData, Settle, Maturity, Name,Value)` computes the spread of the CDS with additional options specified by one or more `Name,Value` pair arguments.

## Input Arguments

### ZeroData

`M`-by-`2` vector of dates and zero rates or `IRCurve` of zero rates.

### ProbData

`P`-by-`2` array of dates and default probabilities.

### Settle

Settlement date is a serial date number or date string. This must be earlier than or equal to the dates in `MarketData`.

### Maturity

`N`-by-`1` vector of serial date numbers or date strings containing the maturity dates.

### Name-Value Pair Arguments

Specify optional comma-separated pairs of `Name,Value` arguments, where `Name` is the argument name and `Value` is the corresponding value. `Name` must appear inside single quotes (`' '`). You can specify several name and value pair arguments in any order as `Name1,Value1,...,NameN,ValueN`.

---

**Note** Any optional input of size `N`-by-`1` is also acceptable as an array of size `1`-by-`N`, or as a single value applicable to all contracts. Single values are internally expanded to an array of size `N`-by-`1`.

---

### Basis

`N`-by-`1` vector of day-count basis of the CDS:

- 0 = actual/actual
- 1 = 30/360 (SIA)
- 2 = actual/360
- 3 = actual/365
- 4 = 30/360 (BMA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/actual (ISDA)
- 13 = BUS/252

For more information, see basis.

**Default:** `2` (actual/360)

**BusDayConvention**

String or `N-by-1` cell array of strings of business day conventions. Values are:

- `actual`
- `follow`
- `modifiedfollow`
- `previous`
- `modifiedprevious`

**Default:** `actual`

**PayAccruedPremium**

`N`-by-`1` vector of Boolean flags, `True`, if accrued premiums are paid upon default, `False` otherwise.

**Default:** `True`

**Period**

`N`-by-`1` vector of number of premiums per year of the CDS. Allowed values are 1, 2, 3, 4, 6, and 12.

**Default:** `4`

**RecoveryRate**

`N`-by-`1` vector of recovery rates, expressed as a decimal from `0` to `1`.

**Default:** `0.4`

**TimeStep**

Positive integer indicating the number of days to take as time step for the numerical integration.

**Default:** `10` (days)

### ZeroBasis

Basis of the zero curve, where the choices are identical to `Basis`.

**Default:** `0` (actual/actual)

### ZeroCompounding

Compounding frequency of the zero curve. Allowed values are:

- 1 — Annual compounding
- 2 — Semiannual compounding
- 3 — Compounding three times per year
- 4 — Quarterly compounding
- 6 — Bimonthly compounding
- 12 — Monthly compounding
- -1 — Continuous compounding

---

**Note** When `ZeroData` is an `IRCurve` object, the arguments `ZeroCompounding` and `ZeroBasis` are implicit in `ZeroData` and are redundant inside this function. In that case, specify these optional arguments when constructing the `IRCurve` object before calling this function.

---

**Default:** `2` (semiannual compounding )

## Output Arguments

### Spread

`N`-by-`1` vector of spreads (in basis points).

### PaymentDates

`N`-by-`numCF` matrix of payment dates.

**PaymentTimes**

`N`-by-`numCF` matrix of accrual fractions.

## Definitions

### CDS Spread

The market, or breakeven, spread value of a CDS can be computed by equating the value of the protection leg with the value of the premium leg:

```
Market Spread * RPV01 = Value of Protection Leg
```

The left side corresponds to the value of the premium leg, and this has been decomposed as the product of the market or breakeven spread times the `RPV01` or ’risky present value of a basis point’ of the contract. The latter is the present value of the premium payments, taking into consideration the default probability. The `Market Spread` can be computed as the ratio of the value of the protection leg, to the `RPV01` of the contract. `cdsspread` returns the resulting spread in basis points.

## Examples

Use `cdsspread` to compute the clean price for a CDS contract:

```
Settle = '17-Jul-2009';
Zero_Time = [.5 1 2 3 4 5]';
Zero_Rate = [1.35 1.43 1.9 2.47 2.936 3.311]'/100;
Zero_Dates = daysadd(Settle,360*Zero_Time,1);
ZeroData = [Zero_Dates Zero_Rate];
ProbData = [daysadd(datenum(Settle),360,1), 0.0247];
Maturity = '20-Sep-2010';

Spread = cdsspread(ZeroData,ProbData,Settle,Maturity)

Spread =

  148.2485
```

## Algorithms

The premium leg is computed as the product of a spread $S$ and the risky present value of a basis point (`RPV01`). The `RPV01` is given by:

$$RPV01 = \sum_{j=1}^{N} Z(t_j)\Delta(t_{j-1}, t_j, B)Q(t_j)$$

when no accrued premium are paid upon default, and it can be approximated by

$$RPV01 \approx \frac{1}{2}\sum_{j=1}^{N} Z(t_j)\Delta(t_{j-1}, t_j, B)(Q(t_{j-1}) + Q(t_j))$$

when accrued premiums are paid upon default. Here, $t_0$ = `0` is the valuation date, and $t_1,\ldots,t_n = T$ are the premium payment dates over the life of the contract, $T$ is the maturity of the contract, $Z(t)$ is the discount factor for a payment received at time $t$, and $\Delta(t_{j-1}, t_j, B)$ is a day count between dates $t_{j-1}$ and $t_j$ corresponding to a basis $B$.

The protection leg of a CDS contract is given by the following formula:

$$ProtectionLeg = \int_0^T Z(\tau)(1-R)dPD(\tau)$$

$$\approx (1-R)\sum_{i=1}^{M} Z(\tau_i)(PD(\tau_i) - PD(\tau_{i-1}))$$

$$= (1-R)\sum_{i=1}^{M} Z(\tau_i)(Q(\tau_{i-1}) - Q(\tau_i))$$

where the integral is approximated with a finite sum over the discretization $\tau_0$ = `0`, $\tau_1,\ldots,\tau_M = T$.

A breakeven spread $S_0$ makes the value of the premium and protection legs equal. It follows that:

$$S_0 = \frac{ProtectionLeg}{RPV01}$$

## References

Beumee, J., D. Brigo, D. Schiemert, and G. Stoyle. "Charting a Course Through the CDS Big Bang," *Fitch Solutions, Quantitative Research*, Global Special Report. April 7, 2009.

Hull, J., and A. White, "Valuing Credit Default Swaps I: No Counterparty Default Risk," *Journal of Derivatives* 8, 29-40.

O'Kane, D. and S. Turnbull, "Valuation of Credit Default Swaps." *Lehman Brothers, Fixed Income Quantitative Credit Research*, April, 2003.

## See Also

| `cdsprice` | `cdsbootstrap`

## Tutorials

- "Credit Default Swap (CDS)" on page 8-2

## Purpose

Yield on certificate of deposit (CD)

## Syntax

```
Yield = cdyield(Price, CouponRate, Settle, Maturity, IssueDate,
Basis)
```

## Arguments

| | |
|---|---|
| `Price` | Clean price of the certificate of deposit per $100 face. If you have a vector of dirty or cash prices of CDs, compute the accrued interest portion using `cdai`. |
| `CouponRate` | Annual interest rate in decimal. |
| `Settle` | Settlement date. `Settle` must be earlier than `Maturity`. |
| `Maturity` | Maturity date. |
| `IssueDate` | Issue date. |
| `Basis` | (Optional) Day-count basis of the instrument.<br>• 0 = actual/actual (default)<br>• 1 = 30/360 (SIA)<br>• 2 = actual/360<br>• 3 = actual/365<br>• 4 = 30/360 (BMA)<br>• 5 = 30/360 (ISDA)<br>• 6 = 30/360 (European)<br>• 7 = actual/365 (Japanese)<br>• 8 = actual/actual (ICMA)<br>• 9 = actual/360 (ICMA)<br>• 10 = actual/365 (ICMA)<br>• 11 = 30/360E (ICMA) |

- 12 = actual/actual (ISDA)
- 13 = BUS/252

For more information, see basis.

Each required input must be some certificates of deposit (`NCDS`)-by-`1` or `1`-by-`NCDS` conforming vector or scalar. The optional `Basis` argument may be either a `NCDS`-by-`1` or a `1`-by-`NCDS` vector, a scalar, or the empty matrix (`[]`).

## Description

`Yield = cdyield(Price, CouponRate, Settle, Maturity, IssueDate, Basis)` computes the yield to maturity of a certificate of deposit given its clean price.

This function assumes that the certificates of deposit pay interest at maturity. Because of the simple interest treatment of these securities, the function is best used for short-term maturities (less than 1 year). The default simple interest calculation is the actual/360 convention.

## Examples

Given a certificate of deposit (CD) with these characteristics, compute the yield on the CD.

```
Price      = 101.125;
CouponRate = 0.05;
Settle     = '02-Jan-02';
Maturity   = '31-Mar-02';
IssueDate = '1-Oct-01';

Yield = cdyield(Price, CouponRate, Settle, Maturity, IssueDate)

Yield =

    0.0039
```

## See Also

`bndprice` | `cdai` | `cdprice` | `stepcpnprice` | `tbillprice`

## Purpose

Generate principal balance schedule for planned amortization class (PAC) or targeted amortization class (TAC) bond

## Syntax

```
[BalanceSchedule, InitialBalance] = cmosched(Principal,
Coupon, OriginalTerm, TermRemaining, PrepaySpeed)
[BalanceSchedule, InitialBalance] = cmosched(Principal,
Coupon, OriginalTerm, TermRemaining, PrepaySpeed,
TranchePrincipal)
```

## Description

`[BalanceSchedule, InitialBalance] = cmosched(Principal, Coupon, OriginalTerm, TermRemaining, PrepaySpeed)` generates a principal balance schedule for planned amortization class (PAC) bonds using two bands of Public Securities Association Prepayment Model (PSA) speeds or targeted amortization class (TAC) bonds using a single PSA speed.

`[BalanceSchedule, InitialBalance] = cmosched(Principal, Coupon, OriginalTerm, TermRemaining, PrepaySpeed, TranchePrincipal)` with a specified tranche principal generates a principal balance schedule for planned amortization class (PAC) bonds using two bands of PSA speeds or targeted amortization class (TAC) bonds using a single PSA speed.

## Input Arguments

### Principal

Principal of the underlying mortgage pool.

### Coupon

Coupon of the underlying mortgage pool.

### OriginalTerm

Original term in months of the underlying mortgage pool.

### TermRemaining

Terms remaining in months of the underlying mortgage pool.

**PrepaySpeed**

PSA speed. For a PAC, the speed is a `1`-by-`2` matrix where the first element is the lower band and the second element is the upper band. For a TAC, the speed is a scalar.

**TranchePrincipal**

(Optional) Principal of the scheduled tranche. If it is unspecified or empty `[]`, the principal of the scheduled tranche is assumed to be the sum of the payment schedule calculated from the PSA prepayment speeds.

## Output Arguments

**BalanceSchedule**

Matrix of size `1`-by-`NUMTERMS`, where `NUMTERMS` is the number of terms remaining. Each column contains the scheduled principal balance for the time period corr-by-esponding to the column number.

**InitialBalance**

Scalar containing the initial principal balance of the scheduled tranche.

## Definitions

### Planned Amortization Class (PAC) Bond

PAC bonds are a type of CMO bond. They are designed to largely eliminate prepayment risk for investors. They do this by transferring essentially all prepayment risk to other bonds in the CMO that are called support bonds.

### Targeted Amortization Class (TAC) Bond

TAC bonds are analogous to PAC bonds, but are structured differently. TAC bonds offer one-sided protection, shielding investors from high prepayment rates up to a specified PSA and do not protect against low prepayment rates.

## Examples

### Calculate the Principal Balance Schedule for a CMO PAC Bond

Define the mortgage pool under consideration and generate a principal balance schedule for planned amortization class (PAC) bonds using two bands of PSA speeds.

Calculate PAC bonds using `cmosched`.

```
Principal = 128687000;
GrossRate = 0.0648;
OriginalTerm = 360;
TermRemaining = 325;
PrepaySpeed = [300 525];
PacPrincipal = 100250000;

[BalanceSchedule, InitialBalance] ...
= cmosched(Principal, GrossRate, OriginalTerm, TermRemaining, ...
PrepaySpeed, PacPrincipal);
```

```
BalanceSchedule =


  1.0e+07 *


  Columns 1 through 10


   9.7996    9.5780    9.3602    9.1461    8.9357    8.7289    8.5257    8.3259    8.1296    7.9366
  Columns 11 through 20


   7.7469    7.5605    7.3773    7.1972    7.0202    6.8463    6.6754    6.5073    6.3422    6.1799


  Columns 21 through 30


   6.0204    5.8637    5.7096    5.5582    5.4094    5.2632    5.1194    4.9782    4.8394    4.7030


  Columns 31 through 40


   4.5689    4.4372    4.3077    4.1804    4.0554    3.9325    3.8118    3.6931    3.5765    3.4619
```

```
  Columns 41 through 50

   3.3494    3.2406    3.1353    3.0334    2.9348    2.8394    2.7470    2.6576    2.5711    2.4873

  Columns 51 through 60

   2.4063    2.3279    2.2520    2.1786    2.1075    2.0387    1.9722    1.9078    1.8455    1.7852

  Columns 61 through 70

   1.7268    1.6703    1.6157    1.5628    1.5117    1.4622    1.4142    1.3679    1.3231    1.2797

  Columns 71 through 80

   1.2377    1.1970    1.1577    1.1197    1.0829    1.0473    1.0129    0.9795    0.9473    0.9161

  Columns 81 through 90

   0.8859    0.8567    0.8285    0.8011    0.7747    0.7491    0.7244    0.7004    0.6773    0.6549

  Columns 91 through 100

   0.6332    0.6122    0.5920    0.5723    0.5534    0.5350    0.5172    0.5001    0.4835    0.4674

  Columns 101 through 110

   0.4518    0.4368    0.4223    0.4082    0.3946    0.3814    0.3687    0.3564    0.3445    0.3330

  Columns 111 through 120

   0.3219    0.3111    0.3007    0.2906    0.2809    0.2715    0.2623    0.2535    0.2450    0.2368

  Columns 121 through 130

   0.2288    0.2211    0.2137    0.2065    0.1995    0.1928    0.1863    0.1800    0.1739    0.1680

  Columns 131 through 140
```

```
 0.1623    0.1568    0.1515    0.1464    0.1414    0.1366    0.1319    0.1275    0.1231    0.1189

Columns 141 through 150

 0.1149    0.1109    0.1072    0.1035    0.0999    0.0965    0.0932    0.0900    0.0869    0.0839

Columns 151 through 160

 0.0811    0.0783    0.0756    0.0730    0.0704    0.0680    0.0657    0.0634    0.0612    0.0591

Columns 161 through 170

 0.0570    0.0550    0.0531    0.0513    0.0495    0.0478    0.0461    0.0445    0.0429    0.0414

Columns 171 through 180

 0.0400    0.0386    0.0372    0.0359    0.0346    0.0334    0.0322    0.0311    0.0300    0.0289

Columns 181 through 190

 0.0279    0.0269    0.0260    0.0250    0.0241    0.0233    0.0224    0.0216    0.0209    0.0201

Columns 191 through 200

 0.0194    0.0187    0.0180    0.0174    0.0167    0.0161    0.0155    0.0150    0.0144    0.0139

Columns 201 through 210

 0.0134    0.0129    0.0124    0.0120    0.0115    0.0111    0.0107    0.0103    0.0099    0.0096

Columns 211 through 220

 0.0092    0.0089    0.0085    0.0082    0.0079    0.0076    0.0073    0.0070    0.0068    0.0065

Columns 221 through 230
```

```
  0.0063    0.0060    0.0058    0.0056    0.0054    0.0052    0.0050    0.0048    0.0046    0.0044

Columns 231 through 240

  0.0042    0.0041    0.0039    0.0037    0.0036    0.0035    0.0033    0.0032    0.0031    0.0029

Columns 241 through 250

  0.0028    0.0027    0.0026    0.0025    0.0024    0.0023    0.0022    0.0021    0.0020    0.0019

Columns 251 through 260

  0.0018    0.0018    0.0017    0.0016    0.0016    0.0015    0.0014    0.0014    0.0013    0.0012

Columns 261 through 270

  0.0012    0.0011    0.0011    0.0010    0.0010    0.0009    0.0009    0.0009    0.0008    0.0008

Columns 271 through 280

  0.0007    0.0007    0.0007    0.0006    0.0006    0.0006    0.0005    0.0005    0.0005    0.0005

Columns 281 through 290

  0.0004    0.0004    0.0004    0.0004    0.0004    0.0003    0.0003    0.0003    0.0003    0.0003

Columns 291 through 300

  0.0002    0.0002    0.0002    0.0002    0.0002    0.0002    0.0002    0.0002    0.0001    0.0001

Columns 301 through 310

  0.0001    0.0001    0.0001    0.0001    0.0001    0.0001    0.0001    0.0001    0.0001    0.0000

Columns 311 through 320

  0.0000    0.0000    0.0000    0.0000    0.0000    0.0000    0.0000    0.0000         0         0
```

```
  Columns 321 through 325

         0          0          0          0          0


InitialBalance =

   100250000
```

## References

Hayre, Lakhbir, ed., *Salomon Smith Barney Guide to Mortgage-Backed and Asset-Backed Securities*, John Wiley and Sons, New York, 2001.

Lyuu, Yuh-Dah, *Financial Engineering and Computation*, Cambridge University Press, 2004.

## See Also

`| cmoschedcf |`

## Tutorials

- "Using Collateralized Mortgage Obligations (CMOs)" on page 5-50

## How To

- "Create PAC and Sequential CMO" on page 5-65

## Purpose

Generate cash flows for scheduled collateralized mortgage obligation (CMO) using PAC or TAC model

## Syntax

```
[Balances, Principal, Interest] =
cmoschedcf(PrincipalPayments, TranchePrincipals,
TrancheCoupons, BalanceSchedule)
```

## Description

`[Balances, Principal, Interest] = cmoschedcf(PrincipalPayments, TranchePrincipals, TrancheCoupons, BalanceSchedule)` generate cash flows for a scheduled CMO such as the planned amortization class (PAC) or targeted amortization class (TAC), given the underlying mortgage pool payments (or payments from another CMO tranche). The output `Balance`, `Principal`, and `Interest` from this function can be used as input into `cmoseqcf` to further divide the PAC, TAC, or support tranche into sequential tranches.

## Input Arguments

### PrincipalPayments

Matrix of size `1`-by-`NUMTERMS`, where `NUMTERMS` is the number of terms remaining. Each column contains the underlying principal payment for the time period corresponding to the row number. Calculate underlying principal payments using `mbscfamounts` or `mbspassthrough`. The underlying principal payments can also be outputs from other CMO cash flow functions.

### TranchePrincipals

Matrix of size `2`-by-`1` specifying the initial principal for the scheduled and the support tranche.

### TrancheCoupons

Matrix of size `2`-by-`1` specifying the coupons for the schedule tranche and the support tranche. The weighted average coupon for the CMO should not exceed the coupon of the underlying mortgage.

### BalanceSchedule

Matrix of size `1` -by-`NUMTERMS`, where `NUMTERMS` is the number of terms remaining. Each element represents the targeted balance schedule for the time period corresponding to that column.

## Output Arguments

### Balance

Matrix of size `2`-by-`NUMTERMS`, where `NUMTERMS` is the number of terms remaining. The first row is the principal balances of the scheduled tranche, and the second row is the principal balances of the support tranche at the time period corresponding to the column.

### Principal

Matrix of size `2`-by-`NUMTERMS`, where `NUMTERMS` is the number of terms remaining. The first row is the principal payments of the scheduled tranche, and the second row is the principal payments of the support tranche at the time period corresponding to the column.

### Interest

Matrix of size `2`-by-`NUMTERMS`, where `NUMTERMS` is the number of terms remaining. The first row is the interest payments of the schedule tranche, and the second row is the interest payments of the support tranche at the time period corresponding to the column.

## Definitions

### Planned Amortization Class (PAC) Tranches

In a PAC CMO, there is a main tranche, known as the schedule tranche, and a support tranche. The main purpose of a schedule tranche is to give investors in the PAC tranche a more certain cash flow.

### Targeted Amortization Class (TAC) Tranches

TACs are like PACs, but principal payment is specified for only one prepayment rate. If prepayment rates are higher or lower, then the principal payment to TAC holders will be higher or lower accordingly.

### Schedule and Support Tranche

The main purpose of a PAC tranche is to give investors in the PAC tranche a more certain cash flow. The PAC tranche receives priority for receiving payments of principal and interest that gives investors in the PAC tranche a steadier income. If prepayments differ from what was expected, then the support tranche gets the variable portion of the payments. While income to the support tranche is more variable, it is also higher yielding. Estimates of the yield, average life, and lockout periods of the PAC tranche is more certain.

## Examples

### Calculate Cash Flows for Each PAC Tranche

Define the mortgage pool under consideration for CMO structuring using `mbscfamounts` or `mbspassthrough`. Calculate the underlying mortgage cash flow, define the PAC schedule and CMO tranches, and calculate the cash flows for each tranche.

Calculate the underlying cash flow using `mbspassthrough`:

```
% Underlying mortgage
MortgagePrincipal = 1000000;
Coupon = 0.12;
Terms = 6; % months

[PrincipalBalance, MonthlyPayments, SchedPrincipalPayments, ...
InterestPayments, Prepayments] = ...
mbspassthrough(MortgagePrincipal, Coupon, Terms, Terms, 0, []);
PrincipalPayments = SchedPrincipalPayments.' + Prepayments.'
```

```
PrincipalPayments =

   1.0e+05 *

    1.6255    1.6417    1.6582    1.6747    1.6915    1.7084
```

Calculate the PAC schedule for CMO using `cmosched`.

```
PrepaySpeed = [100 300];
[BalanceSchedule, InitialBalance] ...
= cmosched(MortgagePrincipal, Coupon, Terms, Terms, PrepaySpeed, [])
```

```
BalanceSchedule =

   1.0e+05 *

    8.3617    6.7180    5.0581    3.3828    1.6955         0


InitialBalance =

   9.9886e+05
```

Define CMO tranches.

```
TranchePrincipals = ...
[InitialBalance; MortgagePrincipal-InitialBalance];
TrancheCoupons = [0.12; 0.12];

TrancheCoupons =

    0.1200
    0.1200
```

Calculate cash flows for each tranche.

```
[Balance, Principal, Interest] = ...
cmoschedcf(PrincipalPayments, TranchePrincipals, ...
TrancheCoupons, BalanceSchedule)

Balance =

   1.0e+05 *

    8.3631    6.7213    5.0632    3.3885    1.6970         0
    0.0114    0.0114    0.0114    0.0114    0.0114    0.0000


Principal =
```

```
   1.0e+05 *

    1.6255    1.6417    1.6582    1.6747    1.6915    1.6970
         0         0         0         0         0    0.0114


Interest =

   1.0e+03 *

    9.9886    8.3631    6.7213    5.0632    3.3885    1.6970
    0.0114    0.0114    0.0114    0.0114    0.0114    0.0114
```

## References

Hayre, Lakhbir, ed., *Salomon Smith Barney Guide to Mortgage-Backed and Asset-Backed Securities*, John Wiley and Sons, New York, 2001.

Lyuu, Yuh-Dah, *Financial Engineering and Computation*, Cambridge University Press, 2004.

## See Also

| `cmoseqcf` | `cmosched` | `mbscfamounts` | `mbspassthrough` |

## Tutorials

- “Using Collateralized Mortgage Obligations (CMOs)” on page 5-50

## How To

- “Create PAC and Sequential CMO” on page 5-65

## Purpose

Generate cash flows for sequential collateralized mortgage obligation (CMO)

## Syntax

```
[balances, principals, interests] =
cmoseqcf(PrincipalPayments, TranchePrincipals,
TrancheCoupons)
[balances, principals, interests] =
cmoseqcf(PrincipalPayments, TranchePrincipals,
TrancheCoupons, HasZ)
```

## Description

`[balances, principals, interests] = cmoseqcf(PrincipalPayments, TranchePrincipals, TrancheCoupons)` generates cash flows for a sequential CMO without a Z-bond, given the underlying mortgage pool payments.

`[balances, principals, interests] = cmoseqcf(PrincipalPayments, TranchePrincipals, TrancheCoupons, HasZ)` generates cash flows for a sequential CMO with a Z-bond, given the underlying mortgage pool payments.

## Input Arguments

### PrincipalPayments

Matrix of size `1`-by-`NUMTERMS`, where `NUMTERMS` is the number of terms remaining. Each row contains the underlying principal payment for the time period corresponding to the row number. The underlying principal payments can be calculated using `mbscfamounts` or `mbspassthrough`. The underlying principal payments can also be outputs from other CMO cash flow functions

### TranchePrincipals

Matrix of size `NUMTRANCHES`-by-`1`, where `NUMTRANCHES` is the number of tranches in the sequential CMO. Each element of the matrix represents the initial principal for each tranche. If the sequential CMO includes a Z-bond (`HasZ` is `true`), the last element of this matrix is the principal of the Z-bond.

### TrancheCoupons

Matrix of size `NUMTRANCHES`-by-`1`, where `NUMTRANCHES` is the number of tranches in the sequential CMO. Each element of the matrix represents the coupon for each tranche. If the sequential CMO includes a Z-bond (`HasZ` is `true`), the last element of this matrix is the coupon of the Z-bond. The weighted average coupon for the CMO should not exceed the coupon of the underlying mortgage.

### HasZ

(Optional) Boolean (`true` or `false`). A value of `true` indicates that the sequential CMO contains a Z-bond, and the last element of `TranchePrincipals` and `TrancheCoupons` will be treated as that of the Z-bond. A value of `false` indicates that there is no Z-bond in the sequential CMO, and the last element of `TranchePrincipals` and `TrancheCoupons` will be treated as an ordinary tranche.

**Default:** `false`

## Output Arguments

### Balance

Matrix of size `NUMTRANCHES`-by-`NUMTERMS`, where `NUMTRANCHES` is the number of terms remaining and `NUMTRANCHES` is the number of tranches. Each element represents the principal balance at the time period corresponding to the column, and for the tranche corresponding to the row.

### Principal

Matrix of size `NUMTRANCHES`-by-`NUMTERMS`, where `NUMTRANCHES` is the number of terms remaining and `NUMTRANCHES` is the number of tranches. Each element represents the principal payments made at the time period corresponding to the column, and to the tranche corresponding to the row.

### Interest

Matrix of size `NUMTRANCHES`-by-`NUMTERMS`, where `NUMTRANCHES` is the number of terms remaining and `NUMTRANCHES` is the number of tranches. Each element represents the interest payments made at the time period corresponding to the column, and to the tranche corresponding to the row.

## Definitions

### Sequential Pay CMO

A sequential pay CMO involves tranches that pay off principal sequentially. For example, consider the following case, where all principal from the underlying mortgage pool is repaid on tranche A first, then tranche B, then tranche C. Note that interest is paid on each tranche as long as the principal for the tranche has not been retired.

### CMO Tranche

Tranche is a term often used to describe a specific class of bonds within an offering wherein each tranche offers varying degrees of risk to the investor.

## Examples

### Calculate Cash Flows for a Sequential Collateralized Mortgage Obligation (CMO)

Define the mortgage pool under consideration for CMO structuring using `mbscfamounts` or `mbspassthrough` and calculate the cash flows with an A and B tranche for a sequential CMO.

Calculate underlying cash flow using `mbspassthrough`:

```
MortgagePrincipal = 1000000;
Coupon = 0.12;
Terms = 6; % months

% Calculate underlying mortgage cash flows
[PrincipalBalance, MonthlyPayments, SchedPrincipalPayments, ...
InterestPayments, Prepayments] = ...
mbspassthrough(MortgagePrincipal, Coupon, Terms, Terms, 0, []);
PrincipalPayments = SchedPrincipalPayments.' + Prepayments.'
```

```
PrincipalPayments =

   1.0e+05 *
```

```
    1.6255    1.6417    1.6582    1.6747    1.6915    1.7084
```

Define CMO tranches, A and B.

```
TranchePrincipals = [500000; 500000];
TrancheCoupons = [0.12; 0.12];
```

Calculate cash flows for each tranche.

```
[Balance, Principal, Interest] = ...
cmoseqcf(PrincipalPayments, TranchePrincipals, TrancheCoupons, false)
```

```
Balance =

   1.0e+05 *

    3.3745    1.7328    0.0746         0         0         0
    5.0000    5.0000    5.0000    3.3999    1.7084    0.0000


Principal =

   1.0e+05 *

    1.6255    1.6417    1.6582    0.0746         0         0
         0         0         0    1.6001    1.6915    1.7084


Interest =

   1.0e+03 *

    5.0000    3.3745    1.7328    0.0746         0         0
    5.0000    5.0000    5.0000    5.0000    3.3999    1.7084
```

## References

Hayre, Lakhbir, ed., *Salomon Smith Barney Guide to Mortgage-Backed and Asset-Backed Securities*, John Wiley and Sons, New York, 2001.

Lyuu, Yuh-Dah, *Financial Engineering and Computation*, Cambridge University Press, 2004.

## See Also

| `cmoschedcf` | `mbscfamounts` | `mbspassthrough` | `cmosched` |

## Tutorials

- "Using Collateralized Mortgage Obligations (CMOs)" on page 5-50

## How To

- "Create PAC and Sequential CMO" on page 5-65

# convfactor

## Purpose

Bond conversion factors

## Syntax

```
CF = convfactor(RefDate, Maturity, CouponRate)
CF = convfactor(RefDate, Maturity, CouponRate,
'ParameterName',ParameterValue ...)
```

## Description

`CF = convfactor(RefDate, Maturity, CouponRate)` computes a conversion factor for a bond futures contract.

`CF = convfactor(RefDate, Maturity, CouponRate, 'ParameterName',ParameterValue ...)` accepts optional inputs as one or more comma-separated parameter-value pairs. `'ParameterName'` is the name of the parameter inside single quotes. `'ParameterValue` is the value corresponding to `'ParameterName'`. Specify parameter/value pairs in any order. Names are case-insensitive. `convfactor` computes a conversion factor for a bond futures contract, given a `Convention` value for a U.S. Treasury bond, German bond, U.K. Gilt, or Japanese Government Bond.

## Input Arguments

### RefDate

Reference dates, for which conversion factor is computed (usually the first day of delivery months).

### Maturity

Maturity date of the underlying bond.

### CouponRate

Annual coupon rate of the underlying bond in decimal.

## Parameter–Value Pairs

Enter the following inputs only as parameter–value pairs.

### Convention

Conversion factor convention. Scalar. Valid values are:

- `1` = U.S. Treasury bond (30-year) and Treasury note (10-year) futures contract
- `2` = U.S. 2-year and 5-year Treasury note futures contract
- `3` = German Bobl, Bund, Buxl, and Schatz
- `4` = U.K. gilts
- `5` = Japanese Government Bonds (JGBs)

**Default:** 1

### FirstCouponDate

Irregular or normal first coupon date.

### RefYield

Reference semiannual yield.

**Default:** 0.06 (6%)

### StartDate

Forward starting date of payments.

## Output Arguments

### CF

`N`-by`1` vector of conversion factors against the 6% yield par-bond.

## Definitions

Conversion factors of U.S. Treasury bonds and other government bonds are based on a bond yielding 6%. Optionally, you can specify other types of bonds and yields using inputs for `RefYield` and `Convention`. For U.S. Treasury bonds, verify the output of `convfactor` by comparing the output against the quotations provided by the Chicago Board of Trade (`http://www.cbot.com`).

For German bonds, verify the output of `convfactor` by comparing the output against the quotations provided by Eurex (`http://www.eurexchange.com`).

For U.K. Gilts, verify the output of `convfactor` by comparing the output against the quotations provided by Euronext (`http://www.euronext.com`).

For Japanese Government Bonds, verify the output of `convfactor` by comparing the output against the quotations provided by the Tokyo Stock Exchange (`http://www.tse.or.jp/english/`).

## Examples

Calculate `CF`, given the following `RefDate`, `Maturity`, and `CouponRate`:

```
RefDate  = {'1-Dec-2002';
               '1-Mar-2003';
               '1-Jun-2003';
               '1-Sep-2003';
               '1-Dec-2003';
               '1-Sep-2003';
               '1-Dec-2002';
               '1-Jun-2003'};

 Maturity = {'15-Nov-2012';
               '15-Aug-2012';
               '15-Feb-2012';
               '15-Feb-2011';
               '15-Aug-2011';
               '15-Aug-2010';
               '15-Aug-2009';
               '15-Feb-2010'};

 CouponRate = [0.04; 0.04375; 0.04875; 0.05; 0.05; 0.0575; 0.06; 0.065];

    CF = convfactor(RefDate, Maturity, CouponRate)
```

This returns:

```
CF =
          0.8539
          0.8858
          0.9259
```

```
          0.9418
          0.9403
          0.9862
          1.0000
          1.0266
```

---

Calculate `cf`, given the following `RefDate`, `Maturity`, and `CouponRate` for a German Bond:

```
cf = convfactor('3/10/2009','1/04/2018', .04,.06,3)
```

This returns:

```
cf =

    0.8659
```

## References

Burghardt, G., T. Belton, M. Lane, and J. Papa, *The Treasury Bond Basis*, McGraw-Hill, 2005.

Krgin, Dragomir, *Handbook of Global Fixed Income Calculations*, John Wiley & Sons, 2002.

## See Also

`tfutbyprice` | `tfutbyyield` | `tfutimprepo` | `bndfutimprepo` | `bndfutprice`

## How To

- “Bond Futures” on page 7-12

# fitFunction

**Purpose** Custom fit interest-rate curve object to bond market data

**Class** `@IRFunctionCurve`

**Syntax**

```
CurveObj = IRFunctionCurve.fitFunction(Type, Settle,
FunctionHandle, Instruments, IRFitOptionsObj)
CurveObj = IRFunctionCurve.fitFunction(Type, Settle,
FunctionHandle, Instruments, IRFitOptionsObj, 'Parameter1',
Value1, 'Parameter2', Value2, ...)
```

**Arguments**

| | |
|---|---|
| `Type` | Type of interest-rate curve for a bond: `zero`, `forward`, or `discount`. |
| `Settle` | Scalar or column vector of settlement dates. `Settle` must be earlier than `Maturity`. |
| `FunctionHandle` | Function handle that defines the interest-rate curve. The function handle takes two numeric vectors (time-to-maturity and a vector of function coefficients) and returns one numeric output (interest rate or discount factor). For more information on defining a function handle, see the MATLAB Programming Fundamentals documentation. |
| `Instruments` | `N`-by-4 data matrix for `Instruments` where the first column is `Settle` date, the second column is `Maturity`, the third column is the clean price, and the fourth column is a `CouponRate` for the bond. |
| `IRFitOptionsObj` | Object constructed from `IRFitOptions`. |

| | |
|---|---|
| `Compounding` | (Optional) Scalar that sets the compounding frequency per year for the `IRFunctionCurve` object:<br>• -1 = Continuous compounding<br>• 1 = Annual compounding<br>• 2 = Semiannual compounding (default)<br>• 3 = Compounding three times per year<br>• 4 = Quarterly compounding<br>• 6 = Bimonthly compounding<br>• 12 = Monthly compounding |
| `Basis` | (Optional) Day-count basis of the bond. A scalar of integers.<br>• 0 = actual/actual (default)<br>• 1 = 30/360 (SIA)<br>• 2 = actual/360<br>• 3 = actual/365<br>• 4 = 30/360 (BMA)<br>• 5 = 30/360 (ISDA)<br>• 6 = 30/360 (European)<br>• 7 = actual/365 (Japanese)<br>• 8 = actual/actual (ICMA)<br>• 9 = actual/360 (ICMA)<br>• 10 = actual/365 (ICMA)<br>• 11 = 30/360E (ICMA)<br>• 12 = actual/actual (ISDA)<br>• 13 = BUS/252<br>For more information, see basis. |

## Instrument Parameters

For each bond `Instrument`, you can specify the following additional instrument parameters as parameter/value pairs. For example, `InstrumentBasis` distinguishes a bond instrument's `Basis` value from the curve's `Basis` value.

| | |
|---|---|
| `InstrumentPeriod` | (Optional) Coupons per year of the bond. A vector of integers. Allowed values are 0, 1, 2 (default), 3, 4, 6, and 12. |
| `InstrumentBasis` | (Optional) Day-count basis of the bond. A vector of integers.<br>• 0 = actual/actual (default)<br>• 1 = 30/360 (SIA)<br>• 2 = actual/360<br>• 3 = actual/365<br>• 4 = 30/360 (BMA)<br>• 5 = 30/360 (ISDA)<br>• 6 = 30/360 (European)<br>• 7 = actual/365 (Japanese)<br>• 8 = actual/actual (ICMA)<br>• 9 = actual/360 (ICMA)<br>• 10 = actual/365 (ICMA)<br>• 11 = 30/360E (ICMA)<br>• 12 = actual/actual (ISDA)<br>• 13 = BUS/252<br>For more information, see basis. |

| | |
|---|---|
| `InstrumentEndMonthRule` | (Optional) End-of-month rule. A vector. This rule applies only when `Maturity` is an end-of-month date for a month having 30 or fewer days. 0 = ignore rule, meaning that a bond's coupon payment date is always the same numerical day of the month. 1 = set rule on (default), meaning that a bond's coupon payment date is always the last actual day of the month. |
| `InstrumentIssueDate` | (Optional) Date when an instrument was issued. |
| `InstrumentFirstCouponDate` | (Optional) Date when a bond makes its first coupon payment; used when bond has an irregular first coupon period. When `FirstCouponDate` and `LastCouponDate` are both specified, `FirstCouponDate` takes precedence in determining the coupon payment structure. If you do not specify a `FirstCouponDate`, the cash flow payment dates are determined from other inputs. |
| `InstrumentLastCouponDate` | (Optional) Last coupon date of a bond before the maturity date; used when bond has an irregular last coupon period. In the absence of a specified `FirstCouponDate`, a specified `LastCouponDate` determines the coupon structure of the bond. The coupon structure of a bond is truncated at the `LastCouponDate`, regardless of where it falls, and is followed only by the bond's maturity cash flow date. If you do not specify a `LastCouponDate`, the cash flow payment dates are determined from other inputs. |
| `InstrumentFace` | (Optional) Face or par value. Default = 100. |

---

**Note** When using `Instrument` parameter/value pairs, you can specify simple interest for a bond by specifying the `InstrumentPeriod` value as `0`. If `InstrumentBasis` and `InstrumentPeriod` are not specified for a bond, the following default values are used: `Basis` is `0` (act/act) and `Period` is `2`.

---

## Description

`CurveObj = IRFunctionCurve.fitFunction(Type, Settle, FunctionHandle, Instruments, IRFitOptionsObj, 'Parameter1', Value1, 'Parameter2', Value2, ...)` fits a bond to a custom fitting function. You must enter the optional arguments for `Basis` and `Compounding` as parameter/value pairs.

## Examples

```
Settle = repmat(datenum('30-Apr-2008'),[6 1]);
Maturity = [datenum('07-Mar-2009');datenum('07-Mar-2011');...
datenum('07-Mar-2013');datenum('07-Sep-2016');...
datenum('07-Mar-2025');datenum('07-Mar-2036')];
CleanPrice = [100.1;100.1;100.8;96.6;103.3;96.3];
CouponRate = [0.0400;0.0425;0.0450;0.0400;0.0500;0.0425];
Instruments = [Settle Maturity CleanPrice CouponRate];
CurveSettle = datenum('30-Apr-2008');
OptOptions = optimset('lsqnonlin');
OptOptions = optimset(OptOptions,'display','iter');
functionHandle = @(t,theta) polyval(theta,t);

CustomModel = IRFunctionCurve.fitFunction('Zero', CurveSettle, ...
functionHandle,Instruments, ...
IRFitOptions([.05 .05 .05],'FitType','price',...
'OptOptions',OptOptions));
```

```
                     Norm of      First-order
 Iteration  Func-count     f(x)          step          optimality   CG-iterations
     0           4         38036.7                       4.92e+004
     1           8         38036.7             10        4.92e+004             0
     2          12         38036.7            2.5        4.92e+004             0
     3          16         38036.7          0.625        4.92e+004             0
```

```
         4          20         38036.7         0.15625        4.92e+004             0
         5          24         30741.5       0.0390625        1.72e+005             0
         6          28         30741.5        0.078125        1.72e+005             0
         7          32         30741.5       0.0195312        1.72e+005             0
         8          36         28713.6      0.00488281        2.33e+005             0
         9          40         20323.3      0.00976562        9.47e+005             0
        10          44         20323.3       0.0195312        9.47e+005             0
        11          48         20323.3      0.00488281        9.47e+005             0
        12          52         20323.3       0.0012207        9.47e+005             0
        13          56         19698.8     0.000305176        1.08e+006             0
        14          60           17493     0.000610352           7e+006             0
        15          64           17493       0.0012207           7e+006             0
        16          68           17493     0.000305176           7e+006             0
        17          72         15455.1     7.62939e-005       2.25e+007             0
        18          76         15455.1     0.000177558        2.25e+007             0
        19          80         13317.1     3.8147e-005        3.18e+007             0
        20          84         12867.9     7.62939e-005       7.84e+007             0
        21          88         11779.8     7.62939e-005       7.58e+006             0
        22          92         11747.6     0.000152588        1.46e+005             0
        23          96         11720.9     0.000305176        2.48e+005             0
        24         100         11667.2     0.000610352        1.48e+005             0
        25         104         11558.5       0.0012207        4.47e+005             0
        26         108         11335.4      0.00244141        1.58e+005             0
        27         112           10864      0.00488281        1.61e+005             0
        28         116         9797.68      0.00976562        6.85e+005             0
        29         120         6884.03       0.0195312        5.79e+005             0
        30         124         6884.03        0.037498        5.79e+005             0
        31         128         3216.51      0.00937449        1.75e+006             0
        32         132         607.317        0.018749        2.94e+006             0
        33         136         12.7284       0.0253662           3e+006             0
        34         140       0.0760939      0.00153457        4.88e+004             0
        35         144       0.0731652     3.58678e-006            24.6             0
        36         148       0.0731652     6.04329e-008          0.0213             0

Local minimum possible.

lsqnonlin stopped because the final change in the sum of squares relative to
```

`its initial value is less than the selected value of the function tolerance.`

## How To

- "@IRFitOptions" on page A-10
- "@IRFunctionCurve" on page A-12

## Purpose

Fit Nelson-Siegel function to bond market data

## Class

`@IRFunctionCurve`

## Syntax

```
CurveObj = IRFunctionCurve.fitNelsonSiegel(Type, Settle, Instruments)
CurveObj = IRFunctionCurve.fitNelsonSiegel(Type, Settle, Instruments,
'Parameter1', Value1, 'Parameter2', Value2, ...)
```

## Arguments

| | |
|---|---|
| `Type` | Type of interest-rate curve for a bond: `zero` or `forward`. |
| `Settle` | Scalar or column vector of settlement dates. |
| `Instruments` | `N`-by-`4` data matrix for `Instruments` where the first column is `Settle` date, the second column is `Maturity`, the third column is the clean price, and the fourth column is a `CouponRate` for the bond. |
| `Compounding` | (Optional) Scalar that sets the compounding frequency per year for the `IRFunctionCurve` object:<br>• -1 = Continuous compounding |
| `Basis` | (Optional) Day-count basis of the interest-rate curve. A scalar of integers.<br>• 0 = actual/actual (default)<br>• 1 = 30/360 (SIA)<br>• 2 = actual/360<br>• 3 = actual/365<br>• 4 = 30/360 (BMA)<br>• 5 = 30/360 (ISDA)<br>• 6 = 30/360 (European) |

| | |
|---|---|
| | • 7 = actual/365 (Japanese)<br>• 8 = actual/actual (ICMA)<br>• 9 = actual/360 (ICMA)<br>• 10 = actual/365 (ICMA)<br>• 11 = 30/360E (ICMA)<br>• 12 = actual/actual (ISDA)<br>• 13 = BUS/252<br>For more information, see basis. |
| `IRFitOptionsObj` | (Optional) Object constructed from `IRFitOptions`. |

## Instrument Parameters

For each bond `Instrument`, you can specify the following additional instrument parameters as parameter/value pairs. For example, `InstrumentBasis` distinguishes a bond instrument's `Basis` value from the curve's `Basis` value.

| | |
|---|---|
| `InstrumentPeriod` | (Optional) Coupons per year of the bond. A vector of integers. Allowed values are 0, 1, 2 (default), 3, 4, 6, and 12. |
| `InstrumentBasis` | (Optional) Day-count basis of the bond. A vector of integers.<br>• 0 = actual/actual (default)<br>• 1 = 30/360 (SIA)<br>• 2 = actual/360<br>• 3 = actual/365<br>• 4 = 30/360 (BMA)<br>• 5 = 30/360 (ISDA)<br>• 6 = 30/360 (European) |

- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/actual (ISDA)
- 13 = BUS/252

For more information, see basis.

| | |
|---|---|
| `InstrumentEndMonthRule` | (Optional) End-of-month rule. A vector. This rule applies only when `Maturity` is an end-of-month date for a month having 30 or fewer days. 0 = ignore rule, meaning that a bond's coupon payment date is always the same numerical day of the month. 1 = set rule on (default), meaning that a bond's coupon payment date is always the last actual day of the month. |
| `InstrumentIssueDate` | (Optional) Date when an instrument was issued. |
| `InstrumentFirstCouponDate` | (Optional) Date when a bond makes its first coupon payment; used when bond has an irregular first coupon period. When `FirstCouponDate` and `LastCouponDate` are both specified, `FirstCouponDate` takes precedence in determining the coupon payment structure. If you do not specify a `FirstCouponDate`, the cash flow payment dates are determined from other inputs. |

| | |
|---|---|
| `InstrumentLastCouponDate` | (Optional) Last coupon date of a bond before the maturity date; used when bond has an irregular last coupon period. In the absence of a specified `FirstCouponDate`, a specified `LastCouponDate` determines the coupon structure of the bond. The coupon structure of a bond is truncated at the `LastCouponDate`, regardless of where it falls, and is followed only by the bond's maturity cash flow date. If you do not specify a `LastCouponDate`, the cash flow payment dates are determined from other inputs. |
| `InstrumentFace` | (Optional) Face or par value. Default = 100. |

---

**Note** When using `Instrument` parameter/value pairs, you can specify simple for a bond by specifying the `InstrumentPeriod` value as `0`. If `InstrumentBasis` and `InstrumentPeriod` are not specified for a bond, the following default values are used: `Basis` is `0` (act/act) and `Period` is `2`.

---

## Description

`CurveObj = IRFunctionCurve.fitNelsonSiegel(Type, Settle, Instruments, 'Parameter1', Value1, 'Parameter2', Value2, ...)` fits a Nelson-Siegel function to market data for a bond. You must enter the optional arguments for `Basis`, `Compounding`, and `IRFitOptionsObj` as parameter/value pairs. After creating a Nelson-Siegel model, you can view the model parameters using:

```
CurveObj.Parameters
```

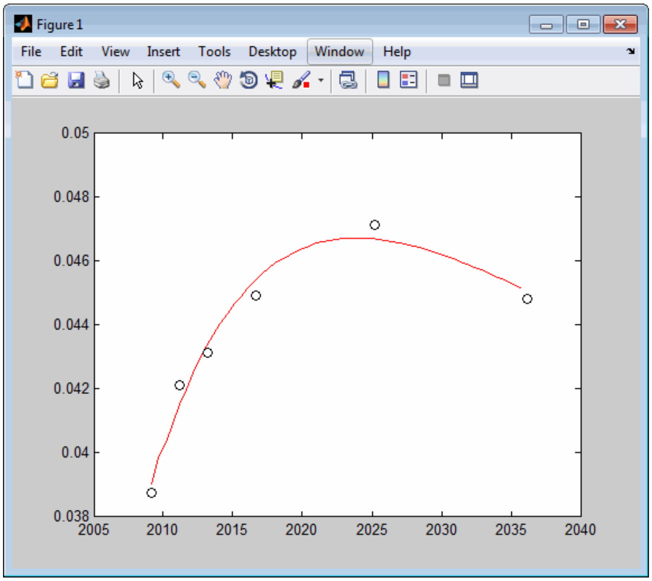
## Examples

```
Settle = repmat(datenum('30-Apr-2008'),[6 1]);
Maturity = [datenum('07-Mar-2009');datenum('07-Mar-2011');...
datenum('07-Mar-2013');datenum('07-Sep-2016');...
datenum('07-Mar-2025');datenum('07-Mar-2036')];

CleanPrice = [100.1;100.1;100.8;96.6;103.3;96.3];
```

```
CouponRate = [0.0400;0.0425;0.0450;0.0400;0.0500;0.0425];
Instruments = [Settle Maturity CleanPrice CouponRate];
PlottingPoints = datenum('07-Mar-2009'):180:datenum('07-Mar-2036');
Yield = bndyield(CleanPrice,CouponRate,Settle,Maturity);

NSModel = IRFunctionCurve.fitNelsonSiegel('Zero',datenum('30-Apr-2008'),Instruments);

NSModel.Parameters

ans =

    0.0000    3.7433    8.6789   16.1655
```

To create the plot:

```
plot(PlottingPoints,NSModel.getParYields(PlottingPoints),'r')
hold on
scatter(Maturity,Yield,'black')
datetick('x')
```



## How To

- “@IRFitOptions” on page A-10
- “@IRFunctionCurve” on page A-12

## Purpose

Fit smoothing spline to bond market data

## Class

`@IRFunctionCurve`

## Syntax

```
CurveObj = IRFunctionCurve.fitSmoothingSpline(Type, Settle,
Instruments, Lambdafun)
CurveObj = IRFunctionCurve.fitSmoothingSpline(Type, Settle,
Instruments, Lambdafun, 'Parameter1', Value1, 'Parameter2',
Value2, ...)
```

## Arguments

---

**Note** You must have a license for Curve Fitting Toolbox software to use the `fitSmoothingSpline` method.

---

| | |
|---|---|
| `Type` | Type of interest-rate curve for a bond: `Forward`, `Zero`, or `Discount`. |
| `Settle` | Scalar or column vector of settlement dates. |
| `Instruments` | `N`-by-`4` data matrix for `Instruments` where the first column is `Settle` date, the second column is `Maturity`, the third column is the clean price, and the fourth column is a `CouponRate` for the bond. |
| `Lambdafun` | Penalty function that takes as its input time and returns a penalty value. Use a function handle to support the penalty function. The function handle for the penalty function which takes one numeric input (time-to-maturity) and returns one numeric output (penalty to be applied to the curvature of the spline). For more information on defining a function handle, see the MATLAB Programming Fundamentals documentation. |

---

**Note** The smoothing spline represents the forward curve. The spline is penalized for curvature by specifying a penalty function. This fit may only be done with a `FitType` of `DurationWeightedPrice`.

---

| | |
|---|---|
| `Knots` | (Optional) Vector of knot locations (times-to-maturity); by default, knots is set to be a vector comprised of `0` and the time to maturity of all input instruments. The default is for the spline type to be cubic but you can specify any spline type by explicitly specifying the knots. User-defined knots can be specified using the following command, where`k` is the order: `augknt(knots,k)`. |
| `Compounding` | (Optional) Scalar that sets the compounding frequency per year for the `IRFunctionCurve` object:<br>• -1 = Continuous compounding (default)<br>• 1 = Annual compounding<br>• 2 = Semiannual compounding<br>• 3 = Compounding three times per year<br>• 4 = Quarterly compounding<br>• 6 = Bimonthly compounding<br>• 12 = Monthly compounding |
| `Basis` | (Optional) Day-count basis of the interest-rate curve. A scalar of integers.<br>• 0 = actual/actual (default)<br>• 1 = 30/360 (SIA)<br>• 2 = actual/360<br>• 3 = actual/365 |

- 4 = 30/360 (BMA)
- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/actual (ISDA)
- 13 = BUS/252

For more information, see basis.

## Instrument Parameters

For each bond `Instrument`, you can specify the following additional instrument parameters as parameter/value pairs. For example, `InstrumentBasis` distinguishes a bond instrument's `Basis` value from the curve's `Basis` value.

| | |
|---|---|
| `InstrumentPeriod` | (Optional) Coupons per year of the bond. A vector of integers. Allowed values are 0, 1, 2 (default), 3, 4, 6, and 12. |
| `InstrumentBasis` | (Optional) Day-count basis of the bond. A vector of integers.<br>• 0 = actual/actual (default)<br>• 1 = 30/360 (SIA)<br>• 2 = actual/360<br>• 3 = actual/365<br>• 4 = 30/360 (BMA) |

- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/actual (ISDA)
- 13 = BUS/252

For more information, see basis.

| | |
|---|---|
| `InstrumentEndMonthRule` | (Optional) End-of-month rule. A vector. This rule applies only when `Maturity` is an end-of-month date for a month having 30 or fewer days. 0 = ignore rule, meaning that a bond's coupon payment date is always the same numerical day of the month. 1 = set rule on (default), meaning that a bond's coupon payment date is always the last actual day of the month. |
| `InstrumentIssueDate` | (Optional) Date when an instrument was issued. |
| `InstrumentFirstCouponDate` | (Optional) Date when a bond makes its first coupon payment; used when bond has an irregular first coupon period. When `FirstCouponDate` and `LastCouponDate` are both specified, `FirstCouponDate` takes precedence in determining the coupon payment structure. If you do not specify a `FirstCouponDate`, the cash flow payment dates are determined from other inputs. |

| | |
|---|---|
| `InstrumentLastCouponDate` | (Optional) Last coupon date of a bond before the maturity date; used when bond has an irregular last coupon period. In the absence of a specified `FirstCouponDate`, a specified `LastCouponDate` determines the coupon structure of the bond. The coupon structure of a bond is truncated at the `LastCouponDate`, regardless of where it falls, and is followed only by the bond's maturity cash flow date. If you do not specify a `LastCouponDate`, the cash flow payment dates are determined from other inputs. |
| `InstrumentFace` | (Optional) Face or par value. Default = 100. |

---

**Note** When using `Instrument` parameter/value pairs, you can specify simple interest for a bond by specifying the `InstrumentPeriod` value as `0`. If `InstrumentBasis` and `InstrumentPeriod` are not specified for a bond, the following default values are used: `Basis` is `0` (act/act) and `Period` is `2`.

---

## Description

`Fcurve = IRFunctionCurve.fitSmoothingSpline(Type, Settle, Instruments, Lambdafun, 'Parameter1', Value1, 'Parameter2', Value2, ...)` fits a smoothing spline to market data for a bond. You must enter the optional arguments for `Basis`, `Compounding`, and `Knots` as parameter/value pairs.

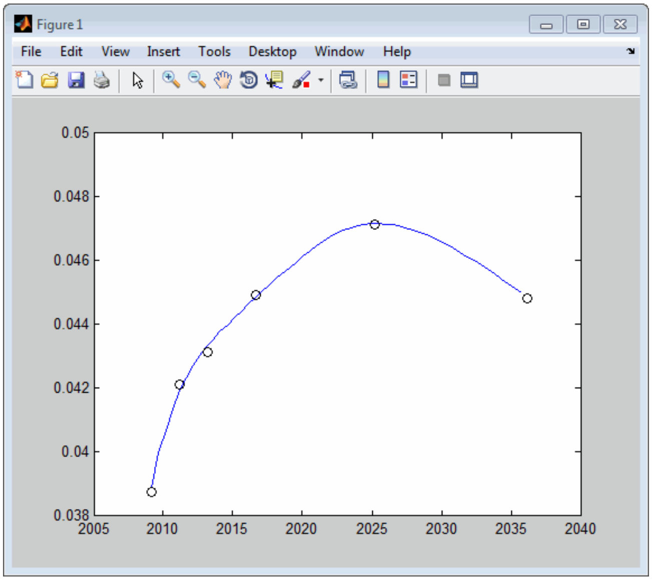
## Examples

```
Settle = repmat(datenum('30-Apr-2008'),[6 1]);
Maturity = [datenum('07-Mar-2009');datenum('07-Mar-2011');...
datenum('07-Mar-2013');datenum('07-Sep-2016');...
datenum('07-Mar-2025');datenum('07-Mar-2036')];

CleanPrice = [100.1;100.1;100.8;96.6;103.3;96.3];
CouponRate = [0.0400;0.0425;0.0450;0.0400;0.0500;0.0425];
Instruments = [Settle Maturity CleanPrice CouponRate];
```

```
PlottingPoints = datenum('07-Mar-2009'):180:datenum('07-Mar-2036');
Yield = bndyield(CleanPrice,CouponRate,Settle,Maturity);

% Use the AUGKNT function to construct the knots for a cubic spline at every 5 years

CustomKnots = augknt(0:5:30,4);
SmoothingModel = IRFunctionCurve.fitSmoothingSpline('Zero',datenum('30-Apr-2008'),...
Instruments,@(t) 1000,'knots', CustomKnots)
```

To create the plot:

```
plot(PlottingPoints,SmoothingModel.getParYields(PlottingPoints),'b')
hold on
scatter(Maturity,Yield,'black')
datetick('x')
```



## How To

- "@IRFunctionCurve" on page A-12

## Purpose

Fit Svensson function to bond market data

## Class

`@IRFunctionCurve`

## Syntax

```
CurveObj = IRFunctionCurve.fitSvensson(Type, Settle,
Instruments)
CurveObj = IRFunctionCurve.fitSvensson(Type, Settle,
Instruments, 'Parameter1', Value1,  'Parameter2',
Value2, ...)
```

## Arguments

| | |
|---|---|
| `Type` | Type of interest-rate curve for a bond: `zero` or `forward`. |
| `Settle` | Scalar or column vector of settlement dates. |
| `Instruments` | `N`-by-`4` data matrix for `Instruments` where the first column is `Settle` date, the second column is `Maturity`, the third column is the clean price, and the fourth column is a `CouponRate` for the bond. |
| `Compounding` | (Optional) Scalar that sets the compounding frequency per year for the `IRFunctionCurve` object:<br>• -1 =  Continuous compounding |

| | |
|---|---|
| `Basis` | (Optional) Day-count basis of the interest-rate curve. A scalar of integers.<br>• 0 = actual/actual (default)<br>• 1 = 30/360 (SIA)<br>• 2 = actual/360<br>• 3 = actual/365<br>• 4 = 30/360 (BMA)<br>• 5 = 30/360 (ISDA)<br>• 6 = 30/360 (European)<br>• 7 = actual/365 (Japanese)<br>• 8 = actual/actual (ICMA)<br>• 9 = actual/360 (ICMA)<br>• 10 = actual/365 (ICMA)<br>• 11 = 30/360E (ICMA)<br>• 12 = actual/actual (ISDA)<br>• 13 = BUS/252<br>For more information, see basis. |
| `IRFitOptionsObj` | (Optional) Object constructed from `IRFitOptions`. |

## Instrument Parameters

For each bond `Instrument`, you can specify the following additional instrument parameters as parameter/value pairs. For example, `InstrumentBasis` distinguishes a bond instrument's `Basis` value from the curve's `Basis` value.

| | |
|---|---|
| `InstrumentPeriod` | (Optional) Coupons per year of the bond. A vector of integers. Allowed values are 0, 1, 2 (default), 3, 4, 6, and 12. |
| `InstrumentBasis` | (Optional) Day-count basis of the bond. A vector of integers.<br>• 0 = actual/actual (default)<br>• 1 = 30/360 (SIA)<br>• 2 = actual/360<br>• 3 = actual/365<br>• 4 = 30/360 (BMA)<br>• 5 = 30/360 (ISDA)<br>• 6 = 30/360 (European)<br>• 7 = actual/365 (Japanese)<br>• 8 = actual/actual (ICMA)<br>• 9 = actual/360 (ICMA)<br>• 10 = actual/365 (ICMA)<br>• 11 = 30/360E (ICMA)<br>• 12 = actual/actual (ISDA)<br>• 13 = BUS/252<br>For more information, see basis. |

| | |
|---|---|
| `InstrumentEndMonthRule` | (Optional) End-of-month rule. A vector. This rule applies only when `Maturity` is an end-of-month date for a month having 30 or fewer days. `0` = ignore rule, meaning that a bond's coupon payment date is always the same numerical day of the month. 1 = set rule on (default), meaning that a bond's coupon payment date is always the last actual day of the month. |
| `InstrumentIssueDate` | (Optional) Date when an instrument was issued. |
| `InstrumentFirstCouponDate` | (Optional) Date when a bond makes its first coupon payment; used when bond has an irregular first coupon period. When `FirstCouponDate` and `LastCouponDate` are both specified, `FirstCouponDate` takes precedence in determining the coupon payment structure. If you do not specify a `FirstCouponDate`, the cash flow payment dates are determined from other inputs. |
| `InstrumentLastCouponDate` | (Optional) Last coupon date of a bond before the maturity date; used when bond has an irregular last coupon period. In the absence of a specified `FirstCouponDate`, a specified `LastCouponDate` determines the coupon structure of the bond. The coupon structure of a bond is truncated at the `LastCouponDate`, regardless of where it falls, and is followed only by the bond's maturity cash flow date. If you do not specify a `LastCouponDate`, the cash flow payment dates are determined from other inputs. |
| `InstrumentFace` | (Optional) Face or par value. Default = 100. |

---

**Note** When using `Instrument` parameter/value pairs, you can specify simple interest for a bond by specifying the `InstrumentPeriod` value as `0`. If `InstrumentBasis` and `InstrumentPeriod` are not specified for a bond, the following default values are used: `Basis` is `0` (act/act) and `Period` is `2`.

---

## Description

`CurveObj = IRFunctionCurve.fitSvensson(Type, Settle, Instruments, 'Parameter1', Value1, 'Parameter2', Value2, ...)` fits the Svensson function to bond market data. You must enter the optional arguments for `Basis`, `Compounding`, and `IRFitOptionsObj` as parameter/value pairs. After creating a Svensson model, you can view the model parameters using:

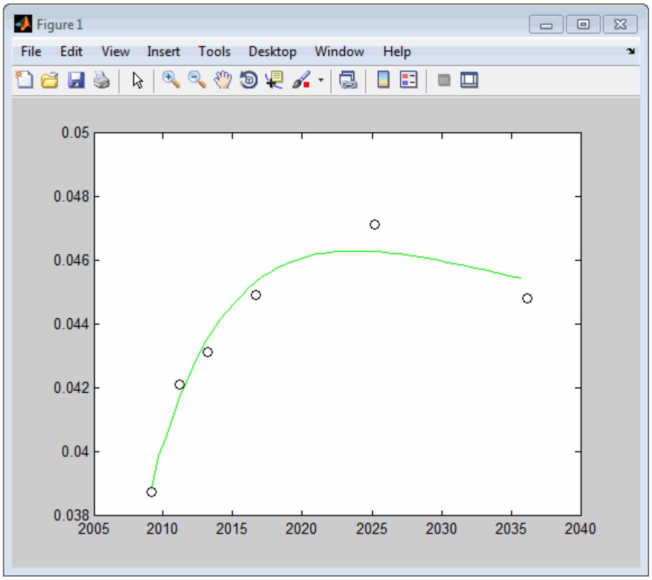
```
CurveObj.Parameters
```

## Examples

```
Settle = repmat(datenum('30-Apr-2008'),[6 1]);
Maturity = [datenum('07-Mar-2009');datenum('07-Mar-2011');...
datenum('07-Mar-2013');datenum('07-Sep-2016');...
datenum('07-Mar-2025');datenum('07-Mar-2036')];

CleanPrice = [100.1;100.1;100.8;96.6;103.3;96.3];
CouponRate = [0.0400;0.0425;0.0450;0.0400;0.0500;0.0425];
Instruments = [Settle Maturity CleanPrice CouponRate];
PlottingPoints = datenum('07-Mar-2009'):180:datenum('07-Mar-2036');
Yield = bndyield(CleanPrice,CouponRate,Settle,Maturity);

SvenssonModel = IRFunctionCurve.fitSvensson('Zero',datenum('30-Apr-2008'),Instruments);

SvenssonModel.Parameters

ans =

   13.6054   -9.8349    0.0373 -184.1093   16.2256  540.9103
```

To create a plot:

```
plot(PlottingPoints,SvenssonModel.getParYields(PlottingPoints),'g')
hold on
scatter(Maturity,Yield,'black')
datetick('x')
```



## How To

- “@IRFitOptions” on page A-10
- “@IRFunctionCurve” on page A-12

**Purpose** Get discount factors for input dates for `IRDataCurve`

**Class** `@IRDataCurve`

**Syntax** `F = getDiscountFactors(CurveObj, InpDates)`

**Arguments**

| | |
|---|---|
| `CurveObj` | Interest-rate curve object that is constructed using `IRDataCurve`. |
| `InpDates` | Vector of input dates using MATLAB date format. The input dates must be after the settle date. |

**Description** `F = getDiscountFactors(CurveObj, InpDates)` returns discount factors for the input dates.

**Examples**

```
Data = [2.09 2.47 2.71 3.12 3.43 3.85 4.57 4.58]/100;
Dates = daysadd(today,[360 2*360 3*360 5*360 7*360 10*360 20*360 30*360],1);
irdc = IRDataCurve('Zero',today,Dates,Data);
irdc.getDiscountFactors(today+30:30:today+720)
ans =

    0.9986
    0.9971
    0.9956
    0.9940
    0.9924
    0.9907
    0.9890
    0.9873
    0.9855
    0.9836
    0.9817
    0.9798
    0.9778
    0.9757
```

```
    0.9736
    0.9715
    0.9693
    0.9671
    0.9649
    0.9626
    0.9602
    0.9578
    0.9554
    0.9529
```

## How To

- “@IRDataCurve” on page A-7

## Purpose

Get discount factors for input dates for `IRFunctionCurve`

## Class

`@IRFunctionCurve`

## Syntax

```
F = getDiscountFactors(CurveObj, InpDates)
```

## Arguments

| | |
|---|---|
| `CurveObj` | Interest-rate curve object that is constructed using the `IRFunctionCurve`. |
| `InpDates` | Vector of input dates using MATLAB date format. The input dates must be after the settle date. |

## Description

`F = getDiscountFactors(CurveObj, InpDates)` returns discount factors for the input dates.

## Examples

```
irfc = IRFunctionCurve('Forward',today,@(t) polyval([-0.0001 0.003 0.02],t));
irfc.getDiscountFactors(today+30:30:today+720)
ans =

    0.9984
    0.9967
    0.9950
    0.9933
    0.9916
    0.9899
    0.9881
    0.9864
    0.9846
    0.9828
    0.9810
    0.9792
    0.9773
    0.9755
    0.9736
    0.9717
```

```
    0.9698
    0.9679
    0.9660
    0.9641
    0.9621
    0.9602
    0.9582
    0.9562
```

## How To

- “@IRFunctionCurve” on page A-12

**Purpose** Get forward rates for input dates for `IRDataCurve`

**Class** `@IRDataCurve`

**Syntax**

```
F = getForwardRates(CurveObj, InpDates)
F = getforwardrates(CurveObj, InpDates, 'Parameter1', Value1,
'Parameter2', Value2, ...)
```

**Arguments**

| | |
|---|---|
| `CurveObj` | Interest-rate curve object that is constructed using `IRDataCurve`. |
| `InpDates` | Vector of input dates using MATLAB date format. The input dates must be after the settle date. |
| `Compounding` | (Optional) Scalar that sets the compounding frequency per year for forward rates are:<br>• -1 = Continuous compounding<br>• 1 = Annual compounding<br>• 2 = Semiannual compounding (default)<br>• 3 = Compounding three times per year<br>• 4 = Quarterly compounding<br>• 6 = Bimonthly compounding<br>• 12 = Monthly compounding |
| `Basis` | (Optional) Day-count basis values for the forward rates:<br>• 0 = actual/actual (default)<br>• 1 = 30/360 (SIA)<br>• 2 = actual/360<br>• 3 = actual/365<br>• 4 = 30/360 (BMA) |

- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/actual (ISDA)
- 13 = BUS/252

For more information, see basis.

## Description

`F = getForwardRates(CurveObj, InpDates, 'Parameter1', Value1, 'Parameter2', Value2, ...)` returns forward rates for the input dates. You must enter the optional arguments for `Basis` and `Compounding` as parameter/value pairs.

## Examples

```
Data = [2.09 2.47 2.71 3.12 3.43 3.85 4.57 4.58]/100;
Dates = daysadd(today,[360 2*360 3*360 5*360 7*360 10*360 20*360 30*360],1);
irdc = IRDataCurve('Zero',today,Dates,Data);
irdc.getForwardRates(today+30:30:today+720)
ans =

    0.0174
    0.0180
    0.0187
    0.0193
    0.0199
    0.0205
    0.0212
    0.0218
    0.0224
```

```
    0.0230
    0.0237
    0.0243
    0.0249
    0.0255
    0.0262
    0.0268
    0.0274
    0.0280
    0.0287
    0.0293
    0.0299
    0.0305
    0.0312
    0.0318
```

## How To

- “@IRDataCurve” on page A-7

# getForwardRates

## Purpose

Get forward rates for input dates for `IRFunctionCurve`

## Class

`@IRFunctionCurve`

## Syntax

```
F = getForwardRates(CurveObj, InpDates)
F = getforwardrates(CurveObj, InpDates,  'Parameter1',
Value1, 'Parameter2', Value2, ...)
```

## Arguments

| | |
|---|---|
| `CurveObj` | Interest-rate curve object that is constructed using `IRFunctionCurve`. |
| `InpDates` | Vector of input dates using MATLAB date format. The input dates must be after the settle date. |
| `Compounding` | (Optional) Scalar that sets the compounding frequency per year for the forward rates are:<br>• -1 =  Continuous compounding<br>• 1 = Annual compounding<br>• 2 = Semiannual compounding (default)<br>• 3 = Compounding three times per year<br>• 4 = Quarterly compounding<br>• 6 = Bimonthly compounding<br>• 12 = Monthly compounding |
| `Basis` | (Optional) Day-count basis for the forward rates:<br>• 0 = actual/actual (default)<br>• 1 = 30/360 (SIA)<br>• 2 = actual/360<br>• 3 = actual/365<br>• 4 = 30/360 (BMA)<br>• 5 = 30/360 (ISDA) |

- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/actual (ISDA)
- 13 = BUS/252

For more information, see basis.

## Description

`F = getForwardRates(CurveObj, InpDates, 'Parameter1', Value1, 'Parameter2', Value2, ...)` returns forward rates for the input dates. You must enter the optional arguments for `Basis` and `Compounding` as parameter/value pairs.

## Examples

```
irfc = IRFunctionCurve('Forward',today,@(t) polyval([-0.0001 0.003 0.02],t));
irfc.getForwardRates(today+30:30:today+720)
ans =

    0.0202
    0.0205
    0.0207
    0.0210
    0.0212
    0.0215
    0.0217
    0.0219
    0.0222
    0.0224
    0.0226
    0.0229
    0.0231
```

```
    0.0233
    0.0235
    0.0238
    0.0240
    0.0242
    0.0244
    0.0247
    0.0249
    0.0251
    0.0253
    0.0255
```

## How To

- “@IRFunctionCurve” on page A-12

## Purpose

Get par yields for input dates for `IRDataCurve`

## Class

`@IRDataCurve`

## Syntax

```
F = getParYields(CurveObj, InpDates)
F = getParYields(CurveObj, InpDates, 'Parameter1',
Value1, 'Parameter2', Value2, ...)
```

## Arguments

| | |
|---|---|
| `CurveObj` | Interest-rate curve object that is constructed using `IRDataCurve`. |
| `InpDates` | Vector of input dates using MATLAB date format. The input dates must be after the settle date. |
| `Compounding` | (Optional) Scalar that sets the compounding frequency per year for the par yield rates are:<br>• -1 = Continuous compounding<br>• 1 = Annual compounding<br>• 2 = Semiannual compounding (default)<br>• 3 = Compounding three times per year<br>• 4 = Quarterly compounding<br>• 6 = Bimonthly compounding<br>• 12 = Monthly compounding |
| `Basis` | (Optional) Day-count basis values for the par yield rates:<br>• 0 = actual/actual (default)<br>• 1 = 30/360 (SIA)<br>• 2 = actual/360<br>• 3 = actual/365<br>• 4 = 30/360 (BMA) |

- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/actual (ISDA)
- 13 = BUS/252

For more information, see basis.

## Description

`F = getParYields(CurveObj, InpDates, 'Parameter1', Value1, 'Parameter2', Value2, ...)` returns par yields for the input dates. You must enter the optional arguments for `Basis` and `Compounding` as parameter/value pairs.

## Examples

```
Data = [2.09 2.47 2.71 3.12 3.43 3.85 4.57 4.58]/100;
Dates = daysadd(today,[360 2*360 3*360 5*360 7*360 10*360 20*360 30*360],1);
irdc = IRDataCurve('Zero',today,Dates,Data);
irdc.getParYields(today+30:30:today+720)ans =

    0.0174
    0.0179
    0.0181
    0.0185
    0.0187
    0.0191
    0.0194
    0.0195
    0.0199
    0.0202
```

```
    0.0205
    0.0208
    0.0212
    0.0215
    0.0218
    0.0221
    0.0224
    0.0228
    0.0231
    0.0233
    0.0236
    0.0239
    0.0242
    0.0245
```

## How To

- “@IRDataCurve” on page A-7

# getParYields

## Purpose

Get par yields for input dates for `IRFunctionCurve`

## Class

`@IRFunctionCurve`

## Syntax

```
F = getParYields(CurveObj, InpDates)
F = getParYields(CurveObj, InpDates, 'Parameter1',
Value1, 'Parameter2', Value2, ...)
```

## Arguments

| | |
|---|---|
| `CurveObj` | Interest-rate curve object that is constructed using `IRFunctionCurve`. |
| `InpDates` | Vector of input dates using MATLAB date format. The input dates must be after the settle date. |
| `Compounding` | (Optional) Scalar that sets the compounding frequency per year for par yield rates are:<br>• -1 = Continuous compounding<br>• 1 = Annual compounding<br>• 2 = Semiannual compounding (default)<br>• 3 = Compounding three times per year<br>• 4 = Quarterly compounding<br>• 6 = Bimonthly compounding<br>• 12 = Monthly compounding |
| `Basis` | (Optional) Day-count basis values for par yield rates:<br>• 0 = actual/actual (default)<br>• 1 = 30/360 (SIA)<br>• 2 = actual/360<br>• 3 = actual/365<br>• 4 = 30/360 (BMA) |

- 5 = 30/360 (ISDA)
- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/actual (ISDA)
- 13 = BUS/252

For more information, see basis.

## Description

`F = getParYields(CurveObj, InpDates, 'Parameter1', Value1, 'Parameter2', Value2, ...)` returns par yields for the input dates. You must enter the optional arguments for `Basis` and `Compounding` as parameter/value pairs.

## Examples

```
irfc = IRFunctionCurve('Forward',today,@(t) polyval([-0.0001 0.003 0.02],t));
irfc.getParYields(today+30:30:today+720)
ans =

    0.0202
    0.0205
    0.0205
    0.0207
    0.0207
    0.0209
    0.0210
    0.0209
    0.0211
    0.0212
    0.0213
```

```
0.0214
0.0216
0.0217
0.0218
0.0220
0.0220
0.0222
0.0223
0.0223
0.0225
0.0226
0.0227
0.0228
```

## How To

- “@IRFunctionCurve” on page A-12

**Purpose** Get zero rates for input dates for `IRDataCurve`

**Class** `@IRDataCurve`

**Syntax**

```
F = getZeroRates(CurveObj, InpDates)
F = getZeroRates(CurveObj, InpDates, 'Parameter1',
Value1, 'Parameter2', Value2, ...)
```

**Arguments**

| | |
|---|---|
| `CurveObj` | Interest-rate curve object that is constructed using `IRDataCurve`. |
| `InpDates` | Vector of input dates using MATLAB date format. The input dates must be after the settle date. |
| `Compounding` | (Optional) Scalar that sets the compounding frequency per year for zero rates are:<br>• -1 = Continuous compounding<br>• 1 = Annual compounding<br>• 2 = Semiannual compounding (default)<br>• 3 = Compounding three times per year<br>• 4 = Quarterly compounding<br>• 6 = Bimonthly compounding<br>• 12 = Monthly compounding |
| `Basis` | (Optional) Day-count basis values for zero rates:<br>• 0 = actual/actual (default)<br>• 1 = 30/360 (SIA)<br>• 2 = actual/360<br>• 3 = actual/365<br>• 4 = 30/360 (BMA)<br>• 5 = 30/360 (ISDA) |

- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/actual (ISDA)
- 13 = BUS/252

For more information, see basis.

## Description

`F = getZeroRates(CurveObj, InpDates, 'Parameter1', Value1, 'Parameter2', Value2, ...)` returns zero rates for the input dates. You must enter the optional arguments for `Basis` and `Compounding` as parameter/value pairs.

## Examples

```
Data = [2.09 2.47 2.71 3.12 3.43 3.85 4.57 4.58]/100;
Dates = daysadd(today,[360 2*360 3*360 5*360 7*360 10*360 20*360 30*360],1);
irdc = IRDataCurve('Zero',today,Dates,Data);
irdc.getZeroRates(today+30:30:today+720)
ans =

    0.0174
    0.0177
    0.0180
    0.0183
    0.0187
    0.0190
    0.0193
    0.0196
    0.0199
    0.0202
    0.0205
```

```
    0.0208
    0.0212
    0.0215
    0.0218
    0.0221
    0.0224
    0.0227
    0.0230
    0.0233
    0.0237
    0.0240
    0.0243
    0.0246
```

## How To

- "@IRDataCurve" on page A-7

# getZeroRates

## Purpose

Get zero rates for input dates for `IRFunctionCurve`

## Class

`@IRFunctionCurve`

## Syntax

```
F = getZeroRates(CurveObj, InpDates)
F = getZeroRates(CurveObj, InpDates, 'Parameter1',
Value1, 'Parameter2', Value2, ...)
```

## Arguments

| | |
|---|---|
| `CurveObj` | Interest-rate curve object that is constructed using `IRFunctionCurve`. |
| `InpDates` | Vector of input dates using MATLAB date format. The input dates must be after the settle date. |
| `Compounding` | (Optional) Scalar that sets the compounding frequency per year for zero rates are:<br>• -1 = Continuous compounding<br>• 1 = Annual compounding<br>• 2 = Semiannual compounding (default)<br>• 3 = Compounding three times per year<br>• 4 = Quarterly compounding<br>• 6 = Bimonthly compounding<br>• 12 = Monthly compounding |
| `Basis` | (Optional) Day-count basis value for zero rates:<br>• 0 = actual/actual (default)<br>• 1 = 30/360 (SIA)<br>• 2 = actual/360<br>• 3 = actual/365<br>• 4 = 30/360 (BMA)<br>• 5 = 30/360 (ISDA) |

- 6 = 30/360 (European)
- 7 = actual/365 (Japanese)
- 8 = actual/actual (ICMA)
- 9 = actual/360 (ICMA)
- 10 = actual/365 (ICMA)
- 11 = 30/360E (ICMA)
- 12 = actual/actual (ISDA)
- 13 = BUS/252

For more information, see basis.

## Description

`F = getZeroRates(CurveObj, InpDates, 'Parameter1', Value1, 'Parameter2', Value2, ...)` returns zero rates for the input dates. You must enter the optional arguments for `Basis` and `Compounding` as parameter/value pairs.

## Examples

```
irfc = IRFunctionCurve('Forward',today,@(t) polyval([-0.0001 0.003 0.02],t));
irfc.getZeroRates(today+30:30:today+720)
ans =

    0.0202
    0.0204
    0.0205
    0.0206
    0.0207
    0.0209
    0.0210
    0.0211
    0.0212
    0.0213
    0.0214
    0.0216
    0.0217
```

```
0.0218
0.0219
0.0220
0.0221
0.0223
0.0224
0.0225
0.0226
0.0227
0.0228
0.0229
```

## How To

- “@IRFunctionCurve” on page A-12

**Purpose** Construct specific options for bootstrapping interest-rate curve object

**Class** `@IRBootstrapOptions`

**Syntax**

```
mybootoptions = IRBootstrapOptions('Param1', Value1)
```

**Arguments**

| | |
|---|---|
| `ConvexityAdjustment` | (Optional) Controls the convexity adjustment to interest-rate futures. This can be specified as a function handle that takes one numeric input (time-to-maturity) and returns one numeric output, `ConvexityAdjustment`. For more information on defining a function handle, see the MATLAB Programming Fundamentals documentation.<br>Alternatively, you can define `ConvexityAdjustment` as an `N`-by-`1` vector of values, where `N` is the number of interest-rate futures.<br>In either case, the `ConvexityAdjustment` is subtracted from the futures rate. |

**Description** `mybootoptions = IRBootstrapOptions('Param1', Value1)` constructs an `IRBootstrapOptionsObj` structure. The `IRBootstrapOptionsObj` is used with the `bootstrap` method.

**Examples**

```
mybootoptions = IRBootstrapOptions('ConvexityAdjustment',repmat(.005,10,1))
```

**How To**

- "@IRDataCurve" on page A-7

# IRDataCurve

**Purpose** Construct interest-rate curve object from dates and data

**Class** `@IRDataCurve`

**Syntax**

```
CurveObj = IRDataCurve(Type, Settle,  Dates, Data)
CurveObj = IRDataCurve(Type, Settle, Dates, Data, 'Parameter1',
Value1, 'Parameter2', Value2, ...)
```

**Arguments**

| | |
|---|---|
| `Type` | Type of interest-rate curve. Acceptable values are `forward`, `zero`, or `discount`. |
| `Settle` | Scalar of settlement dates. |
| `Dates` | Dates corresponding to rate data. |
| `Data` | Interest-rate data for the curve object. |
| `Compounding` | (Optional) Scalar that sets the compounding frequency per year for the `IRDataCurve` object:<br>- -1 =  Continuous compounding<br>- 1 = Annual compounding<br>- 2 = Semiannual compounding (default)<br>- 3 = Compounding three times per year<br>- 4 = Quarterly compounding<br>- 6 = Bimonthly compounding<br>- 12 = Monthly compounding |

| | |
|---|---|
| `Basis` | (Optional) Day-count basis of the interest-rate curve. A scalar of integers.<br>• 0 = actual/actual (default)<br>• 1 = 30/360 (SIA)<br>• 2 = actual/360<br>• 3 = actual/365<br>• 4 = 30/360 (BMA)<br>• 5 = 30/360 (ISDA)<br>• 6 = 30/360 (European)<br>• 7 = actual/365 (Japanese)<br>• 8 = actual/actual (ICMA)<br>• 9 = actual/360 (ICMA)<br>• 10 = actual/365 (ICMA)<br>• 11 = 30/360E (ICMA)<br>• 12 = actual/actual (ISDA)<br>• 13 = BUS/252<br>For more information, see basis. |
| `InterpMethod` | (Optional) Values are:<br>• `'linear'` — Linear interpolation (default).<br>• `'constant'` — Piecewise constant interpolation.<br>• `'pchip'` — Piecewise cubic Hermite interpolation.<br>• `'spline'` — Cubic spline interpolation. |

## Description

`CurveObj = IRDataCurve(Type, Settle, Dates, Data, 'Parameter1', Value1, 'Parameter2', Value2, ...)` constructs an interest-rate curve with the specified `Dates` and `Data`. You must enter the optional arguments for `Basis`, `Compounding`, and `InterpMethod` as parameter/value pairs.

Alternatively, an `IRDataCurve` object can be bootstrapped from market data using the `bootstrap` method.

After an `IRDataCurve` curve object is constructed, you can use the following methods to determine the forward rates, zero rates, and discount factors. In addition, you can use the `toRateSpec` method to convert the interest-rate curve object to a `RateSpec` structure.

| Method | Description |
|---|---|
| `getForwardRates` | Returns forward rates for input dates. |
| `getZeroRates` | Returns zero rates for input dates. |
| `getDiscountFactors` | Returns discount factors for input dates. |
| `getParYields` | Returns par yields for input dates. |
| `toRateSpec` | Converts to be a `RateSpec` object; this structure is identical to the `RateSpec` |
| `bootstrap` | Bootstraps an interest rate curve from market data. |

## Examples

```
Data = [2.09 2.47 2.71 3.12 3.43 3.85 4.57 4.58]/100;
Dates = daysadd(today,[360 2*360 3*360 5*360 7*360 10*360 20*360 30*360],1);
irdc = IRDataCurve('Zero',today,Dates,Data)

irdc =

  Properties:
```

```
           Dates: [8x1 double]
            Data: [8x1 double]
    InterpMethod: 'linear'
            Type: 'Zero'
          Settle: 733599
     Compounding: 2
           Basis: 0
```

## How To

- “@IRCurve” on page A-4

## Purpose

Construct specific options for fitting interest-rate curve object

## Class

`@IRFitOptions`

## Syntax

```
myfitoptions = IRFitOptions(InitialGuess)
myfitoptions = IRFitOptions(InitialGuess, 'Parameter1', Value1)
```

## Arguments

| | |
|---|---|
| `InitialGuess` | Initial guess for the parameters of the curve function. Vector of values for the starting point of the optimization. |
| `FitType` | (Optional) `Price`, `Yield`, or `DurationWeightedPrice` determines which is minimized in the curve fitting process. The default is `DurationWeightedPrice`. |
| `UpperBound` | (Optional) Lower bound for the parameters of the curve function. |
| `LowerBound` | (Optional) Upper bound for the parameters of the curve function. |
| `OptOptions` | (Optional) Optimization structure based on the output from the Optimization Toolbox function `optimset`. This optimization structure is evaluated by `lsqnonlin`. |

## Description

`myfitoptions = IRFitOptions('Param1', Value1)` constructs the `IRFitOptions` structure with an initial guess or with an initial guess and bounds. You must enter the optional arguments for `FitType`, `UpperBound`, `LowerBound`, and `OptOptions` as parameter/value pairs.

---

**Note** `IRFitOptions` constructor must be used with `fitFunction` method when building a custom fitting function.

---

## Examples

```
myfitoptions = IRFitOptions([7 2 1 0],'FitType','yield')

myfitoptions =

  Properties:
         FitType: 'yield'
    InitialGuess: [7 2 1 0]
      UpperBound: []
      LowerBound: []
      OptOptions: []
```

## How To

- "@IRFunctionCurve" on page A-12

**Purpose** Construct interest-rate curve object from function handle or function and fit to market data

**Class** `@IRFunctionCurve`

**Syntax**

```
CurveObj = IRFunctionCurve(Type, Settle, FunctionHandle)
CurveObj = IRFunctionCurve(Type, Settle, FunctionHandle,
'Parameter1', Value1, 'Parameter2', Value2, ...)
```

**Arguments**

| | |
|---|---|
| `Type` | Type of interest-rate curve: `zero`, `forward`, or `discount`. |
| `Settle` | Scalar of settlement dates. |
| `FunctionHandle` | Function handle that defines the interest-rate curve. The function handle requires one numeric input (time-to-maturity) and returns one numeric output (interest rate or discount factor). For more information on defining a function handle, see the MATLAB Programming Fundamentals documentation. |
| `Compounding` | (Optional) Scalar that sets the compounding frequency per year for the `IRFunctionCurve` object:<br>• -1 = Continuous compounding<br>• 1 = Annual compounding<br>• 2 = Semiannual compounding (default)<br>• 3 = Compounding three times per year<br>• 4 = Quarterly compounding<br>• 6 = Bimonthly compounding |

| | |
|---|---|
| | • 12 = Monthly compounding |
| `Basis` | (Optional) Day-count basis of the bond. A scalar of integers.<br>• 0 = actual/actual (default)<br>• 1 = 30/360 (SIA)<br>• 2 = actual/360<br>• 3 = actual/365<br>• 4 = 30/360 (BMA)<br>• 5 = 30/360 (ISDA)<br>• 6 = 30/360 (European)<br>• 7 = actual/365 (Japanese)<br>• 8 = actual/actual (ICMA)<br>• 9 = actual/360 (ICMA)<br>• 10 = actual/365 (ICMA)<br>• 11 = 30/360E (ICMA)<br>• 12 = actual/actual (ISDA)<br>• 13 = BUS/252<br>For more information, see basis. |

## Description

`CurveObj = IRFunctionCurve(Type, Settle, FunctionHandle, 'Parameter1', Value1, 'Parameter2', Value2, ...)` constructs an interest-rate curve object directly by specifying a function handle. You must enter the optional arguments for `Basis` and `Compounding` as parameter/value pairs.

After you use the `IRFunctionCurve` constructor to create an `IRFunctionCurve` object, you can fit the bond using the following methods.

| Method | Description |
|---|---|
| `getForwardRates` | Returns forward rates for input dates. |
| `getZeroRates` | Returns zero rates for input dates. |
| `getDiscountFactors` | Returns discount factors for input dates. |
| `getParYields` | Returns par yields for input dates. |
| `toRateSpec` | Converts to be a `RateSpec` object.<br>This `RateSpec` structure is identical to the `RateSpec` produced by the Financial Instruments Toolbox function `intenvset`. |

Alternatively, you can construct an `IRFunctionCurve` object using the following static methods.

| Static Method | Description |
|---|---|
| `fitNelsonSiegel` | Fits a Nelson-Siegel function to market data. |
| `fitSvensson` | Fits a Svensson function to market data. |
| `fitSmoothingSpline` | Fits a smoothing spline function to market data. |
| `fitFunction` | Fits a custom function to market data. |

## Examples

```
irfc = IRFunctionCurve('Forward',today,@(t) polyval([-0.0001 0.003 0.02],t))

irfc =

  Properties:
    FunctionHandle: @(t)polyval([-0.0001,0.003,0.02],t)
              Type: 'Forward'
```

```
           Settle: 733599
      Compounding: 2
            Basis: 0
```

## How To

- “@IRCurve” on page A-4

# liborduration

## Purpose

Duration of LIBOR-based interest-rate swap

## Syntax

```
[PayFixDuration GetFixDuration] = liborduration(SwapFixRate,
Tenor, Settle)
```

## Arguments

| | |
|---|---|
| `SwapFixRate` | Scalar or column vector of swap fixed rates in decimal. |
| `Tenor` | Scalar or column vector indicating life of the swap in years. Fractional numbers are rounded upward. |
| `Settle` | Scalar or column vector of settlement dates. |

## Description

`[PayFixDuration GetFixDuration] = liborduration(SwapFixRate, Tenor, Settle)` computes the duration of LIBOR-based interest-rate swaps.

`PayFixDuration` is the modified duration, in years, realized when entering pay-fix side of the swap.

`GetFixDuration` is the modified duration, in years, realized when entering receive-fix side of the swap.

## Examples

Given the data

```
SwapFixRate = 0.0383;
Tenor = 7;
Settle = datenum('11-Oct-2002');
```

compute the swap durations.

```
[PayFixDuration GetFixDuration] = liborduration(SwapFixRate,...
Tenor, Settle)

PayFixDuration =

   -4.7567
```

```
GetFixDuration =

    4.7567
```

**See Also** `liborfloat2fixed` | `liborprice`

# liborfloat2fixed

## Purpose

Compute par fixed-rate of swap given 3-month LIBOR data

## Syntax

```
[FixedSpec, ForwardDates, ForwardRates] =
liborfloat2fixed(ThreeMonthRates, Settle, Tenor, StartDate,
Interpolation, ConvexAdj, RateParam, InArrears, Sigma,
FixedCompound, FixedBasis)
```

## Arguments

| | |
|---|---|
| `ThreeMonthRates` | Three-month Eurodollar futures data or forward rate agreement data. (A forward rate agreement stipulates that a certain interest rate applies to a certain principal amount for a given future time period.) An `n`-by-`3` matrix in the form of `[month year IMMQuote]`. The floating rate is assumed to compound quarterly and to accrue on an actual/360 basis. |
| `Settle` | Settlement date of the swap. Scalar. |
| `Tenor` | Life of the swap. Scalar. |
| `StartDate` | (Optional) Scalar value to denote reference date for valuation of (forward) swap. This in effect allows forward swap valuation. Default = `Settle`. |
| `Interpolation` | (Optional) Interpolation method to determine applicable forward rate for months when no Eurodollar data is available. Default is `'linear'` or 1. Other possible values are `'Nearest'` or 0, and `'Cubic'` or 2. |
| `ConvexAdj` | (Optional) Default = 0 (off). 1 = on. Denotes whether futures/forward convexity adjustment is required. Pertains to forward rate adjustments when those rates are taken from Eurodollar futures data. |

| | |
|---|---|
| `RateParam` | (Optional) Short-rate model's parameters (Hull-White) `[a S]`, where the short-rate process is:<br>$dr = \left[\theta(t) - ar\right]dt + Sdz.$<br>Default = `[0.05 0.015]`. |
| `InArrears` | (Optional) Default = 0 (off). Set to 1 for on. If on, the routine does an automatic a convexity adjustment to forward rates. |
| `Sigma` | (Optional) Overall annual volatility of caplets. |
| `FixedCompound` | (Optional) Scalar value. Compounding or frequency of payment on the fixed side. Also, the reset frequency. Default = 4 (quarterly). Other values are 1, 2, and 12. |
| `FixedBasis` | (Optional) Scalar value. Basis of the fixed side.<br>• 0 = actual/actual (default)<br>• 1 = 30/360 (SIA)<br>• 2 = actual/360<br>• 3 = actual/365<br>• 4 = 30/360 (BMA)<br>• 5 = 30/360 (ISDA)<br>• 6 = 30/360 (European)<br>• 7 = actual/365 (Japanese)<br>• 8 = actual/actual (ICMA)<br>• 9 = actual/360 (ICMA)<br>• 10 = actual/365 (ICMA)<br>• 11 = 30/360E (ICMA)<br>• 12 = actual/actual (ISDA)<br>For more information, see basis. |

## Description

`[FixedSpec, ForwardDates, ForwardRates] = liborfloat2fixed(ThreeMonthRates, Settle, Tenor, StartDate, Interpolation, ConvexAdj, RateParam, InArrears, Sigma, FixedCompound, FixedBasis)` computes forward rates, dates, and the swap fixed rate.

`FixedSpec` specifies the structure of the fixed-rate side of the swap:

- `Coupon`: Par-swap rate
- `Settle`: Start date
- `Maturity`: End date
- `Period`: Frequency of payment
- `Basis`: Accrual basis

`ForwardDates` are dates corresponding to `ForwardRates` (all third Wednesdays of the month, spread 3 months apart). The first element is the third Wednesday immediately after `Settle`.

`ForwardRates` are forward rates corresponding to the forward dates, quarterly compounded, and on the actual/360 basis.

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**Note** To preserve input integrity, `Tenor` is rounded upward to the closest integer. Currently traded tenors are 2, 5, and 10 years.

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The function assumes that floating-rate observations occur quarterly on the third Wednesday of a delivery month. The first delivery month is the month of the first third Wednesday after `Settle`. Floating-side payments occur on the third-month anniversaries of observation dates.

## Examples

Use the supplied `EDdata.xls` file as input to a `liborfloat2fixed` computation.

```
[EDFutData, textdata] = xlsread('EDdata.xls');
Settle                = datenum('15-Oct-2002');
Tenor                 = 2;
```

```
[FixedSpec, ForwardDates, ForwardRates] =...
liborfloat2fixed(EDFutData(:,1:3), Settle, Tenor)

FixedSpec =

      Coupon: 0.0222
      Settle: '16-Oct-2002'
    Maturity: '16-Oct-2004'
      Period: 4
       Basis: 1

ForwardDates =

      731505
      731596
      731687
      731778
      731869
      731967
      732058
      732149

ForwardRates =

    0.0177
    0.0166
    0.0170
    0.0188
    0.0214
    0.0248
    0.0279
    0.0305
```

## See Also

`liborduration` | `liborprice`

# liborprice

## Purpose

Price swap given swap rate

## Syntax

```
Price = liborprice(ThreeMonthRates, Settle, Tenor,
SwapRate, StartDate, Interpolation, ConvexAdj, RateParam,
InArrears, Sigma, FixedCompound, FixedBasis)
```

## Arguments

| | |
|---|---|
| `ThreeMonthRates` | Three-month Eurodollar futures data or forward rate agreement data. (A forward rate agreement stipulates that a certain interest rate applies to a certain principal amount for a given future time period.) An `n-by-3` matrix in the form of `[month year IMMQuote]`. The floating rate is assumed to compound quarterly and to accrue on an actual/360 basis. |
| `Settle` | Settlement date of swap. Scalar. |
| `Tenor` | Life of the swap. Scalar. |
| `SwapRate` | Swap rate in decimal. |
| `StartDate` | (Optional) Scalar value to denote reference date for valuation of (forward) swap. This in effect allows forward swap valuation. Default = `Settle`. |
| `Interpolation` | (Optional) Interpolation method to determine applicable forward rate for months when no Eurodollar data is available. Default is `'linear'` or 1. Other possible values are `'Nearest'` or 0, and `'Cubic'` or 2. |
| `ConvexAdj` | (Optional) Default = 0 (off). 1 = on. Denotes whether futures/forward convexity adjustment is required. Pertains to forward rate adjustments when those rates are taken from Eurodollar futures data. |

| | |
|---|---|
| `RateParam` | (Optional) Short-rate model's parameters (Hull-White) `[a S]`, where the short-rate process is:<br>$dr = \left[\theta(t) - ar\right]dt + Sdz.$<br>Default = `[0.05 0.015]`. |
| `InArrears` | (Optional) Default = 0 (off). Set to 1 for on. If on, the routine does an automatic convexity adjustment to forward rates. |
| `Sigma` | (Optional) Overall annual volatility of caplets. |
| `FixedCompound` | (Optional) Scalar value. Compounding or frequency of payment on the fixed side. Also, the reset frequency. Default = 4 (quarterly). Other values are 1, 2, and 12. |
| `FixedBasis` | (Optional) Scalar value. Basis of the fixed side.<br>• 0 = actual/actual (default)<br>• 1 = 30/360 (SIA)<br>• 2 = actual/360<br>• 3 = actual/365<br>• 4 = 30/360 (BMA)<br>• 5 = 30/360 (ISDA)<br>• 6 = 30/360 (European)<br>• 7 = actual/365 (Japanese)<br>• 8 = actual/actual (ICMA)<br>• 9 = actual/360 (ICMA)<br>• 10 = actual/365 (ICMA)<br>• 11 = 30/360E (ICMA)<br>• 12 = actual/actual (ISDA)<br>For more information, see basis. |

## Description

`Price = liborprice(ThreeMonthRates, Settle, Tenor, SwapRate, StartDate, Interpolation, ConvexAdj, RateParam, InArrears, Sigma, FixedCompound, FixedBasis)` computes the price per $100 notional value of a swap given the swap rate. A positive result indicates that fixed side is more valuable than the floating side.

`Price` is the present value of the difference between floating and fixed-rate sides of the swap per $100 notional.

## Examples

This example shows that a swap paying the par swap rate has a value of 0.

Load the input data.

```
[EDFutData, textdata] = xlsread('EDdata.xls');
Settle = datenum('15-Oct-2002');
Tenor = 2;
```

Compute the fixed rate from the Eurodollar data.

```
FixedSpec = liborfloat2fixed(EDFutData(:,1:3), Settle, Tenor)

Coupon: 0.0222
Settle: '16-Oct-2002'
Maturity: '16-Oct-2004'
Period: 4
Basis: 1
```

Compute the price of a par swap.

```
Price = liborprice(EDFutData(:,1:3), Settle, Tenor, FixedSpec.Coupon)

Price =

  4.1633e-015
```

MATLAB computes a value for `Price` that is effectively equal to 0.

## See Also

`liborduration` | `liborfloat2fixed`

# mbscfamounts

## Purpose

Cash flow and time mapping for mortgage pool

## Syntax

```
[CFlowAmounts, CFlowDates, TFactors, Factors, Payment, Principal,
Interest, Prepayment] = mbscfamounts(Settle, Maturity, IssueDate, GrossRa
CouponRate, Delay, PrepaySpeed, PrepayMatrix)
```

## Arguments

| | |
|---|---|
| `Settle` | Settlement date. A serial date number or date string. `Settle` must be earlier than `Maturity`. |
| `Maturity` | Maturity date. A serial date number or date string. |
| `IssueDate` | Issue date. A serial date number or date string. |
| `GrossRate` | Gross coupon rate (including fees), in decimal. |
| `CouponRate` | (Optional) Net coupon rate, in decimal. Default = `GrossRate`. |
| `Delay` | (Optional) Delay in days. Default is `0` (no delay). |
| `PrepaySpeed` | (Optional) Relation of the conditional payment rate (CPR) to the benchmark model. Default = `0`. If you input a customized prepayment matrix, set `PrepaySpeed` to `[]`. |
| `PrepayMatrix` | (Optional) Used only when `PrepaySpeed` is unspecified. Customized prepayment vector. A `NaN`-padded matrix of size `max(TermRemaining)`-by-`NMBS`. Each column corresponds to each mortgage-backed security, and each row corresponds to each month after settlement. |

All inputs (except `PrepayMatrix`) are number of mortgage-backed securities (`NMBS`)-by-`1` vectors.

## Description

`[CFlowAmounts, CFLowDates, TFactors, Factors, Payment, Principal, Interest, Prepayment] = mbscfamounts(Settle, Maturity, IssueDate, GrossRate, CouponRate, Delay, PrepaySpeed, PrepayMatrix)` computes cash flows between settle and maturity dates, the corresponding time factors in months from settle, and the mortgage factor (the fraction of loan principal outstanding).

`CFlowAmounts` is a vector of cash flows starting from `Settle` through end of the last month (`Maturity`).

`CFlowDates` indicates when cash flows occur, including at `Settle`. A negative number at `Settle` indicates accrued interest is due.

`TFactors` is a vector of times in months from `Settle`, corresponding to each cash flow.

`Factors` is a vector of mortgage factors (the fraction of the balance still outstanding at the end of each month).

`Payment` is a `NMBS`-by-`P` matrix of total monthly payment.

`Principal` is a `NMBS`-by-`P` matrix of principal portion of the payment

`Interest` is a `NMBS`-by-`P` matrix of interest portion of the payment.

`Prepayment` is a `NMBS`-by-`P` matrix of unscheduled payment of principal.

## Examples

### Calculate Cash Flow Amounts and Dates, Time Factors, and Mortgage Factors for a Single Mortgage

Given a mortgage with the following characteristics, compute the cash flow amounts and dates, the time factors, and the mortgage factors.

Define the mortgage characteristics.

```
Settle      = datenum('17-April-2002');
Maturity    = datenum('1-Jan-2030');
IssueDate   = datenum('1-Jan-2000');
GrossRate   = 0.08125;
CouponRate  = 0.075;
Delay       = 14;
PrepaySpeed = 100;
```

Use `mbscfamonts` to evaluate the mortgage.

```
[CFlowAmounts, CFLowDates, TFactors, Factors] = ...
mbscfamounts(Settle, Maturity, IssueDate, GrossRate, ...
CouponRate, Delay, PrepaySpeed)
```

```
CFlowAmounts =

  Columns 1 through 7

   -0.0033    0.0118    0.0120    0.0121    0.0120    0.0119    0.0119
  Columns 8 through 14

    0.0118    0.0117    0.0117    0.0116    0.0115    0.0115    0.0114

  Columns 15 through 21

    0.0114    0.0113    0.0112    0.0112    0.0111    0.0110    0.0110

  Columns 22 through 28

    0.0109    0.0109    0.0108    0.0107    0.0107    0.0106    0.0106

  Columns 29 through 35

    0.0105    0.0105    0.0104    0.0103    0.0103    0.0102    0.0102

  Columns 36 through 42

    0.0101    0.0101    0.0100    0.0099    0.0099    0.0098    0.0098

  Columns 43 through 49

    0.0097    0.0097    0.0096    0.0096    0.0095    0.0095    0.0094

  Columns 50 through 56
```

```
  0.0094    0.0093    0.0093    0.0092    0.0092    0.0091    0.0090

Columns 57 through 63

  0.0090    0.0089    0.0089    0.0088    0.0088    0.0087    0.0087

Columns 64 through 70

  0.0087    0.0086    0.0086    0.0085    0.0085    0.0084    0.0084

Columns 71 through 77

  0.0083    0.0083    0.0082    0.0082    0.0081    0.0081    0.0080

Columns 78 through 84

  0.0080    0.0079    0.0079    0.0079    0.0078    0.0078    0.0077

Columns 85 through 91

  0.0077    0.0076    0.0076    0.0075    0.0075    0.0075    0.0074

Columns 92 through 98

  0.0074    0.0073    0.0073    0.0073    0.0072    0.0072    0.0071

Columns 99 through 105

  0.0071    0.0070    0.0070    0.0070    0.0069    0.0069    0.0068

Columns 106 through 112

  0.0068    0.0068    0.0067    0.0067    0.0066    0.0066    0.0066

Columns 113 through 119
```

```
    0.0065    0.0065    0.0065    0.0064    0.0064    0.0063    0.0063

  Columns 120 through 126

    0.0063    0.0062    0.0062    0.0062    0.0061    0.0061    0.0061

  Columns 127 through 133

    0.0060    0.0060    0.0059    0.0059    0.0059    0.0058    0.0058

  Columns 134 through 140

    0.0058    0.0057    0.0057    0.0057    0.0056    0.0056    0.0056

  Columns 141 through 147

    0.0055    0.0055    0.0055    0.0054    0.0054    0.0054    0.0053

  Columns 148 through 154

    0.0053    0.0053    0.0052    0.0052    0.0052    0.0052    0.0051

  Columns 155 through 161

    0.0051    0.0051    0.0050    0.0050    0.0050    0.0049    0.0049

  Columns 162 through 168

    0.0049    0.0048    0.0048    0.0048    0.0048    0.0047    0.0047

  Columns 169 through 175

    0.0047    0.0046    0.0046    0.0046    0.0046    0.0045    0.0045

  Columns 176 through 182

    0.0045    0.0044    0.0044    0.0044    0.0044    0.0043    0.0043
```

```
  Columns 183 through 189

    0.0043    0.0043    0.0042    0.0042    0.0042    0.0041    0.0041

  Columns 190 through 196

    0.0041    0.0041    0.0040    0.0040    0.0040    0.0040    0.0039

  Columns 197 through 203

    0.0039    0.0039    0.0039    0.0038    0.0038    0.0038    0.0038

  Columns 204 through 210

    0.0037    0.0037    0.0037    0.0037    0.0036    0.0036    0.0036

  Columns 211 through 217

    0.0036    0.0035    0.0035    0.0035    0.0035    0.0035    0.0034

  Columns 218 through 224

    0.0034    0.0034    0.0034    0.0033    0.0033    0.0033    0.0033

  Columns 225 through 231

    0.0033    0.0032    0.0032    0.0032    0.0032    0.0031    0.0031

  Columns 232 through 238

    0.0031    0.0031    0.0031    0.0030    0.0030    0.0030    0.0030

  Columns 239 through 245

    0.0030    0.0029    0.0029    0.0029    0.0029    0.0029    0.0028
```

```
  Columns 246 through 252

    0.0028    0.0028    0.0028    0.0028    0.0027    0.0027    0.0027

  Columns 253 through 259

    0.0027    0.0027    0.0026    0.0026    0.0026    0.0026    0.0026

  Columns 260 through 266

    0.0025    0.0025    0.0025    0.0025    0.0025    0.0024    0.0024

  Columns 267 through 273

    0.0024    0.0024    0.0024    0.0024    0.0023    0.0023    0.0023

  Columns 274 through 280

    0.0023    0.0023    0.0023    0.0022    0.0022    0.0022    0.0022

  Columns 281 through 287

    0.0022    0.0021    0.0021    0.0021    0.0021    0.0021    0.0021

  Columns 288 through 294

    0.0020    0.0020    0.0020    0.0020    0.0020    0.0020    0.0020

  Columns 295 through 301

    0.0019    0.0019    0.0019    0.0019    0.0019    0.0019    0.0018

  Columns 302 through 308

    0.0018    0.0018    0.0018    0.0018    0.0018    0.0017    0.0017

  Columns 309 through 315
```

```
    0.0017    0.0017    0.0017    0.0017    0.0017    0.0016    0.0016

  Columns 316 through 322

    0.0016    0.0016    0.0016    0.0016    0.0016    0.0015    0.0015

  Columns 323 through 329

    0.0015    0.0015    0.0015    0.0015    0.0015    0.0014    0.0014

  Columns 330 through 334

    0.0014    0.0014    0.0014    0.0014    0.0014


CFLowDates =

  Columns 1 through 6

      731323      731337      731368      731398      731429      731460

  Columns 7 through 12

      731490      731521      731551      731582      731613      731641

  Columns 13 through 18

      731672      731702      731733      731763      731794      731825

  Columns 19 through 24

      731855      731886      731916      731947      731978      732007

  Columns 25 through 30

      732038      732068      732099      732129      732160      732191
```

```
Columns 31 through 36

   732221      732252      732282      732313      732344      732372

Columns 37 through 42

   732403      732433      732464      732494      732525      732556

Columns 43 through 48

   732586      732617      732647      732678      732709      732737

Columns 49 through 54

   732768      732798      732829      732859      732890      732921

Columns 55 through 60

   732951      732982      733012      733043      733074      733102

Columns 61 through 66

   733133      733163      733194      733224      733255      733286

Columns 67 through 72

   733316      733347      733377      733408      733439      733468

Columns 73 through 78

   733499      733529      733560      733590      733621      733652

Columns 79 through 84

   733682      733713      733743      733774      733805      733833
```

```
Columns 85 through 90

   733864      733894      733925      733955      733986      734017

Columns 91 through 96

   734047      734078      734108      734139      734170      734198

Columns 97 through 102

   734229      734259      734290      734320      734351      734382

Columns 103 through 108

   734412      734443      734473      734504      734535      734563

Columns 109 through 114

   734594      734624      734655      734685      734716      734747

Columns 115 through 120

   734777      734808      734838      734869      734900      734929

Columns 121 through 126

   734960      734990      735021      735051      735082      735113

Columns 127 through 132

   735143      735174      735204      735235      735266      735294

Columns 133 through 138

   735325      735355      735386      735416      735447      735478

Columns 139 through 144
```

```
    735508      735539      735569      735600      735631      735659

Columns 145 through 150

    735690      735720      735751      735781      735812      735843

Columns 151 through 156

    735873      735904      735934      735965      735996      736024

Columns 157 through 162

    736055      736085      736116      736146      736177      736208

Columns 163 through 168

    736238      736269      736299      736330      736361      736390

Columns 169 through 174

    736421      736451      736482      736512      736543      736574

Columns 175 through 180

    736604      736635      736665      736696      736727      736755

Columns 181 through 186

    736786      736816      736847      736877      736908      736939

Columns 187 through 192

    736969      737000      737030      737061      737092      737120

Columns 193 through 198
```

```
   737151      737181      737212      737242      737273      737304

Columns 199 through 204

   737334      737365      737395      737426      737457      737485

Columns 205 through 210

   737516      737546      737577      737607      737638      737669

Columns 211 through 216

   737699      737730      737760      737791      737822      737851

Columns 217 through 222

   737882      737912      737943      737973      738004      738035

Columns 223 through 228

   738065      738096      738126      738157      738188      738216

Columns 229 through 234

   738247      738277      738308      738338      738369      738400

Columns 235 through 240

   738430      738461      738491      738522      738553      738581

Columns 241 through 246

   738612      738642      738673      738703      738734      738765

Columns 247 through 252

   738795      738826      738856      738887      738918      738946
```

```
Columns 253 through 258

   738977      739007      739038      739068      739099      739130

Columns 259 through 264

   739160      739191      739221      739252      739283      739312

Columns 265 through 270

   739343      739373      739404      739434      739465      739496

Columns 271 through 276

   739526      739557      739587      739618      739649      739677

Columns 277 through 282

   739708      739738      739769      739799      739830      739861

Columns 283 through 288

   739891      739922      739952      739983      740014      740042

Columns 289 through 294

   740073      740103      740134      740164      740195      740226

Columns 295 through 300

   740256      740287      740317      740348      740379      740407

Columns 301 through 306

   740438      740468      740499      740529      740560      740591
```

```
  Columns 307 through 312

    740621      740652      740682      740713      740744      740773

  Columns 313 through 318

    740804      740834      740865      740895      740926      740957

  Columns 319 through 324

    740987      741018      741048      741079      741110      741138

  Columns 325 through 330

    741169      741199      741230      741260      741291      741322

  Columns 331 through 334

    741352      741383      741413      741444


TFactors =

  Columns 1 through 7

         0    0.9333    1.9333    2.9333    3.9333    4.9333    5.9333

  Columns 8 through 14

    6.9333    7.9333    8.9333    9.9333   10.9333   11.9333   12.9333

  Columns 15 through 21

   13.9333   14.9333   15.9333   16.9333   17.9333   18.9333   19.9333

  Columns 22 through 28
```

```
 20.9333   21.9333   22.9333   23.9333   24.9333   25.9333   26.9333

Columns 29 through 35

 27.9333   28.9333   29.9333   30.9333   31.9333   32.9333   33.9333

Columns 36 through 42

 34.9333   35.9333   36.9333   37.9333   38.9333   39.9333   40.9333

Columns 43 through 49

 41.9333   42.9333   43.9333   44.9333   45.9333   46.9333   47.9333

Columns 50 through 56

 48.9333   49.9333   50.9333   51.9333   52.9333   53.9333   54.9333

Columns 57 through 63

 55.9333   56.9333   57.9333   58.9333   59.9333   60.9333   61.9333

Columns 64 through 70

 62.9333   63.9333   64.9333   65.9333   66.9333   67.9333   68.9333

Columns 71 through 77

 69.9333   70.9333   71.9333   72.9333   73.9333   74.9333   75.9333

Columns 78 through 84

 76.9333   77.9333   78.9333   79.9333   80.9333   81.9333   82.9333

Columns 85 through 91

 83.9333   84.9333   85.9333   86.9333   87.9333   88.9333   89.9333
```

```
Columns 92 through 98

 90.9333   91.9333   92.9333   93.9333   94.9333   95.9333   96.9333

Columns 99 through 105

 97.9333   98.9333   99.9333  100.9333  101.9333  102.9333  103.9333

Columns 106 through 112

104.9333  105.9333  106.9333  107.9333  108.9333  109.9333  110.9333

Columns 113 through 119

111.9333  112.9333  113.9333  114.9333  115.9333  116.9333  117.9333

Columns 120 through 126

118.9333  119.9333  120.9333  121.9333  122.9333  123.9333  124.9333

Columns 127 through 133

125.9333  126.9333  127.9333  128.9333  129.9333  130.9333  131.9333

Columns 134 through 140

132.9333  133.9333  134.9333  135.9333  136.9333  137.9333  138.9333

Columns 141 through 147

139.9333  140.9333  141.9333  142.9333  143.9333  144.9333  145.9333

Columns 148 through 154

146.9333  147.9333  148.9333  149.9333  150.9333  151.9333  152.9333
```

```
Columns 155 through 161

153.9333  154.9333  155.9333  156.9333  157.9333  158.9333  159.9333

Columns 162 through 168

160.9333  161.9333  162.9333  163.9333  164.9333  165.9333  166.9333

Columns 169 through 175

167.9333  168.9333  169.9333  170.9333  171.9333  172.9333  173.9333

Columns 176 through 182

174.9333  175.9333  176.9333  177.9333  178.9333  179.9333  180.9333

Columns 183 through 189

181.9333  182.9333  183.9333  184.9333  185.9333  186.9333  187.9333

Columns 190 through 196

188.9333  189.9333  190.9333  191.9333  192.9333  193.9333  194.9333

Columns 197 through 203

195.9333  196.9333  197.9333  198.9333  199.9333  200.9333  201.9333

Columns 204 through 210

202.9333  203.9333  204.9333  205.9333  206.9333  207.9333  208.9333

Columns 211 through 217

209.9333  210.9333  211.9333  212.9333  213.9333  214.9333  215.9333

Columns 218 through 224
```

```
216.9333  217.9333  218.9333  219.9333  220.9333  221.9333  222.9333

Columns 225 through 231

223.9333  224.9333  225.9333  226.9333  227.9333  228.9333  229.9333

Columns 232 through 238

230.9333  231.9333  232.9333  233.9333  234.9333  235.9333  236.9333

Columns 239 through 245

237.9333  238.9333  239.9333  240.9333  241.9333  242.9333  243.9333

Columns 246 through 252

244.9333  245.9333  246.9333  247.9333  248.9333  249.9333  250.9333

Columns 253 through 259

251.9333  252.9333  253.9333  254.9333  255.9333  256.9333  257.9333

Columns 260 through 266

258.9333  259.9333  260.9333  261.9333  262.9333  263.9333  264.9333

Columns 267 through 273

265.9333  266.9333  267.9333  268.9333  269.9333  270.9333  271.9333

Columns 274 through 280

272.9333  273.9333  274.9333  275.9333  276.9333  277.9333  278.9333

Columns 281 through 287
```

```
  279.9333  280.9333  281.9333  282.9333  283.9333  284.9333  285.9333

  Columns 288 through 294

  286.9333  287.9333  288.9333  289.9333  290.9333  291.9333  292.9333

  Columns 295 through 301

  293.9333  294.9333  295.9333  296.9333  297.9333  298.9333  299.9333

  Columns 302 through 308

  300.9333  301.9333  302.9333  303.9333  304.9333  305.9333  306.9333

  Columns 309 through 315

  307.9333  308.9333  309.9333  310.9333  311.9333  312.9333  313.9333

  Columns 316 through 322

  314.9333  315.9333  316.9333  317.9333  318.9333  319.9333  320.9333

  Columns 323 through 329

  321.9333  322.9333  323.9333  324.9333  325.9333  326.9333  327.9333

  Columns 330 through 334

  328.9333  329.9333  330.9333  331.9333  332.9333


Factors =

  Columns 1 through 7

    1.0000    0.9944    0.9887    0.9828    0.9769    0.9711    0.9653
```

```
  Columns 8 through 14

    0.9595    0.9538    0.9481    0.9424    0.9368    0.9311    0.9255

  Columns 15 through 21

    0.9199    0.9144    0.9089    0.9034    0.8979    0.8925    0.8871

  Columns 22 through 28

    0.8817    0.8763    0.8710    0.8657    0.8604    0.8552    0.8499

  Columns 29 through 35

    0.8447    0.8396    0.8344    0.8293    0.8242    0.8191    0.8140

  Columns 36 through 42

    0.8090    0.8040    0.7990    0.7941    0.7892    0.7842    0.7794

  Columns 43 through 49

    0.7745    0.7697    0.7649    0.7601    0.7553    0.7506    0.7458

  Columns 50 through 56

    0.7411    0.7365    0.7318    0.7272    0.7226    0.7180    0.7134

  Columns 57 through 63

    0.7089    0.7044    0.6999    0.6954    0.6910    0.6865    0.6821

  Columns 64 through 70

    0.6777    0.6734    0.6690    0.6647    0.6604    0.6561    0.6519

  Columns 71 through 77
```

```
  0.6476    0.6434    0.6392    0.6350    0.6309    0.6267    0.6226

Columns 78 through 84

  0.6185    0.6144    0.6104    0.6063    0.6023    0.5983    0.5943

Columns 85 through 91

  0.5903    0.5864    0.5825    0.5785    0.5747    0.5708    0.5669

Columns 92 through 98

  0.5631    0.5593    0.5555    0.5517    0.5479    0.5442    0.5405

Columns 99 through 105

  0.5368    0.5331    0.5294    0.5257    0.5221    0.5185    0.5149

Columns 106 through 112

  0.5113    0.5077    0.5042    0.5006    0.4971    0.4936    0.4901

Columns 113 through 119

  0.4866    0.4832    0.4797    0.4763    0.4729    0.4695    0.4661

Columns 120 through 126

  0.4628    0.4594    0.4561    0.4528    0.4495    0.4462    0.4430

Columns 127 through 133

  0.4397    0.4365    0.4333    0.4301    0.4269    0.4237    0.4205

Columns 134 through 140
```

```
    0.4174    0.4143    0.4111    0.4080    0.4049    0.4019    0.3988

  Columns 141 through 147

    0.3958    0.3927    0.3897    0.3867    0.3837    0.3808    0.3778

  Columns 148 through 154

    0.3748    0.3719    0.3690    0.3661    0.3632    0.3603    0.3574

  Columns 155 through 161

    0.3546    0.3517    0.3489    0.3461    0.3433    0.3405    0.3377

  Columns 162 through 168

    0.3350    0.3322    0.3295    0.3267    0.3240    0.3213    0.3186

  Columns 169 through 175

    0.3160    0.3133    0.3106    0.3080    0.3054    0.3027    0.3001

  Columns 176 through 182

    0.2975    0.2950    0.2924    0.2898    0.2873    0.2847    0.2822

  Columns 183 through 189

    0.2797    0.2772    0.2747    0.2722    0.2698    0.2673    0.2649

  Columns 190 through 196

    0.2624    0.2600    0.2576    0.2552    0.2528    0.2504    0.2480

  Columns 197 through 203

    0.2457    0.2433    0.2410    0.2386    0.2363    0.2340    0.2317
```

```
 Columns 204 through 210

   0.2294    0.2271    0.2249    0.2226    0.2204    0.2181    0.2159

 Columns 211 through 217

   0.2137    0.2115    0.2093    0.2071    0.2049    0.2027    0.2005

 Columns 218 through 224

   0.1984    0.1962    0.1941    0.1920    0.1899    0.1877    0.1856

 Columns 225 through 231

   0.1836    0.1815    0.1794    0.1773    0.1753    0.1732    0.1712

 Columns 232 through 238

   0.1692    0.1671    0.1651    0.1631    0.1611    0.1591    0.1572

 Columns 239 through 245

   0.1552    0.1532    0.1513    0.1493    0.1474    0.1455    0.1436

 Columns 246 through 252

   0.1416    0.1397    0.1378    0.1359    0.1341    0.1322    0.1303

 Columns 253 through 259

   0.1285    0.1266    0.1248    0.1229    0.1211    0.1193    0.1175

 Columns 260 through 266

   0.1157    0.1139    0.1121    0.1103    0.1085    0.1068    0.1050
```

```
Columns 267 through 273

  0.1032    0.1015    0.0998    0.0980    0.0963    0.0946    0.0929

Columns 274 through 280

  0.0912    0.0895    0.0878    0.0861    0.0844    0.0827    0.0811

Columns 281 through 287

  0.0794    0.0778    0.0761    0.0745    0.0728    0.0712    0.0696

Columns 288 through 294

  0.0680    0.0664    0.0648    0.0632    0.0616    0.0600    0.0584

Columns 295 through 301

  0.0568    0.0553    0.0537    0.0522    0.0506    0.0491    0.0476

Columns 302 through 308

  0.0460    0.0445    0.0430    0.0415    0.0400    0.0385    0.0370

Columns 309 through 315

  0.0355    0.0340    0.0325    0.0311    0.0296    0.0281    0.0267

Columns 316 through 322

  0.0252    0.0238    0.0223    0.0209    0.0195    0.0180    0.0166

Columns 323 through 329

  0.0152    0.0138    0.0124    0.0110    0.0096    0.0082    0.0068

Columns 330 through 334
```

```
    0.0055    0.0041    0.0027    0.0014         0
```

The result is contained in four 334-element row vectors.

### Compute Cash Flow Amounts and Dates, Time Factors, and Mortgage Factors for a Mortgage Portfolio

Given a portfolio of mortgage-backed securities, use `mbscfamounts` to compute the cash flows and other factors from the portfolio.

Define characteristics for a mortgage portfolio.

```
Settle   = datenum(['13-Jan-2000';'17-Apr-2002';'17-May-2002']);
Maturity    = datenum('1-Jan-2030');
IssueDate   = datenum('1-Jan-2000');
GrossRate   = 0.08125;
CouponRate  = [0.075; 0.07875; 0.0775];
Delay       = 14;
PrepaySpeed = 100;
```

Use `mbscfamonts` to evaluate the mortgage.

```
[CFlowAmounts, CFlowDates, TFactors, Factors] = ...
mbscfamounts(Settle, Maturity, IssueDate, GrossRate, ...
CouponRate, Delay, PrepaySpeed)
```

```
CFlowAmounts =


  Columns 1 through 10


  -0.0033    0.0118    0.0120    0.0121    0.0120    0.0119    0.0119    0.0118    0.0117    0.0117
  Columns 11 through 20


   0.0116    0.0115    0.0115    0.0114    0.0114    0.0113    0.0112    0.0112    0.0111    0.0110


  Columns 21 through 30
```

```
  0.0110    0.0109    0.0109    0.0108    0.0107    0.0107    0.0106    0.0106    0.0105    0.0105

Columns 31 through 40

  0.0104    0.0103    0.0103    0.0102    0.0102    0.0101    0.0101    0.0100    0.0099    0.0099

Columns 41 through 50

  0.0098    0.0098    0.0097    0.0097    0.0096    0.0096    0.0095    0.0095    0.0094    0.0094

Columns 51 through 60

  0.0093    0.0093    0.0092    0.0092    0.0091    0.0090    0.0090    0.0089    0.0089    0.0088

Columns 61 through 70

  0.0088    0.0087    0.0087    0.0087    0.0086    0.0086    0.0085    0.0085    0.0084    0.0084

Columns 71 through 80

  0.0083    0.0083    0.0082    0.0082    0.0081    0.0081    0.0080    0.0080    0.0079    0.0079

Columns 81 through 90

  0.0079    0.0078    0.0078    0.0077    0.0077    0.0076    0.0076    0.0075    0.0075    0.0075

Columns 91 through 100

  0.0074    0.0074    0.0073    0.0073    0.0073    0.0072    0.0072    0.0071    0.0071    0.0070

Columns 101 through 110

  0.0070    0.0070    0.0069    0.0069    0.0068    0.0068    0.0068    0.0067    0.0067    0.0066

Columns 111 through 120

  0.0066    0.0066    0.0065    0.0065    0.0065    0.0064    0.0064    0.0063    0.0063    0.0063
```

```
  Columns 121 through 130

    0.0062    0.0062    0.0062    0.0061    0.0061    0.0061    0.0060    0.0060    0.0059    0.0059

  Columns 131 through 140

    0.0059    0.0058    0.0058    0.0058    0.0057    0.0057    0.0057    0.0056    0.0056    0.0056

  Columns 141 through 150

    0.0055    0.0055    0.0055    0.0054    0.0054    0.0054    0.0053    0.0053    0.0053    0.0052

  Columns 151 through 160

    0.0052    0.0052    0.0052    0.0051    0.0051    0.0051    0.0050    0.0050    0.0050    0.0049

  Columns 161 through 170

    0.0049    0.0049    0.0048    0.0048    0.0048    0.0048    0.0047    0.0047    0.0047    0.0046

  Columns 171 through 180

    0.0046    0.0046    0.0046    0.0045    0.0045    0.0045    0.0044    0.0044    0.0044    0.0044

  Columns 181 through 190

    0.0043    0.0043    0.0043    0.0043    0.0042    0.0042    0.0042    0.0041    0.0041    0.0041

  Columns 191 through 200

    0.0041    0.0040    0.0040    0.0040    0.0040    0.0039    0.0039    0.0039    0.0039    0.0038

  Columns 201 through 210

    0.0038    0.0038    0.0038    0.0037    0.0037    0.0037    0.0037    0.0036    0.0036    0.0036
```

```
  Columns 211 through 220

    0.0036    0.0035    0.0035    0.0035    0.0035    0.0035    0.0034    0.0034    0.0034    0.0034

  Columns 221 through 230

    0.0033    0.0033    0.0033    0.0033    0.0033    0.0032    0.0032    0.0032    0.0032    0.0031

  Columns 231 through 240

    0.0031    0.0031    0.0031    0.0031    0.0030    0.0030    0.0030    0.0030    0.0030    0.0029

  Columns 241 through 250

    0.0029    0.0029    0.0029    0.0029    0.0028    0.0028    0.0028    0.0028    0.0028    0.0027

  Columns 251 through 260

    0.0027    0.0027    0.0027    0.0027    0.0026    0.0026    0.0026    0.0026    0.0026    0.0025

  Columns 261 through 270

    0.0025    0.0025    0.0025    0.0025    0.0024    0.0024    0.0024    0.0024    0.0024    0.0024

  Columns 271 through 280

    0.0023    0.0023    0.0023    0.0023    0.0023    0.0023    0.0022    0.0022    0.0022    0.0022

  Columns 281 through 290

    0.0022    0.0021    0.0021    0.0021    0.0021    0.0021    0.0021    0.0020    0.0020    0.0020

  Columns 291 through 300

    0.0020    0.0020    0.0020    0.0020    0.0019    0.0019    0.0019    0.0019    0.0019    0.0019

  Columns 301 through 310
```

```
    0.0018    0.0018    0.0018    0.0018    0.0018    0.0018    0.0017    0.0017    0.0017    0.0017

  Columns 311 through 320

    0.0017    0.0017    0.0017    0.0016    0.0016    0.0016    0.0016    0.0016    0.0016    0.0016

  Columns 321 through 330

    0.0015    0.0015    0.0015    0.0015    0.0015    0.0015    0.0015    0.0014    0.0014    0.0014

  Columns 331 through 334

    0.0014    0.0014    0.0014    0.0014


CFLowDates =

  Columns 1 through 8

      731323      731337      731368      731398      731429      731460      731490      731521

  Columns 9 through 16

      731551      731582      731613      731641      731672      731702      731733      731763

  Columns 17 through 24

      731794      731825      731855      731886      731916      731947      731978      732007

  Columns 25 through 32

      732038      732068      732099      732129      732160      732191      732221      732252

  Columns 33 through 40

      732282      732313      732344      732372      732403      732433      732464      732494
```

```
Columns 41 through 48

  732525      732556      732586      732617      732647      732678      732709      732737

Columns 49 through 56

  732768      732798      732829      732859      732890      732921      732951      732982

Columns 57 through 64

  733012      733043      733074      733102      733133      733163      733194      733224

Columns 65 through 72

  733255      733286      733316      733347      733377      733408      733439      733468

Columns 73 through 80

  733499      733529      733560      733590      733621      733652      733682      733713

Columns 81 through 88

  733743      733774      733805      733833      733864      733894      733925      733955

Columns 89 through 96

  733986      734017      734047      734078      734108      734139      734170      734198

Columns 97 through 104

  734229      734259      734290      734320      734351      734382      734412      734443

Columns 105 through 112

  734473      734504      734535      734563      734594      734624      734655      734685
```

```
  Columns 113 through 120

    734716      734747      734777      734808      734838      734869      734900      734929

  Columns 121 through 128

    734960      734990      735021      735051      735082      735113      735143      735174

  Columns 129 through 136

    735204      735235      735266      735294      735325      735355      735386      735416

  Columns 137 through 144

    735447      735478      735508      735539      735569      735600      735631      735659

  Columns 145 through 152

    735690      735720      735751      735781      735812      735843      735873      735904

  Columns 153 through 160

    735934      735965      735996      736024      736055      736085      736116      736146

  Columns 161 through 168

    736177      736208      736238      736269      736299      736330      736361      736390

  Columns 169 through 176

    736421      736451      736482      736512      736543      736574      736604      736635

  Columns 177 through 184

    736665      736696      736727      736755      736786      736816      736847      736877

  Columns 185 through 192
```

```
   736908      736939      736969      737000      737030      737061      737092      737120

Columns 193 through 200

   737151      737181      737212      737242      737273      737304      737334      737365

Columns 201 through 208

   737395      737426      737457      737485      737516      737546      737577      737607

Columns 209 through 216

   737638      737669      737699      737730      737760      737791      737822      737851

Columns 217 through 224

   737882      737912      737943      737973      738004      738035      738065      738096

Columns 225 through 232

   738126      738157      738188      738216      738247      738277      738308      738338

Columns 233 through 240

   738369      738400      738430      738461      738491      738522      738553      738581

Columns 241 through 248

   738612      738642      738673      738703      738734      738765      738795      738826

Columns 249 through 256

   738856      738887      738918      738946      738977      739007      739038      739068

Columns 257 through 264
```

```
     739099      739130      739160      739191      739221      739252      739283      739312

  Columns 265 through 272

     739343      739373      739404      739434      739465      739496      739526      739557

  Columns 273 through 280

     739587      739618      739649      739677      739708      739738      739769      739799

  Columns 281 through 288

     739830      739861      739891      739922      739952      739983      740014      740042

  Columns 289 through 296

     740073      740103      740134      740164      740195      740226      740256      740287

  Columns 297 through 304

     740317      740348      740379      740407      740438      740468      740499      740529

  Columns 305 through 312

     740560      740591      740621      740652      740682      740713      740744      740773

  Columns 313 through 320

     740804      740834      740865      740895      740926      740957      740987      741018

  Columns 321 through 328

     741048      741079      741110      741138      741169      741199      741230      741260

  Columns 329 through 334

     741291      741322      741352      741383      741413      741444
```

```
TFactors =

  Columns 1 through 10

         0    0.9333    1.9333    2.9333    3.9333    4.9333    5.9333    6.9333    7.9333    8.9333

  Columns 11 through 20

    9.9333   10.9333   11.9333   12.9333   13.9333   14.9333   15.9333   16.9333   17.9333   18.9333

  Columns 21 through 30

   19.9333   20.9333   21.9333   22.9333   23.9333   24.9333   25.9333   26.9333   27.9333   28.9333

  Columns 31 through 40

   29.9333   30.9333   31.9333   32.9333   33.9333   34.9333   35.9333   36.9333   37.9333   38.9333

  Columns 41 through 50

   39.9333   40.9333   41.9333   42.9333   43.9333   44.9333   45.9333   46.9333   47.9333   48.9333

  Columns 51 through 60

   49.9333   50.9333   51.9333   52.9333   53.9333   54.9333   55.9333   56.9333   57.9333   58.9333

  Columns 61 through 70

   59.9333   60.9333   61.9333   62.9333   63.9333   64.9333   65.9333   66.9333   67.9333   68.9333

  Columns 71 through 80

   69.9333   70.9333   71.9333   72.9333   73.9333   74.9333   75.9333   76.9333   77.9333   78.9333

  Columns 81 through 90
```

```
  79.9333   80.9333   81.9333   82.9333   83.9333   84.9333   85.9333   86.9333   87.9333   88.9333

  Columns 91 through 100

  89.9333   90.9333   91.9333   92.9333   93.9333   94.9333   95.9333   96.9333   97.9333   98.9333

  Columns 101 through 110

  99.9333  100.9333  101.9333  102.9333  103.9333  104.9333  105.9333  106.9333  107.9333  108.9333

  Columns 111 through 120

 109.9333  110.9333  111.9333  112.9333  113.9333  114.9333  115.9333  116.9333  117.9333  118.9333

  Columns 121 through 130

 119.9333  120.9333  121.9333  122.9333  123.9333  124.9333  125.9333  126.9333  127.9333  128.9333

  Columns 131 through 140

 129.9333  130.9333  131.9333  132.9333  133.9333  134.9333  135.9333  136.9333  137.9333  138.9333

  Columns 141 through 150

 139.9333  140.9333  141.9333  142.9333  143.9333  144.9333  145.9333  146.9333  147.9333  148.9333

  Columns 151 through 160

 149.9333  150.9333  151.9333  152.9333  153.9333  154.9333  155.9333  156.9333  157.9333  158.9333

  Columns 161 through 170

 159.9333  160.9333  161.9333  162.9333  163.9333  164.9333  165.9333  166.9333  167.9333  168.9333

  Columns 171 through 180
```

```
169.9333  170.9333  171.9333  172.9333  173.9333  174.9333  175.9333  176.9333  177.9333  178.9333

 Columns 181 through 190

179.9333  180.9333  181.9333  182.9333  183.9333  184.9333  185.9333  186.9333  187.9333  188.9333

 Columns 191 through 200

189.9333  190.9333  191.9333  192.9333  193.9333  194.9333  195.9333  196.9333  197.9333  198.9333

 Columns 201 through 210

199.9333  200.9333  201.9333  202.9333  203.9333  204.9333  205.9333  206.9333  207.9333  208.9333

 Columns 211 through 220

209.9333  210.9333  211.9333  212.9333  213.9333  214.9333  215.9333  216.9333  217.9333  218.9333

 Columns 221 through 230

219.9333  220.9333  221.9333  222.9333  223.9333  224.9333  225.9333  226.9333  227.9333  228.9333

 Columns 231 through 240

229.9333  230.9333  231.9333  232.9333  233.9333  234.9333  235.9333  236.9333  237.9333  238.9333

 Columns 241 through 250

239.9333  240.9333  241.9333  242.9333  243.9333  244.9333  245.9333  246.9333  247.9333  248.9333

 Columns 251 through 260

249.9333  250.9333  251.9333  252.9333  253.9333  254.9333  255.9333  256.9333  257.9333  258.9333

 Columns 261 through 270

259.9333  260.9333  261.9333  262.9333  263.9333  264.9333  265.9333  266.9333  267.9333  268.9333
```

```
  Columns 271 through 280

 269.9333  270.9333  271.9333  272.9333  273.9333  274.9333  275.9333  276.9333  277.9333  278.9333

  Columns 281 through 290

 279.9333  280.9333  281.9333  282.9333  283.9333  284.9333  285.9333  286.9333  287.9333  288.9333

  Columns 291 through 300

 289.9333  290.9333  291.9333  292.9333  293.9333  294.9333  295.9333  296.9333  297.9333  298.9333

  Columns 301 through 310

 299.9333  300.9333  301.9333  302.9333  303.9333  304.9333  305.9333  306.9333  307.9333  308.9333

  Columns 311 through 320

 309.9333  310.9333  311.9333  312.9333  313.9333  314.9333  315.9333  316.9333  317.9333  318.9333

  Columns 321 through 330

 319.9333  320.9333  321.9333  322.9333  323.9333  324.9333  325.9333  326.9333  327.9333  328.9333

  Columns 331 through 334

 329.9333  330.9333  331.9333  332.9333


Factors =

  Columns 1 through 10

   1.0000    0.9944    0.9887    0.9828    0.9769    0.9711    0.9653    0.9595    0.9538    0.9481

  Columns 11 through 20
```

```
  0.9424    0.9368    0.9311    0.9255    0.9199    0.9144    0.9089    0.9034    0.8979    0.8925

 Columns 21 through 30

  0.8871    0.8817    0.8763    0.8710    0.8657    0.8604    0.8552    0.8499    0.8447    0.8396

 Columns 31 through 40

  0.8344    0.8293    0.8242    0.8191    0.8140    0.8090    0.8040    0.7990    0.7941    0.7892

 Columns 41 through 50

  0.7842    0.7794    0.7745    0.7697    0.7649    0.7601    0.7553    0.7506    0.7458    0.7411

 Columns 51 through 60

  0.7365    0.7318    0.7272    0.7226    0.7180    0.7134    0.7089    0.7044    0.6999    0.6954

 Columns 61 through 70

  0.6910    0.6865    0.6821    0.6777    0.6734    0.6690    0.6647    0.6604    0.6561    0.6519

 Columns 71 through 80

  0.6476    0.6434    0.6392    0.6350    0.6309    0.6267    0.6226    0.6185    0.6144    0.6104

 Columns 81 through 90

  0.6063    0.6023    0.5983    0.5943    0.5903    0.5864    0.5825    0.5785    0.5747    0.5708

 Columns 91 through 100

  0.5669    0.5631    0.5593    0.5555    0.5517    0.5479    0.5442    0.5405    0.5368    0.5331

 Columns 101 through 110
```

```
  0.5294    0.5257    0.5221    0.5185    0.5149    0.5113    0.5077    0.5042    0.5006    0.4971

Columns 111 through 120

  0.4936    0.4901    0.4866    0.4832    0.4797    0.4763    0.4729    0.4695    0.4661    0.4628

Columns 121 through 130

  0.4594    0.4561    0.4528    0.4495    0.4462    0.4430    0.4397    0.4365    0.4333    0.4301

Columns 131 through 140

  0.4269    0.4237    0.4205    0.4174    0.4143    0.4111    0.4080    0.4049    0.4019    0.3988

Columns 141 through 150

  0.3958    0.3927    0.3897    0.3867    0.3837    0.3808    0.3778    0.3748    0.3719    0.3690

Columns 151 through 160

  0.3661    0.3632    0.3603    0.3574    0.3546    0.3517    0.3489    0.3461    0.3433    0.3405

Columns 161 through 170

  0.3377    0.3350    0.3322    0.3295    0.3267    0.3240    0.3213    0.3186    0.3160    0.3133

Columns 171 through 180

  0.3106    0.3080    0.3054    0.3027    0.3001    0.2975    0.2950    0.2924    0.2898    0.2873

Columns 181 through 190

  0.2847    0.2822    0.2797    0.2772    0.2747    0.2722    0.2698    0.2673    0.2649    0.2624

Columns 191 through 200

  0.2600    0.2576    0.2552    0.2528    0.2504    0.2480    0.2457    0.2433    0.2410    0.2386
```

```
  Columns 201 through 210

    0.2363    0.2340    0.2317    0.2294    0.2271    0.2249    0.2226    0.2204    0.2181    0.2159

  Columns 211 through 220

    0.2137    0.2115    0.2093    0.2071    0.2049    0.2027    0.2005    0.1984    0.1962    0.1941

  Columns 221 through 230

    0.1920    0.1899    0.1877    0.1856    0.1836    0.1815    0.1794    0.1773    0.1753    0.1732

  Columns 231 through 240

    0.1712    0.1692    0.1671    0.1651    0.1631    0.1611    0.1591    0.1572    0.1552    0.1532

  Columns 241 through 250

    0.1513    0.1493    0.1474    0.1455    0.1436    0.1416    0.1397    0.1378    0.1359    0.1341

  Columns 251 through 260

    0.1322    0.1303    0.1285    0.1266    0.1248    0.1229    0.1211    0.1193    0.1175    0.1157

  Columns 261 through 270

    0.1139    0.1121    0.1103    0.1085    0.1068    0.1050    0.1032    0.1015    0.0998    0.0980

  Columns 271 through 280

    0.0963    0.0946    0.0929    0.0912    0.0895    0.0878    0.0861    0.0844    0.0827    0.0811

  Columns 281 through 290

    0.0794    0.0778    0.0761    0.0745    0.0728    0.0712    0.0696    0.0680    0.0664    0.0648
```

```
  Columns 291 through 300

    0.0632    0.0616    0.0600    0.0584    0.0568    0.0553    0.0537    0.0522    0.0506    0.0491

  Columns 301 through 310

    0.0476    0.0460    0.0445    0.0430    0.0415    0.0400    0.0385    0.0370    0.0355    0.0340

  Columns 311 through 320

    0.0325    0.0311    0.0296    0.0281    0.0267    0.0252    0.0238    0.0223    0.0209    0.0195

  Columns 321 through 330

    0.0180    0.0166    0.0152    0.0138    0.0124    0.0110    0.0096    0.0082    0.0068    0.0055

  Columns 331 through 334

    0.0041    0.0027    0.0014         0
```

Each output is a 3-by-361 element matrix padded with `NaN`s wherever elements are missing.

### Calculate Payment, Principal, Interest, and Prepayment for a Single Mortgage

Given a mortgage with the following characteristics, compute payments, principal, interest, and prepayment.

Define the mortgage characteristics.

```
Settle      = datenum('17-April-2002');
Maturity    = datenum('1-Jan-2030');
IssueDate   = datenum('1-Jan-2000');
GrossRate   = 0.08125;
CouponRate  = 0.075;
Delay       = 14;
PrepaySpeed = 100;
```

Use `mbscfamonts` to evaluate the mortgage.

```
[Payment, Principal, Interest, Prepayment] = ...
mbscfamounts(Settle, Maturity, IssueDate, GrossRate, ...
CouponRate, Delay, PrepaySpeed)
```

```
Payment =

  Columns 1 through 8

   -0.0033    0.0118    0.0120    0.0121    0.0120    0.0119    0.0119    0.0118

  Columns 9 through 16

    0.0117    0.0117    0.0116    0.0115    0.0115    0.0114    0.0114    0.0113

  Columns 17 through 24

    0.0112    0.0112    0.0111    0.0110    0.0110    0.0109    0.0109    0.0108

  Columns 25 through 32

    0.0107    0.0107    0.0106    0.0106    0.0105    0.0105    0.0104    0.0103

  Columns 33 through 40

    0.0103    0.0102    0.0102    0.0101    0.0101    0.0100    0.0099    0.0099

  Columns 41 through 48

    0.0098    0.0098    0.0097    0.0097    0.0096    0.0096    0.0095    0.0095

  Columns 49 through 56

    0.0094    0.0094    0.0093    0.0093    0.0092    0.0092    0.0091    0.0090
```

```
Columns 57 through 64

  0.0090    0.0089    0.0089    0.0088    0.0088    0.0087    0.0087    0.0087

Columns 65 through 72

  0.0086    0.0086    0.0085    0.0085    0.0084    0.0084    0.0083    0.0083

Columns 73 through 80

  0.0082    0.0082    0.0081    0.0081    0.0080    0.0080    0.0079    0.0079

Columns 81 through 88

  0.0079    0.0078    0.0078    0.0077    0.0077    0.0076    0.0076    0.0075

Columns 89 through 96

  0.0075    0.0075    0.0074    0.0074    0.0073    0.0073    0.0073    0.0072

Columns 97 through 104

  0.0072    0.0071    0.0071    0.0070    0.0070    0.0070    0.0069    0.0069

Columns 105 through 112

  0.0068    0.0068    0.0068    0.0067    0.0067    0.0066    0.0066    0.0066

Columns 113 through 120

  0.0065    0.0065    0.0065    0.0064    0.0064    0.0063    0.0063    0.0063

Columns 121 through 128

  0.0062    0.0062    0.0062    0.0061    0.0061    0.0061    0.0060    0.0060

Columns 129 through 136
```

```
  0.0059    0.0059    0.0059    0.0058    0.0058    0.0058    0.0057    0.0057

Columns 137 through 144

  0.0057    0.0056    0.0056    0.0056    0.0055    0.0055    0.0055    0.0054

Columns 145 through 152

  0.0054    0.0054    0.0053    0.0053    0.0053    0.0052    0.0052    0.0052

Columns 153 through 160

  0.0052    0.0051    0.0051    0.0051    0.0050    0.0050    0.0050    0.0049

Columns 161 through 168

  0.0049    0.0049    0.0048    0.0048    0.0048    0.0048    0.0047    0.0047

Columns 169 through 176

  0.0047    0.0046    0.0046    0.0046    0.0046    0.0045    0.0045    0.0045

Columns 177 through 184

  0.0044    0.0044    0.0044    0.0044    0.0043    0.0043    0.0043    0.0043

Columns 185 through 192

  0.0042    0.0042    0.0042    0.0041    0.0041    0.0041    0.0041    0.0040

Columns 193 through 200

  0.0040    0.0040    0.0040    0.0039    0.0039    0.0039    0.0039    0.0038

Columns 201 through 208
```

```
  0.0038    0.0038    0.0038    0.0037    0.0037    0.0037    0.0037    0.0036

Columns 209 through 216

  0.0036    0.0036    0.0036    0.0035    0.0035    0.0035    0.0035    0.0035

Columns 217 through 224

  0.0034    0.0034    0.0034    0.0034    0.0033    0.0033    0.0033    0.0033

Columns 225 through 232

  0.0033    0.0032    0.0032    0.0032    0.0032    0.0031    0.0031    0.0031

Columns 233 through 240

  0.0031    0.0031    0.0030    0.0030    0.0030    0.0030    0.0030    0.0029

Columns 241 through 248

  0.0029    0.0029    0.0029    0.0029    0.0028    0.0028    0.0028    0.0028

Columns 249 through 256

  0.0028    0.0027    0.0027    0.0027    0.0027    0.0027    0.0026    0.0026

Columns 257 through 264

  0.0026    0.0026    0.0026    0.0025    0.0025    0.0025    0.0025    0.0025

Columns 265 through 272

  0.0024    0.0024    0.0024    0.0024    0.0024    0.0024    0.0023    0.0023

Columns 273 through 280

  0.0023    0.0023    0.0023    0.0023    0.0022    0.0022    0.0022    0.0022
```

```
  Columns 281 through 288

    0.0022    0.0021    0.0021    0.0021    0.0021    0.0021    0.0021    0.0020

  Columns 289 through 296

    0.0020    0.0020    0.0020    0.0020    0.0020    0.0020    0.0019    0.0019

  Columns 297 through 304

    0.0019    0.0019    0.0019    0.0019    0.0018    0.0018    0.0018    0.0018

  Columns 305 through 312

    0.0018    0.0018    0.0017    0.0017    0.0017    0.0017    0.0017    0.0017

  Columns 313 through 320

    0.0017    0.0016    0.0016    0.0016    0.0016    0.0016    0.0016    0.0016

  Columns 321 through 328

    0.0015    0.0015    0.0015    0.0015    0.0015    0.0015    0.0015    0.0014

  Columns 329 through 334

    0.0014    0.0014    0.0014    0.0014    0.0014    0.0014


Principal =

  Columns 1 through 6

      731323      731337      731368      731398      731429      731460

  Columns 7 through 12
```

```
   731490      731521      731551      731582      731613      731641

Columns 13 through 18

   731672      731702      731733      731763      731794      731825

Columns 19 through 24

   731855      731886      731916      731947      731978      732007

Columns 25 through 30

   732038      732068      732099      732129      732160      732191

Columns 31 through 36

   732221      732252      732282      732313      732344      732372

Columns 37 through 42

   732403      732433      732464      732494      732525      732556

Columns 43 through 48

   732586      732617      732647      732678      732709      732737

Columns 49 through 54

   732768      732798      732829      732859      732890      732921

Columns 55 through 60

   732951      732982      733012      733043      733074      733102

Columns 61 through 66
```

```
   733133      733163      733194      733224      733255      733286

Columns 67 through 72

   733316      733347      733377      733408      733439      733468

Columns 73 through 78

   733499      733529      733560      733590      733621      733652

Columns 79 through 84

   733682      733713      733743      733774      733805      733833

Columns 85 through 90

   733864      733894      733925      733955      733986      734017

Columns 91 through 96

   734047      734078      734108      734139      734170      734198

Columns 97 through 102

   734229      734259      734290      734320      734351      734382

Columns 103 through 108

   734412      734443      734473      734504      734535      734563

Columns 109 through 114

   734594      734624      734655      734685      734716      734747

Columns 115 through 120

   734777      734808      734838      734869      734900      734929
```

```
Columns 121 through 126

    734960      734990      735021      735051      735082      735113

Columns 127 through 132

    735143      735174      735204      735235      735266      735294

Columns 133 through 138

    735325      735355      735386      735416      735447      735478

Columns 139 through 144

    735508      735539      735569      735600      735631      735659

Columns 145 through 150

    735690      735720      735751      735781      735812      735843

Columns 151 through 156

    735873      735904      735934      735965      735996      736024

Columns 157 through 162

    736055      736085      736116      736146      736177      736208

Columns 163 through 168

    736238      736269      736299      736330      736361      736390

Columns 169 through 174

    736421      736451      736482      736512      736543      736574
```

```
  Columns 175 through 180

     736604      736635      736665      736696      736727      736755

  Columns 181 through 186

     736786      736816      736847      736877      736908      736939

  Columns 187 through 192

     736969      737000      737030      737061      737092      737120

  Columns 193 through 198

     737151      737181      737212      737242      737273      737304

  Columns 199 through 204

     737334      737365      737395      737426      737457      737485

  Columns 205 through 210

     737516      737546      737577      737607      737638      737669

  Columns 211 through 216

     737699      737730      737760      737791      737822      737851

  Columns 217 through 222

     737882      737912      737943      737973      738004      738035

  Columns 223 through 228

     738065      738096      738126      738157      738188      738216

  Columns 229 through 234
```

```
    738247      738277      738308      738338      738369      738400

Columns 235 through 240

    738430      738461      738491      738522      738553      738581

Columns 241 through 246

    738612      738642      738673      738703      738734      738765

Columns 247 through 252

    738795      738826      738856      738887      738918      738946

Columns 253 through 258

    738977      739007      739038      739068      739099      739130

Columns 259 through 264

    739160      739191      739221      739252      739283      739312

Columns 265 through 270

    739343      739373      739404      739434      739465      739496

Columns 271 through 276

    739526      739557      739587      739618      739649      739677

Columns 277 through 282

    739708      739738      739769      739799      739830      739861

Columns 283 through 288
```

```
     739891      739922      739952      739983      740014      740042

   Columns 289 through 294

     740073      740103      740134      740164      740195      740226

   Columns 295 through 300

     740256      740287      740317      740348      740379      740407

   Columns 301 through 306

     740438      740468      740499      740529      740560      740591

   Columns 307 through 312

     740621      740652      740682      740713      740744      740773

   Columns 313 through 318

     740804      740834      740865      740895      740926      740957

   Columns 319 through 324

     740987      741018      741048      741079      741110      741138

   Columns 325 through 330

     741169      741199      741230      741260      741291      741322

   Columns 331 through 334

     741352      741383      741413      741444


 Interest =
```

```
Columns 1 through 8

      0    0.9333    1.9333    2.9333    3.9333    4.9333    5.9333    6.9333

Columns 9 through 16

  7.9333    8.9333    9.9333   10.9333   11.9333   12.9333   13.9333   14.9333

Columns 17 through 24

 15.9333   16.9333   17.9333   18.9333   19.9333   20.9333   21.9333   22.9333

Columns 25 through 32

 23.9333   24.9333   25.9333   26.9333   27.9333   28.9333   29.9333   30.9333

Columns 33 through 40

 31.9333   32.9333   33.9333   34.9333   35.9333   36.9333   37.9333   38.9333

Columns 41 through 48

 39.9333   40.9333   41.9333   42.9333   43.9333   44.9333   45.9333   46.9333

Columns 49 through 56

 47.9333   48.9333   49.9333   50.9333   51.9333   52.9333   53.9333   54.9333

Columns 57 through 64

 55.9333   56.9333   57.9333   58.9333   59.9333   60.9333   61.9333   62.9333

Columns 65 through 72

 63.9333   64.9333   65.9333   66.9333   67.9333   68.9333   69.9333   70.9333

Columns 73 through 80
```

```
 71.9333   72.9333   73.9333   74.9333   75.9333   76.9333   77.9333   78.9333

Columns 81 through 88

 79.9333   80.9333   81.9333   82.9333   83.9333   84.9333   85.9333   86.9333

Columns 89 through 96

 87.9333   88.9333   89.9333   90.9333   91.9333   92.9333   93.9333   94.9333

Columns 97 through 104

 95.9333   96.9333   97.9333   98.9333   99.9333  100.9333  101.9333  102.9333

Columns 105 through 112

103.9333  104.9333  105.9333  106.9333  107.9333  108.9333  109.9333  110.9333

Columns 113 through 120

111.9333  112.9333  113.9333  114.9333  115.9333  116.9333  117.9333  118.9333

Columns 121 through 128

119.9333  120.9333  121.9333  122.9333  123.9333  124.9333  125.9333  126.9333

Columns 129 through 136

127.9333  128.9333  129.9333  130.9333  131.9333  132.9333  133.9333  134.9333

Columns 137 through 144

135.9333  136.9333  137.9333  138.9333  139.9333  140.9333  141.9333  142.9333

Columns 145 through 152
```

```
143.9333  144.9333  145.9333  146.9333  147.9333  148.9333  149.9333  150.9333

Columns 153 through 160

151.9333  152.9333  153.9333  154.9333  155.9333  156.9333  157.9333  158.9333

Columns 161 through 168

159.9333  160.9333  161.9333  162.9333  163.9333  164.9333  165.9333  166.9333

Columns 169 through 176

167.9333  168.9333  169.9333  170.9333  171.9333  172.9333  173.9333  174.9333

Columns 177 through 184

175.9333  176.9333  177.9333  178.9333  179.9333  180.9333  181.9333  182.9333

Columns 185 through 192

183.9333  184.9333  185.9333  186.9333  187.9333  188.9333  189.9333  190.9333

Columns 193 through 200

191.9333  192.9333  193.9333  194.9333  195.9333  196.9333  197.9333  198.9333

Columns 201 through 208

199.9333  200.9333  201.9333  202.9333  203.9333  204.9333  205.9333  206.9333

Columns 209 through 216

207.9333  208.9333  209.9333  210.9333  211.9333  212.9333  213.9333  214.9333

Columns 217 through 224

215.9333  216.9333  217.9333  218.9333  219.9333  220.9333  221.9333  222.9333
```

```
  Columns 225 through 232

  223.9333  224.9333  225.9333  226.9333  227.9333  228.9333  229.9333  230.9333

  Columns 233 through 240

  231.9333  232.9333  233.9333  234.9333  235.9333  236.9333  237.9333  238.9333

  Columns 241 through 248

  239.9333  240.9333  241.9333  242.9333  243.9333  244.9333  245.9333  246.9333

  Columns 249 through 256

  247.9333  248.9333  249.9333  250.9333  251.9333  252.9333  253.9333  254.9333

  Columns 257 through 264

  255.9333  256.9333  257.9333  258.9333  259.9333  260.9333  261.9333  262.9333

  Columns 265 through 272

  263.9333  264.9333  265.9333  266.9333  267.9333  268.9333  269.9333  270.9333

  Columns 273 through 280

  271.9333  272.9333  273.9333  274.9333  275.9333  276.9333  277.9333  278.9333

  Columns 281 through 288

  279.9333  280.9333  281.9333  282.9333  283.9333  284.9333  285.9333  286.9333

  Columns 289 through 296

  287.9333  288.9333  289.9333  290.9333  291.9333  292.9333  293.9333  294.9333
```

```
  Columns 297 through 304

  295.9333  296.9333  297.9333  298.9333  299.9333  300.9333  301.9333  302.9333

  Columns 305 through 312

  303.9333  304.9333  305.9333  306.9333  307.9333  308.9333  309.9333  310.9333

  Columns 313 through 320

  311.9333  312.9333  313.9333  314.9333  315.9333  316.9333  317.9333  318.9333

  Columns 321 through 328

  319.9333  320.9333  321.9333  322.9333  323.9333  324.9333  325.9333  326.9333

  Columns 329 through 334

  327.9333  328.9333  329.9333  330.9333  331.9333  332.9333


Prepayment =

  Columns 1 through 8

    1.0000    0.9944    0.9887    0.9828    0.9769    0.9711    0.9653    0.9595

  Columns 9 through 16

    0.9538    0.9481    0.9424    0.9368    0.9311    0.9255    0.9199    0.9144

  Columns 17 through 24

    0.9089    0.9034    0.8979    0.8925    0.8871    0.8817    0.8763    0.8710

  Columns 25 through 32
```

```
    0.8657    0.8604    0.8552    0.8499    0.8447    0.8396    0.8344    0.8293

  Columns 33 through 40

    0.8242    0.8191    0.8140    0.8090    0.8040    0.7990    0.7941    0.7892

  Columns 41 through 48

    0.7842    0.7794    0.7745    0.7697    0.7649    0.7601    0.7553    0.7506

  Columns 49 through 56

    0.7458    0.7411    0.7365    0.7318    0.7272    0.7226    0.7180    0.7134

  Columns 57 through 64

    0.7089    0.7044    0.6999    0.6954    0.6910    0.6865    0.6821    0.6777

  Columns 65 through 72

    0.6734    0.6690    0.6647    0.6604    0.6561    0.6519    0.6476    0.6434

  Columns 73 through 80

    0.6392    0.6350    0.6309    0.6267    0.6226    0.6185    0.6144    0.6104

  Columns 81 through 88

    0.6063    0.6023    0.5983    0.5943    0.5903    0.5864    0.5825    0.5785

  Columns 89 through 96

    0.5747    0.5708    0.5669    0.5631    0.5593    0.5555    0.5517    0.5479

  Columns 97 through 104

    0.5442    0.5405    0.5368    0.5331    0.5294    0.5257    0.5221    0.5185
```

```
  Columns 105 through 112

    0.5149    0.5113    0.5077    0.5042    0.5006    0.4971    0.4936    0.4901

  Columns 113 through 120

    0.4866    0.4832    0.4797    0.4763    0.4729    0.4695    0.4661    0.4628

  Columns 121 through 128

    0.4594    0.4561    0.4528    0.4495    0.4462    0.4430    0.4397    0.4365

  Columns 129 through 136

    0.4333    0.4301    0.4269    0.4237    0.4205    0.4174    0.4143    0.4111

  Columns 137 through 144

    0.4080    0.4049    0.4019    0.3988    0.3958    0.3927    0.3897    0.3867

  Columns 145 through 152

    0.3837    0.3808    0.3778    0.3748    0.3719    0.3690    0.3661    0.3632

  Columns 153 through 160

    0.3603    0.3574    0.3546    0.3517    0.3489    0.3461    0.3433    0.3405

  Columns 161 through 168

    0.3377    0.3350    0.3322    0.3295    0.3267    0.3240    0.3213    0.3186

  Columns 169 through 176

    0.3160    0.3133    0.3106    0.3080    0.3054    0.3027    0.3001    0.2975
```

```
  Columns 177 through 184

    0.2950    0.2924    0.2898    0.2873    0.2847    0.2822    0.2797    0.2772

  Columns 185 through 192

    0.2747    0.2722    0.2698    0.2673    0.2649    0.2624    0.2600    0.2576

  Columns 193 through 200

    0.2552    0.2528    0.2504    0.2480    0.2457    0.2433    0.2410    0.2386

  Columns 201 through 208

    0.2363    0.2340    0.2317    0.2294    0.2271    0.2249    0.2226    0.2204

  Columns 209 through 216

    0.2181    0.2159    0.2137    0.2115    0.2093    0.2071    0.2049    0.2027

  Columns 217 through 224

    0.2005    0.1984    0.1962    0.1941    0.1920    0.1899    0.1877    0.1856
  Columns 225 through 232

    0.1836    0.1815    0.1794    0.1773    0.1753    0.1732    0.1712    0.1692

  Columns 233 through 240

    0.1671    0.1651    0.1631    0.1611    0.1591    0.1572    0.1552    0.1532

  Columns 241 through 248

    0.1513    0.1493    0.1474    0.1455    0.1436    0.1416    0.1397    0.1378

  Columns 249 through 256
```

```
  0.1359    0.1341    0.1322    0.1303    0.1285    0.1266    0.1248    0.1229

Columns 257 through 264

  0.1211    0.1193    0.1175    0.1157    0.1139    0.1121    0.1103    0.1085

Columns 265 through 272

  0.1068    0.1050    0.1032    0.1015    0.0998    0.0980    0.0963    0.0946

Columns 273 through 280

  0.0929    0.0912    0.0895    0.0878    0.0861    0.0844    0.0827    0.0811

Columns 281 through 288

  0.0794    0.0778    0.0761    0.0745    0.0728    0.0712    0.0696    0.0680

Columns 289 through 296

  0.0664    0.0648    0.0632    0.0616    0.0600    0.0584    0.0568    0.0553

Columns 297 through 304

  0.0537    0.0522    0.0506    0.0491    0.0476    0.0460    0.0445    0.0430

Columns 305 through 312

  0.0415    0.0400    0.0385    0.0370    0.0355    0.0340    0.0325    0.0311

Columns 313 through 320

  0.0296    0.0281    0.0267    0.0252    0.0238    0.0223    0.0209    0.0195

Columns 321 through 328

  0.0180    0.0166    0.0152    0.0138    0.0124    0.0110    0.0096    0.0082
```

```
  Columns 329 through 334

    0.0068    0.0055    0.0041    0.0027    0.0014         0
```

## References

[1] *PSA Uniform Practices*, SF-4

## See Also

`mbspassthrough` | `mbsnoprepay` | `cmoseqcf` | `cmoschedcf` | `cmosched`

# mbsconvp

## Purpose

Convexity of mortgage pool given price

## Syntax

```
Convexity = mbsconvp(Price, Settle, Maturity, IssueDate, GrossRate,
CouponRate, Delay, PrepaySpeed, PrepayMatrix)
```

## Arguments

| | |
|---|---|
| `Price` | Clean price for every $100 face value. |
| `Settle` | Settlement date. A serial date number or date string. `Settle` must be earlier than `Maturity`. |
| `Maturity` | Maturity date. A serial date number or date string. |
| `IssueDate` | Issue date. A serial date number or date string. |
| `GrossRate` | Gross coupon rate (including fees), in decimal. |
| `CouponRate` | Net coupon rate, in decimal. Default = `GrossRate`. |
| `Delay` | Delay in days. |
| `PrepaySpeed` | (Optional) Relation of the conditional payment rate (CPR) to the benchmark model. Default = 0. Set `PrepaySpeed` to `[]` if you input a customized prepayment matrix. |
| `PrepayMatrix` | (Optional) Used only when `PrepaySpeed` is unspecified. Customized prepayment vector. A `NaN`-padded matrix of size `max(TermRemaining)`-by-`NMBS`. Each column corresponds to each mortgage-backed security, and each row corresponds to each month after settlement. |

All inputs (except `PrepayMatrix`) are number of mortgage-backed securities (`NMBS`) by 1 vectors.

## Description

```
Convexity = mbsconvp(Price, Settle, Maturity, IssueDate,
GrossRate, CouponRate, Delay, PrepaySpeed, PrepayMatrix)
```

computes mortgage-backed security convexity, given time information, price at settlement, and optionally, a prepayment model.

---

**Note** If you specify the PSA or FHA model, it will be seasoned with how long the debt has been outstanding (the loan's age).

---

## Examples

Given a mortgage-backed security with the following characteristics, compute the convexity of the security.

```
Price      = 101;
Settle     = '15-Apr-2002';
Maturity   = '1 Jan 2030';
IssueDate  = '1-Jan-2000';
GrossRate  = 0.08125;
CouponRate = 0.075;
Delay = 14;
Speed = 100;

Convexity = mbsconvp(Price, Settle, Maturity, IssueDate,...
GrossRate, CouponRate, Delay, Speed)

Convexity =

   71.6299
```

## References

[1] *PSA Uniform Practices*, SF-49

## See Also

`mbsconvy` | `mbsdurp` | `mbspassthrough` | `mbsnoprepay` | `mbsdury`

## Purpose

Convexity of mortgage pool given yield

## Syntax

```
Convexity = mbsconvy(Yield, Settle, Maturity, IssueDate, GrossRate,
CouponRate, Delay, PrepaySpeed, PrepayMatrix)
```

## Arguments

| | |
|---|---|
| `Yield` | Mortgage yield, compounded monthly (in decimal). |
| `Settle` | Settlement date. A serial date number or date string. `Settle` must be earlier than `Maturity`. |
| `Maturity` | Maturity date. A serial date number or date string. |
| `IssueDate` | Issue date. A serial date number or date string. |
| `GrossRate` | Gross coupon rate (including fees), in decimal. |
| `CouponRate` | Net coupon rate, in decimal. Default = `GrossRate`. |
| `Delay` | Delay in days. |
| `PrepaySpeed` | (Optional) Relation of the conditional payment rate (CPR) to the benchmark model. Default = 0. Set `PrepaySpeed` to `[]` if you input a customized prepayment matrix. |
| `PrepayMatrix` | (Optional) Used only when `PrepaySpeed` is unspecified. Customized prepayment vector. A `NaN`-padded matrix of size `max(TermRemaining)`-by-`NMBS`. Each column corresponds to each mortgage-backed security, and each row corresponds to each month after settlement. |

All inputs (except `PrepayMatrix`) are number of mortgage-backed securities (`NMBS`) by `1` vectors.

## Description

`Convexity = mbsconvy(Yield, Settle, Maturity, IssueDate, GrossRate, CouponRate, Delay, PrepaySpeed, PrepayMatrix)` computes mortgage-backed security convexity, given time information, semiannual mortgage yield, and optionally, a prepayment model.

---

**Note** If you specify the PSA or FHA model, it will be seasoned with how long the debt has been outstanding (the loan's age).

---

## Examples

Given a mortgage-backed security with the following characteristics, compute the convexity of the security.

```
Yield      = 0.07125;
Settle     = '15-Apr-2002';
Maturity   = '1 Jan 2030';
IssueDate  = '1-Jan-2000';
GrossRate  = 0.08125;
Speed      = 100;
CouponRate = 0.075;
Delay = 14;

Convexity = mbsconvy(Yield, Settle, Maturity, IssueDate, ...
GrossRate, CouponRate, Delay, Speed)

Convexity =

  72.8263
```

## References

[1] *PSA Uniform Practices*, SF-49

## See Also

`mbsconvp` | `mbsdurp` | `mbsdury`

# mbsdurp

## Purpose

Duration of mortgage pool given price

## Syntax

```
[YearDuration, ModDuration] = mbsdurp(Price, Settle, Maturity,
IssueDate, GrossRate, CouponRate, Delay, PrepaySpeed,
PrepayMatrix)
```

## Arguments

| | |
|---|---|
| `Price` | Clean price for every $100 face value. |
| `Settle` | Settlement date. A serial date number or date string. `Settle` must be earlier than or equal to `Maturity`. |
| `Maturity` | Maturity date. A serial date number or date string. |
| `IssueDate` | Issue date. A serial date number or date string. |
| `GrossRate` | Gross coupon rate (including fees), in decimal. |
| `CouponRate` | Net coupon rate, in decimal. Default = `GrossRate`. |
| `Delay` | Delay in days. |
| `PrepaySpeed` | (Optional) Relation of the conditional payment rate (CPR) to the benchmark model. Default = 0. Set `PrepaySpeed` to `[]` if you input a customized prepayment matrix. |
| `PrepayMatrix` | (Optional) Used only when `PrepaySpeed` is unspecified. Customized prepayment vector. A `NaN`-padded matrix of size `max(TermRemaining)`-by-`NMBS`. Each column corresponds to each mortgage-backed security, and each row corresponds to each month after settlement. |

All inputs (except `PrepayMatrix`) are number of mortgage-backed securities (`NMBS`) by `1` vectors.

## Description

```
[YearDuration, ModDuration] = mbsdurp(Price, Settle,
Maturity, IssueDate, GrossRate, CouponRate, Delay,
```

`PrepaySpeed, PrepayMatrix)` computes the mortgage-backed security Macaulay (`YearDuration`) and modified (`ModDuration`) durations, given time information, price at settlement, and optionally, a prepayment model.

---

**Note** If you specify the PSA or FHA model, it will be seasoned with how long the debt has been outstanding (the loan's age).

---

## Examples

Given a mortgage-backed security with the following characteristics, compute the Macaulay and modified durations of the security.

```
Price = 101;
Settle = datenum('15-Apr-2002');
Maturity = datenum('1 Jan 2030');
IssueDate = datenum('1-Jan-2000');
GrossRate = 0.08125;
CouponRate = 0.075;;
Delay = 14;
Speed = 100;

[YearDuration, ModDuration] = mbsdurp(Price, Settle, Maturity,...
IssueDate, GrossRate, CouponRate, Delay, Speed)

YearDuration =

    6.4380

ModDuration =

    6.2080
```

## References

[1] *PSA Uniform Practices*, SF-49

## See Also

`mbsconvp` | `mbsconvy` | `mbsdury`

# mbsdury

## Purpose

Duration of mortgage pool given yield

## Syntax

```
[YearDuration, ModDuration] = mbsdury(Yield, Settle, Maturity,
IssueDate, GrossRate, CouponRate, Delay, PrepaySpeed,
PrepayMatrix)
```

## Arguments

| | |
|---|---|
| `Yield` | Mortgage yield, compounded monthly, in decimal. |
| `Settle` | Settlement date. A serial date number or date string. `Settle` must be earlier than `Maturity`. |
| `Maturity` | Maturity date. A serial date number or date string. |
| `IssueDate` | Issue date. A serial date number or date string. |
| `GrossRate` | Gross coupon rate (including fees), in decimal. |
| `CouponRate` | Net coupon rate, in decimal. Default = `GrossRate`. |
| `Delay` | Delay in days. |
| `PrepaySpeed` | (Optional) Relation of the conditional payment rate (CPR) to the benchmark model. Default = 0. Set `PrepaySpeed` to `[]` if you input a customized prepayment matrix. |
| `PrepayMatrix` | (Optional) Used only when `PrepaySpeed` is unspecified. Customized prepayment vector. A `NaN`-padded matrix of size `max(TermRemaining)`-by-`NMBS`. Each column corresponds to each mortgage-backed security, and each row corresponds to each month after settlement. |

All inputs (except `PrepayMatrix`) are number of mortgage-backed securities (`NMBS`) by `1` vectors.

## Description

`[YearDuration, ModDuration] = mbsdurvy(Yield, Settle, Maturity, IssueDate, GrossRate, CouponRate, Delay, PrepaySpeed, PrepayMatrix)` computes the mortgage-backed security

Macaulay (`YearDuration`) and Modified (`ModDuration`) durations, given time information, yield to maturity, and optionally, a prepayment model.

---

**Note** If you specify the PSA or FHA model, it will be seasoned with how long the debt has been outstanding (the loan's age).

---

## Examples

Given a mortgage-backed security with the following characteristics, compute the Macaulay and Modified durations of the security.

```
Yield = 0.07298413;
Settle = '15-Apr-2002';
Maturity = '1 Jan 2030';
IssueDate = '1-Jan-2000';
GrossRate = 0.08125;
Speed = 100;
CouponRate = 0.075;
Delay = 14;

[YearDuration, ModDuration] = mbsdury(Yield, Settle, Maturity,...
IssueDate, GrossRate, CouponRate, Delay, Speed)

YearDuration =

    6.4380

ModDuration =

    6.2080
```

## References

[1] *PSA Uniform Practices*, SF-49

## See Also

`mbsconvp` | `mbsconvy` | `mbsdurp`

# mbsnoprepay

## Purpose

End-of-month mortgage cash flows and balances without prepayment

## Syntax

```
[Balance, Interest, Payment, Principal] =
mbsnoprepay(OriginalBalance, GrossRate, Term)
```

## Arguments

| | |
|---|---|
| `OriginalBalance` | Original face value in dollars. |
| `GrossRate` | Gross coupon rate (including fees), in decimal. |
| `Term` | Term of the mortgage in months. |

All inputs are number of mortgage-backed securities (`NMBS`)-by-`1` vectors.

## Description

`[Balance, Interest, Payment, Principal] = mbsnoprepay(OriginalBalance, GrossRate, Term)` computes end-of-month mortgage balance, interest payments, principal payments, and cash flow payments with zero prepayment rate.

The function returns amortizing cash flows and balances over a specified term with no prepayment. When the lengths of pass-throughs are not the same, MATLAB software pads the shorter ones with `NaN`.

`Balance` lists the end-of-month balances over the life of the pass-through.

`Interest` lists all end-of-month interest payments over the life of the pass-through.

`Payment` lists all end-of-month payments over the life of the pass-through.

`Principal` lists all scheduled end-of-month principal payments over the life of the pass-through.

All outputs are `Term`-by-`1` vectors.

## Examples

Given mortgage pools with the following characteristics, compute an amortization schedule.

```
OriginalBalance = 400000000;
CouponRate = 0.08125;
Term = [357; 355]; % Three- and five-month old mortgage pools.

[Balance, Interest, Payment, Principal] = ...
mbsnoprepay(OriginalBalance, CouponRate, Term);
```

## Purpose

Price given option-adjusted spread

## Syntax

```
Price = mbsoas2price(ZeroCurve, OAS, Settle, Maturity, IssueDate,
GrossRate, CouponRate, Delay, Interpolation, PrepaySpeed,
PrepayMatrix)
```

## Arguments

| | |
|---|---|
| `ZeroCurve` | A matrix of three columns:<br>• Column 1: Serial date numbers.<br>• Column 2: Spot rates with maturities corresponding to the dates in Column 1, in decimal (for example, 0.075).<br>• Column 3: Compounding of the rates in Column 2. (This is the agency spot rate on the settlement date.) |
| `OAS` | Option-adjusted spreads in basis points. |
| `Settle` | Settlement date (scalar only). A serial date number or date string. Date when option-adjusted spread is calculated. `Settle` must be earlier than `Maturity`. |
| `Maturity` | Maturity date. Scalar or vector in serial date number or date string format. |
| `IssueDate` | Issue date. A serial date number or date string. |
| `GrossRate` | Gross coupon rate (including fees), in decimal. |
| `CouponRate` | (Optional) Net coupon rate, in decimal. Default = `GrossRate`. |
| `Delay` | (Optional) Delay (in days) between payment from homeowner and receipt by bondholder. Default = 0 (no delay between payment and receipt). |

| | |
|---|---|
| `Interpolation` | Interpolation method. Computes the corresponding spot rates for the bond's cash flow. Available methods are (0) nearest, (1) linear, and (2) cubic spline. Default = 1. See `interp1` for more information. |
| `PrepaySpeed` | (Optional) Relation of the conditional payment rate (CPR) to the benchmark model. Default = end of month's CPR. Set `PrepaySpeed` to `[]` if you input a customized prepayment matrix. |
| `PrepayMatrix` | (Optional) Customized prepayment matrix. A matrix of size `max(TermRemaining)`-by-`NMBS`. Missing values are padded with `NaN`s. Each column corresponds to a mortgage-backed security, and each row corresponds to each month after settlement. |

All inputs (except `PrepayMatrix`) are number of mortgage-backed securities (`NMBS`) by `1` vectors.

## Description

`Price = mbsoas2price(ZeroCurve, OAS, Settle, Maturity, IssueDate, GrossRate, CouponRate, Delay, Interpolation, PrepaySpeed, PrepayMatrix)` computes the clean price of a pass-through security for each $100 face value of outstanding principal.

## Examples

Given an option-adjusted spread, a spot curve, and a prepayment assumption, compute theoretical price of a mortgage pool.

Create the bonds matrix.

```
Bonds = [datenum('11/21/2002')  0        100  0  2  1;
         datenum('02/20/2003')  0        100  0  2  1;
         datenum('07/31/2004')  0.03     100  2  3  1;
         datenum('08/15/2007')  0.035    100  2  3  1;
         datenum('08/15/2012')  0.04875  100  2  3  1;
         datenum('02/15/2031')  0.05375  100  2  3  1];
```

Choose a settlement date.

```
Settle = datenum('20-Aug-2002');
```

Assume these clean prices for the bonds.

```
Prices =  [ 98.97467;
            98.58044;
           100.10534;
            98.18054;
           101.38136;
            99.25411];
```

Use this formula to compute spot compounding for the bonds.

```
SpotCompounding = 2*ones(size(Prices));
```

Use `compute the` zero curve.

```
[ZeroRatesP, CurveDatesP] = zbtprice(Bonds, Prices, Settle);
ZeroCurve = [CurveDatesP, ZeroRatesP, SpotCompounding];
```

Assign parameters.

```
OAS           = [26.0502; 28.6348; 31.2222];
Maturity      = datenum('02-Jan-2030');
IssueDate     = datenum('02-Jan-2000');
GrossRate     = 0.08125;
CouponRate    = 0.075;
Delay         = 14;
Interpolation = 1;
PrepaySpeed   = [0 50 100];
```

Calculate the price from the option-adjusted spread.

```
Price = mbsoas2price(ZeroCurve, OAS, Settle, Maturity, ...
IssueDate, GrossRate, CouponRate, Delay, Interpolation, ...
PrepaySpeed)
```

```
Price =

   95.0000
   95.0000
   95.0000
```

## See Also

`mbsprice2oas` | `mbsyield2oas` | `mbsoas2yield`

# mbsoas2yield

**Purpose** Yield given option-adjusted spread

**Syntax**

```
[MYield, BEMBSYield] = mbsoas2yield(ZeroCurve, OAS, Settle,
Maturity, IssueDate, GrossRate, CouponRate, Delay, Interpolation,
PrepaySpeed, PrepayMatrix)
```

**Arguments**

| | |
|---|---|
| `ZeroCurve` | A matrix of three columns:<br>• Column 1: Serial date numbers.<br>• Column 2: Spot rates with maturities corresponding to the dates in Column 1, in decimal (for example, 0.075).<br>• Column 3: Compounding of the rates in Column 2. (This is the agency spot rate on the settlement date.) |
| `OAS` | Option-adjusted spreads in basis points. |
| `Settle` | Settlement date (scalar only). A serial date number or date string. Date when option-adjusted spread is calculated. `Settle` must be earlier than `Maturity`. |
| `Maturity` | Maturity date. Scalar or vector in serial date number or date string format. |
| `IssueDate` | Issue date. A serial date number or date string. |
| `GrossRate` | Gross coupon rate (including fees), in decimal. |
| `CouponRate` | (Optional) Net coupon rate, in decimal. Default = `GrossRate`. |
| `Delay` | (Optional) Delay (in days) between payment from homeowner and receipt by bondholder. Default = 0 (no delay between payment and receipt). |

| | |
|---|---|
| `Interpolation` | Interpolation method. Computes the corresponding spot rates for the bond's cash flow. Available methods are (0) nearest, (1) linear, and (2) cubic spline. Default = 1. See `interp1` for more information. |
| `PrepaySpeed` | (Optional) Relation of the conditional payment rate (CPR) to the benchmark model. Default = end of month's CPR. Set `PrepaySpeed` to `[]` if you input a customized prepayment matrix. |
| `PrepayMatrix` | (Optional) Customized prepayment matrix. A matrix of size `max(TermRemaining)`-by-`NMBS`. Missing values are padded with `NaN`s. Each column corresponds to a mortgage-backed security, and each row corresponds to each month after settlement. |

All inputs (except `PrepayMatrix`) are number of mortgage-backed securities (`NMBS`) by `1` vectors.

## Description

`[MYield, BEMBSYield] = mbsoas2yield(ZeroCurve, OAS, Settle, Maturity, IssueDate, GrossRate, CouponRate, Delay, Interpolation, PrepaySpeed, PrepayMatrix)` computes the mortgage and bond-equivalent yields of a pass-through security.

`MYield` is the yield to maturity of the mortgage-backed security (the mortgage yield). This yield is compounded monthly (12 times per year). For example:

0.075 (7.5%)

`BEMBSYield` is the corresponding bond equivalent yield of the mortgage-backed security. This yield is compounded semiannually (two times per year). For example:

0.0761 (7.61%)

## Examples

Given an option-adjusted spread, a spot curve, and a prepayment assumption, compute the theoretical yield to maturity of a mortgage pool.

Create the bonds matrix.

```
Bonds = [datenum('11/21/2002')   0       100  0  2  1;
          datenum('02/20/2003')   0       100  0  2  1;
          datenum('07/31/2004')   0.03    100  2  3  1;
          datenum('08/15/2007')   0.035   100  2  3  1;
          datenum('08/15/2012')   0.04875 100  2  3  1;
          datenum('02/15/2031')   0.05375 100  2  3  1];
```

Choose a settlement date.

```
Settle = datenum('08/20/2002');
```

Assume these clean prices for the bonds.

```
Prices =  [ 98.97467;
             98.58044;
            100.10534;
             98.18054;
            101.38136;
             99.25411];
```

Use this formula to compute spot compounding for the bonds.

```
SpotCompounding = 2*ones(size(Prices));
```

Compute the zero curve.

```
[ZeroRatesP, CurveDatesP] = zbtprice(Bonds, Prices, Settle);
ZeroCurve = [CurveDatesP, ZeroRatesP, SpotCompounding];
```

Assign parameters.

```
OAS           = [26.0502; 28.6348; 31.2222];
Maturity      = datenum('02-Jan-2030');
```

```
IssueDate     = datenum('02-Jan-2000');
GrossRate     = 0.08125;
CouponRate    = 0.075;
Delay         = 14;
Interpolation = 1;
PrepaySpeed   = [0 50 100];
```

Compute the mortgage yield and bond equivalent mortgage yield.

```
[MYield BEMBSYield] = mbsoas2yield(ZeroCurve, OAS, Settle, ...
Maturity, IssueDate, GrossRate, CouponRate, Delay, ...
Interpolation, PrepaySpeed)

MYield =

    0.0802
    0.0814
    0.0828

BEMBSYield =

    0.0816
    0.0828
    0.0842
```

## See Also

`mbsprice2oas` | `mbsyield2oas` | `mbsoas2price`

# mbspassthrough

## Purpose

Mortgage pool cash flows and balances with prepayment

## Syntax

```
[Balance, Payment, Principal, Interest, Prepayment] =
mbspassthrough(OriginalBalance, GrossRate, OriginalTerm,
TermRemaining,  PrepaySpeed, PrepayMatrix)
```

## Arguments

| | |
|---|---|
| `OriginalBalance` | Original balance value in dollars (balance at the beginning of each `TermRemaining`). |
| `GrossRate` | Gross coupon rate (including fees), in decimal. |
| `OriginalTerm` | Term of the mortgage in months. |
| `TermRemaining` | (Optional) Number of full months between settlement and maturity. |
| `PrepaySpeed` | (Optional) Relation of the conditional payment rate (CPR) to the benchmark model. Default = 0 (no prepayment). Set `PrepaySpeed` to `[]` if you input a customized prepayment matrix. |
| `PrepayMatrix` | (Optional) Used only when `PrepaySpeed` is unspecified. Customized prepayment vector. A `NaN`-padded matrix of size `max(TermRemaining)`-by-`NMBS`. Each column corresponds to each mortgage-backed security, and each row corresponds to each month after settlement. |

All inputs (except `PrepayMatrix`) are number of mortgage-backed securities (`NMBS`) by `1` vectors.

## Description

`[Balance, Payment, Principal, Prepayment, Interest] = mbspassthrough(OriginalBalance, GrossRate, OriginalTerm, TermRemaining, PrepaySpeed, PrepayMatrix)` computes the cash flow of principal, interest, and prepayment of pass-through securities.

All outputs are `TermRemaining`-by-`1` vectors of end-of-month values.

`Balance` is a matrix for the principal balance at end of month.

`Payment` is a matrix for the total monthly payment.

`Principal` is a matrix for the principal portion of the payment.

`Interest` is a matrix for the interest portion of the payment.

`Prepayment` is a matrix that indicates any unscheduled principal payment.

By default, the securities are seasoned. The applicable CPR depends upon `TermRemaining` based on a 30-year prepayment model (PSA or FHA). You may supply a different CPR vector of size `TermRemaining`-by-`1`.

## Examples

Compute the cash flows and balances of a 3-month old mortgage pool with original term of 360 months, assuming a prepayment speed of 100.

```
OriginalBalance = 100000;
GrossRate = 0.08125;
OriginalTerm = 360;
TermRemaining = 357;
PrepaySpeed = 100;

[Balance, Payment, Principal, Interest, Prepayment] =...
mbspassthrough(OriginalBalance, GrossRate, OriginalTerm,...
TermRemaining, PrepaySpeed);
```

## See Also

`mbswal`

## Purpose

Mortgage-backed security price given yield

## Syntax

```
[Price, AccrInt] = mbsprice(Yield, Settle, Maturity, IssueDate,
GrossRate, CouponRate, Delay, PrepaySpeed, PrepayMatrix)
```

## Arguments

| | |
|---|---|
| `Yield` | Mortgage yield, compounded monthly (in decimal). |
| `Settle` | Settlement date. A serial date number or date string. `Settle` must be earlier than `Maturity`. |
| `Maturity` | Maturity date. A serial date number or date string. |
| `IssueDate` | Issue date. A serial date number or date string. |
| `GrossRate` | Gross coupon rate (including fees), in decimal. |
| `CouponRate` | (Optional) Net coupon rate, in decimal. Default = `GrossRate`. |
| `Delay` | (Optional) Delay (in days) between payment from homeowner and receipt by bondholder. Default = 0 (no delay between payment and receipt. |
| `PrepaySpeed` | (Optional) Relation of the conditional payment rate (CPR) to the benchmark model. Default = 0 (no prepayment). Set `PrepaySpeed` to `[]` if you input a customized prepayment matrix. |
| `PrepayMatrix` | (Optional) Customized prepayment matrix. A matrix of size `max(TermRemaining)`-by-`NMBS`. Missing values are padded with `NaN`s. Each column corresponds to a mortgage-backed security, and each row corresponds to each month after settlement. |

All inputs (except `PrepayMatrix`) are number of mortgage-backed securities (`NMBS`) by `1` vectors.

## Description

```
[Price, AccrInt] = mbsprice(Yield, Settle, Maturity,
IssueDate, GrossRate, CouponRate, Delay, PrepaySpeed,
```

`PrepayMatrix)` computes a mortgage-backed security price, given time information, mortgage yield at settlement, and optionally, a prepayment model.

All outputs are scalar values.

`Price` is the clean price for every $100 face value of the securities.

`AccrInt` is the accrued interest of the mortgage-backed securities.

## Examples

**Example 1**. Given a mortgage-backed security with the following characteristics, compute the price and the accrued interest due on the security.

```
Yield = 0.0725;
Settle = datenum('15-Apr-2002');
Maturity = datenum('1 Jan 2030');
IssueDate = datenum('1-Jan-2000');
GrossRate = 0.08125;
CouponRate = 0.075;
Delay = 14;
Speed = 100;

[Price AccrInt] = mbsprice(Yield, Settle, Maturity, IssueDate,...
GrossRate, CouponRate, Delay, Speed)

Price =

  101.3147


AccrInt =

    0.2917
```

**Example 2**. Given a portfolio of mortgage-backed securities, compute the clean prices and accrued interest.

```
Yield = 0.075;
```

```
Settle = datenum(['13-Feb-2000';'17-Apr-2002';'17-May-2002';...
'13-Jan-2000']);
Maturity  = datenum('1-Jan-2030');
IssueDate = datenum('1-Jan-2000');
GrossRate = 0.08125;
CouponRate = [0.075; 0.07875; 0.0775; 0.08125];
Delay = 14;
Speed = 100;

[Price AccrInt] = mbsprice(Yield, Settle, Maturity, IssueDate,...
GrossRate, CouponRate, Delay, Speed)

Price =

   99.7085
  102.0678
  101.2792
  104.0175

AccrInt =

    0.2500
    0.3500
    0.3444
    0.2708
```

## References

[1] *PSA Uniform Practices*, SF-49

## See Also

`mbsyield`

## Purpose

Option-adjusted spread given price

## Syntax

```
OAS = mbsprice2oas(ZeroCurve, Price, Settle, Maturity, IssueDate,
GrossRate, CouponRate, Delay, Interpolation  PrepaySpeed,
PrepayMatrix)
```

## Arguments

| | |
|---|---|
| `ZeroCurve` | A matrix of three columns:<br>• Column 1: Serial date numbers.<br>• Column 2: Spot rates with maturities corresponding to the dates in Column 1, in decimal (for example, 0.075).<br>• Column 3: Compounding of the rates in Column 2. Values are 1 (annual), 2 (semiannual, 3 (three times per year), 4 (quarterly), 6 (bimonthly), 12 (monthly), and -1 (continuous). |
| `Price` | Clean price for every $100 face value of bond issue. |
| `Settle` | Settlement date (scalar only). A serial date number or date string. Date when option-adjusted spread is calculated. `Settle` must be earlier than `Maturity`. |
| `Maturity` | Maturity date. Scalar or vector in serial date number or date string format. |
| `IssueDate` | Issue date. A serial date number or date string. |
| `GrossRate` | Gross coupon rate (including fees), in decimal. |
| `CouponRate` | (Optional) Net coupon rate, in decimal. Default = `GrossRate`. |

| | |
|---|---|
| `Delay` | (Optional) Delay (in days) between payment from homeowner and receipt by bondholder. Default = 0 (no delay between payment and receipt. |
| `Interpolation` | Interpolation method. Computes the corresponding spot rates for the bond's cash flow. Available methods are (0) nearest, (1) linear, and (2) cubic spline. Default = 1. See `interp1` for more information. |
| `PrepaySpeed` | (Optional) Relation of the conditional payment rate (CPR) to the benchmark model. Default = end of month's CPR. Set `PrepaySpeed` to `[]` if you input a customized prepayment matrix. |
| `PrepayMatrix` | (Optional) Customized prepayment matrix. A matrix of size `max(TermRemaining)`-by-`NMBS`. Missing values are padded with `NaN`s. Each column corresponds to a mortgage-backed security, and each row corresponds to each month after settlement. |

All inputs (except `PrepayMatrix`) are number of mortgage-backed securities (`NMBS`) by `1` vectors.

## Description

`OAS = mbsprice2oas(ZeroCurve, Price, Settle, Maturity, IssueDate, GrossRate, CouponRate, Delay, Interpolation, PrepaySpeed, PrepayMatrix)` computes the monthly option-adjusted spread in basis points.

## Examples

Calculate the option-adjusted spread of a 30-year fixed-rate mortgage with about a 28-year weighted average maturity remaining, given assumptions of 0, 50, and 100 PSA prepayments.

Create the bonds matrix.

```
Bonds = [datenum('11/21/2002')   0         100  0  2  1;
         datenum('02/20/2003')   0         100  0  2  1;
```

```
        datenum('07/31/2004')   0.03     100  2  3  1;
        datenum('08/15/2007')   0.035    100  2  3  1;
        datenum('08/15/2012')   0.04875  100  2  3  1;
        datenum('02/15/2031')   0.05375  100  2  3  1];
```

Choose a settlement date.

```
Settle= datenum('20-Aug-2002');
```

Assume these clean prices for the bonds.

```
Prices =  [ 98.97467;
            98.58044;
           100.10534;
            98.18054;
           101.38136;
            99.25411];
```

Use this formula to compute spot compounding for the bonds.

```
SpotCompounding = 2*ones(size(Prices));
```

Compute the zero curve.

```
[ZeroRatesP, CurveDatesP] = zbtprice(Bonds, Prices, Settle);
ZeroCurve = [CurveDatesP, ZeroRatesP, SpotCompounding];
```

Assign parameters.

```
Price         = 95;
Maturity      = datenum('02-Jan-2030');
IssueDate     = datenum('02-Jan-2000');
GrossRate     = 0.08125;
CouponRate    = 0.075;
Delay         = 14;
Interpolation = 1;
PrepaySpeed   = [0; 50; 100];
Interpolation = 1;
```

Compute the option-adjusted spread.

```
OAS = mbsprice2oas(ZeroCurve, Price, Settle, Maturity, ...
IssueDate, GrossRate, CouponRate, Delay, Interpolation, ...
PrepaySpeed)

OAS =

  26.0502
  28.6348
  31.2222
```

## See Also

`mbsoas2price` | `mbsoas2yield` | `mbsyield2oas`

## Purpose

Implied PSA prepayment speeds given price

## Syntax

```
[ImpSpdOnPrc, ImpSpdOnDur, ImpSpdOnCnv] = mbsprice2speed(Price,
Settle, Maturity, IssueDate, GrossRate, PrepayMatrix, CouponRate,
Delay)
```

## Arguments

| | |
|---|---|
| `Price` | Clean price for every $100 face value. |
| `Settle` | Settlement date. A serial date number or date string. `Settle` must be earlier than `Maturity`. |
| `Maturity` | Maturity date. A serial date number or date string. |
| `IssueDate` | Issue date. A serial date number or date string. |
| `GrossRate` | Gross coupon rate (including fees), in decimal. |
| `PrepayMatrix` | Customized prepayment matrix. A matrix of size `max(TermRemaining)`-by-`NMBS`. Missing values are padded with `NaN`s. Each column corresponds to a mortgage-backed security, and each row corresponds to each month after settlement. |
| `CouponRate` | (Optional) Net coupon rate, in decimal. Default = `GrossRate`. |
| `Delay` | (Optional) Delay (in days) between payment from homeowner and receipt by bondholder. Default = 0 (no delay between payment and receipt. |

All inputs (except `PrepayMatrix`) are number of mortgage-backed securities (`NMBS`)-by-`1` vectors.

## Description

`[ImpSpdOnPrc, ImpSpdOnDur, ImpSpdOnCnv] = mbsprice2speed(Price, Settle, Maturity, IssueDate, GrossRate, PrepayMatrix, CouponRate, Delay)` computes PSA prepayment speeds implied by pool prices and projected (user-defined) prepayment vectors. The calculated PSA speed produces the same price, modified duration, or modified convexity, depending upon the output requested.

`ImpSpdOnPrc` calculates the equivalent PSA benchmark prepayment speed for the pass-through to carry the same price.

`ImpSpdOnDur` calculates the equivalent PSA benchmark prepayment speed for the pass-through to carry the same modified duration.

`ImpSpdOnCnv` calculates the equivalent PSA benchmark prepayment speed for the pass-through to carry the same modified convexity.

All outputs are `NMBS`-by-`1` vectors.

## Examples

Calculate the equivalent PSA benchmark prepayment speeds for a mortgage pool with these characteristics and prepayment matrix.

```
Price        = 101;
Settle       = datenum('1-Jan-2000');
Maturity     = datenum('1-Jan-2030');
IssueDate    = datenum('1-Jan-2000');
GrossRate    = 0.08125;
PrepayMatrix = 0.005*ones(360,1);
CouponRate   = 0.075;
Delay        = 14;

[ImpSpdOnPrc, ImpSpdOnDur, ImpSpdOnCnv] = ...
mbsprice2speed(Price,Settle, Maturity, IssueDate, ...
GrossRate, PrepayMatrix, CouponRate, Delay)

ImpSpdOnPrc =

  118.5980

ImpSpdOnDur =

  118.3946

ImpSpdOnCnv =
```

```
   109.5115
```

## References

[1] *PSA Uniform Practices*, SF-49

## See Also

`mbsprice` | `mbsyield2speed`

# mbswal

## Purpose

Weighted average life of mortgage pool

## Compatibility

PSA

## Syntax

```
WAL =  mbswal(Settle, Maturity, IssueDate, GrossRate, CouponRate,
Delay, PrepaySpeed, PrepayMatrix)
```

## Arguments

| | |
|---|---|
| `Settle` | Settlement date. A serial date number or date string. `Settle` must be earlier than `Maturity`. |
| `Maturity` | Maturity date. A serial date number or date string. |
| `IssueDate` | Issue date. A serial date number or date string. |
| `GrossRate` | Gross coupon rate (including fees), in decimal. |
| `CouponRate` | (Optional) Net coupon rate, in decimal. Default = `GrossRate`. |
| `Delay` | (Optional) Delay (in days) between payment from homeowner and receipt by bondholder. Default = 0 (no delay between payment and receipt). |
| `PrepaySpeed` | (Optional) Relation of the conditional payment rate (CPR) to the benchmark model. Default = end of month's CPR. Set `PrepaySpeed` to `[]` if you input a customized prepayment matrix. |
| `PrepayMatrix` | (Optional) Customized prepayment matrix. A matrix of size `max(TermRemaining)`-by-`NMBS`. Missing values are padded with `NaN`s. Each column corresponds to a mortgage-backed security, and each row corresponds to each month after settlement. |

All inputs (except `PrepayMatrix`) are number of mortgage-backed securities (`NMBS`) by `1` vectors.

## Description

`WAL = mbswal(Settle, Maturity, IssueDate, GrossRate, CouponRate, Delay, PrepaySpeed, PrepayMatrix)` computes the weighted average life, in number of years, of a mortgage pool, as measured from the settlement date.

## Examples

Given a pass-through security with the following characteristics, compute the weighted average life of the security.

```
Settle = datenum('15-Apr-2002');
Maturity = datenum('1 Jan 2030');
IssueDate = datenum('1-Jan-2000');
GrossRate = 0.08125;
CouponRate = 0.075;
Delay = 14;
Speed = 100;

WAL = mbswal(Settle, Maturity, IssueDate, GrossRate, ...
CouponRate, Delay, Speed)

WAL =

   10.5477
```

## References

[1] *PSA Uniform Practices*, SF-49

## See Also

`mbspassthrough`

# mbsyield

## Purpose

Mortgage-backed security yield given price

## Syntax

```
[MYield, BEMBSYield] = mbsyield(Price, Settle, Maturity, IssueDate,
GrossRate, CouponRate, Delay,  PrepaySpeed, PrepayMatrix)
```

## Arguments

| | |
|---|---|
| `Price` | Clean price for every $100 face value. |
| `Settle` | Settlement date. A serial date number or date string. `Settle` must be earlier than `Maturity`. |
| `Maturity` | Maturity date. A serial date number or date string. |
| `IssueDate` | Issue date. A serial date number or date string. |
| `GrossRate` | Gross coupon rate (including fees), in decimal. |
| `CouponRate` | (Optional) Net coupon rate, in decimal. Default = `GrossRate`. |
| `Delay` | (Optional) Delay (in days) between payment from homeowner and receipt by bondholder. Default = 0 (no delay between payment and receipt. |
| `PrepaySpeed` | (Optional) Relation of the conditional payment rate (CPR) to the benchmark model. Default = 0 (no prepayment). Set `PrepaySpeed` to `[]` if you input a customized prepayment matrix. |
| `PrepayMatrix` | (Optional) Customized prepayment matrix. A matrix of size `max(TermRemaining)`-by-`NMBS`. Missing values are padded with `NaN`s. Each column corresponds to a mortgage-backed security, and each row corresponds to each month after settlement. |

All inputs (except `PrepayMatrix`) are number of mortgage-backed securities (`NMBS`) by `1` vectors.

## Description

`[MYield, BEMBSYield] = mbsyield(Price, Settle, Maturity, IssueDate, GrossRate, CouponRate, Delay, PrepaySpeed, PrepayMatrix)` computes a mortgage-backed security yield to maturity and the bond equivalent yield, given time information, price at settlement, and optionally, a prepayment model.

`MYield` is the yield to maturity of the mortgage-backed security (the mortgage yield). This yield is compounded monthly (12 times a year).

`BEMBSYield` is the corresponding bond equivalent yield of the mortgage-backed security. This yield is compounded semiannually (two times a year).

## Examples

**Example 1**. Given a mortgage-backed security with the following characteristics, compute the mortgage yield and the bond equivalent yield of the security.

```
Price = 102;
Settle = '15-Apr-2002';
Maturity = '1 Jan 2030';
IssueDate = '1-Jan-2000';
GrossRate = 0.08125;
CouponRate = 0.075;
Delay = 14;
Speed = 100;

[MYield, BEMBSYield] = mbsyield(Price, Settle, Maturity, ...
IssueDate, GrossRate, CouponRate, Delay,  Speed)

MYield =

    0.0715

BEMBSYield =

    0.0725
```

**Example 2**. Given a portfolio of mortgage-backed securities, compute the mortgage yields and the bond equivalent yields.

```
Price = 102;
```

```
Settle = datenum(['13-Feb-2000';'17-Apr-2002';'17-May-2002';...
'13-Jan-2000']);
Maturity  = datenum('1-Jan-2030');
IssueDate = datenum('1-Jan-2000');
GrossRate = 0.08125;
CouponRate = [0.075; 0.07875; 0.0775; 0.08125];
Delay = 14;
Speed = 100;

[MYield, BEMBSYield] = mbsyield(Price, Settle, Maturity,...
IssueDate, GrossRate, CouponRate, Delay,  Speed)

MYield =

    0.0717
    0.0751
    0.0739
    0.0779

BEMBSYield =

    0.0728
    0.0763
    0.0750
    0.0791
```

## References

[1] *PSA Uniform Practices*, SF-49

## See Also

`mbsprice`

## Purpose

Option-adjusted spread given yield

## Syntax

```
OAS = mbsyield2oas(ZeroCurve, Yield, Settle, Maturity, IssueDate,
GrossRate, CouponRate, Delay, Interpolation  PrepaySpeed,
PrepayMatrix)
```

## Arguments

| | |
|---|---|
| `ZeroCurve` | A matrix of three columns:<br>• Column 1: Serial date numbers.<br>• Column 2: Spot rates with maturities corresponding to the dates in Column 1, in decimal (for example, 0.075).<br>• Column 3: Compounding of the rates in Column 2. Values are 1 (annual), 2 (semiannual, 3 (three times per year), 4 (quarterly), 6 (bimonthly), 12 (monthly), and -1 (continuous). |
| `Yield` | Mortgage yield, compounded monthly (in decimal). |
| `Settle` | Settlement date (scalar only). A serial date number or date string. Date when option-adjusted spread is calculated. `Settle` must be earlier than `Maturity`. |
| `Maturity` | Maturity date. Scalar or vector in serial date number or date string format. |
| `IssueDate` | Issue date. A serial date number or date string. |
| `GrossRate` | Gross coupon rate (including fees), in decimal. |
| `CouponRate` | (Optional) Net coupon rate, in decimal. Default = `GrossRate`. |
| `Delay` | (Optional) Delay (in days) between payment from homeowner and receipt by bondholder. Default = 0 (no delay between payment and receipt). |

| | |
|---|---|
| `Interpolation` | Interpolation method. Computes the corresponding spot rates for the bond's cash flow. Available methods are (0) nearest, (1) linear, and (2) cubic spline. Default = 1. See `interp1` for more information. |
| `PrepaySpeed` | (Optional) Relation of the conditional payment rate (CPR) to the benchmark model. Default = end of month's CPR. Set `PrepaySpeed` to `[]` if you input a customized prepayment matrix. |
| `PrepayMatrix` | (Optional) Customized prepayment matrix. A matrix of size `max(TermRemaining)`-by-`NMBS`. Missing values are padded with `NaN`s. Each column corresponds to a mortgage-backed security, and each row corresponds to each month after settlement. |

All inputs (except `PrepayMatrix`) are number of mortgage-backed securities (`NMBS`) by `1` vectors.

## Description

`OAS = mbsyield2oas(ZeroCurve, Yield, Settle, Maturity, IssueDate, GrossRate, CouponRate, Delay, Interpolation, PrepaySpeed, PrepayMatrix)` computes the option-adjusted spread in basis points.

## Examples

Calculate the option-adjusted spread of a 30-year, fixed-rate mortgage pool with about 28-year weighted average maturity left, given assumptions of 0, 50, and 100 PSA prepayments.

Create a bonds matrix.

```
Bonds = [datenum('11/21/2002')  0       100  0  2  1;
         datenum('02/20/2003')  0       100  0  2  1;
         datenum('07/31/2004')  0.03    100  2  3  1;
         datenum('08/15/2007')  0.035   100  2  3  1;
         datenum('08/15/2012')  0.04875 100  2  3  1;
         datenum('02/15/2031')  0.05375 100  2  3  1];
```

Choose a settlement date.

```
Settle = datenum('08/20/2002');
```

Assume these clean prices for the bonds.

```
Prices =  [ 98.97467;
            98.58044;
           100.10534;
            98.18054;
           101.38136;
            99.25411];
```

Use this formula to compute spot compounding for the bonds.

```
SpotCompounding = 2*ones(size(Prices));
```

Compute the zero curve.

```
[ZeroRatesP, CurveDatesP] = zbtprice(Bonds, Prices, Settle);
ZeroCurve = [CurveDatesP, ZeroRatesP, SpotCompounding];
```

Assign parameters.

```
Price         = 95;
Maturity      = datenum('02-Jan-2030');
IssueDate     = datenum('02-Jan-2000');
GrossRate     = 0.08125;
CouponRate    = 0.075;
Delay         = 14;
Interpolation = 1;
PrepaySpeed   = [0 50 100];
```

Compute the yield, and from the yield, compute the option-adjusted spread.

```
[mbsyld, beyld] = mbsyield(Price, Settle, ...
Maturity, IssueDate, GrossRate, CouponRate, Delay, PrepaySpeed);
```

```
OAS = mbsyield2oas(ZeroCurve, mbsyld, Settle, ...
Maturity, IssueDate, GrossRate, CouponRate, Delay, ...
Interpolation, PrepaySpeed)

OAS =

   26.0502
   28.6348
   31.2222
```

## See Also

`mbsoas2price` | `mbsoas2yield` | `mbsprice2oas`

## Purpose

Implied PSA prepayment speeds given yield

## Syntax

```
[ImpSpdOnYld, ImpSpdOnDur, ImpSpdOnCnv] = mbsyield2speed(Yield,
Settle, Maturity, IssueDate, GrossRate, PrepayMatrix, CouponRate,
Delay)
```

## Arguments

| | |
|---|---|
| `Yield` | Mortgage yield, compounded monthly, in decimal. |
| `Settle` | Settlement date. A serial date number or date string. `Settle` must be earlier than `Maturity`. |
| `Maturity` | Maturity date. A serial date number or date string. |
| `IssueDate` | Issue date. A serial date number or date string. |
| `GrossRate` | Gross coupon rate (including fees), in decimal. |
| `PrepayMatrix` | Customized prepayment matrix. A matrix of size `max(TermRemaining)`-by-`NMBS`. Missing values are padded with `NaN`s. Each column corresponds to a mortgage-backed security, and each row corresponds to each month after settlement. |
| `CouponRate` | (Optional) Net coupon rate, in decimal. Default = `GrossRate`. |
| `Delay` | (Optional) Delay (in days) between payment from homeowner and receipt by bondholder. Default = 0 (no delay between payment and receipt). |

All inputs (except `PrepayMatrix`) are number of mortgage-backed securities (`NMBS`) by `1` vectors.

## Description

`[ImpSpdOnYld, ImpSpdOnDur, ImpSpdOnCnv] = mbsyield2speed(Yield, Settle, Maturity, IssueDate, GrossRate, PrepayMatrix, CouponRate, Delay)` computes PSA prepayment speeds implied by pool yields and projected (user-defined) prepayment vectors. The calculated PSA speed produces the same yield, modified duration, or modified convexity, depending upon the output requested.

`ImpSpdOnPrc` calculates the equivalent PSA benchmark prepayment speed for the pass-through to carry the same price.

`ImpSpdOnDur` calculates the equivalent PSA benchmark prepayment speed for the pass-through to carry the same modified duration.

`ImpSpdOnCnv` calculates the equivalent PSA benchmark prepayment speed for the pass-through to carry the same modified convexity.

All outputs are `NMBS`-by-`1` vectors.

## Examples

Calculate the equivalent PSA benchmark prepayment speeds for a security with these characteristics and prepayment matrix.

```
Yield = 0.065;
Settle = datenum('1-Jan-2000');
Maturity = datenum('1-Jan-2030');
IssueDate = datenum('1-Jan-2000');
GrossRate = 0.08125;
PrepayMatrix = 0.005*ones(360,1);
CouponRate = 0.075;
Delay = 14;

[ImpSpdOnYld, ImpSpdOnDur, ImpSpdOnCnv] = ...
mbsyield2speed(Yield, Settle, Maturity, IssueDate, GrossRate, ...
PrepayMatrix, CouponRate, Delay)

ImpSpdOnYld =

  117.7644

ImpSpdOnDur =

  116.7436

ImpSpdOnCnv =
```

```
108.3309
```

## References

[1] *PSA Uniform Practices*, SF-49

## See Also

`mbsyield` | `mbsprice2speed`

# psaspeed2default

## Purpose

Benchmark default

## Syntax

```
[ADRPSA, MDRPSA] = psaspeed2default(DefaultSpeed)
```

## Arguments

| | |
|---|---|
| `DefaultSpeed` | Annual speed relative to the benchmark. PSA benchmark = 100. |

## Description

`[ADRPSA, MDRPSA] = psaspeed2default(DefaultSpeed)` computes the benchmark default on the performing balance of mortgage-backed securities per PSA benchmark speed.

`ADRPSA` is the PSA default rate, in decimal (360-by-1).

`MDRPSA` is the PSA monthly default rate, in decimal (360-by-1).

## Examples

Given a mortgage-backed security with annual speed set at the PSA default benchmark, compute the default rates.

```
DefaultSpeed = 100;

[ADRPSA, MDRPSA] = psaspeed2default(DefaultSpeed);
```

## See Also

`psaspeed2rate`

## Purpose

Single monthly mortality rate given PSA speed

## Syntax

```
[CPRPSA, SMMPSA]= psaspeed2rate(PSASpeed)
```

## Arguments

| | |
|---|---|
| `PSASpeed` | Any value > 0 representing the annual speed relative to the benchmark. PSA benchmark = 100. |

## Description

`[CPRPSA, SMMPSA]= psaspeed2rate(PSASpeed)` calculates vectors of PSA prepayments, each containing 360 prepayment elements, to represent the 360 months in a 30-year mortgage pool.

`CPRPSA` is the PSA conditional prepayment rate, in decimal [360-by-1].

`SMMPSA` is the PSA single monthly mortality rate, in decimal [360-by-1].

## Examples

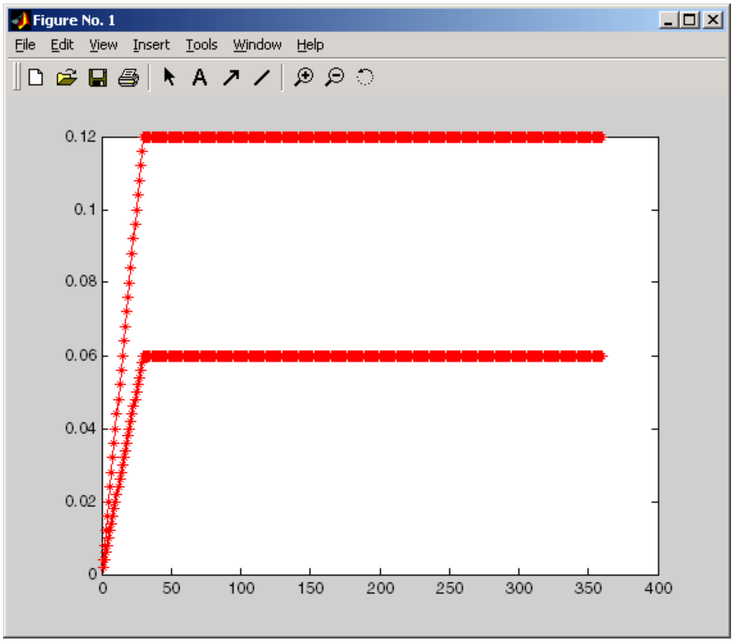
Given a mortgage-backed security with annual speed set at the PSA default benchmark, compute the prepayment and mortality rates.

```
PSASpeed = [100 200];
```

```
[CPRPSA, SMMPSA]= psaspeed2rate(PSASpeed);
```

View a plot of the output.

```
psaspeed2rate(PSASpeed)
```



**See Also**
`psaspeed2default`

## Purpose

Cash flow amounts and times for bonds and stepped coupons

## Syntax

```
[CFlows, CDates, CTimes] = stepcpncfamounts(Settle, Maturity,
ConvDates, CouponRates, Period, Basis, EndMonthRule, Face)
```

## Arguments

| | |
|---|---|
| `Settle` | Settlement date. A scalar or vector of serial date numbers. `Settle` must be earlier than `Maturity`. |
| `Maturity` | Maturity date. A scalar or vector of serial date numbers. |
| `ConvDates` | Matrix of serial date numbers representing conversion dates after `Settle`. Size = number of instruments by maximum number of conversions. Fill unspecified entries with `NaN`. |
| `CouponRates` | Matrix indicating the coupon rates for each bond in decimal form. Size = number of instruments by maximum number of conversions + 1. First column of this matrix contains rates applicable between `Settle` and the first conversion date (date in the first column of `ConvDates`). Fill unspecified entries with `NaN`. See Note below. |
| `Period` | (Optional) Coupons per year of the bond. A vector of integers. Allowed values are 0, 1, 2, 3, 4, 6, and 12. Default = 2. |

| | |
|---|---|
| `Basis` | (Optional) Day-count basis of the instrument. A vector of integers.<br>• 0 = actual/actual (default)<br>• 1 = 30/360 (SIA)<br>• 2 = actual/360<br>• 3 = actual/365<br>• 4 = 30/360 (BMA)<br>• 5 = 30/360 (ISDA)<br>• 6 = 30/360 (European)<br>• 7 = actual/365 (Japanese)<br>• 8 = actual/actual (ICMA)<br>• 9 = actual/360 (ICMA)<br>• 10 = actual/365 (ICMA)<br>• 11 = 30/360E (ICMA)<br>• 12 = actual/actual (ISDA)<br>• 13 = BUS/252<br>For more information, see basis. |
| `EndMonthRule` | (Optional) End-of-month rule. A vector. This rule applies only when `Maturity` is an end-of-month date for a month having 30 or fewer days. 0 = ignore rule, meaning that a bond's coupon payment date is always the same numerical day of the month. 1 = set rule on (default), meaning that a bond's coupon payment date is always the last actual day of the month. |
| `Face` | (Optional) Face value of each bond in the portfolio. Default = 100. |

All arguments must be scalars or number of bonds (`NUMBONDS`)-by-1 vectors, except for `ConvDates` and `CouponRates`.

---

**Note** `ConvDates` has the same number of rows as `CouponRates` to reflect the same number of bonds. However, `ConvDates` has one less column than `CouponRates`. This situation is illustrated by

```
Settle---------ConvDate1-----------ConvDate2------------Maturity


        Rate1               Rate2                  Rate3
```

---

## Description

`[CFlows, CDates, CTimes] = stepcpncfamounts(Settle, Maturity, ConvDates, CouponRates, Period, Basis, EndMonthRule, Face)` returns matrices of cash flow amounts, cash flow dates, and time factors for a portfolio of `NUMBONDS` stepped-coupon bonds.

`CFlows` is a matrix of cash flow amounts. The first entry in each row vector is a negative number indicating the accrued interest due at settlement. If no accrued interest is due, the first column is 0.

`CDates` is a matrix of cash flow dates in serial date number form. At least two columns are always present, one for settlement and one for maturity.

`CTimes` is a matrix of time factors for the SIA semiannual price/yield conversion.

```
DiscountFactor = (1 + Yield/2).^(-TFactor)
```

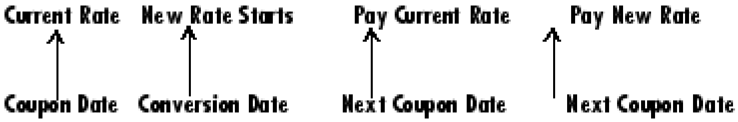
Time factors are in units of semiannual coupon periods. In computing time factors, use SIA actual/actual conventions for all time factor calculations.

**Note** For bonds with fixed coupons, use `cfamounts`. If you use a fixed-coupon bond with `stepcpncfamounts`, MATLAB software generates an error.

## Examples

This example generates stepped cash flows for three different bonds, all paying interest semiannually. Their life span is about 18 to 19 years each:

- Bond A has two conversions, but the first one occurs on the settlement date and immediately expires.
- Bond B has three conversions, with conversion dates exactly on the coupon dates.
- Bond C has three conversions, with some conversion dates not on the coupon dates. It has the longest maturity. This case illustrates that only cash flows for full periods after conversion dates are affected, as illustrated below.



The following table illustrates the interest rate characteristics of this bond portfolio.

| Bond A Dates | Bond A Rates | Bond B Dates | Bond B Rates | Bond C Dates | Bond C Rates |
|---|---|---|---|---|---|
| Settle (02-Aug-92) | 7.5% | Settle (02-Aug-92) | 7.5% | Settle (02-Aug-92) | 2.5% |
| First Conversion (02-Aug-92) | 8.875% | First Conversion (15-Jun-97)) | 8.875% | First Conversion (14-Jun-97)) | 5.0% |

| **Bond A Dates** | **Bond A Rates** | **Bond B Dates** | **Bond B Rates** | **Bond C Dates** | **Bond C Rates** |
|---|---|---|---|---|---|
| Second Conversion (15-Jun-03) | 9.25% | Second Conversion (15-Jun-01) | 9.25% | Second Conversion (14-Jun-01) | 7.5% |
| Maturity (15-Jun-10) | `NaN` | Third Conversion (15-Jun-05) | 10.0% | Third Conversion (14-Jun-05) | 10.0% |
| | | Maturity (15-Jun-10) | `NaN` | Maturity (15-Jun-11) | `NaN` |

```
Settle   = datenum('02-Aug-1992');

ConvDates = [datenum('02-Aug-1992'), datenum('15-Jun-2003'),...
             nan;
            datenum('15-Jun-1997'), datenum('15-Jun-2001'),...
            datenum('15-Jun-2005');
            datenum('14-Jun-1997'), datenum('14-Jun-2001'),...
            datenum('14-Jun-2005')];

Maturity = [datenum('15-Jun-2010');
            datenum('15-Jun-2010');
            datenum('15-Jun-2011')];

CouponRates = [0.075 0.08875 0.0925 nan;
               0.075 0.08875 0.0925 0.1;
               0.025 0.05    0.0750 0.1];
Basis = 1;
Period = 2;
EndMonthRule = 1;
Face = 100;
```

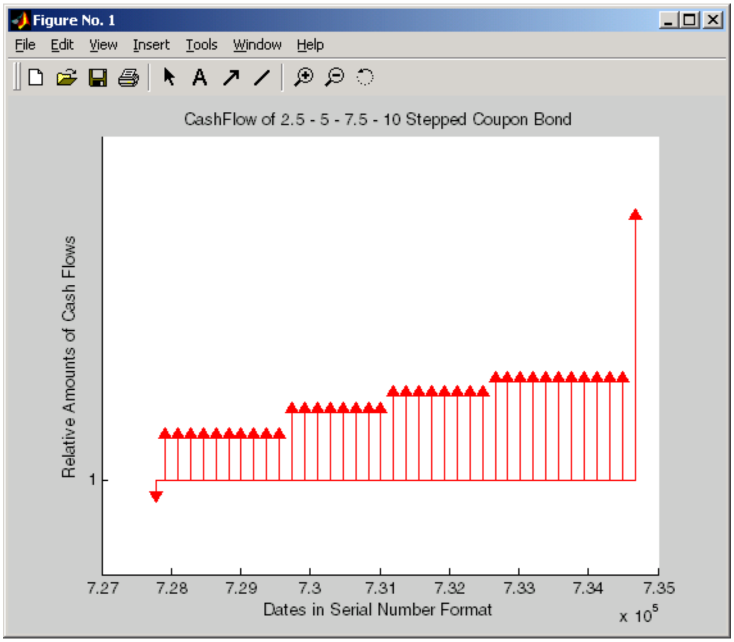
Call `stepcpncfamounts` to compute cash flows and timings.

```
[CFlows, CDates, CTimes] = stepcpncfamounts(Settle, Maturity, ...
```

```
ConvDates, CouponRates);
```

Visualize the third bond cash flows (2.5 - 5 - 7.5 - 10). (`cfplot` is available at `/finance/findemos/cfplot.m`.)

```
cfplot(CDates(3,:),CFlows(3,:));
xlabel('Dates in Serial Number Format')
ylabel('Relative Amounts of Cash Flows')
title('CashFlow of 2.5 - 5 - 7.5 - 10 Stepped Coupon Bond')
```



## See Also

`stepcpnprice` | `stepcpnyield`

## Purpose

Price bond with stepped coupons

## Syntax

```
[Price, AccruedInterest] = stepcpnprice(Yield, Settle, Maturity,
ConvDates, CouponRates, Period, Basis, EndMonthRule, Face)
```

## Arguments

| | |
|---|---|
| `Yield` | Scalar or vector containing yield to maturity of instruments. |
| `Settle` | Settlement date. A scalar or vector of serial date numbers. `Settle` must be earlier than `Maturity`. |
| `Maturity` | Maturity date. A scalar or vector of serial date numbers. |
| `ConvDates` | Matrix of serial date numbers representing conversion dates after `Settle`. Size = number of instruments by maximum number of conversions. Fill unspecified entries with `NaN`. |
| `CouponRates` | Matrix indicating the coupon rates for each bond in decimal form. Size = number of instruments by maximum number of conversions + 1. First column of this matrix contains rates applicable between `Settle` and the first conversion date (date in the first column of `ConvDates`). Fill unspecified entries with `NaN`. See Note below. |
| `Period` | (Optional) Coupons per year of the bond. A vector of integers. Allowed values are 0, 1, 2, 3, 4, 6, and 12. Default = 2. |

| | |
|---|---|
| `Basis` | (Optional) Day-count basis of the instrument. A vector of integers.<br>• 0 = actual/actual (default)<br>• 1 = 30/360 (SIA)<br>• 2 = actual/360<br>• 3 = actual/365<br>• 4 = 30/360 (BMA)<br>• 5 = 30/360 (ISDA)<br>• 6 = 30/360 (European)<br>• 7 = actual/365 (Japanese)<br>• 8 = actual/actual (ICMA)<br>• 9 = actual/360 (ICMA)<br>• 10 = actual/365 (ICMA)<br>• 11 = 30/360E (ICMA)<br>• 12 = actual/actual (ISDA)<br>• 13 = BUS/252<br>For more information, see basis. |
| `EndMonthRule` | (Optional) End-of-month rule. A vector. This rule applies only when `Maturity` is an end-of-month date for a month having 30 or fewer days. 0 = ignore rule, meaning that a bond's coupon payment date is always the same numerical day of the month. 1 = set rule on (default), meaning that a bond's coupon payment date is always the last actual day of the month. |
| `Face` | (Optional) Face value of each bond in the portfolio. Default = 100. |

All arguments must be scalars or number of bonds (`NUMBONDS`)-by-1 vectors, except for `ConvDates` and `CouponRates`.

---

**Note** `ConvDates` has the same number of rows as `CouponRate` to reflect the same number of bonds. However, `ConvDates` has one less column than `CouponRate`. This situation is illustrated by

```
Settle---------ConvDate1-----------ConvDate2------------Maturity


        Rate1               Rate2                 Rate3
```

---

## Description

`[Price, AccruedInterest] = stepcpnprice(Yield, Settle, Maturity, ConvDates, CouponRates, Period, Basis, EndMonthRule, Face)` computes the price of bonds with stepped coupons given the yield to maturity. The function supports any number of conversion dates.

`Price` is a `NUMBONDS`-by-1 vector of clean prices.

`AccruedInterest` is a `NUMBONDS`-by-1 vector of accrued interest payable at settlement dates.

---

**Note** For bonds with fixed coupons, use `bndprice`. If you use a fixed-coupon bond with `stepcpnprice`, you will receive the error: `incorrect number of inputs`.

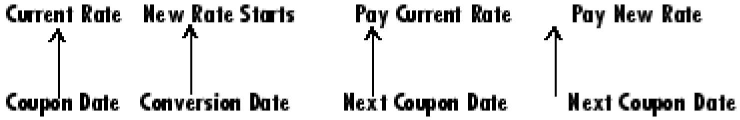
---

## Examples

Compute the price and accrued interest due on a portfolio of stepped-coupon bonds having a yield of 7.221%, given three conversion scenarios:

- Bond A has two conversions, the first one falling on the settle date and immediately expiring.

- Bond B has three conversions, with conversion dates exactly on the coupon dates.
- Bond C has three conversions, with one or more conversion dates not on coupon dates. This case illustrates that only cash flows for full periods after conversion dates are affected, as illustrated below.



The following table illustrates the interest rate characteristics of this bond portfolio.

| Bond A Dates | Bond A Rates | Bond B Dates | Bond B Rates | Bond C Dates | Bond C Rates |
|---|---|---|---|---|---|
| Settle (02-Aug-92) | 7.5% | Settle (02-Aug-92) | 7.5% | Settle (02-Aug-92) | 7.5% |
| First Conversion (02-Aug-92) | 8.875% | First Conversion (15-Jun-97) | 8.875% | First Conversion (14-Jun-97) | 8.875% |
| Second Conversion (15-Jun-03) | 9.25% | Second Conversion (15-Jun-01) | 9.25% | Second Conversion (14-Jun-01) | 9.25% |
| Maturity (15-Jun-10) | `NaN` | Third Conversion (15-Jun-05) | 10.0% | Third Conversion (14-Jun-05) | 10.0% |
|  |  | Maturity (15-Jun-10) | `NaN` | Maturity (15-Jun-10) | `NaN` |

```
Yield = 0.07221;
Settle   = datenum('02-Aug-1992');
ConvDates = [datenum('02-Aug-1992'), datenum('15-Jun-2003'),...
             nan;
            datenum('15-Jun-1997'), datenum('15-Jun-2001'),...
```

```
            datenum('15-Jun-2005');
            datenum('14-Jun-1997'), datenum('14-Jun-2001'),...
            datenum('14-Jun-2005')];
Maturity = datenum('15-Jun-2010');

CouponRates = [0.075 0.08875 0.0925 nan;
               0.075 0.08875 0.0925 0.1;
               0.075 0.08875 0.0925 0.1];
Basis = 1;
Period = 2;
EndMonthRule = 1;
Face = 100;

[Price, AccruedInterest] = ...
stepcpnprice(Yield, Settle, Maturity, ConvDates, CouponRates, ...
Period, Basis, EndMonthRule, Face)

Price =

  117.3824
  113.4339
  113.4339

AccruedInterest =

    1.1587
    0.9792
    0.9792
```

## References

This function adheres to *SIA Fixed Income Securities Formulas for Price, Yield, and Accrued Interest*, Volume 1, 3rd edition, pp. 120 - 123, on zero-coupon instruments pricing.

## See Also

`bndprice` | `cdprice` | `stepcpncfamounts` | `stepcpnyield` | `tbillprice` | `zeroprice`

## Purpose

Yield to maturity of bond with stepped coupons

## Syntax

```
Yield = stepcpnyield(Price, Settle, Maturity, ConvDates,
CouponRate, Period, Basis, EndMonthRule, Face)
```

## Arguments

| | |
|---|---|
| `Price` | Vector containing price of the bonds. |
| `Settle` | Settlement date. A vector of serial date numbers. `Settle` must be earlier than `Maturity`. |
| `Maturity` | Maturity date. A vector of serial date numbers. |
| `ConvDates` | Matrix of serial date numbers representing conversion dates after `Settle`. Size = number of instruments by maximum number of conversions. Fill unspecified entries with `NaN`. |
| `CouponRates` | Matrix indicating the coupon rates for each bond in decimal form. Size = number of instruments by maximum number of conversions + 1. First column of this matrix contains rates applicable between `Settle` and the first conversion date (date in the first column of `ConvDates`). Fill unspecified entries with `NaN`. See Note below. |
| `Period` | (Optional) Coupons per year of the bond. A vector of integers. Allowed values are 0, 1, 2, 3, 4, 6, and 12. Default = 2. |

| | |
|---|---|
| `Basis` | (Optional) Day-count basis of the instrument. A vector of integers.<br>• 0 = actual/actual (default)<br>• 1 = 30/360 (SIA)<br>• 2 = actual/360<br>• 3 = actual/365<br>• 4 = 30/360 (BMA)<br>• 5 = 30/360 (ISDA)<br>• 6 = 30/360 (European)<br>• 7 = actual/365 (Japanese)<br>• 8 = actual/actual (ICMA)<br>• 9 = actual/360 (ICMA)<br>• 10 = actual/365 (ICMA)<br>• 11 = 30/360E (ICMA)<br>• 12 = actual/actual (ISDA)<br>• 13 = BUS/252<br>For more information, see basis. |
| `EndMonthRule` | (Optional) End-of-month rule. A vector. This rule applies only when `Maturity` is an end-of-month date for a month having 30 or fewer days. 0 = ignore rule, meaning that a bond's coupon payment date is always the same numerical day of the month. 1 = set rule on (default), meaning that a bond's coupon payment date is always the last actual day of the month. |
| `Face` | (Optional) Face value of each bond in the portfolio. Default = 100. |

All arguments must be number of bonds (`NUMBONDS`)-by-1 vectors, except for `ConvDates` and `CouponRate`.

---

**Note** `ConvDates` has the same number of rows as `CouponRate` to reflect the same number of bonds. However, `ConvDates` has one less column than `CouponRate`. This situation is illustrated by

```
Settle---------ConvDate1-----------ConvDate2------------Maturity


      Rate1            Rate2              Rate3
```

---

## Description

`Yield = stepcpnyield(Price, Settle, Maturity, ConvDates, CouponRate, Period, Basis, EndMonthRule, Face)` computes the yield to maturity of bonds with stepped coupons given the price. The function supports any number of conversion dates.

`Yield` is a `NUMBONDS`-by-`1` vector of yields to maturity in decimal form.

---

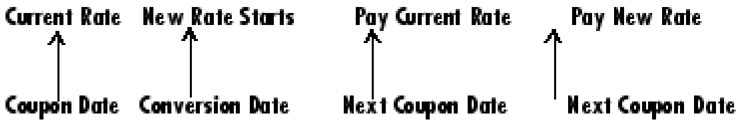
**Note** For bonds with fixed coupons, use `bndyield`. You will receive the error `incorrect number of inputs` if you use a fixed-coupon bond with `stepcpnyield`.

---

## Examples

Find the yield to maturity of three stepped-coupon bonds of known price, given three conversion scenarios:

- Bond A has two conversions, the first one falling on the settle date and immediately expiring.
- Bond B has three conversions, with conversion dates exactly on the coupon dates.
- Bond C has three conversions, with one or more conversion dates not on coupon dates. This case illustrates that only cash flows for full periods after conversion dates are affected, as illustrated below.



The following table illustrates the interest rate characteristics of this bond portfolio.

| Bond A Dates | Bond A Rates | Bond B Dates | Bond B Rates | Bond C Dates | Bond C Rates |
|---|---|---|---|---|---|
| Settle (02-Aug-92) | 7.5% | Settle (02-Aug-92) | 7.5% | Settle (02-Aug-92) | 7.5% |
| First Conversion (02-Aug-92) | 8.875% | First Conversion (15-Jun-97) | 8.875% | First Conversion (14-Jun-97) | 8.875% |
| Second Conversion (15-Jun-03) | 9.25% | Second Conversion (15-Jun-01) | 9.25% | Second Conversion (14-Jun-01) | 9.25% |
| Maturity (15-Jun-10) | `NaN` | Third Conversion (15-Jun-05) | 10.0% | Third Conversion (14-Jun-05) | 10.0% |
| | | Maturity (15-Jun-10) | `NaN` | Maturity (15-Jun-10) | `NaN` |

```
format long
Price = [117.3824; 113.4339; 113.4339];
Settle = datenum('02-Aug-1992');

ConvDates = [datenum('02-Aug-1992'), datenum('15-Jun-2003'), nan;
datenum('15-Jun-1997'), datenum('15-Jun-2001'), datenum('15-Jun-2005');
datenum('14-Jun-1997'), datenum('14-Jun-2001'), datenum('14-Jun-2005')];

Maturity = datenum('15-Jun-2010');

CouponRates = [0.075 0.08875 0.0925 nan;
```

```
               0.075 0.08875 0.0925 0.1;
               0.075 0.08875 0.0925 0.1];
Basis = 1;
Period = 2;
EndMonthRule = 1;
Face = 100;

Yield = stepcpnyield(Price, Settle, Maturity, ConvDates, ...
CouponRates, Period, Basis, EndMonthRule, Face)

Yield =

0.07221440204915
0.07221426780036
0.07221426780036
```

## References

This function adheres to *SIA Fixed Income Securities Formulas for Price, Yield, and Accrued Interest*, Volume 1, 3rd edition, pp. 120 - 123, on zero-coupon instruments pricing.

## See Also

`bndprice` | `cdprice` | `stepcpncfamounts` | `stepcpnprice` | `tbillprice` | `zeroprice`

# tbilldisc2yield

## Purpose

Convert Treasury bill discount to equivalent yield

## Syntax

```
[BEYield MMYield] = tbilldisc2yield(Discount, Settle, Maturity)
```

## Arguments

| | |
|---|---|
| `Discount` | Discount rate of Treasury bills in decimal. The discount rate basis is actual/360. |
| `Settle` | Settlement date. `Settle` must be earlier than `Maturity`. |
| `Maturity` | Maturity date. |

Inputs must either be a scalar or a vector of size equal to the number of Treasury bills (`NTBILLS`) by `1` or `1`-by-`NTBILLS`.

## Description

`[BEYield MMYield] = tbilldisc2yield(Yield, Settle, Maturity)` converts the discount rate on Treasury bills into their respective money-market or bond-equivalent yields.

`BEYield` is an `NTBILLS`-by-`1` vector of bond-equivalent yields. The bond-equivalent yield basis is actual/365.

`MMYield` is an `NTBILLS`-by-`1` vector of money-market yields. The money-market yield basis is actual/360.

## Examples

Given a Treasury bill with these characteristics, compute the bond-equivalent and money-market yields.

```
Discount = 0.0497;
Settle = '01-Oct-02';
Maturity = '31-Mar-03';

[BEYield MMYield] = tbilldisc2yield(Discount, Settle, Maturity)

BEYield =
```

```
    0.0517

MMYield =

    0.0510
```

## References

This function adheres to *SIA Fixed Income Securities Formulas for Price, Yield, and Accrued Interest*, Volume 1, 3rd edition, pp. 44 - 45 (on Treasury bills), and *Money Market and Bond Calculation* by Stigum and Robinson.

## See Also

`tbillyield2disc` | `zeroyield`

## Purpose

Price Treasury bill

## Syntax

`Price = tbillprice(Rate, Settle, Maturity, Type)`

## Arguments

| | |
|---|---|
| `Rate` | Bond-equivalent yield, money-market yield, or discount rate in decimal. |
| `Settle` | Settlement date. `Settle` must be earlier than `Maturity`. |
| `Maturity` | Maturity date. |
| `Type` | (Optional) Rate type. Determines how to interpret values entered in `Rate`. 1 = money market (default). 2 = bond-equivalent. 3 = discount rate. |

All arguments must be a scalar or some Treasury bills (`NTBILLS`) by 1 or `1`-by-`NTBILLS` vector.

**Note** The bond-equivalent yield basis is actual/365. The money-market yield basis is actual/360. The discount rate basis is actual/360.

## Description

`Price = tbillprice(Rate, Settle, Maturity, Type)` computes the price of a Treasury bill given a yield or discount rate.

`Price` is an `NTBILLS`-by-`1` vector of T-bill prices for every $100 face.

## Examples

**Example 1**. Given a Treasury bill with these characteristics, compute the price of the Treasury bill using the bond-equivalent yield as input.

```
Rate = 0.045;
Settle = '01-Oct-02';
Maturity = '31-Mar-03';
```

```
Type = 2;

Price = tbillprice(Rate, Settle, Maturity, Type)

Price =

   97.8172
```

**Example 2**. Use `tbillprice` to price a portfolio of Treasury bills.

```
Rate = [0.045; 0.046];
Settle = {'02-Jan-02'; '01-Mar-02'};
Maturity = {'30-June-02'; '30-June-02'};
Type = [2 3];

Price = tbillprice(Rate, Settle, Maturity, Type)

Price =

   97.8408
   98.4539
```

## References

This function adheres to *SIA Fixed Income Securities Formulas for Price, Yield, and Accrued Interest*, Volume 1, 3rd edition, pp. 44 - 45 (on Treasury bills), and *Money Market and Bond Calculation* by Stigum and Robinson.

## See Also

`tbillyield` | `zeroprice`

## Purpose

Break-even discount of repurchase agreement

## Syntax

```
TBEDiscount = tbillrepo(RepoRate, InitialDiscount, PurchaseDate,
SaleDate, Maturity)
```

## Arguments

| | |
|---|---|
| `RepoRate` | The annualized, 360-day based repurchase rate, in decimal. |
| `InitialDiscount` | Discount on the Treasury bill on the day of purchase, in decimal. |
| `PurchaseDate` | Date the Treasury bill is purchased. |
| `SaleDate` | Date the Treasury bill repurchase term is due. |
| `Maturity` | Treasury bill maturity date. |

All arguments must be a scalar or some Treasury bills (`NTBILLS`) by `1` or a `1`-by-`NTBILLS` vector.

All dates must be in serial date number format.

## Description

`TBEDiscount = tbillrepo(RepoRate, InitialDiscount, PurchaseDate, SaleDate, Maturity)` computes the true break-even discount of a repurchase agreement. `TBEDiscount` can be a scalar or vector of size `NTBills`-by-`1`.

## Examples

Compute the true break-even discount on a Treasury bill repurchase agreement.

```
RepoRate = [0.045; 0.0475];
InitialDiscount = 0.0475;
PurchaseDate = '3-Jan-2002';
SaleDate = '3-Feb-2002';
Maturity = '3-Apr-2002';
```

```
TBEDiscount = tbillrepo(RepoRate, InitialDiscount,...
PurchaseDate, SaleDate, Maturity)

TBEdiscount =

    0.0491
    0.0478
```

## References

This function adheres to *SIA Fixed Income Securities Formulas for Price, Yield, and Accrued Interest*, Volume 1, 3rd edition, pp. 44 - 45 (on Treasury bills), and *Money Market and Bond Calculation* by Stigum and Robinson.

## Purpose

Value of one basis point

## Syntax

`[Val01Disc, Val01MMY, Val01BEY] = tbillval01(Settle, Maturity)`

## Arguments

| | |
|---|---|
| `Settle` | Settlement date of Treasury bills. `Settle` must be earlier than `Maturity`. |
| `Maturity` | Maturity date of Treasury bills. |

## Description

`[Val01Disc, Val01MMY, Val01BEY] = tbillval01(Settle, Maturity)` calculates the value of one basis point of $100 Treasury bill face value on the discount rate, money-market yield, or bond-equivalent yield.

`Val01Disc` is the value of one basis point of discount rate.

`Val01MMY` is the value of one basis point of money-market yield.

`Val01BEY` is the value of one basis point of bond-equivalent yield.

All outputs are of size equal to the number of Treasury bills (`NTBILLS`) by `1`.

## Examples

Given a Treasury bill with these settle and maturity dates, compute the value of one basis point.

```
Settle = '01-Mar-03';
Maturity = '30-June-03';
[Val01Disc, Val01MMY, Val01BEY] = tbillval01(Settle, Maturity)

Val01Disc =

    0.0034

Val01MMY =
```

```
    0.0034

Val01BEY =

    0.0033
```

## References

This function adheres to *SIA Fixed Income Securities Formulas for Price, Yield, and Accrued Interest*, Volume 1, 3rd edition, pp 108 - 115, on zero coupon instrument pricing.

## See Also

`tbilldisc2yield` | `tbillprice` | `tbillyield` | `tbillyield2disc`

## Purpose

Yield on Treasury bill

## Syntax

`[MMYield, BEYield, Discount] = tbillyield(Price, Settle, Maturity)`

## Arguments

| | |
|---|---|
| `Price` | Price of Treasury bills for every $100 face value. |
| `Settle` | Settlement date. `Settle` must be earlier than `Maturity`. |
| `Maturity` | Maturity date. |

All arguments must be a scalar or some Treasury bills (`NTBILLS`) by `1` or `1`-by-`NTBILLS` vector.

## Description

`[MMYield, BEYield, Discount] = tbillyield(Price, Settle, Maturity)` computes the yield of U.S. Treasury bills given `Price`, `Settle`, and `Maturity`. `MMYield` is the money-market yields of the Treasury bills. `BEYield` is the bond equivalent yields of the Treasury bills. `Discount` is the discount rates of the Treasury bills.

All outputs are `NTBILLS`-by-`1` vectors.

---

**Note** The money-market yield basis is actual/360. The bond-equivalent yield basis is actual/365. The discount rate basis is actual/360.

---

## Examples

Given a Treasury bill with these characteristics, compute the money-market and bond-equivalent yields and the discount rate.

```
Price = 98.75;
Settle = '01-Oct-02';
Maturity = '31-Mar-03';

[MMYield, BEYield, Discount] = tbillyield(Price, Settle,...
```

```
Maturity)

MMYield =

    0.0252

BEYield =

    0.0255

Discount =

    0.0249
```

## References

This function adheres to *SIA Fixed Income Securities Formulas for Price, Yield, and Accrued Interest*, Volume 1, 3rd edition, pp. 44 - 45 (on Treasury bills), and *Money Market and Bond Calculation* by Stigum and Robinson.

## See Also

`tbilldisc2yield` | `tbillprice` | `tbillyield2disc` | `zeroyield`

## Purpose

Convert Treasury bill yield to equivalent discount

## Syntax

```
Discount = tbillyield2disc(Yield, Settle, Maturity, Type)
```

## Arguments

| | |
|---|---|
| `Yield` | Yield of Treasury bills in decimal. |
| `Settle` | Settlement date. `Settle` must be earlier than `Maturity`. |
| `Maturity` | Maturity date. |
| `Type` | (Optional) Yield type. Determines how to interpret values entered in `Yield`. 1 = money market (default). 2 = bond-equivalent. |

Inputs must either be a scalar or a vector of size equal to the number of Treasury bills (`NTBILLS`) by `1` or `1`-by-`NTBILLS`.

---

**Note** The money-market yield basis is actual/360. The bond-equivalent yield basis is actual/365. The discount rate basis is actual/360.

---

## Description

`Discount = tbillyield2disc(Yield, Settle, Maturity, Type)` converts the yield on some Treasury bills into their respective discount rates.

`Discount` is a `NTBILLS`-by-`1` vector of T-bill discount rates.

## Examples

Given a Treasury bill with these characteristics, compute the discount rate on a money-market basis.

```
Yield = 0.0497;
Settle = '01-Oct-02';
Maturity = '31-Mar-03';

Discount = tbillyield2disc(Yield, Settle, Maturity)
```

```
Discount =

    0.0485
```

Now recompute the discount on a bond-equivalent basis.

```
Discount = tbillyield2disc(Yield, Settle, Maturity, 2)

Discount =

    0.0478
```

## References

This function adheres to *SIA Fixed Income Securities Formulas for Price, Yield, and Accrued Interest*, Volume 1, 3rd edition, pp. 44 - 45 (on Treasury bills), and *Money Market and Bond Calculation* by Stigum and Robinson.

## See Also

`tbilldisc2yield`

## Purpose

Future prices of Treasury bonds given spot price

## Syntax

```
QtdFutPrice = tfutbyprice(SpotCurve, Price, SettleFut, MatFut,
ConvFactor, CouponRate, Maturity, Interpolation)
```

## Arguments

| | |
|---|---|
| `SpotCurve` | Treasury spot curve; a number of futures (`NFUT`) by `3` matrix in the form of `[SpotDates SpotRates Compounding]`.<br>Allowed compounding values are -1, 1, 2 (default), 3, 4, and 12. |
| `Price` | Scalar or vector containing prices of Treasury bonds or notes per $100 notional. Use `bndprice` for theoretical value of bond. |
| `SettleFut` | Scalar or vector of identical elements containing settlement date of futures contract. |
| `MatFut` | Scalar or vector containing maturity dates (or anticipated delivery dates) of futures contract. |
| `ConvFactor` | Conversion factor. See `convfactor`. |
| `CouponRate` | Scalar or vector containing underlying bond annual coupon in decimal. |
| `Maturity` | Scalar or vector containing underlying bond maturity. |
| `Interpolation` | (Optional) Interpolation method. Available methods are (0) nearest, (1) linear, and (2) cubic. Default = 1. See `interp1` for more information. |

Inputs (except `SpotCurve`) must either be a scalar or a vector of size equal to the number of Treasury futures (`NFUT`) by `1` or `1`-by-`NFUT`.

## Description

`QtdFutPrice = tfutbyprice(SpotCurve, Price, SettleFut, MatFut, ConvFactor, CouponRate, Maturity, Interpolation)` computes future prices of Treasury notes and bonds given the spot price.

In addition, you can use the Financial Instruments Toolbox method `getZeroRates` for an `IRDataCurve` object with a `Dates` property to create a vector of dates and data acceptable for `tfutbyprice`. For more information, see "Converting an IRDataCurve or IRFunctionCurve Object" on page 9-37.

## Examples

Determine the future price of two Treasury bonds based upon a spot rate curve constructed from data for November 14, 2002.

```
% Constructing spot curve from Nov 14, data
Bonds = [datenum('02/13/2003'),        0;
         datenum('05/15/2003'),        0;
         datenum('10/31/2004'),  0.02125;
         datenum('11/15/2007'),     0.03;
         datenum('11/15/2012'),     0.04;
         datenum('02/15/2031'),  0.05375];

Yields  = [1.20; 1.25; 1.86; 2.99; 4.02; 4.93]/100;

Settle = datenum('11/15/2002');

[ZeroRates, CurveDates] = ...
zbtyield(Bonds, Yields, Settle);

SpotCurve  = [CurveDates, ZeroRates];

% Calculating a particular bond's future quoted price
RefDate    = [datenum('1-Dec-2002'); datenum('1-Mar-2003')];
MatFut     = [datenum('15-Dec-2002'); datenum('15-Mar-2003')];
Maturity   = [datenum('15-Aug-2009');datenum('15-Aug-2010')];
CouponRate = [0.06;0.0575];
ConvFactor = convfactor(RefDate, Maturity, CouponRate);
Price = [114.416; 113.171];
Interpolation = 1;

QtdFutPrice = tfutbyprice(SpotCurve, Price, Settle, ...
MatFut, ConvFactor, CouponRate, Maturity, Interpolation)
```

```
QtdFutPrice =

  114.0409
  113.4029
```

## See Also

`convfactor` | `tfutbyyield`

# tfutbyyield

## Purpose

Future prices of Treasury bonds given current yield

## Syntax

```
QtdFutPrice = tfutbyyield(SpotCurve, Yield, SettleFut, MatFut,
ConvFactor, CouponRate, Maturity, Interpolation)
```

## Arguments

| | |
|---|---|
| `SpotCurve` | Treasury spot curve. A number of futures (`NFUT`)-by-`3` matrix in the form of `[SpotDates SpotRates Compounding]`.<br>Allowed compounding values are -1, 1, 2 (default), 3, 4, and 12. |
| `Yield` | Scalar or vector containing yield to maturity of bonds. Use `bndyield` for theoretical value of bond yield. |
| `SettleFut` | Scalar or vector of identical elements containing settlement date of futures contract. |
| `MatFut` | Scalar or vector containing maturity dates (or anticipated delivery dates) of futures contract. |
| `ConvFactor` | Conversion factor. See `convfactor`. |
| `CouponRate` | Scalar or vector containing underlying bond annual coupon in decimal. |
| `Maturity` | Scalar or vector containing underlying bond maturity. |
| `Interpolation` | (Optional) Interpolation method. Available methods are (0) nearest, (1) linear, and (2) cubic. Default = 1. See `interp1` for more information. |

Inputs (except `SpotCurve`) must either be a scalar or a vector of size equal to the number of Treasury futures (`NFUT`) by `1` or `1`-by-`NFUT`.

## Description

```
QtdFutPrice = tfutbyyield(SpotCurve, Yield, SettleFut,
MatFut, ConvFactor, CouponRate, Maturity, Interpolation)
```

computes future prices of Treasury notes and bonds given current yields of Treasury bonds/notes.

In addition, you can use the Financial Instruments Toolbox method `getZeroRates` for an `IRDataCurve` object with a `Dates` property to create a vector of dates and data acceptable for `tfutbyyield`. For more information, see "Converting an IRDataCurve or IRFunctionCurve Object" on page 9-37.

## Examples

Determine the future price of two Treasury bonds based upon a spot rate curve constructed from data for November 14, 2002.

```
% Constructing spot curve from Nov 14, data
Bonds = [datenum('02/13/2003'),       0;
         datenum('05/15/2003'),       0;
         datenum('10/31/2004'),  0.02125;
         datenum('11/15/2007'),     0.03;
         datenum('11/15/2012'),     0.04;
         datenum('02/15/2031'),  0.05375];

Yields  = [1.20; 1.25; 1.86; 2.99; 4.02; 4.93]/100;

Settle = datenum('11/15/2002');

[ZeroRates, CurveDates] = ...
zbtyield(Bonds, Yields, Settle);

SpotCurve  = [CurveDates, ZeroRates];

% Calculating a particular bond's future quoted price
RefDate    = [datenum('1-Dec-2002'); datenum('1-Mar-2003')];
MatFut     = [datenum('15-Dec-2002'); datenum('15-Mar-2003')];
Maturity   = [datenum('15-Aug-2009');datenum('15-Aug-2010')];
CouponRate = [0.06;0.0575];
ConvFactor = convfactor(RefDate, Maturity, CouponRate);
Yield = [0.03576; 0.03773];
Interpolation = 1;
```

```
QtdFutPrice = tfutbyyield(SpotCurve, Yield, Settle, ...
MatFut, ConvFactor, CouponRate, Maturity, Interpolation)

QtdFutPrice =

  114.0416
  113.4034
```

## See Also

`convfactor` | `tfutbyprice`

## Purpose

Implied repo rates for Treasury bond future given price

## Syntax

```
ImpliedRepo = tfutimprepo(ReinvestData, Price, QtdFutPrice, Settle,
MatFut, ConvFactor, CouponRate, Maturity)
```

## Arguments

| | |
|---|---|
| `ReinvestData` | Number of futures (`NFUT`) by `2` matrix of rates and bases for the reinvestment of intervening coupons in the form of `[ReinvestRate ReinvestBasis]`. `ReinvestRate` is the simple reinvestment rate, in decimal. Specify `ReinvestBasis` as 0 = not reinvested, 2 = actual/360, or 3 = actual/365. |
| `Price` | Current bond price per $100 notional. |
| `QtdFutPrice` | Quoted bond futures price per $100 notional. |
| `Settle` | Settlement/valuation date of futures contract. |
| `MatFut` | Maturity date (or anticipated delivery dates) of futures contract. |
| `ConvFactor` | Conversion factor. See `convfactor`. |
| `CouponRate` | Underlying bond annual coupon, in decimal. |
| `Maturity` | Underlying bond maturity date. |

Inputs (except `ReinvestData`) must either be a scalar or a vector of size equal to the number of Treasury futures (`NFUT`) by `1` or `1`-by-`NFUT`.

## Description

`ImpliedRepo = tfutimprepo(ReinvestData, Price, QtdFutPrice, Settle, MatFut, ConvFactor, CouponRate, Maturity)` computes the implied repo rate that prevents arbitrage of Treasury bond futures, given the clean price at the settlement and delivery dates.

`ImpliedRepo` is the implied annual repo rate, in decimal, with an actual/360 basis.

## Examples

Compute the implied repo rate given the following set of data.

```
ReinvestData = [0.018  3];
Price = [114.4160; 113.1710];
QtdFutPrice = [114.1201; 113.7090];
Settle = datenum('11/15/2002');
MatFut = [datenum('15-Dec-2002'); datenum('15-Mar-2003')];
ConvFactor = [1; 0.9854];
CouponRate = [0.06; 0.0575];
Maturity = [datenum('15-Aug-2009'); datenum('15-Aug-2010')];

ImpliedRepo = tfutimprepo(ReinvestData, Price, QtdFutPrice, ...
Settle, MatFut, ConvFactor, CouponRate, Maturity)

ImpliedRepo =

    0.0200
    0.0200
```

## See Also

`tfutpricebyrepo` | `tfutyieldbyrepo`

## Purpose

Implied repo rates given Treasury bond future price

## Syntax

```
[QtdFutPrice AccrInt] = tfutpricebyrepo(RepoData, ReinvestData,
Price, Settle, MatFut, ConvFactor, CouponRate, Maturity)
```

## Arguments

| | |
|---|---|
| `RepoData` | Number of futures (`NFUT`) by `2` matrix of simple term repo/funding rates in decimal and their bases in the form of `[RepoRate RepoBasis]`. Specify `RepoBasis` as 2 = actual/360 or 3 = actual/365. |
| `ReinvestData` | Number of futures (`NFUT`) by `2` matrix of rates and bases for the reinvestment of intervening coupons in the form of `[ReinvestRate ReinvestBasis]`. `ReinvestRate` is the simple reinvestment rate, in decimal. Specify `ReinvestBasis` as 0 = not reinvested, 2 = actual/360, or 3 = actual/365. |
| `Price` | Quoted clean prices of Treasury bonds per $100 notional at `Settle`. |
| `Settle` | Settlement/valuation date of futures contract. |
| `MatFut` | Maturity date (or anticipated delivery dates) of futures contract. |
| `ConvFactor` | Conversion factor. See `convfactor`. |
| `CouponRate` | Underlying bond annual coupon, in decimal. |
| `Maturity` | Underlying bond maturity date. |

Inputs (except `RepoData` and `ReinvestData`) must either be a scalar or a vector of size equal to the number of Treasury futures (`NFUT`) by `1` or `1`-by-`NFUT`.

## Description

```
[QtdFutPrice AccrInt] = tfutpricebyrepo(RepoData,
ReinvestData, Price, Settle, MatFut, ConvFactor,
```

`CouponRate, Maturity)` computes the theoretical futures bond price given the settlement price, the repo/funding rates, and the reinvestment rate.

`QtdFutPrice` is the quoted futures price, per $100 notional.

`AccrInt` is the accrued interest due at the delivery date, per $100 notional.

## Examples

Compute the quoted futures price and accrued interest due on the target delivery date, given the following data.

```
RepoData     = [0.020  2];
ReinvestData = [0.018  3];
Price        = [114.416; 113.171];
Settle       = datenum('11/15/2002');
MatFut       = [datenum('15-Dec-2002'); datenum('15-Mar-2003')];
ConvFactor   = [1 ; 0.9854];
CouponRate   = [0.06;0.0575];
Maturity     = [datenum('15-Aug-2009'); datenum('15-Aug-2010')];

[QtdFutPrice AccrInt] = tfutpricebyrepo(RepoData, ...
ReinvestData, Price, Settle, MatFut, ConvFactor, CouponRate, ...
Maturity)

QtdFutPrice =

  114.1201
  113.7090

AccrInt =

    1.9891
    0.4448
```

## See Also

`tfutimprepo` | `tfutyieldbyrepo`

## Purpose

Implied repo rates given Treasury bond future yield

## Syntax

`FwdYield = tfutyieldbyrepo(RepoData, ReinvestData, Yield, Settle, MatFut, ConvFactor, CouponRate, Maturity)`

## Arguments

| | |
|---|---|
| `RepoData` | Number of futures (`NFUT`) by `2` matrix of simple term repo/funding rates in decimal and their bases in the form of `[RepoRate RepoBasis]`.<br>Specify `RepoBasis` as 2 = actual/360 or 3 = actual/365. |
| `ReinvestData` | Number of futures (`NFUT`) by `2` matrix of rates and bases for the reinvestment of intervening coupons in the form of `[ReinvestRate ReinvestBasis]`.<br>`ReinvestRate` is the simple reinvestment rate, in decimal. Specify `ReinvestBasis` as 0 = not reinvested, 2 = actual/360, or 3 = actual/365. |
| `Yield` | Yield to maturity of Treasury bonds per $100 notional at `Settle`. |
| `Settle` | Settlement/valuation date of futures contract. |
| `MatFut` | Maturity date (or anticipated delivery dates) of futures contract. |
| `ConvFactor` | Conversion factor. See `convfactor`. |
| `CouponRate` | Underlying bond annual coupon, in decimal. |
| `Maturity` | Underlying bond maturity date. |

Inputs (except `RepoData` and `ReinvestData`) must either be a scalar or a vector of size equal to the number of Treasury futures (`NFUT`) by `1` or `1`-by-`NFUT`.

## Description

`FwdYield = tfutyieldbyrepo(RepoData, ReinvestData, Yield, Settle, MatFut, ConvFactor, CouponRate, Maturity)` computes the theoretical futures bond yield given the settlement yield, the repo/funding rate, and the reinvestment rate.

`FwdYield` is the forward yield to maturity, in decimal, compounded semiannually.

## Examples

Compute the quoted futures bond yield, given the following data:

```
RepoData     = [0.020  2];
ReinvestData = [0.018  3];
Yield        = [0.0215; 0.0257];
Settle       = datenum('11/15/2002');
MatFut       = [datenum('15-Dec-2002'); datenum('15-Mar-2003')];
ConvFactor   = [1; 0.9854];
CouponRate   = [0.06; 0.0575];
Maturity     = [datenum('15-Aug-2009'); datenum('15-Aug-2010')];

FwdYield = tfutyieldbyrepo(RepoData, ReinvestData, Yield,...
Settle, MatFut, ConvFactor, CouponRate, Maturity)

FwdYield =

    0.0221
    0.0282
```

## See Also

`tfutimprepo` | `tfutpricebyrepo`

## Purpose

Convert `IRDataCurve` object to `RateSpec`

## Class

`@IRDataCurve`

## Syntax

`F = toratespec(CurveObj, InpDates)`

## Arguments

| | |
|---|---|
| `CurveObj` | Interest-rate curve object that is constructed using `IRDataCurve`. |
| `InpDates` | Vector of input dates using MATLAB date format. The input dates must be after the settle date. |

## Description

`F = toratespec(CurveObj, InpDates)` returns a `RateSpec` object that is identical to the `RateSpec` structure created by the Financial Instruments Toolbox function `intenvset`.

## Examples

This example creates an `IRDataCurve` object from the `IRDataCurve` constructor using `Dates` and `Data` and then is converted to a `RateSpec` structure using the `toRateSpec` method:

```
Data = [2.09 2.47 2.71 3.12 3.43 3.85 4.57 4.58]/100;
Dates = daysadd(today,[360 2*360 3*360 5*360 7*360 10*360 20*360 30*360],1);
irdc = IRDataCurve('Forward',today,Dates,Data)
irdc.toRateSpec(today+30:30:today+365)


irdc =


  IRDataCurve handle


  Properties:
           Dates: [8x1 double]
            Data: [8x1 double]
    InterpMethod: 'linear'
            Type: 'Forward'
          Settle: 733596
```

```
       Compounding: 2
             Basis: 0

    Methods, Events, Superclasses

  ans =

             FinObj: 'RateSpec'
        Compounding: 2
               Disc: [12x1 double]
              Rates: [12x1 double]
           EndTimes: [12x1 double]
         StartTimes: [12x1 double]
           EndDates: [12x1 double]
         StartDates: 733596
      ValuationDate: 733596
              Basis: 0
       EndMonthRule: 1
```

## How To

- “@IRDataCurve” on page A-7

## Purpose

Convert `IRFunctionCurve` object to `RateSpec`

## Class

`@IRFunctionCurve`

## Syntax

`F = toRateSpec(CurveObj, InpDates)`

## Arguments

| | |
|---|---|
| `CurveObj` | Interest-rate curve object that is constructed using `IRFunctionCurve`. |
| `InpDates` | Vector of input dates using MATLAB date format. The input dates must be after the settle date. |

## Description

`F = toRateSpec(CurveObj, InpDates)` returns a `RateSpec` object that is identical to the `RateSpec` structure created by the Financial Instruments Toolbox function `intenvset`.

## Examples

This example creates an `IRFunctionCurve` object using the `IRFunctionCurve` constructor and then a `RateSpec` structure is created using the `toRateSpec` method:

```
irfc = IRFunctionCurve('Forward',today,@(t) polyval([-0.0001 0.003 0.02],t));
irfc.toRateSpec(today+30:30:today+365)

ans =

          FinObj: 'RateSpec'
     Compounding: 2
            Disc: [12x1 double]
           Rates: [12x1 double]
        EndTimes: [12x1 double]
      StartTimes: [12x1 double]
        EndDates: [12x1 double]
      StartDates: 733596
   ValuationDate: 733596
           Basis: 0
```

```
EndMonthRule: 1
```

## How To

- “@IRFunctionCurve” on page A-12

## Purpose

Price zero-coupon instruments given yield

## Syntax

```
Price = zeroprice(Yield, Settle, Maturity, Period, Basis,
EndMonthRule)
```

## Arguments

| | |
|---|---|
| `Yield` | Scalar or vector containing yield to maturity of instruments. |
| `Settle` | Settlement date. A vector of serial date numbers or date strings. `Settle` must be earlier than `Maturity`. |
| `Maturity` | Maturity date. A vector of serial date numbers or date strings. |
| `Period` | (Optional) Scalar or vector specifying number of quasi-coupons per year. Default = 2. |
| `Basis` | (Optional) Day-count basis of the bond. A vector of integers.<br>• 0 = actual/actual (default)<br>• 1 = 30/360 (SIA)<br>• 2 = actual/360<br>• 3 = actual/365<br>• 4 = 30/360 (BMA)<br>• 5 = 30/360 (ISDA)<br>• 6 = 30/360 (European)<br>• 7 = actual/365 (Japanese)<br>• 8 = actual/actual (ICMA)<br>• 9 = actual/360 (ICMA)<br>• 10 = actual/365 (ICMA)<br>• 11 = 30/360E (ICMA) |

- 12 = actual/actual (ISDA)
- 13 = BUS/252

For more information, see basis.

| | |
|---|---|
| `EndMonthRule` | (Optional) End-of-month rule. A vector. This rule applies only when `Maturity` is an end-of-month date for a month having 30 or fewer days. 0 = ignore rule, meaning that a bond's coupon payment date is always the same numerical day of the month. 1 = set rule on (default), meaning that a bond's coupon payment date is always the last actual day of the month. |

## Description

`Price = zeroprice(Yield, Settle, Maturity, Period, Basis, EndMonthRule)` calculates the prices for a portfolio of general short and long term zero-coupon instruments given the yield of the instruments. `Price` is a column vector containing a price for each zero-coupon instrument.

When there is less than one quasi-coupon, the function uses a simple yield based upon "Period times Number of Days in quasi coupon period" day-year. The default period is 2 and the default number of days is 180, which makes the user-supplied yield a simple yield on a 360-day year.

For longer term computations (more than one quasi-coupon), use the bond equivalent yield based upon present value (or compounding).

## Formulas

To compute the price when there is 1 or 0 quasi-coupon periods to redemption, `zeroprice` uses the formula

$$Price = \frac{RV}{1 + \left(\frac{DSR}{E} \times \frac{Y}{M}\right)}$$

.

*Quasi-coupon periods* are the coupon periods that would exist if the bond were paying interest at a rate other than zero.

When there is more than one quasi-coupon period to the redemption date, `zeroprice` uses the formula

$$Price = \frac{RV}{\left(1 + \frac{Y}{M}\right)^{N_q - 1 + \frac{DSC}{E}}}$$

.

The elements of the equations are defined as follows.

| Variable | Definition |
|---|---|
| *DSC* | Number of days from settlement date to next quasi-coupon date as if the security paid periodic interest. |
| *DSR* | Number of days from settlement date to the redemption date (call date, put date, and so on). |
| *E* | Number of days in quasi-coupon period. |
| *M* | Number of quasi-coupon periods per year (standard for the particular security involved). |
| *Nq* | Number of quasi-coupon periods between settlement date and redemption date. If this number contains a fractional part, raise it to the next whole number. |
| *Price* | Dollar price per $100 par value. |
| *RV* | Redemption value. |
| *Y* | Annual yield (decimal) when held to redemption. |

## Examples

**Example 1**. Compute the price of a short-term zero-coupon instrument.

```
Settle = '24-Jun-1993';
Maturity = '1-Nov-1993';
Period = 2;
```

```
Basis = 0;
Yield = 0.04;

Price = zeroprice(Yield, Settle, Maturity, Period, Basis)

Price =

   98.6066
```

**Example 2**. Compute the prices of a portfolio of two zero-coupon instruments, one short-term, and the other long-term.

```
Settle = '24-Jun-1993';
Maturity = ['01-Nov-1993'; '15-Jan-2024'];
Basis = [0; 1];
Yield = [0.04; 0.1];

Price = zeroprice(Yield, Settle, Maturity, [], Basis)

Price =

   98.6066
    5.0697
```

## References

[1] Mayle, Jan. *Standard Securities Calculation Methods*. New York: Securities Industry Association, Inc. Vol. 1, 3rd ed., 1993, ISBN 1-882936-01-9. Vol. 2, 1994, ISBN 1-882936-02-7.

## See Also

`bndprice` | `cdprice` | `tbillprice` | `zeroyield`

## Purpose

Yield of zero-coupon instruments given price

## Syntax

```
Yield = zeroyield(Price, Settle, Maturity, Period, Basis,
EndMonthRule)
```

## Arguments

| | |
|---|---|
| `Price` | Scalar or vector containing prices of instruments. |
| `Settle` | Settlement date. A vector of serial date numbers or date strings. `Settle` must be earlier than `Maturity`. |
| `Maturity` | Maturity date. A vector of serial date numbers or date strings. |
| `Period` | (Optional) Scalar or vector specifying number of quasi-coupons per year. Default = 2. |
| `Basis` | (Optional) Day-count basis of the bond. A vector of integers.<br>• 0 = actual/actual (default)<br>• 1 = 30/360 (SIA)<br>• 2 = actual/360<br>• 3 = actual/365<br>• 4 = 30/360 (BMA)<br>• 5 = 30/360 (ISDA)<br>• 6 = 30/360 (European)<br>• 7 = actual/365 (Japanese)<br>• 8 = actual/actual (ICMA)<br>• 9 = actual/360 (ICMA)<br>• 10 = actual/365 (ICMA)<br>• 11 = 30/360E (ICMA)<br>• 12 = actual/actual (ISDA) |

| | |
|---|---|
| | • 13 = BUS/252<br>For more information, see basis. |
| `EndMonthRule` | (Optional) End-of-month rule. A vector. This rule applies only when `Maturity` is an end-of-month date for a month having 30 or fewer days. 0 = ignore rule, meaning that a bond's coupon payment date is always the same numerical day of the month. 1 = set rule on (default), meaning that a bond's coupon payment date is always the last actual day of the month. |

## Description

`Yield = zeroyield(Price, Settle, Maturity, Period, Basis, EndMonthRule)` calculates the bond-equivalent yield for a portfolio of general short and long term zero-coupon instruments given the price of the instruments. `Yield` is a column vector containing a yield for each zero-coupon instrument.

When the maturity date is fewer than 182 days away and the basis is actual/365, the function uses a simple-interest algorithm. If maturity is more than 182 days away, the function uses present value calculations.

When the basis is actual/360, the simple interest algorithm gives the money-market yield for short (1 to 6 months to maturity) Treasury bills.

The present value algorithm always gives the bond equivalent yield of the zero-coupon instrument. The algorithm is equivalent to calling `bndyield` with the zero-coupon information within one basis point.

## Formulas

To compute the yield when there is zero or one quasi-coupon periods to redemption, `zeroyield` uses the formula

$$Yield = \left(\frac{RV - P}{P}\right) \times \left(\frac{M \times E}{DSR}\right)$$

.

*Quasi-coupon periods* are the coupon periods which would exist if the bond was paying interest at a rate other than zero. The first term calculates the yield on invested dollars. The second term converts this yield to a per annum basis.

When there is more than one quasi-coupon period to the redemption date, `zeroyield` uses the formula

$$Yield = \left( \left( \frac{RV}{P} \right)^{\frac{1}{Nq - 1 + \frac{DSC}{E}}} - 1 \right) \times M$$

.

The elements of the equations are defined as follows.

| Variable | Definition |
|---|---|
| *DSC* | Number of days from the settlement date to next quasi-coupon date as if the security paid periodic interest. |
| *DSR* | Number of days from the settlement date to redemption date (call date, put date, and so on). |
| *E* | Number of days in quasi-coupon period. |
| *M* | Number of quasi-coupon periods per year (standard for the particular security involved). |
| *Nq* | Number of quasi-coupon periods between the settlement date and redemption date. If this number contains a fractional part, raise it to the next whole number. |
| *P* | Dollar price per $100 par value. |
| *RV* | Redemption value. |
| *Yield* | Annual yield (decimal) when held to redemption. |

## Examples

**Example 1**. Compute the yield of a short-term zero-coupon instrument.

```
Settle   = '24-Jun-1993';
Maturity = '1-Nov-1993';
Basis    = 0;
Price    = 95;

Yield = zeroyield(Price, Settle, Maturity, [], Basis)

Yield =

    0.1490
```

**Example 2**. Recompute the yield of the same instrument using a different day-count basis.

```
Settle   = '24-Jun-1993';
Maturity = '1-Nov-1993';
Basis    = 1;
Price    = 95;

Yield = zeroyield(Price, Settle, Maturity, [], Basis)

Yield =

    0.1492
```

**Example 3**. Compute the yield of a long-term zero-coupon instrument.

```
Settle   = '24-Jun-1993';
Maturity = '15-Jan-2024';
Basis    = 0;
Price    = 9;

Yield = zeroyield(Price, Settle, Maturity, [], Basis)

Yield =
```

```
   0.0804
```

## References

[1] Mayle, Jan. *Standard Securities Calculation Methods.* New York: Securities Industry Association, Inc. Vol. 1, 3rd ed., 1993, ISBN 1-882936-01-9. Vol. 2, 1994, ISBN 1-882936-02-7.

## See Also

`bndyield` | `cdyield` | `tbillyield` | `zeroprice`

# Derivatives Pricing Options

# Pricing Options Structure

**In this section...**

## Introduction

The MATLAB `Options` structure provides additional input to most pricing functions. The `Options` structure

- Tells pricing functions how to use the interest-rate tree to calculate instrument prices.
- Determines what additional information the Command Window displays along with instrument prices.
- Tells pricing functions which method to use in pricing barrier options.

The pricing options structure is primarily used in the pricing of interest-rate-based financial derivatives. However, the `BarrierMethod` field in the structure allows you to use it in pricing equity barrier options as well.

You provide pricing options in an optional `Options` argument passed to a pricing function. (See, for example, `bondbyhjm`, `bdtprice`, `barrierbycrr`, `barrierbyeqp`, or `barrierbyitt`.)

## Default Structure

If you do not specify the `Options` argument in the call to a pricing function, the function uses a default structure. To observe the default structure, use `derivset` without any arguments.

```
Options = derivset

Options =

    Diagnostics: 'off'
       Warnings: 'on'
      ConstRate: 'on'
```

```
  BarrierMethod: 'unenhanced'
```

The `Options` structure has four fields: `Diagnostics`, `Warnings`, `ConstRate`, and `BarrierMethod`.

### Diagnostics Field

`Diagnostics` indicates whether additional information is displayed if the tree is modified. The default value for this option is `'off'`. If `Diagnostics` is set to `'on'` and `ConstRate` is set to `'off'`, the pricing functions display information such as the number of nodes in the last level of the tree generated for pricing purposes.

### Warnings Field

`Warnings` indicates whether to display warning messages when the input tree is not adequate for accurately pricing the instruments. The default value for this option is `'on'`. If both `ConstRate` and `Warnings` are `'on'`, a warning is displayed if any of the instruments in the input portfolio has a cash flow date between tree dates. If `ConstRate` is `'off'`, and `Warnings` is `'on'`, a warning is displayed if the tree is modified to match the cash flow dates on the instruments in the portfolio.

### ConstRate Field

`ConstRate` indicates whether the interest rates should be assumed constant between tree dates. By default this option is `'on'`, which is not an arbitrage-free assumption. Consequently the pricing functions return an approximate price for instruments featuring cash flows between tree dates. Instruments featuring cash flows only on tree nodes are not affected by this option and return exact (arbitrage-free) prices. When `ConstRate` is `'off'`, the pricing function finds the cash flow dates for all instruments in the portfolio. If these cash flows do not align exactly with the tree dates, a new tree is generated and used for pricing. This new tree features the same volatility and initial rate specifications of the input tree but contains tree nodes for each date in which at least one instrument in the portfolio has a cash flow. Keep in mind that the number of nodes in a tree grows exponentially with the number of tree dates. Consequently, setting `ConstRate 'off'` dramatically increases the memory and processor demands on the computer.

## BarrierMethod Field

When using binomial trees to price barrier options, you may require a large number of tree steps to achieve an accurate result when tree nodes do not align with the barrier level. With the `BarrierMethod` field, the toolbox provides an enhancement method that improves the accuracy of the results without having to use large trees.

The `BarrierMethod` field can be set to `'unenhanced'` (default) or `'interp'`. If you specify `'unenhanced'`, no correction calculation is used. Otherwise, if you specify `'interp'`, the toolbox provides an enhanced valuation by interpolating between nodes on barrier boundaries.

You specify the barrier method in the last input argument, `Options`, of the functions `barrierbycrr`, `barrierbyeqp`, `crrprice`, or `eqpprice`. `Options` is a structure that you create with the function `derivset`. Using `derivset`, you specify whether to use the enhanced or the unenhanced method.

For more information about this algorithm, see Derman, E., I. Kani, D. Ergener and I. Bardhan, “Enhanced Numerical Methods for Options with Barriers,” *Financial Analysts Journal*, (Nov. - Dec. 1995), pp. 65-74.

# Customizing the Structure

Customize the Options structure by passing property name/property value pairs to the `derivset` function.

As an example, consider an Options structure with `ConstRate 'off'` and `Diagnostics 'on'`.

```
Options = derivset('ConstRate', 'off', 'Diagnostics', 'on')

Options =

  Diagnostics: 'on'
     Warnings: 'on'
    ConstRate: 'off'
BarrierMethod: 'unenhanced'
```

To obtain the value of a specific property from the Options structure, use `derivget`.

```
CR = derivget(Options, 'ConstRate')

CR =
Off
```

---

**Note** Use `derivset` and `derivget` to construct the Options structure. These functions are guaranteed to remain unchanged, while the implementation of the structure itself may be modified in the future.

---

Now observe the effects of setting `ConstRate 'off'`. Obtain the tree dates from the HJM tree.

```
TreeDates = [HJMTree.TimeSpec.ValuationDate;...
HJMTree.TimeSpec.Maturity]

TreeDates =

     730486
```

```
     730852
     731217
     731582
     731947

datedisp(TreeDates)

01-Jan-2000
01-Jan-2001
01-Jan-2002
01-Jan-2003
01-Jan-2004
```

All instruments in `HJMInstSet` settle on January 1, 2000, and all have cash flows once a year, with the exception of the second bond, which features a period of 2. This bond has cash flows twice a year, with every other cash flow consequently falling between tree dates. You can extract this bond from the portfolio to compare how its price differs by setting `ConstRate` to `'on'` and `'off'`.

```
BondPort = instselect(HJMInstSet, 'Index', 2);

instdisp(BondPort)

Index Type CouponRate Settle      Maturity     Period Basis...
1     Bond 0.04       01-Jan-2000 01-Jan-2004  2      NaN...
```

First price the bond with `ConstRate` `'on'` (default).

```
format long
[BondPrice, BondPriceTree] = hjmprice(HJMTree, BondPort)
Warning: Not all cash flows are aligned with the tree. Result will
be approximated.

BondPrice =

  97.52801411736377

BondPriceTree =
FinObj: 'HJMPriceTree'
```

```
 PBush: {1x5 cell}
AIBush: {[0]  [1x1x2 double] ... [1x4x2 double]  [1x8 double]}
  tObs: [0 1 2 3 4]
```

Now recalculate the price of the bond setting `ConstRate 'off'`.

```
OptionsNoCR = derivset('ConstR', 'off')

OptionsNoCR =

Diagnostics: 'off'
   Warnings: 'on'
  ConstRate: 'off'

[BondPriceNoCR, BondPriceTreeNoCR] = hjmprice(HJMTree,...
BondPort, OptionsNoCR)
Warning: Not all cash flows are aligned with the tree. Rebuilding
tree.

BondPriceNoCR =

  97.53342361674437

BondPriceTreeNoCR =

FinObj: 'HJMPriceTree'
 PBush: {1x9 cell}
AIBush: {1x9 cell}
  tObs: [0 0.5000 1 1.5000 2 2.5000 3 3.5000 4]
```

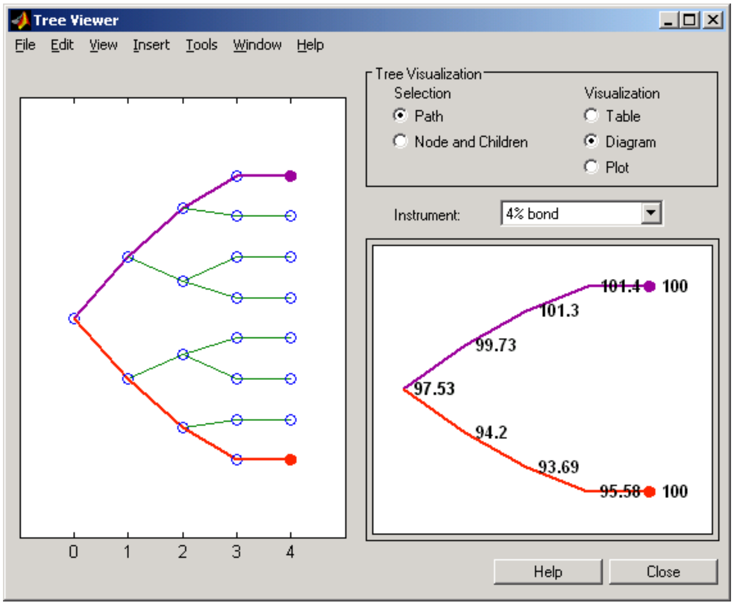
As indicated in the last warning, because the cash flows of the bond did not align with the tree dates, a new tree was generated for pricing the bond. This pricing method returns more accurate results since it guarantees that the process is arbitrage-free. It also takes longer to calculate and requires more memory. The `tObs` field of the price tree structure indicates the increased memory usage. `BondPriceTree.tObs` has only five elements, while `BondPriceTreeNoCR.tObs` has nine. While this may not seem like a large difference, it has a dramatic effect on the number of states in the last node.

```
size(BondPriceTree.PBush{end})
```

```
ans =

     1  8

size(BondPriceTreeNoCR.PBush{end})

ans =

     1  128
```

The differences become more obvious by examining the price trees with `treeviewer`.

```
treeviewer(BondPriceTree, BondPort)
```



Tree Visualization

Selection

● Path

○ Node and Children

Visualization

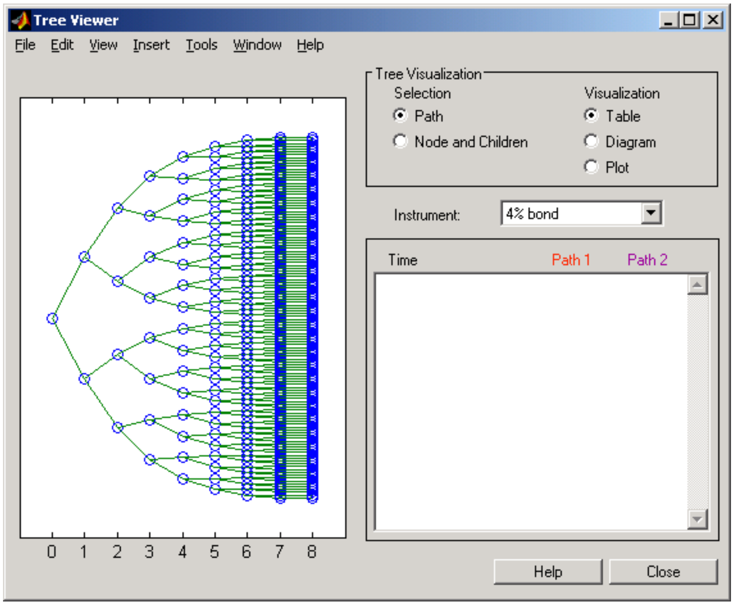
○ Table

● Diagram

○ Plot

Instrument: 4% bond

```
treeviewer(BondPriceTreeNoCR, BondPort)
```



Tree Visualization

Selection
● Path
○ Node and Children

Visualization
● Table
○ Diagram
○ Plot

Instrument: 4% bond

```
All = [Delta ./ Price, Gamma ./ Price, Vega ./ Price, Price]

All =

         -2.76         10.43          0.00         98.72
         -3.56         16.64         -0.00         97.53
       -166.18      13235.59        700.96          0.05
         -2.76         10.43          0.00         98.72
         -0.01          0.03             0        100.55
         46.95       1090.63         14.91          6.28
       -969.85     173969.77       1926.72          0.05
        -76.39        287.00          0.00          3.690
```

# Bibliography

## Black-Derman-Toy (BDT) Modeling

A description of the Black-Derman-Toy interest-rate model can be found in:

Black, Fischer, Emanuel Derman, and William Toy, “A One Factor Model of Interest Rates and its Application to Treasury Bond Options,” *Financial Analysts Journal*, January - February 1990.

# Heath-Jarrow-Morton (HJM) Modeling

An introduction to Heath-Jarrow-Morton modeling, used extensively in Financial Instruments Toolbox software, can be found in:

Jarrow, Robert A., *Modelling Fixed Income Securities and Interest Rate Options*, McGraw-Hill, 1996, ISBN 0-07-912253-1.

# Hull-White (HW) and Black-Karasinski (BK) Modeling

A description of the Hull-White model and its Black-Karasinski modification can be found in:

Hull, John C., *Options, Futures, and Other Derivatives*, Prentice-Hall, 1997, ISBN 0-13-186479-3.

You can find additional information about the Hull-White single-factor model used in this toolbox in these papers:

Hull, J., and A. White, “Numerical Procedures for Implementing Term Structure Models I: Single-Factor Models,” *Journal of Derivatives,* 1994.

Hull, J., and A. White, “Using Hull-White Interest Rate Trees,” *Journal of Derivatives*, 1996.

# Cox-Ross-Rubinstein (CRR) Modeling

To learn about the Cox-Ross-Rubinstein model, see:

Cox, J. C., S. A. Ross, and M. Rubinstein, “Option Pricing: A Simplified Approach,” *Journal of Financial Economics*, Number 7, 1979, pp. 229-263.

## Implied Trinomial Tree (ITT) Modeling

To learn about the Implied Trinomial Tree model, see:

Chriss, Neil A., E. Derman, and I. Kani, “Implied trinomial trees of the volatility smile,” *Journal of Derivatives*, 1996.

# Leisen-Reimer Tree (LR) Modeling

To learn about the Leisen-Reimer model, see:

Leisen D.P., M. Reimer, “Binomial Models for Option Valuation – Examining and Improving Convergence,” *Applied Mathematical Finance*, Number 3, 1996, pp. 319-346.

## Equal Probabilities Tree (EQP) Modeling

To learn about the Equal Probabilities model, see:

Chriss, Neil A.,*Black Scholes and Beyond: Option Pricing Models*, McGraw-Hill, 1996, ISBN 0-7863-1025-1.

# Closed-Form Solutions Modeling

To learn about the Bjerksund-Stensland 2002 model, see:

Bjerksund, P. and G. Stensland, *Closed-Form Approximation of American Options*, Scandinavian Journal of Management, 1993, Vol. 9, Suppl., pp. S88-S99.

Bjerksund, P. and G. Stensland, *Closed Form Valuation of American Options*, Discussion paper 2002 (http://brage.bibsys.no/nhh/bitstream/URN:NBN:no-bibsys_brage_22301/1/bjerksund%20p

## Financial Derivatives

You can find information on the creation of financial derivatives and their role in the marketplace in numerous sources. Among those consulted in the development of Financial Instruments Toolbox software are:

Chance, Don. M., *An Introduction to Derivatives*, The Dryden Press, 1998, ISBN 0-030-024483-8.

Fabozzi, Frank J., *Treasury Securities and Derivatives*, Frank J. Fabozzi Associates, 1998, ISBN 1-883249-23-6.

Wilmott, Paul, *Derivatives: The Theory and Practice of Financial Engineering*, John Wiley and Sons, 1998, ISBN 0-471-983-89-6.

## Fitting Interest-Rate Curve Functions

Nelson, C.R., Siegel, A.F., "Parsimonious modelling of yield curves," *Journal of Business*, Number 60, 1987, pp 473-89.

Svensson, L.E.O., "Estimating and interpreting forward interest rates: Sweden 1992-4," International Monetary Fund, IMF Working Paper, 1994, p. 114.

Fisher, M., Nychka, D., Zervos, D., "Fitting the term structure of interest rates with smoothing splines," Board of Governors of the Federal Reserve System, Federal Reserve Board Working Paper, 1995.

Anderson, N., Sleath, J., "New estimates of the UK real and nominal yield curves," *Bank of England Quarterly Bulletin*, November, 1999, pp 384-92.

Waggoner, D., "Spline Methods for Extracting Interest Rate Curves from Coupon Bond Prices," Federal Reserve Board Working Paper, 1997, p. 10.

"Zero-coupon yield curves: technical documentation," *BIS Papers*, Bank for International Settlements, Number 25, October, 2005.

Bolder, D.J., Gusba,S, "Exponentials, Polynomials, and Fourier Series: More Yield Curve Modelling at the Bank of Canada," *Working Papers*, Bank of Canada, 2002, p. 29.

Bolder, D.J., Streliski, D., "Yield Curve Modelling at the Bank of Canada," *Technical Reports*, Number 84, 1999, Bank of Canada.

## Bootstrapping a Swap Curve

Hagan, P., West, G., "Interpolation Methods for Curve Construction," *Applied Mathematical Finance*, Vol. 13, Number 2, 2006.

Ron, Uri, "A Practical Guide to Swap Curve Construction," *Working Papers*, Bank of Canada, 2000, p. 17.

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Burghardt, G., T. Belton, M. Lane, and J. Papa, *The Treasury Bond Basis*, McGraw-Hill, 2005.

Krgin, Dragomir, *Handbook of Global Fixed Income Calculations*, John Wiley & Sons, 2002.

## Credit Derivatives

Beumee, J., D. Brigo, D. Schiemert, and G. Stoyle. “Charting a Course Through the CDS Big Bang,” *Fitch Solutions, Quantitative Research*, Global Special Report. April 7, 2009.

Hull, J., and A. White, “Valuing Credit Default Swaps I: No Counterparty Default Risk,” *Journal of Derivatives*8, 29-40.

O’Kane, D. and S. Turnbull, “Valuation of Credit Default Swaps.” *Lehman Brothers, Fixed Income Quantitative Credit Research*, April, 2003.

# Examples

Use this list to find examples in the documentation.

# Instrument Portfolio Examples

"Creating Instruments or Properties" on page 1-18
"instfind Examples" on page 1-21
"instselect Examples" on page 1-24

# Interest Rate Environment Examples

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# HJM Examples

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## BDT Examples

# Rate Specification Creation

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# Treeviewer Examples

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# Creating Equity Derivatives

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# Closed-Form Solution Examples

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## Agency Option Adjusted Spreads

"Computing the Agency OAS for Bonds" on page 6-3

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“Treasury Bill Repurchase Agreements” on page 6-8
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## Using Zero-Coupon Bonds

"Pricing Treasury Notes" on page 6-13
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# Stepped-Coupon Bonds

## Pricing and Hedging

# Bond Futures

"Analysis of Bond Futures" on page 7-14

## Credit Default Swaps

# Glossary

**American option**

An option that can be exercised any time until its expiration date. Contrast with **European option** on page Glossary-4.

**arbitrary cash flow instrument**

A set of generic cash flow amounts for which a price needs to be established.

**Asian option**

An option whose payoff depends upon the average price of the underlying asset over a certain period of time.

**asset-or-nothing option**

A digital option that pays the value of the underlying security if the option expires in the money.

**barrier option**

An option that is activated or deactivated only if the price of the underlying asset crosses a barrier. See also **knock-in** on page Glossary-6 and **knock-out** on page Glossary-6. If the option fails to execute, the seller may pay to the purchaser a predetermined **rebate** on page Glossary-8.

**barrier option**

An option that is activated or deactivated only if the price of the underlying asset crosses a barrier. See also **knock-in** on page Glossary-6 and **knock-out** on page Glossary-6. If the option fails to execute, the seller may pay to the purchaser a predetermined **rebate** on page Glossary-8.

**basket option**

An option that provides a payoff dependent on the value of a portfolio of assets.

**beta**

The price volatility of a financial instrument relative to the price volatility of a market or index as a whole. Beta is most commonly used with respect to equities. A high-beta instrument is riskier than a low-beta instrument.

**binomial model**

A method in which the probability over time of each possible price or rate follows a binomial distribution. The basic assumption is that prices or rates can move to only two values (one higher and one lower) over any short time period. See also **trinomial model** on page Glossary-10.

**Black-Derman-Toy (BDT) model**

A model for pricing interest rate derivatives where all security prices and rates depend upon the short rate (annualized one-period interest rate).

**bond**

A long-term debt security with fixed interest payments and fixed maturity date.

**bond option**

The right to sell a bond back to the issuer (put) or to redeem a bond from its current owner (call) at a specific price and on a specific date.

**bushy tree**

A tree of prices or interest rates in which the number of branches increases exponentially relative to observation times; branches never recombine. Opposite of a **recombining tree** on page Glossary-8.

**call**

1. An option to buy a certain quantity of a stock or commodity for a specified price within a specified time. See also **put** on page Glossary-7.

2. A demand to submit bonds to the issuer for redemption before the maturity date.

**call swaption**

Allows the option buyer to enter into an interest rate swap in which the buyer of the option pays the fixed rate and receives the floating rate.

**callable bond**

A bond that allows the issuer to buy back the bond at a predetermined price at specified future dates. The bond contains an embedded call option; that is, the holder has sold a call option to the issuer. See also **puttable bond** on page Glossary-8.

**cap**

Interest-rate option that guarantees that the rate on a floating-rate loan will not exceed a certain level.

**caplet**

An interim cap component in a multiperiod interest-rate cap agreement.

**cash-or-nothing option**

A digital option that pays some fixed amount of cash if the option expires in the money.

**compound option**

An option on an option, such as a call on a call, a put on a put, a call on a put, or a put on a call.

**delta**

The rate of change of the price of a derivative security relative to the price of the underlying asset; that is, the first derivative of the curve that relates the price of the derivative to the price of the underlying security.

**derivative**

A financial instrument that is based on some underlying asset. For example, an option is a derivative instrument based on the right to buy or sell an underlying instrument.

**deterministic model**

An interest rate model in which the values of the rates in the next time step are determined solely by the values of the rates in the current time step.

**digital option**

An option whose payout is fixed after the underlying stock exceeds the predetermined threshold or strike price.

**discount factor**

Coefficient used to compute the present value of future cash flows.

**dollar sensitivity**

Sensitivity reported as a dollar price change instead of a percentage price change.

**down-and-in**

A type of **barrier option** on page Glossary-1 that becomes active if the barrier is reached from above. See also **knock-in** on page Glossary-6.

**down-and-out**

A type of **barrier option** on page Glossary-1 that becomes deactivated if the barrier is reached from above. See also **knock-out** on page Glossary-6.

**European option**

An option that can be exercised only on its expiration date. Contrast with **American option** on page Glossary-1.

**ex-dividend date**

Date when a declared dividend belongs to the seller rather than the buyer.

**exercise price**

The price set for buying an asset (call) or selling an asset (put). The strike price.

**exotic option**

Any nonstandard option. Opposite of **vanilla option** on page Glossary-10.

**fixed lookback option**

Strike price is fixed at purchase. The underlying is priced at its highest or lowest level, depending whether it is a call or put, during the life of the option rather than expiring at market.

**fixed-rate note**

A long-term debt security with preset interest rate and maturity, by which the interest must be paid. The principal may or may not be paid at maturity.

**floating lookback option**

Strike price is fixed at maturity. For a call, the price is fixed at the lowest price during the life of the option; for a put it is fixed at the highest price.

**floating-rate note**

A security similar to a bond, but in which the note's interest rate is reset periodically, relative to a reference index rate, to reflect fluctuations in market interest rates.

**floor**

Interest-rate option that guarantees that the rate on a floating-rate loan will not fall below a certain level.

**floorlet**

One of the interim period floors in a multiple period floor agreement.

**forward curve**

The curve of forward interest rates vs. maturity dates for bonds.

**forward rate**

The future interest rate of a bond inferred from the term structure, especially from the yield curve of zero-coupon bonds, calculated from the growth factor of an investment in a zero held until maturity.

**gamma**

The rate of change of delta for a derivative security relative to the price of the underlying asset; that is, the second derivative of the option price relative to the security price.

**gap option**

A digital option in which one strike decides if the option is in or out of money and another strike decides the size the size of the payoff.

**Heath-Jarrow-Morton (HJM) model**

A model of the interest rate term structure that works with a type of interest rate tree called a **bushy tree** on page Glossary-2.

**hedge**

A securities transaction that reduces or offsets the risk on an existing investment position.

**instrument set**

A collection of financial assets. A portfolio.

**inverse discount**

A factor by which the present value of an asset is multiplied to find its future value. The reciprocal of the discount factor.

**irregular coupon**

A bond interest payment for more or less than six-months' interest. The first coupon on many bonds is irregular because payment is other than six months from the dated date.

**knock-in**

A **barrier option** on page Glossary-1 that is activated when the price of the underlying asset achieves a designated target. There are two types: **up-and-in** on page Glossary-10 and **down-and-in** on page Glossary-4.

**knock-out**

A **barrier option** on page Glossary-1 that is deactivated when the price of the underlying asset achieves a designated target. There are two types: **up-and-out** on page Glossary-10 and **down-and-out** on page Glossary-4.

**Lambda**

The percentage change in an option price divided by the percentage change in an underlying price.

**least-squares method**

A mathematical method of determining the best fit of a curve to a series of observations by choosing the curve that minimizes the sum of the squares of all deviations from the curve.

**long rate**

The yield on a zero-coupon Treasury bond.

**lookback option**

An option that reduces uncertainties associated with the timing of market entry. Lookback options can be either **fixed lookback option** on page Glossary-4 and **floating lookback option** on page Glossary-5.

**mean reversion**

The tendency of a variable to return to its mean value after reaching a point of excessive positive or negative valuation relative to the mean.

**option**

A right to buy or sell specific securities or commodities at a stated price (exercise or strike price) within a specified time. An option is a type of derivative.

**per-dollar sensitivity**

The dollar **sensitivity** on page Glossary-8 divided by the corresponding instrument price.

**portfolio**

A collection of financial assets. Also called an instrument set.

**price tree structure**

A MATLAB structure that holds all pricing information.

**price vector**

A vector of instrument prices.

**pricing options structure**

A MATLAB structure that defines how the price tree is used to find the price of instruments in the portfolio, and how much additional information is displayed in the command window when the pricing function is called.

**put**

An option to sell a stipulated amount of stock or securities within a specified time and at a fixed exercise price. See also **call** on page Glossary-2.

**put swaption**

Allows the option buyer to enter into an interest rate swap in which the buyer of the option receives the fixed rate and pays the floating rate.

**puttable bond**

A bond that allows the holder to redeem the bond at a predetermined price at specified future dates. The bond contains an embedded put option; that is, the holder has bought a put option. See also **callable bond** on page Glossary-3.

**rainbow option**

A single option linked to two or more underlying assets. In order for the option to pay off, all the underlying assets must move in the intended direction.

**rate specification**

A MATLAB structure that holds all information needed to identify completely the evolution of interest rates.

**rebate**

A predetermined amount of money paid to the purchaser of a **barrier option** on page Glossary-1 if the option fails to execute.

**recombining tree**

A tree of prices or interest rates whose branches recombine over time. Opposite of a **bushy tree** on page Glossary-2.

**self-financing hedge**

A trading strategy whereby the value of a portfolio after rebalancing is equal to its value at any previous time.

**sensitivity**

The “what if” relationship between variables; the degree to which changes in one variable cause changes in another variable. A specific synonym is volatility. See also **dollar sensitivity** on page Glossary-4.

**short rate**

The annualized one-period interest rate.

**sinking fund bond**

A sinking fund bond is a coupon bond with a sinking fund provision. This provision obligates the issuer to amortize portions of the principal prior to maturity, affecting bond prices since the time of the principal repayment changes.

**spot curve, spot yield curve**

See **zero curve, zero-coupon yield curve** on page Glossary-11.

**spot rate**

The current interest rate appropriate for discounting a cash flow of some given maturity.

**spread**

For options, a combination of call or put options on the same stock with differing exercise prices or maturity dates.

**stepped coupon bond**

A step-up and step-down bond is a debt security with a predetermined coupon structure over time.

**stochastic model**

Involving or containing a random variable or variables; involving chance or probability.

**strike**

Exercise a put or call option.

**strike price**

See **exercise price** on page Glossary-4.

**supershare option**

A digital option that pays out a proportion of the assets underlying a portfolio if the asset lies between a lower and an upper bound at the expiry of the option.

**swap**

A contract between two parties to exchange cash flows in the future according to some formula.

**swaption**

An option on an interest rate swap. It grants the option buyer the right to enter into an interest rate swap at a future date.

**time specification**

A MATLAB structure that represents the mapping between times and dates for interest rate quoting.

**trinomial model**

A method in which the basic assumption is that prices or rates can move to one of three possible values over any short time period. At any time step the price or rate direction can be upward, neutral, or downward. See also **binomial model** on page Glossary-2.

**under-determined system**

A set of simultaneous equations in which the number of independent variables exceeds the number of equations in the set, leading to an infinite number of solutions.

**up-and-in**

A type of **barrier option** on page Glossary-1 that becomes active if the barrier is reached from below. See also **knock-in** on page Glossary-6.

**up-and-out**

A type of **barrier option** on page Glossary-1 that becomes deactivated if the barrier is reached from below. See also **knock-out** on page Glossary-6.

**vanilla option**

A common option, such as a put or call. Opposite of **exotic option** on page Glossary-4.

**vanilla swap**

A **swap** on page Glossary-9 agreement to exchange a fixed rate for a floating rate.

**vega**

The rate of change in the price of a derivative security relative to the volatility of the underlying security. When vega is large, the security is sensitive to small changes in volatility.

**volatility specification**
A MATLAB structure that specifies the forward rate volatility process.

**yields**
The zero coupon rate.

**yield curve**
The zero curve.

**yield volatility**
The zero coupon volatilities.

**zero curve, zero-coupon yield curve**
A yield curve for zero-coupon bonds; zero rates versus maturity dates. Since the maturity and duration (Macaulay duration) are identical for zeros, the zero curve is a pure depiction of supply/demand conditions for loanable funds across a continuum of durations and maturities. Also known as spot curve or spot yield curve.

**zero-coupon bond, or zero**
A bond that, instead of carrying a coupon, is sold at a discount from its face value, pays no interest during its life, and pays the principal only at maturity.

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